EME5943
현대제어시스템

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Syllabus

What is your expectation?

Syllabus

- **Course description**
  - Basic principles of feedback and its use as a tool for altering the dynamics of systems and managing uncertainty
  - Concepts and tools of control theory and the robust control schemes for dynamic systems (i.e. robot, vehicle)

- **Key themes**
  - input/output response
  - (modeling) and model reduction
  - linear versus nonlinear models
  - feedback principles
  - nonlinear/force control

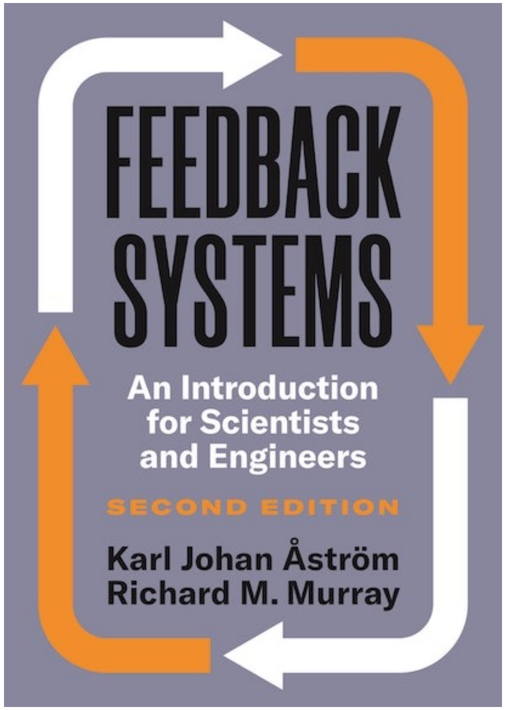
Syllabus

- **Lecture plan**
  - Introduction
  - Modeling / State-space representation
  - Dynamic behavior
    (including Lyapunov stability for nonlinear system)
  - Linear systems
    (with reviewing linear algebra)
  - Transfer function / feedback principles

  - State feedback (including controllability)
  - Output feedback (including observability)
  - Robust control schemes
    (including sliding mode and feedback linearization)
  - Force control
    (including impedance control)

Syllabus

- **Textbook**
  - Karl J. Åström and Richard M. Murray, *Feedback Systems: An Introduction for Scientists and Engineers*, 2nd Edition, Princeton University Press
  - e-book available !



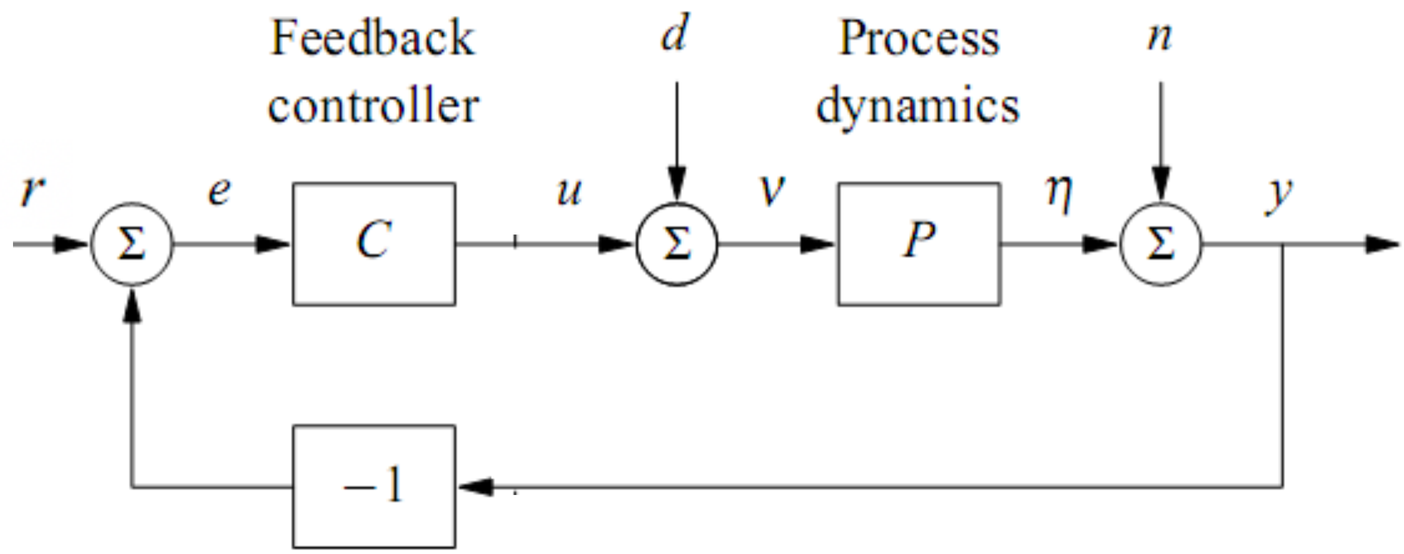
- **Rating**
  - 5% Attendance
  - 20% Homework
  - 35% Midterm exam
  - 40% Final exam

