Lecture5

Sung-Min Hong (smhong@gist.ac.kr)

Semiconductor Device Simulation Lab.

School of Electrical Engineering and Computer Science

Gwangju Institute of Science and Technology

GIST Lecture on September 17, 2018

Poisson equation

- Fixed-source case

$$\frac{d}{dx}\left[\epsilon(x)\frac{d}{dx}\phi(x)\right] = -\rho(x)$$

  – The net charge density, $\rho(x)$, is given by

$$\rho(x) = qp(x) - qn(x) + qN_{dop}^{+}(x)$$

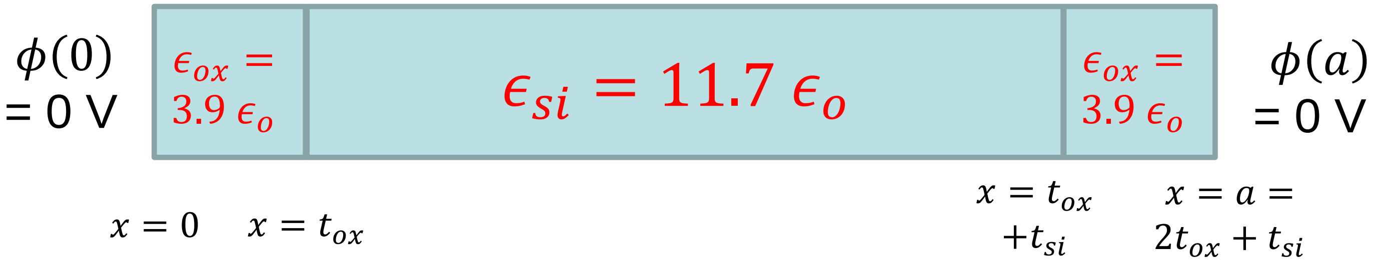
$p(x)$: Hole density, $n(x)$: Electron density, $N_{dop}^{+}(x)$: Net doping density

  – Calculating $p(x)$ and $n(x)$ is not a trivial task.
  – Let us assume that all mobile carriers are depleted.

$$\frac{d}{dx}\left[\epsilon(x)\frac{d}{dx}\phi(x)\right] = -qN_{dop}^{+}(x)$$

Double-gate MOS

- Real engineering problem
  - A silicon layer (whose thickness is t_{si}) surrounded by two oxide layers (whose thickness is t_{ox})

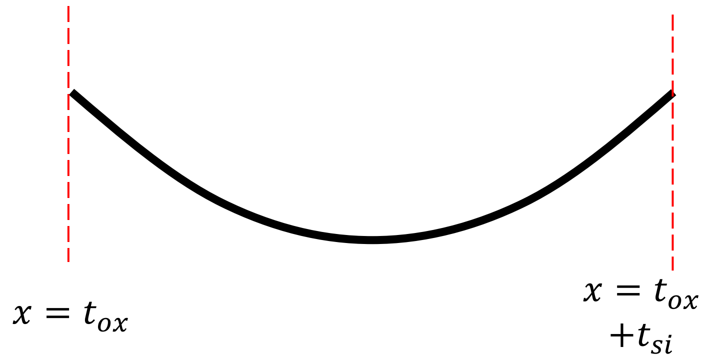


  - The silicon layer is doped with p-type dopants. The doping density is N_{acc}. Since the p-type dopant provides a hole, the dopant itself is negatively charged. $N_{dop}^{+} = -N_{acc}$.

Analytic solution (1)

- Qualitative analysis
  - Due to the mirror symmetry, the electrostatic potential is also mirror symmetric.
  - Inside the oxide layer, the electrostatic potential must be linear.
  - Inside the silicon layer, the Poisson equation reads ($N_{acc} > 0$)

$$\frac{d}{dx}\left[\frac{d}{dx}\phi(x)\right] = \frac{qN_{acc}}{\epsilon_{si}}$$



Analytic solution (2)

- Solution
  - Integrating the Poisson equation inside the silicon layer,

$$\left.\frac{d\phi}{dx}\right|_{x=t_{ox}+t_{si}} - \left.\frac{d\phi}{dx}\right|_{x=t_{ox}} = \frac{qN_{acc}}{\epsilon_{si}} t_{si}$$

  - We know that

$$\left.\frac{d\phi}{dx}\right|_{x=t_{ox}} = \frac{\phi(t_{ox})}{3t_{ox}}$$

  - The electrostatic potential at $x = t_{ox}$ is given by

$$\phi(t_{ox}) = -\frac{3t_{ox}qN_{acc}t_{si}}{2\epsilon_{si}}$$

Scaling

- A suitable form
  - The original form

$$\frac{d}{dx}\left[\epsilon(x)\frac{d}{dx}\phi(x)\right] = qN_{acc}(x)$$

  - However, the values of $\epsilon(x)$ and dx in the SI unit is very small.
  - Equivalently, we will use the following form:

$$(\Delta x)\frac{d}{dx}\left[\frac{\epsilon(x)}{\epsilon_0}\frac{d}{dx}\phi(x)\right] = (\Delta x)\frac{qN_{acc}(x)}{\epsilon_0}$$

