

Calculating electrostatic potential with Newton method

Introduction

I calculated the electrostatic potential, ϕ , with equation

$$N^+ + n_{int} e^{-\frac{q\phi}{k_B T}} - n_{int} e^{\frac{q\phi}{k_B T}} = 0$$

by using the Newton's method.

The parameters I used is

$$k_B = 1.380648521 \times 10^{-23} \text{ m}^2 \text{ kg s}^{-2} \text{ K}^{-1}$$

$$q = 1.602192 \times 10^{-19} \text{ C}$$

$$N^+ = \pm 10^{10} \sim 10^{18} \text{ cm}^{-3}$$

$$n_{int} = 10^{18} \text{ cm}^{-3}$$

$$T = 300 \text{ K}$$

Result

First, I compared numerical solution with analytical solution when loop repetition time $i = 1000$.

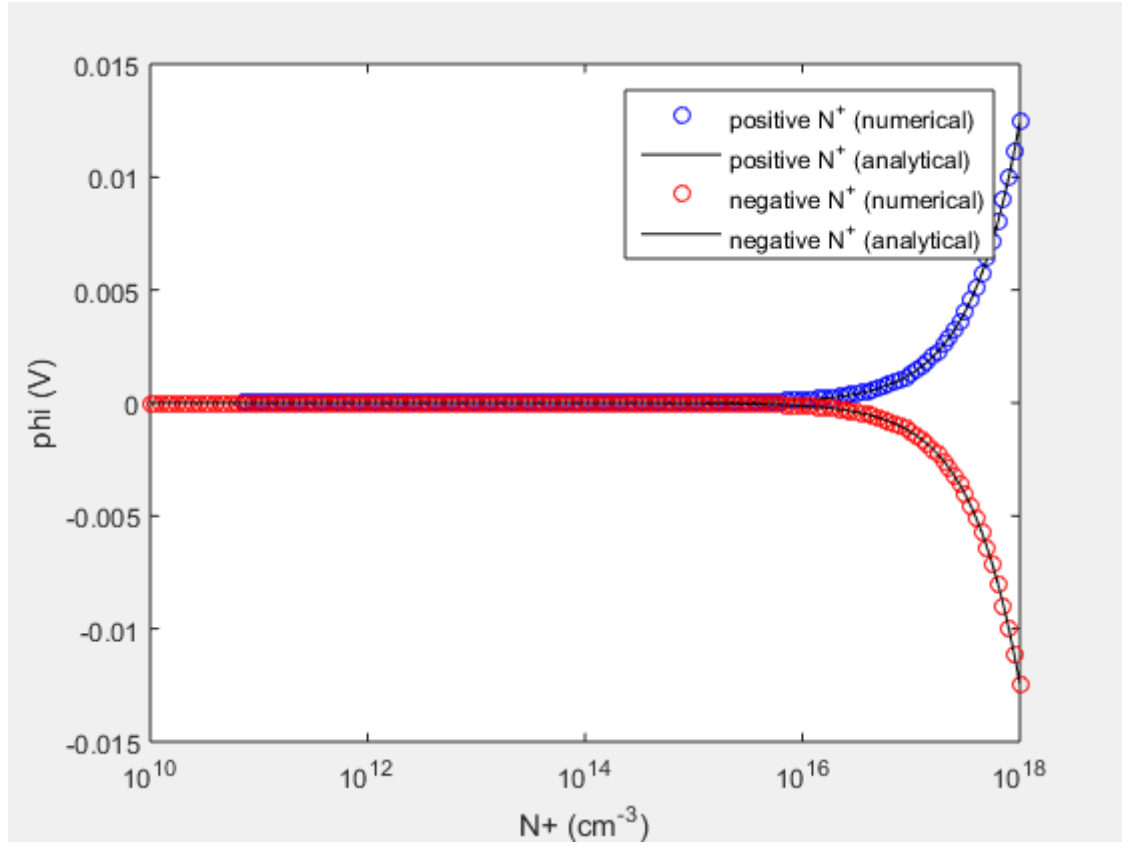


Figure 1. electrostatic potential, ϕ , by using the Newton method (repetition time $i = 1000$).

We can see that numerically calculated value is almost same with analytically calculated value.

I also estimated the error calculated with the equation

$$error = \frac{\text{Analytical solution} - \text{Numerical solution}}{\text{Analytical solution}} \times 100(\%)$$