현대제어시스템

Introduction

What is control?

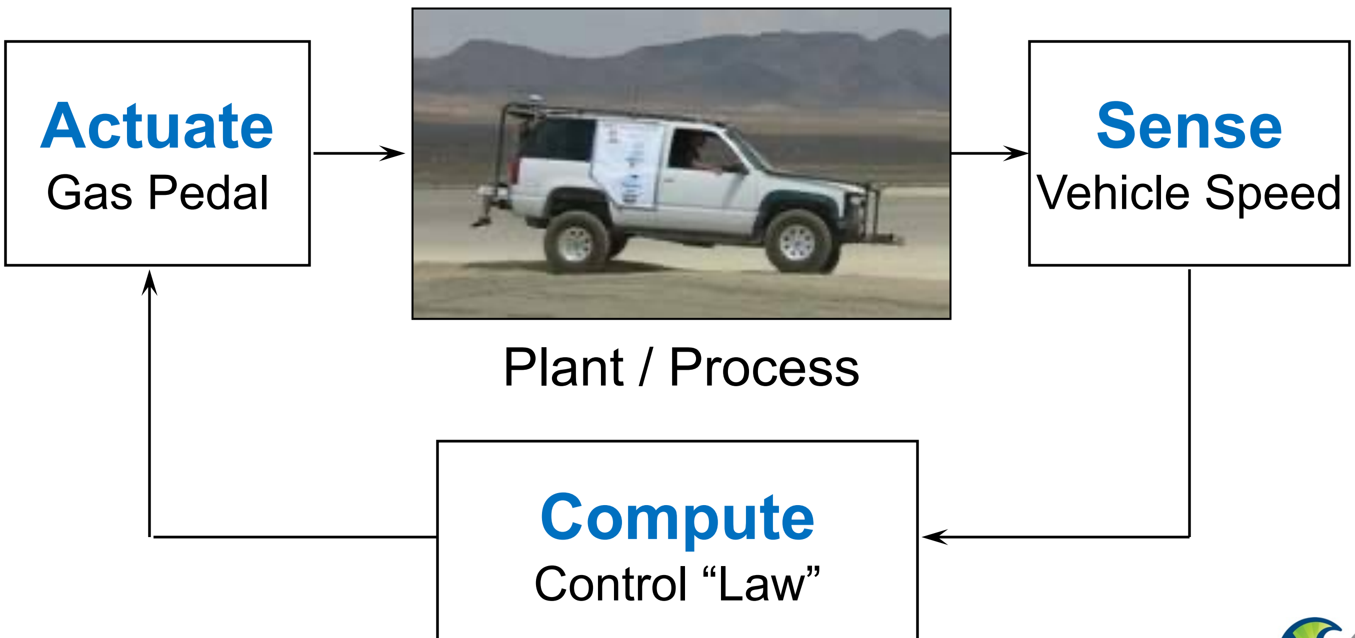
- **Control**
  - To design engineered systems with desired behaviors



What is control?