  - The discretized version at $x = x_i$ is

$$\frac{\epsilon(x_{i+0.5})}{\epsilon_0}\phi_{i+1} - \frac{\epsilon(x_{i+0.5}) + \epsilon(x_{i-0.5})}{\epsilon_0}\phi_i + \frac{\epsilon(x_{i-0.5})}{\epsilon_0}\phi_{i-1} = (\Delta x)^2\frac{qN_{acc}(x_i)}{\epsilon_0}$$

MATLAB example (1)

- Step-by-step procedure
  - First, set up the structure.

```
q = 1.602192e-19; % Elementary charge, C
eps0 = 8.854187817e-12; % Vacuum permittivity, F/m
Deltax = 0.1e-9; % 0.1 nm spacing
N = 61; % 6 nm thick
interface1 = 6; % At x=0.5 nm
interface2 = 56; % At x=5.5 nm
eps_si = 11.7; eps_ox = 3.9; % Relative permittivity
Nacc = 1e24; % 1e18 /cm^3
```

MATLAB example (2)

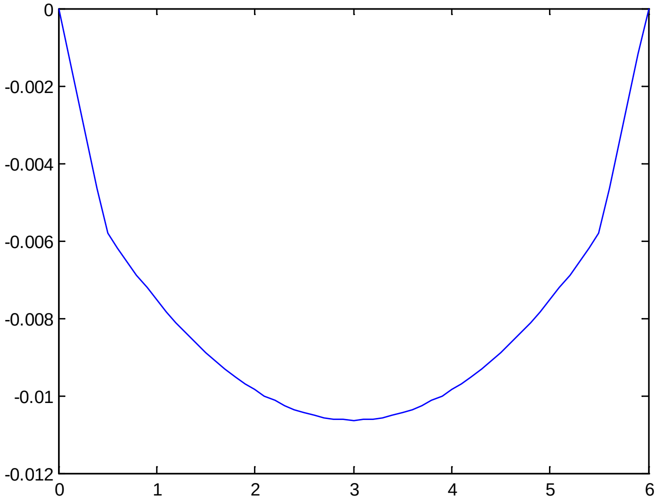
- Step-by-step procedure (continued)
  - Next, set the matrix, A. (Five cases)

```
A = zeros(N,N);
A(1,1) = 1.0;
for ii=2:N-1
   if     (ii< interface1) A(ii,ii-1) = eps_ox; A(ii,ii) = -2*eps_ox;      A(ii,ii+1) = eps_ox;
   elseif (ii==interface1) A(ii,ii-1) = eps_ox; A(ii,ii) = -eps_ox-eps_si; A(ii,ii+1) = eps_si;
   elseif (ii< interface2) A(ii,ii-1) = eps_si; A(ii,ii) = -2*eps_si;      A(ii,ii+1) = eps_si;
   elseif (ii==interface2) A(ii,ii-1) = eps_si; A(ii,ii) = -eps_si-eps_ox; A(ii,ii+1) = eps_ox;
   elseif (ii> interface2) A(ii,ii-1) = eps_ox; A(ii,ii) = -2*eps_ox;      A(ii,ii+1) = eps_ox;
   end
end
A(N,N) = 1.0;
```

MATLAB example (3)

- The vector, b, contains the doping effect.

```
b = zeros(N,1);
for ii=interface1:interface2
   if     (ii==interface1) b(ii,1) = Deltax*Deltax*q*Nacc/eps0*0.5;
   elseif (ii==interface2) b(ii,1) = Deltax*Deltax*q*Nacc/eps0*0.5;
   else                    b(ii,1) = Deltax*Deltax*q*Nacc/eps0;
   end
end
```



0 V? What does it mean?

- Electrostatic potential, $\phi(\mathbf{r})$
  - Let us assume that it is 0 V at a certain point.Then, what is its meaning?
  - Misconception) That point has the same electrostatic potential with the ground.
  - We have to realize that the applied voltages at contacts are <u>*NOT*</u> the electrostatic potential.
- What's the matter with $\phi(\mathbf{r})$?
  - In the computational electronics, it is very important to understand the meaning of $\phi(\mathbf{r})$ exactly.

Ambiguity

- Global shift of the potential
  - Since the electric field is given by $\mathbf{E}(\mathbf{r}) = -\nabla\phi(\mathbf{r})$, a global shift of the potential does not introduce different physics.
- We must answer two questions:
  - Which quatity is described by the electrostatic potential? (Especially, in the semiconductor device simulator)
  - What is the reference of the electrostatic potential?

Widely adopted convention

- Answer #1
  - By the electrostatic potential, we want to point out the intrinsic Fermi level of the reference material.
  - For example, when the reference material is silicon,

$$E_i(\mathbf{r}) = -q\phi(\mathbf{r})$$

$E_i(\mathbf{r})$: Intrinsic Fermi level of silicon in this example

- Answer #2
  - The reference energy is the Fermi level at equilibrium.
  - Therefore, the Fermi potential at equilibrium is 0 V.

Diagram

- First, there is the Fermi level, E_F.
  - Now, the conduction band minimum (E_C) and the valence band maximum (E_V) are identified.
  - Using them, the intrinsic Fermi level (E_i) is found.
  - Question: $E_i > 0$?
  - Question: $\phi > 0$?