Lecture22

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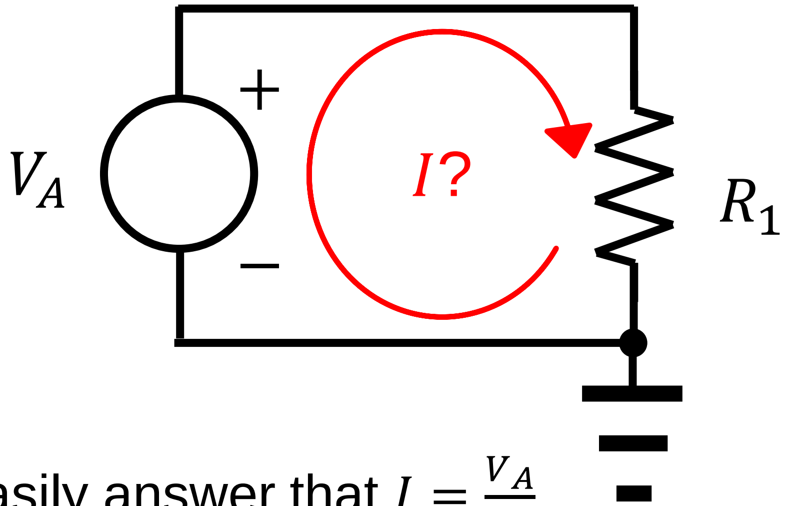
GIST Lecture on November 28, 2018

Calculation of current

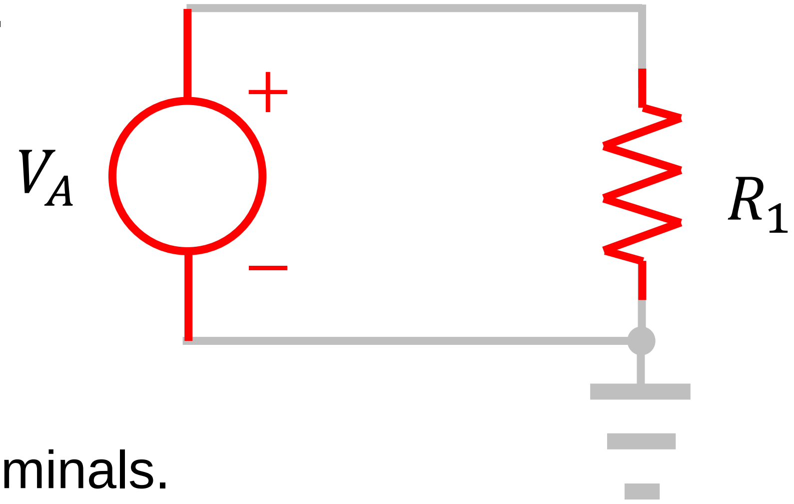
- Consider a simple problem.
  - What is the current?



  - Of course, you can easily answer that $I = \frac{V_A}{R_1}$.
  - But, how can we teach our computer to solve this problem?

Elements

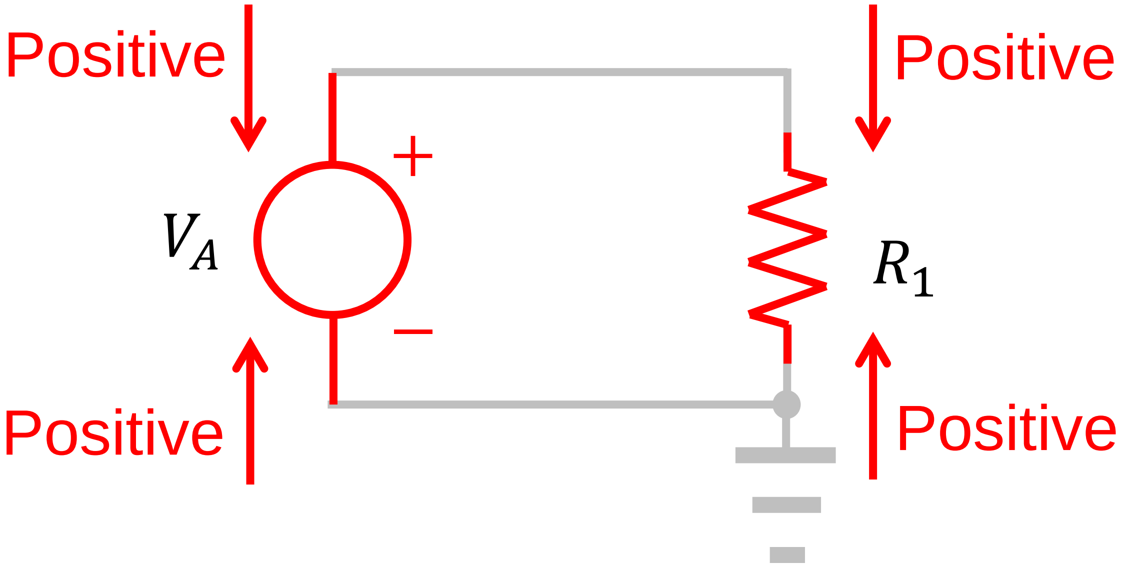
- Resistors, capacitors, etc
  - A circuit is made by connecting the elements.
  - They can have multiple terminals.
  - A resistor has two terminals.
  - A diode has two terminals.