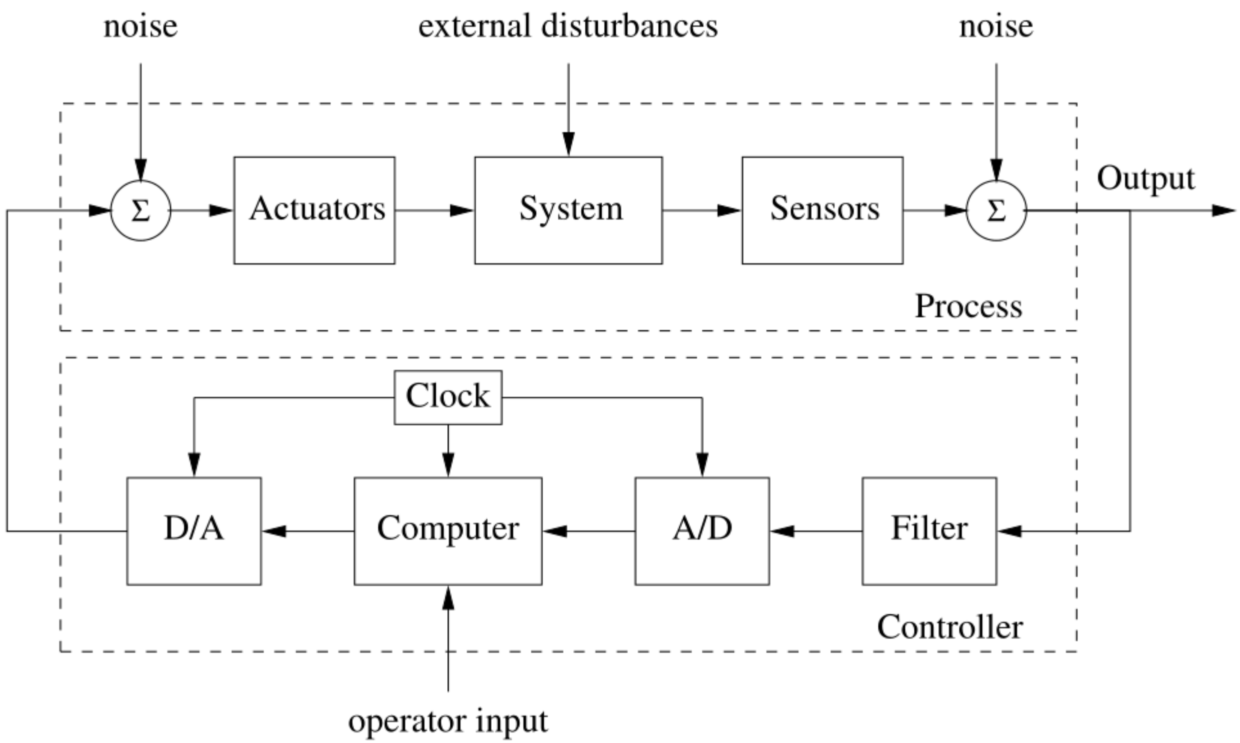
- **Control**
  - To design engineered systems with desired behaviors
  - Using algorithms & feedback in engineered systems
    with the information in both analogue & digital representations

Control **= Sensing + Computation + Actuation**



What is control?

$$\textit{Control} = \underbrace{\textbf{Sensing} + \textbf{Computation} + \textbf{Actuation}}_{\text{In Control Loop (feedback)}}$$



- Behavior
- Desired behaviors
- Stability

What is control?

