

An electrostatic potential in eq. 1 is numerically calculated by using the Newton method.

$$N^+ + n_{int} e^{-\frac{q\phi}{k_B T}} - n_{int} e^{\frac{q\phi}{k_B T}} = 0 \quad \text{eq. 1}$$

$n_{int}$  is the intrinsic carrier density of Si,  $N^+$  is the dopant density,  $\phi$  is the electrostatic potential,  $k_B$  is the Boltzmann constant,  $q$  is the electron charge, and  $T$  is the temperature.  $n_{int}$  of Si at 300 K is  $10^{10} \text{ cm}^{-3}$ .  $N^+$  is from  $10^{10} \text{ cm}^{-3}$  to  $10^{18} \text{ cm}^{-3}$  for positive/negative values.

The first trial solution for the Newton method is 10 and 1000 time repeated. The numerically and analytically calculated data are shown in figure 1. The electrostatic potential varies from -0.5 V to 0.5V. There is no distinctive difference between the numerical data and analytical data. In order to precisely make a comparison, the difference is plotted in figure 2. The differences are within  $10^{-16} \text{ V}$ . In this case the numerical data can be regarded the exactly same as the analytical data within fifteen decimal point.

In conclusion, the numerical data is the same as the analytical data in this case within fifteen significant digits.

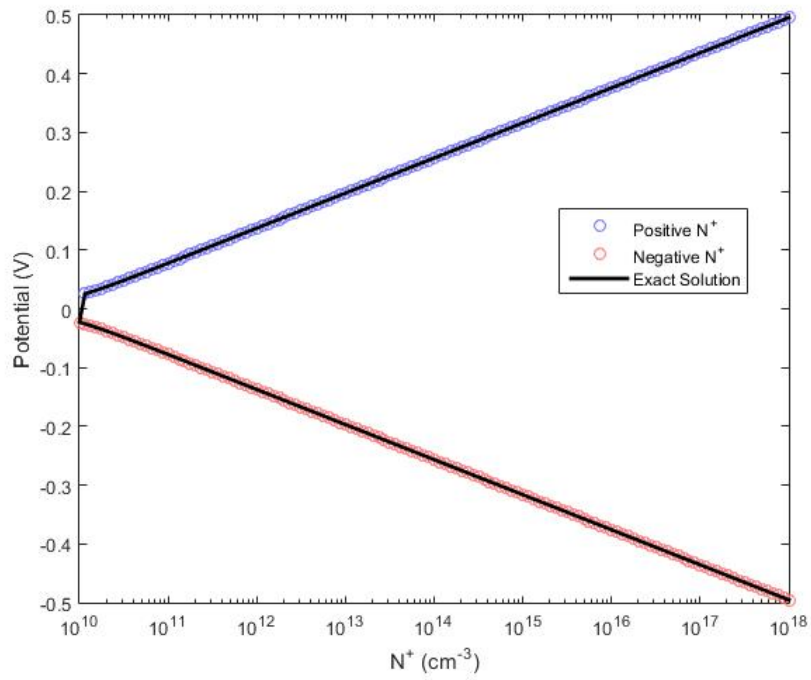


Figure 1. The electrostatic potential as a function of  $N^+$ . The circular symbols are numerically calculated data using the Newton method and the solid line is analytical data.

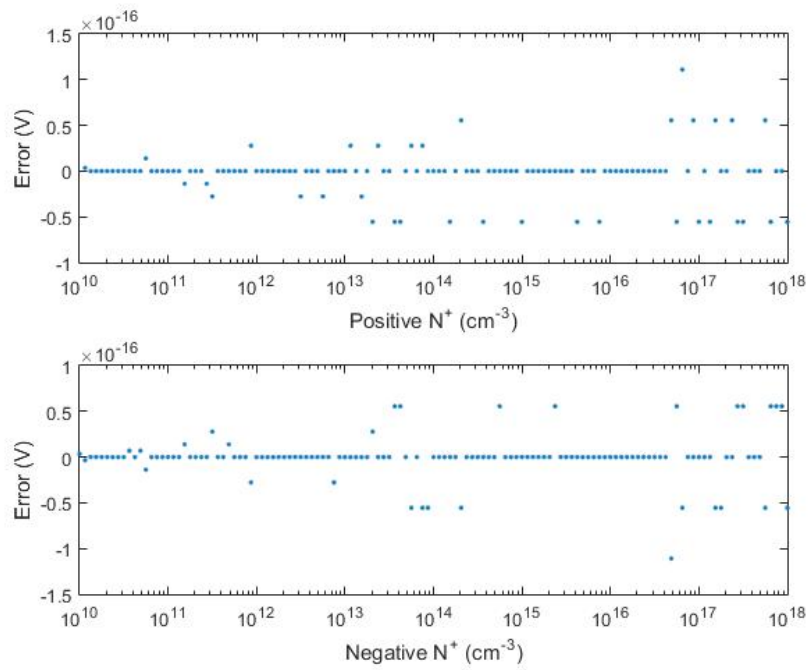


Figure 2. The potential difference between numerical data and analytical data.