  - A MOSFET has three (or four) terminals.

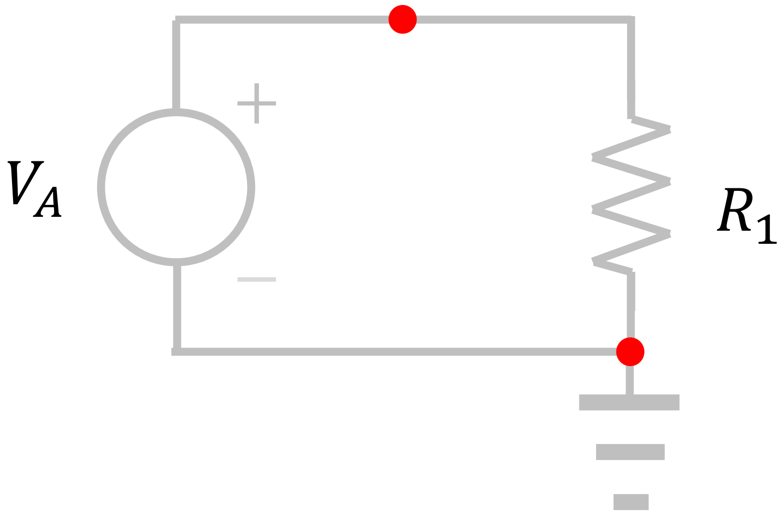
Convention for current

- Terminal current
  - Conventionally, an in-coming current is regarded as a positive one.



Nodes

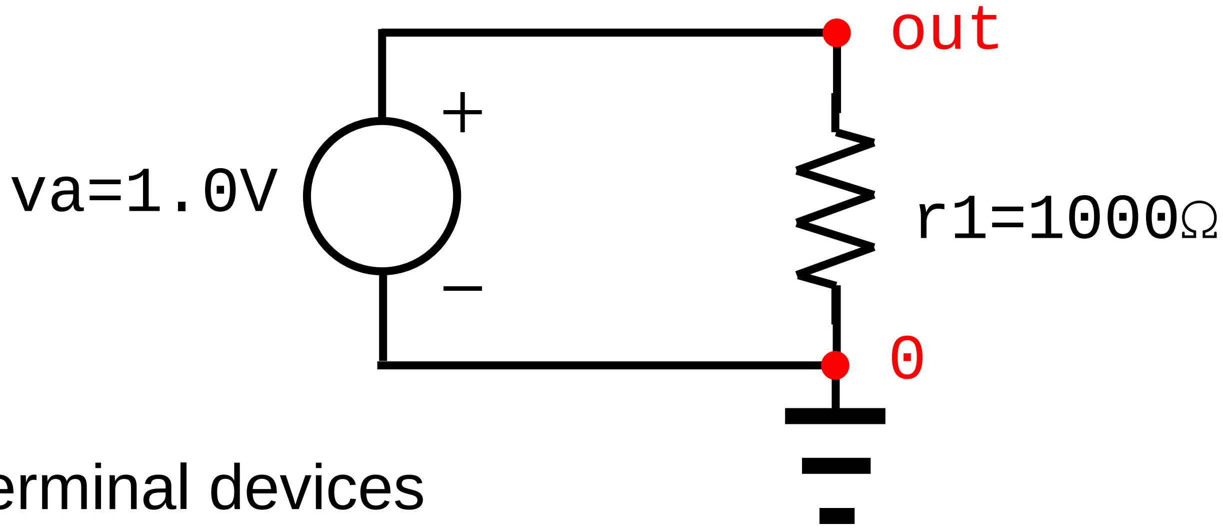
- A point to which multiple terminals are tied.
  - Usually, a dot is used to represent a node.
  - There is a special node, GND.