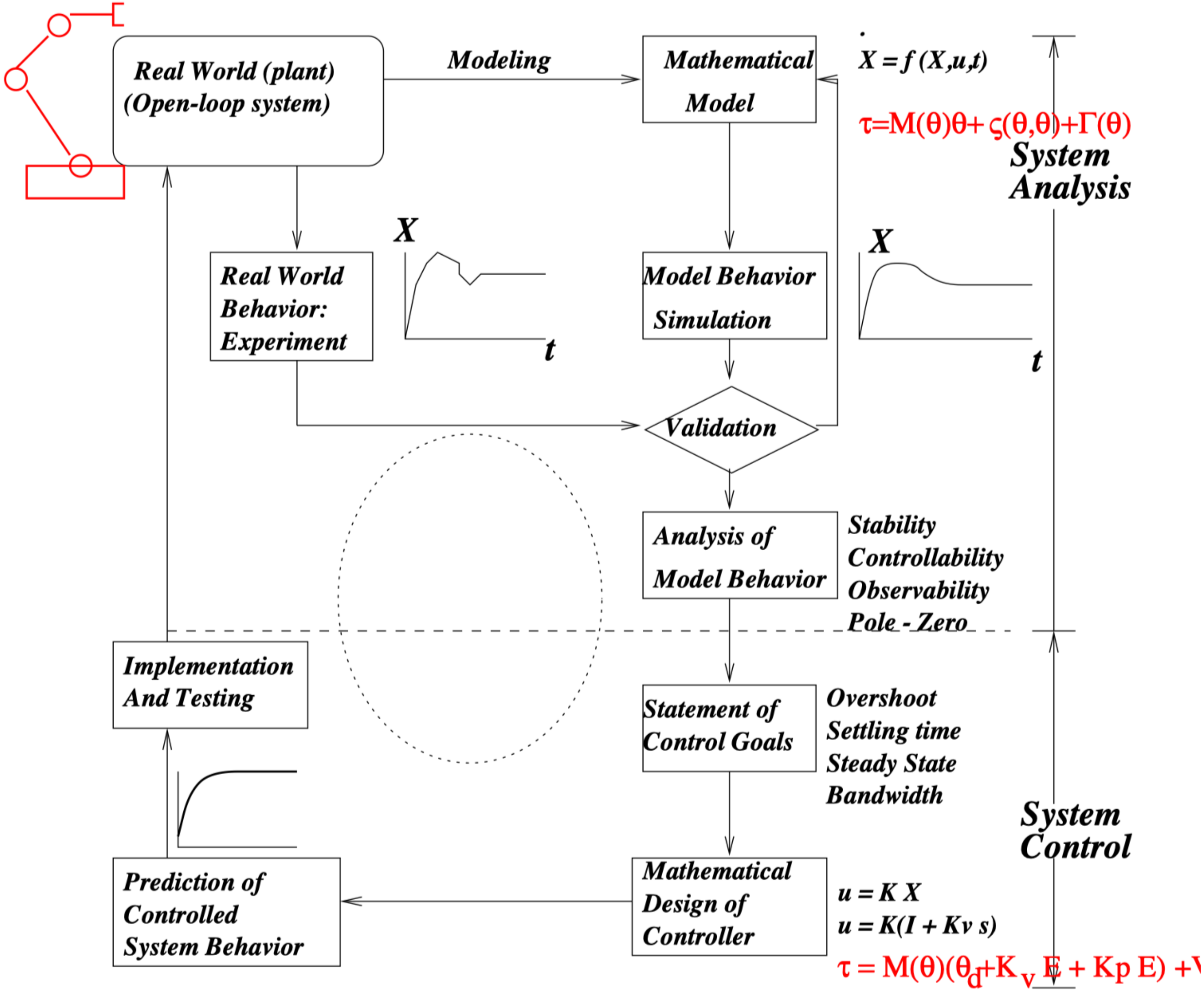
▪ Modeling for plant/process

- What?
  - ✓ The process by which a physical system (plant / process) is simplified to obtain a mathematically expressible form
  - ✓ Mathematical model
    - The resulting simplified version of real system

- Why?
  - ✓ Cost, time, risk …

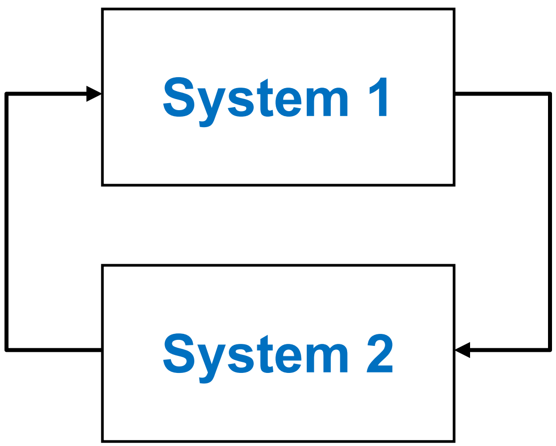
- How?
  - ✓ By using physical laws
    - Newton's law, Kirchhof's laws, …
  - ✓ By experimentally…

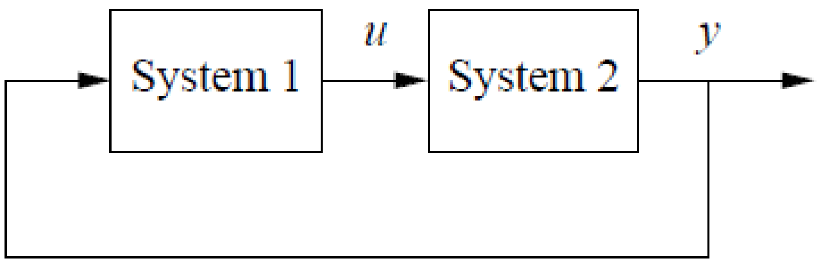
What is control?



