STATS 237 Assignment 3

Due Date: Saturday, August 12, 2017

1. Assume $dS_t = a(b - S_t)dt + \sigma dW_t$, with $a = 0.5$, $b = 104$, $r = 0.05$, $\sigma = 10$, $T = 1$, $S_0 = 100$ and $K = 105$. For each of these options:

   (a) put option with strike K,

   (b) option with payoff $(K - S_T)_+ + (S_T - K)_+$,

   find the price of these options with then without using antithetic variables. Did you achieve variance reduction?
   Note: You may need a large number N of simulations to see their difference. Try larger N (e.g. $N = 100000$) if that does not take very long time to run.

2. Suppose that we have $(S_{i,t})$, $i = 1, \ldots, m$ stocks with dynamics under the risk neutral measure:
$$\frac{dS_{i,t}}{S_{i,t}} = rdt + \sigma_i dW_{i,t}, \quad S_{i,0} = 100.$$
   Suppose that $(W_{i,t})_{i=1,\ldots,m}$ is a m-dimensional Brownian motion with $E[W_{i,t}W_{j,t}] = \rho t$ for $i \neq j$. We want to price the call option on this basket with payoff $(S_{1,T} + S_{2,T} + \cdots + S_{m,T} - K)_+$, where $K = \sum_{i=1}^{m} S_{i,0} = 100m$. For numerical application, consider $m = 10$, $r = 0.04$, $\sigma_i = 0.15$, $T = 1$ and $\rho = 0.2$.

   (a) Compute the price of such an option by a Monte-Carlo method.

   (b) What can you say about the convergence of the Monte-Carlo estimate? Verify your answer by trying the number of Monte-Carlo iteration $N_{MC} = 10^2, 10^4$ and 10^6.

   (c) Explain why we could use the control variate $(m(\Pi_{i=1}^{m} S_{i,T})^{1/m} - K)_+$ to minimize the variance.

   (d) Implement this variance reduction technique. In particular, you will compute
$$E^Q[(m(\Pi_{i=1}^{m} S_{i,T})^{1/m} - K)_+].$$