How to describe a circuit

- Of course, we can draw a circuit schematic. What else?
- A netlist for this circuit looks like:

```
va out 0 1.0
r1 out 0 1000
```



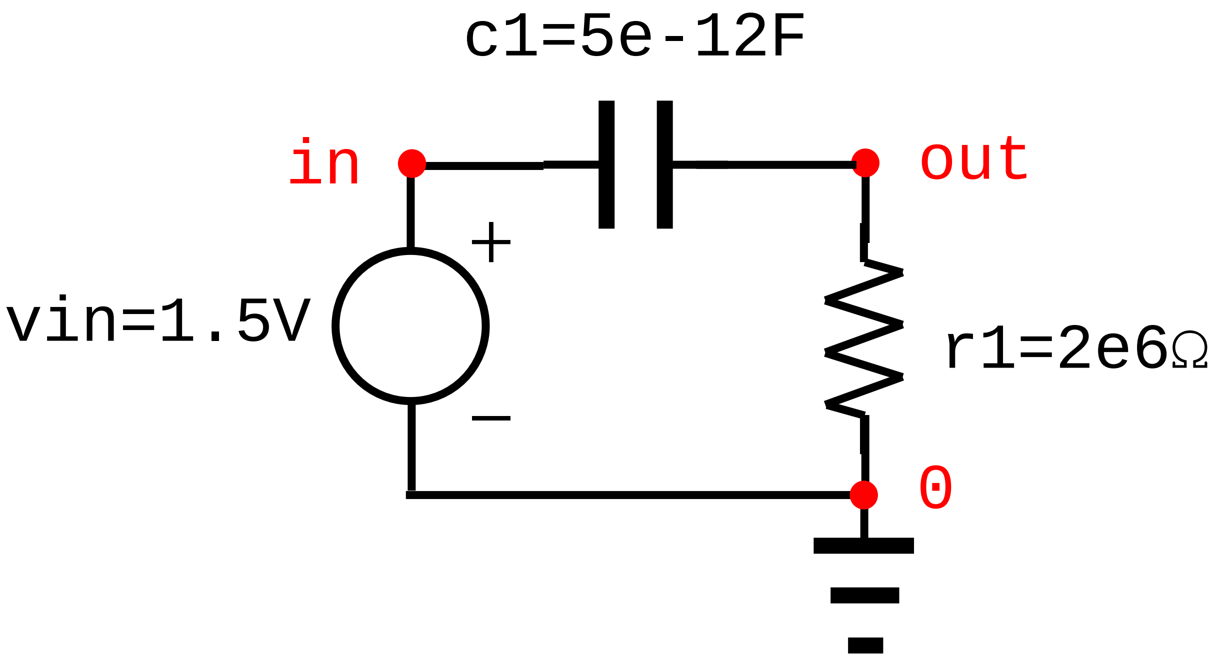
- Format for two-terminal devices

```
elementlabel node1 node2 value
```

RC filter

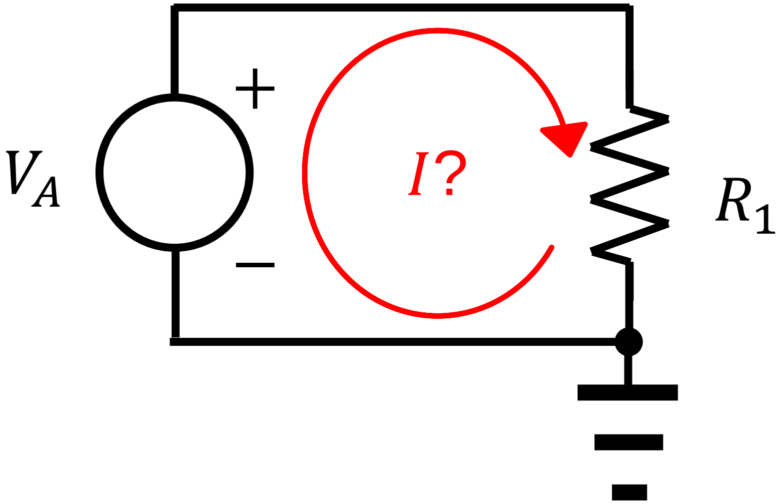
- A netlist for this circuit looks like:

```
c1 in out 5e-12
r1 out 0 2e6
vin in 0 1.5
```