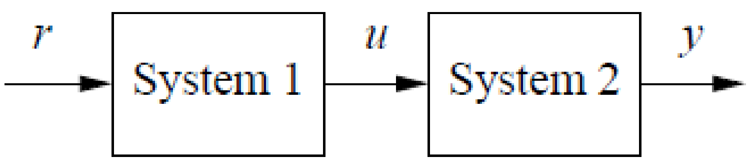
What is feedback?

- **Feedback = mutual interconnection of two (or more) systems**
  - System 1 affects system 2
  - System 2 affects system 1
  - Cause and effect is tricky; systems are mutually dependent





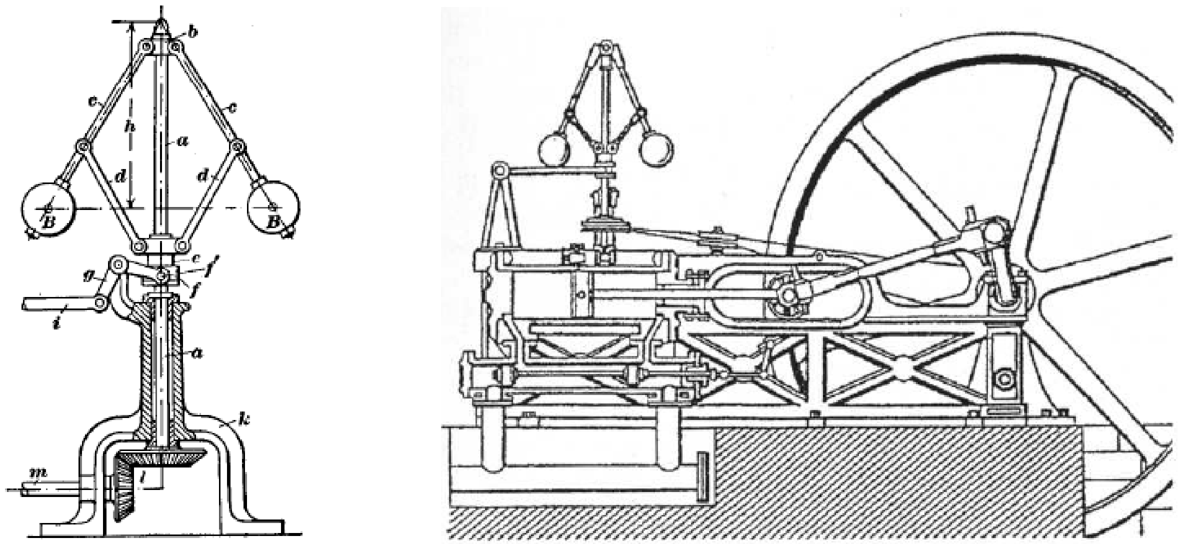
Closed loop



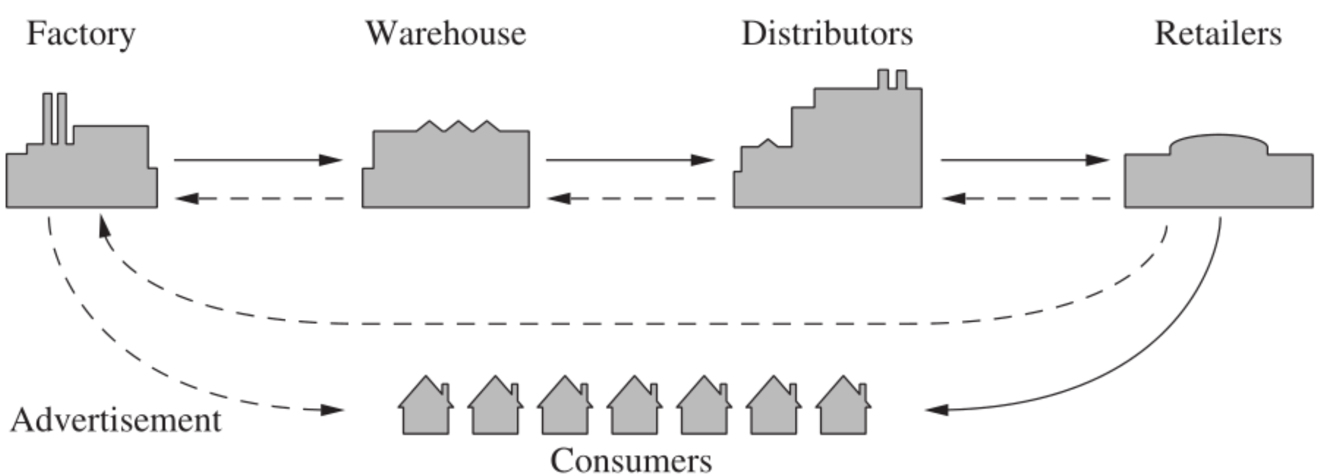
Open loop

What is feedback?

