Lecture16

Sung-Min Hong (smhong@gist.ac.kr)

Semiconductor Device Simulation Lab.

School of Electrical Engineering and Computer Science

Gwangju Institute of Science and Technology

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Review

- Our goal
  - Reduction of the dimensionality
- Our achievement
  - An explicit equation in the energy space

$$\frac{\partial f(x,\epsilon,\phi)}{\partial t} + v\cos\phi\,\frac{\partial f(x,\epsilon,\phi)}{\partial x} + F\left(v\cos\phi\,\frac{\partial f(x,\epsilon,\phi)}{\partial \epsilon} - \sin\phi\,\frac{1}{\hbar k}\,\frac{\partial f(x,\epsilon,\phi)}{\partial \phi}\right) = \hat{S}$$

- Next step
  - Fourier harmonics expansion

BTE simulator (4)

- Derivation
  - Pham's Fourier harmonic, $Y_m(\phi)$, is defined as

$$Y_m(\phi) = c_m \cos(m\phi + \varphi_m)$$

$$c_m = \sqrt{\frac{1}{(1+\delta_{m,0})\pi}}$$

  - The phase, φ_m, is $\frac{\pi}{2}$ for negative m. Otherwise, it is zero.
  - Multiplying Y_m,

$$Zd\epsilon \frac{\partial f}{\partial t} Y_m d\phi + vZd\epsilon \frac{\partial f}{\partial x} \cos\phi\, Y_m d\phi + vFZd\epsilon \frac{\partial f}{\partial \epsilon} \cos\phi\, Y_m d\phi - F \frac{1}{\hbar k} Zd\epsilon \frac{\partial f}{\partial \phi} \sin\phi\, Y_m d\phi = Zd\epsilon \hat{S} Y_m d\phi$$

BTE simulator (5)

- Derivation
  - Note that $\cos\phi\, Y_m = \frac{1}{c_1} Y_1 Y_m$ and $\sin\phi\, Y_m = \frac{1}{c_{-1}} Y_{-1} Y_m$
  - By integration over the angle,

$$Zd\epsilon \frac{\partial}{\partial t} f_m(x,\epsilon,t) + vZd\epsilon \frac{\partial}{\partial x} \sum_{m'} \frac{1}{c_1} f_{m'}(x,\epsilon,t) \Upsilon_{m',m,1}$$

$$+ vFZd\epsilon \frac{\partial}{\partial \epsilon} \sum_{m'} \frac{1}{c_1} f_{m'}(x,\epsilon,t) \Upsilon_{m',m,1}$$

$$- F \frac{1}{\hbar k} Zd\epsilon \sum_{m'} \frac{-m'}{c_{-1}} f_{m'}(x,\epsilon,t) \Upsilon_{-m',m,-1} = Zd\epsilon \hat{S}_m$$

  - Here, $\Upsilon_{m,m',m''}$ is the integral of the triple product.

BTE simulator (6)

- Derivation
  - The H-transformation is introduced. $H = \epsilon - qV$

$$Zd\epsilon \frac{\partial}{\partial t} f_m(x,\epsilon,t) + vZdH \frac{\partial}{\partial x} \sum_{m\prime} \frac{1}{c_1} f_{m\prime}(x,H,t) \Upsilon_{m\prime,m,1}$$

$$- \left(q \frac{\partial V}{\partial x} \right) \frac{1}{\hbar k} ZdH \sum_{m\prime} \frac{-m'}{c_{-1}} f_{m\prime}(x,H,t) \Upsilon_{-m\prime,m,-1} = ZdH \hat{S}_m$$

  - Let us explicitly write the above equation for a given m.

BTE simulator (7)

- Derivation
  - When $m = 0$,

$$Zd\epsilon \frac{\partial}{\partial t} f_0(x, \epsilon, t) + \frac{\partial}{\partial x} \frac{1}{c_1} vZdH f_1(x, H, t) \Upsilon_{1,0,1} = ZdH \hat{S}_0$$

  - Where is the last term?
  - Stabilization scheme is employed.
  - For a general even number,

$$Zd\epsilon \frac{\partial}{\partial t} f_m(x, \epsilon, t) + \frac{\partial}{\partial x} vZdH \sum_{m'} \frac{1}{c_1} f_{m'}(x, H, t) \Upsilon_{m',m,1}$$

$$+ \left(q \frac{\partial V}{\partial x} \right) \frac{1}{\hbar k} ZdH \sum_{m'} \frac{-m}{c_{-1}} f_{m'}(x, H, t) \Upsilon_{-m,m',-1} = ZdH \hat{S}_m$$

BTE simulator (8)

- Derivation
  - When $m = 1$,

$$Zd\epsilon \frac{\partial}{\partial t} f_1(x,\epsilon,t) + vZdH \frac{\partial}{\partial x} \frac{1}{c_1} f_0(x,H,t) \Upsilon_{0,1,1} = ZdH \hat{S}_1$$

  - For a general odd number,

$$Zd\epsilon \frac{\partial}{\partial t} f_m(x,\epsilon,t) + vZdH \frac{\partial}{\partial x} \sum_{m\prime} \frac{1}{c_1} f_{m\prime}(x,H,t) \Upsilon_{m\prime,m,1}$$
$$- \left(q \frac{\partial V}{\partial x} \right) \frac{1}{\hbar k} ZdH \sum_{m\prime} \frac{-m'}{c_{-1}} f_{m\prime}(x,H,t) \Upsilon_{-m\prime,m,-1} = ZdH \hat{S}_m$$

BTE simulator (9)

- Derivation
  - The lowest expansion reads

$$Zd\epsilon \frac{\partial}{\partial t} f_0(x,\epsilon,t) + \frac{\partial}{\partial x}\frac{1}{c_1} vZdH f_1(x,H,t)\Upsilon_{1,0,1} = ZdH\hat{S}_0$$

$$Zd\epsilon \frac{\partial}{\partial t} f_1(x,\epsilon,t) + vZdH \frac{\partial}{\partial x}\frac{1}{c_1} f_0(x,H,t)\Upsilon_{0,1,1} = ZdH\hat{S}_1$$

Homework#10

- Due: AM08:00, November 12
  - Do NOT upload your HW in this time. E-mail submission only.
- Problem#1
  - Solve the following equations under a fixed H.

$$\frac{\partial}{\partial x}\frac{1}{c_1}vZf_1(x,H)\Upsilon_{1,0,1} = 0$$

$$vZ\frac{\partial}{\partial x}\frac{1}{c_1}f_0(x,H)\Upsilon_{0,1,1} = -Z\frac{f_1(x,H)}{\tau}$$

  - Consider both of the graphene ($\epsilon = \hbar v_F k$) and a parabolic band ($\epsilon = \frac{\hbar^2}{2m}k^2$).