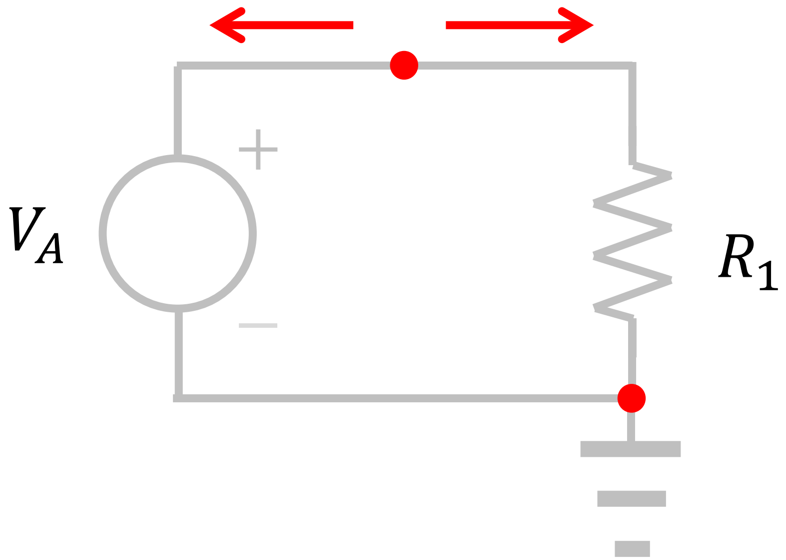
Today's goal

- Solve a simple problem by a numerical means.
  - Identifying the governing equation



Circuit analysis (1)

- Kirchhoff's current law (KCL)!
  - At any node in an electrical circuit, the sum of currents flowing into that node is equal to the sum of currents flowing out of that node.



Circuit analysis (2)

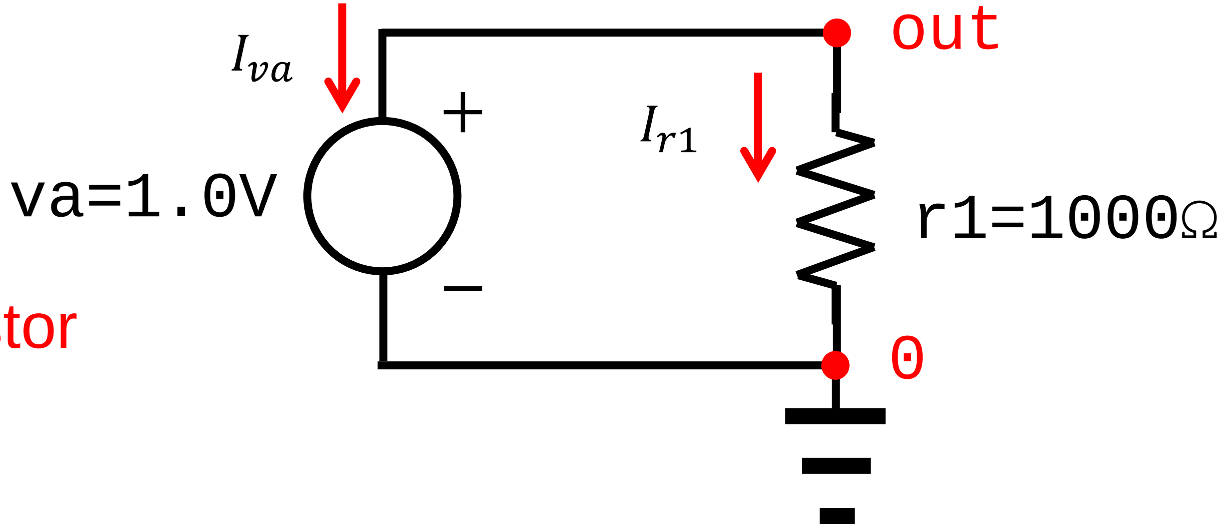
- Our simple problem
  - Three equations:

Voltage source

$$V(out) - 0.0 = 1.0$$

$$I_{r1} = \frac{V(out)}{1000}$$ Resistor

$$I_{va} + I_{r1} = 0$$ KCL



Implementation?