- **Many examples**



Centrifugal governor



Supply chain

What is feedback?

- **Many interesting properties**
  - Resilient toward external influences
  - Linear behavior out of nonlinear components
  - Insensitive both to external disturbances
    & to variation in its individual elements

- **Potential disadvantages**
  - Dynamic instabilities
  - Unwanted sensor noise

What is feedforward?

- **Feedback: reactive**
  - There must be an error before corrective actions are taken.

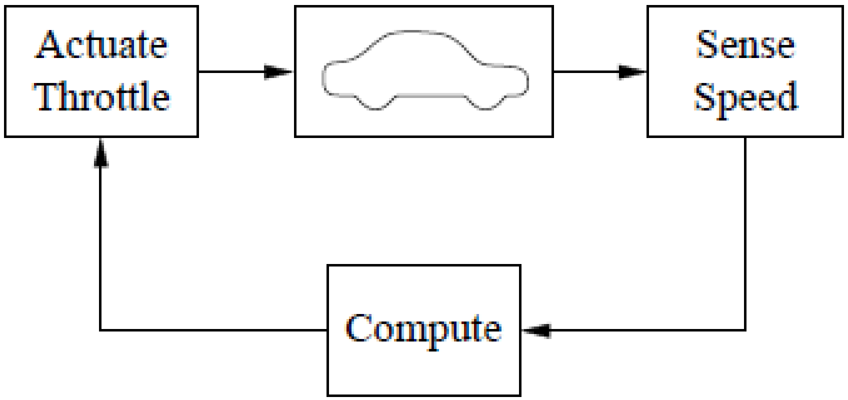
- **Feedforward**
  - Possible to measure a disturbance before it enters the system
  - Taking corrective action before the disturbance has influenced
  - Reducing the effect of the disturbance

Feedback	Feedforward
Closed loop	Open loop
Acts on deviations	Acts on plans
Robust to model uncertainty	Sensitive to model uncertainty
Risk for instability	No risk for instability

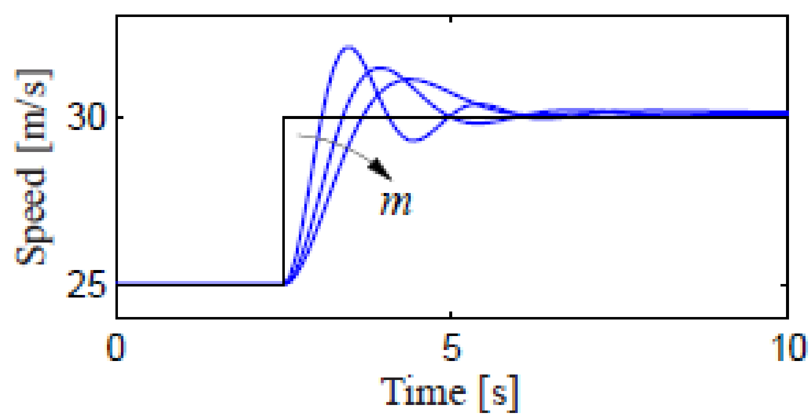
Feedback properties

- **Robustness to uncertainty**
  - Making system insensitive to variation

    ex) Cruise control



Robust performance
w.r.t. uncertainty

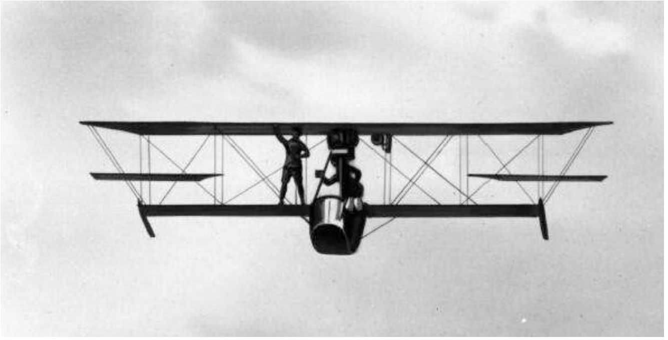


Feedback properties

- **Design of dynamics**
  - Changing dynamics of system

Wright Flyer (1903)

Unstable



Sperry autopilot (1914)

Stable !