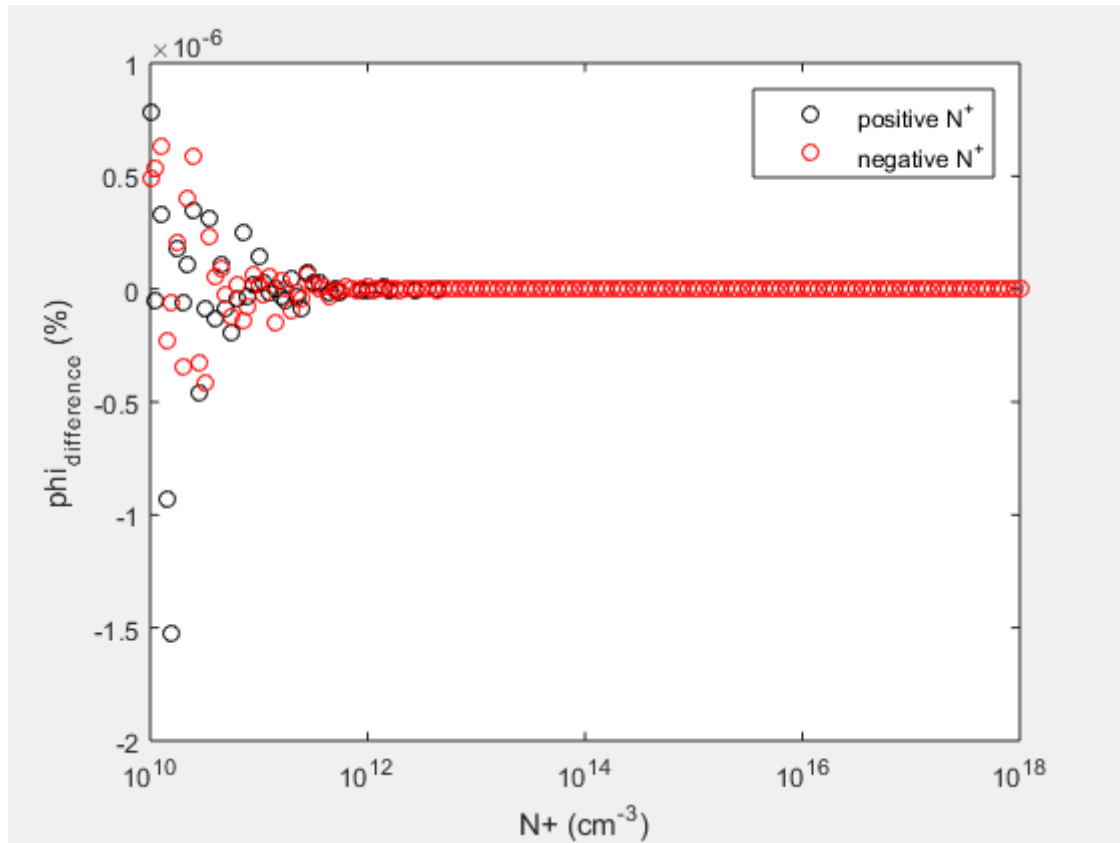


Figure 2. Calculated error.

Here we can clearly see that error is less than 10^{-5} % in every N^+ . However, error of small N^+ is comparatively high than large N^+ . This is because potential of low N^+ is significantly low. Therefore, numerical calculation cannot follow the resolution.

Finally, I also see the numerical solution approach to the analytical solution if we increase the repetition time i .

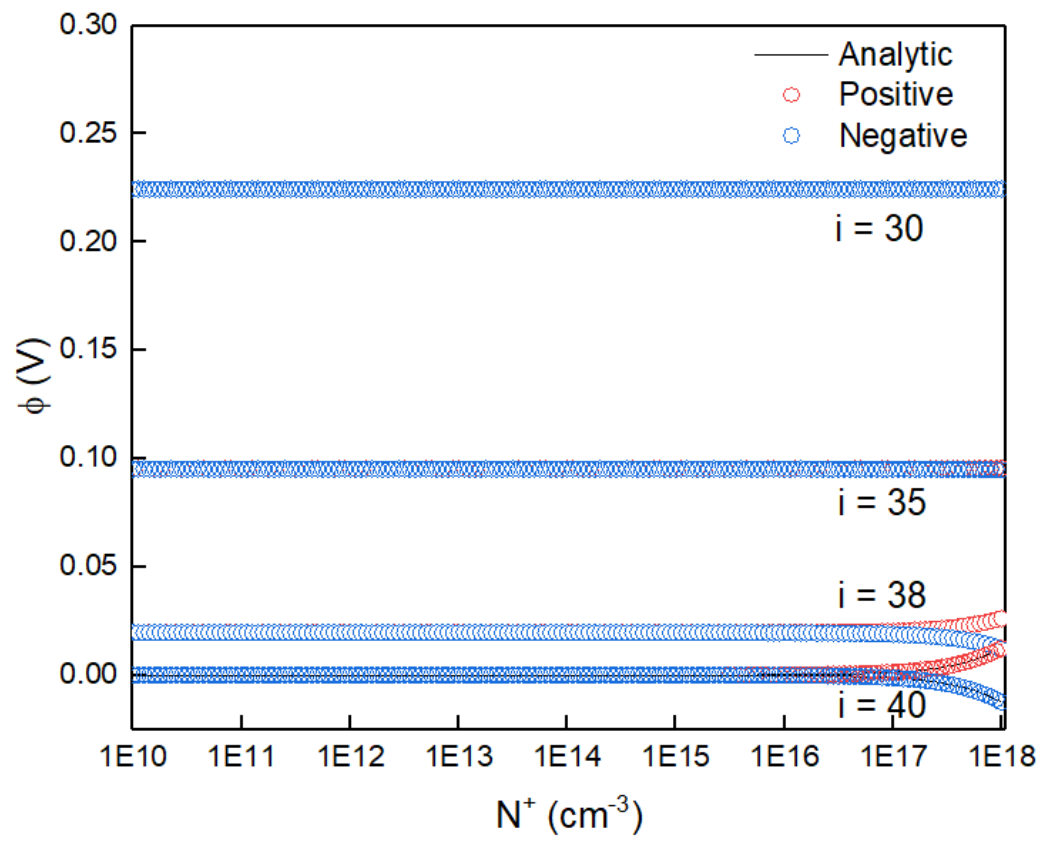


Figure 3. numerical solution approach to the analytical solution