- Solution vector, $[I_{va} \quad I_{r1} \quad V(out)]^T$
  - Then, the system is written as

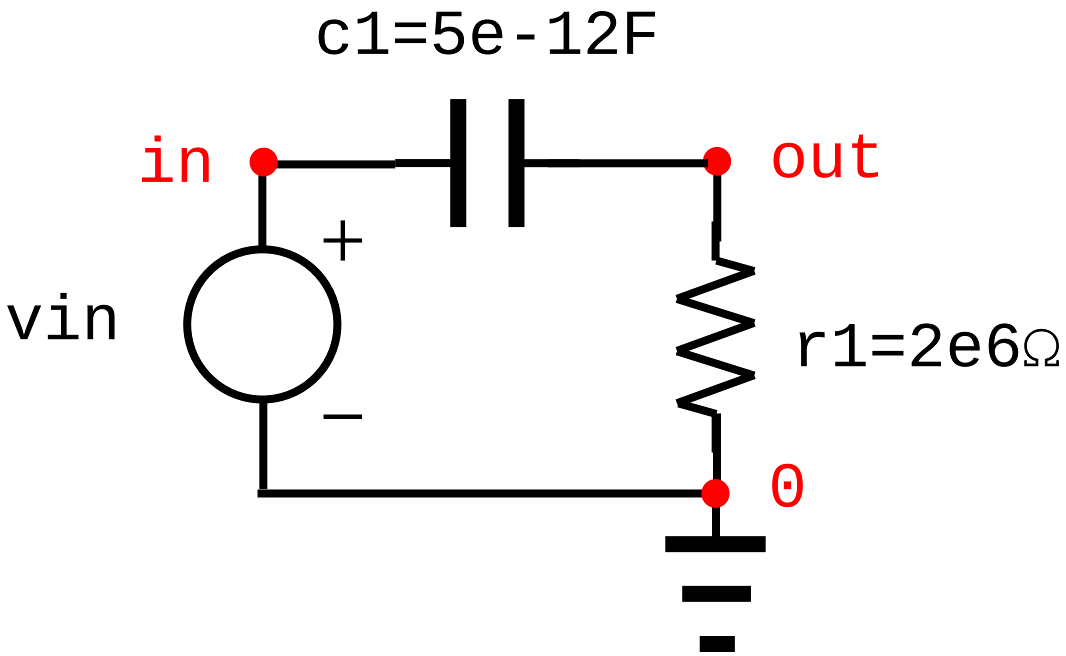
$$\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & -0.001 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} I_{va} \\ I_{r1} \\ V(out) \end{bmatrix} = \begin{bmatrix} 1.0 \\ 0 \\ 0 \end{bmatrix}$$

  - The solution is

$$\begin{bmatrix} I_{va} \\ I_{r1} \\ V(out) \end{bmatrix} = \begin{bmatrix} -0.001 \\ +0.001 \\ 1.0 \end{bmatrix}$$

Today's goal

- Solve a simple transient problem by a numerical means.



Frequency domain

- At a frequency, f, the impedance of the RC part is
$$Z(\omega) = R + \frac{1}{j\omega C} \quad (\omega = 2\pi f)$$
  - Therefore,
$$I(\omega) = \frac{V(\omega)}{Z(\omega)} = \frac{V(\omega)}{R + \frac{1}{j\omega C}} = \frac{j\omega C + \omega^2 R\, C^2}{1 + (\omega RC)^2} V(\omega)$$
  - For example, when $V(t) = V_0 \cos \omega t$,
$$I(t) = \frac{\omega^2 R\, C^2}{1 + (\omega RC)^2} V_0 \cos \omega t - \frac{\omega C}{1 + (\omega RC)^2} V_0 \sin \omega t$$

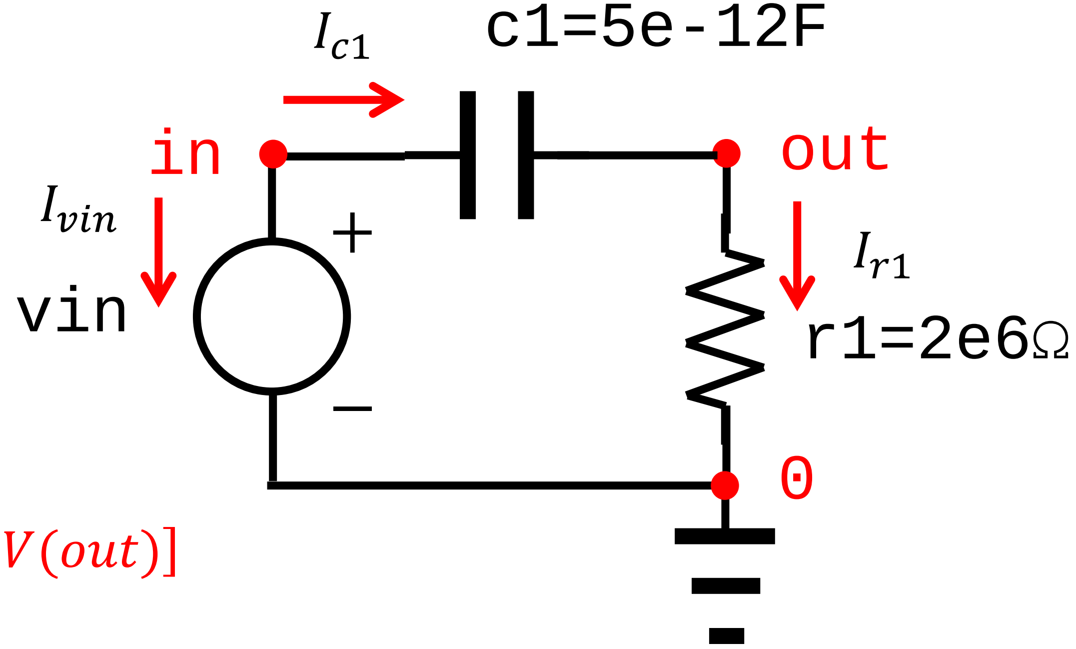
Circuit analysis

- Five equations
  - Two KCL's

$$I_{vin} + I_{c1} = 0$$
$$-I_{c1} + I_{r1} = 0$$

  - Three equations

$$V(in) - 0.0 = \cos \omega t$$
$$I_{c1} = 5 \times 10^{-12} \frac{d}{dt}[V(in) - V(out)]$$
$$I_{r1} = \frac{V(out) - 0.0}{2 \times 10^{6}}$$



Implementation?

- Solution vector, $[I_{vin} \quad I_{c1} \quad I_{r1} \quad V(in) \quad V(out)]^T$
  - Then, the system is written as

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & -R^{-1} \\ 1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} I_{vin} \\ I_{c1} \\ I_{r1} \\ V(in) \\ V(out) \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -C & C \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \frac{d}{dt} \begin{bmatrix} I_{vin} \\ I_{c1} \\ I_{r1} \\ V(in) \\ V(out) \end{bmatrix}$$

$$= \begin{bmatrix} \cos \omega t \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Backward Euler

- An implicit method
  - Uniform time discretization, $t_i = i\Delta t$
  - The time derivative at t_i is assumed to be

$$\frac{d}{dt}\begin{bmatrix} I_{vin} \\ I_{c1} \\ I_{r1} \\ V(in) \\ V(out) \end{bmatrix}_{t=t_i} = \frac{1}{\Delta t}\begin{bmatrix} I_{vin} \\ I_{c1} \\ I_{r1} \\ V(in) \\ V(out) \end{bmatrix}_{t=t_i} - \frac{1}{\Delta t}\begin{bmatrix} I_{vin} \\ I_{c1} \\ I_{r1} \\ V(in) \\ V(out) \end{bmatrix}_{t=t_{i-1}}$$

Discretized form

- By using the backward Euler method,
  - Then, the system is written as

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & -C/\Delta t & C/\Delta t \\ 0 & 0 & 1 & 0 & -R^{-1} \\ 1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} I_{vin} \\ I_{c1} \\ I_{r1} \\ V(in) \\ V(out) \end{bmatrix}_{t=t_i} + \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & C/\Delta t & -C/\Delta t \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} I_{vin} \\ I_{c1} \\ I_{r1} \\ V(in) \\ V(out) \end{bmatrix}_{t=t_{i-1}} = \begin{bmatrix} \cos \omega t \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

MATLAB (1)

- RC filter
  - First, define some constants.

```
R = 2e6; % Ohm
C = 5e-12; % F
freq = 1e0; % Hz
deltat = 1/freq/100; % 0.01 of a period
```

  - The system matrix

```
A = zeros(5,5);
A(1,:) = [0 0 0 1 0];
A(2,:) = [0 1 0 -C/deltat C/deltat];
A(3,:) = [0 0 1 0 -1/R];
A(4,:) = [1 1 0 0 0];
A(5,:) = [0 -1 1 0 0];
```

MATLAB (2)

- RC filter (Continued)

```
b = zeros(5,1);
solution = [0 0 0 1 0]';
N = 1000;
for ii=1:N
    t = ii*deltat;
    solution_old = solution;
    b(1,1) = cos(2*pi*freq*t);
    b(2,1) = -C/deltat*(solution_old(4,1)-solution_old(5,1));
    solution = A \ b;
end
```