- **Negative feedback**

c.f) Positive feedback
Spreading panic → Stampede

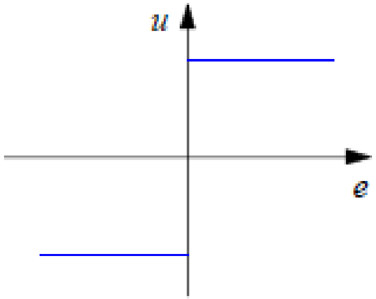
Simple forms of feedback

- **On-Off control**

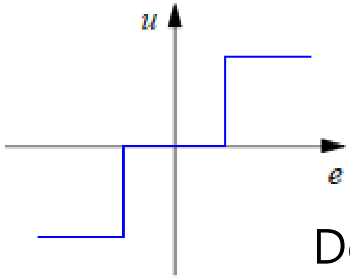
$$u = \begin{cases} u_{\max} & \text{if } e > 0 \\ u_{\min} & \text{if } e < 0. \end{cases} \qquad e = r - y$$

Simple; no parameters to be tune

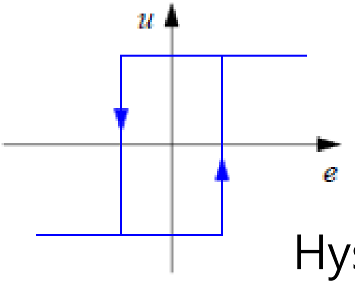
- Problems
  - ✓ Oscillation



  - ✓ No definition at zero



Dead zone



Hysteresis

Simple forms of feedback

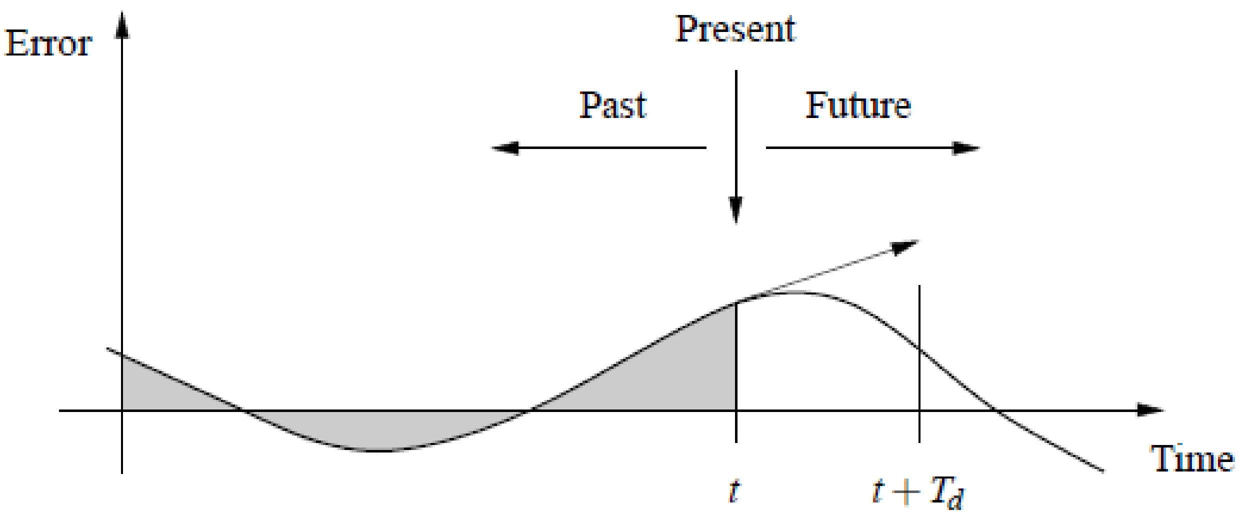
- **PID control**

$$u = \begin{cases} u_{\max} & \text{if } e \geq e_{\max} \\ k_p e & \text{if } e_{\min} < e < e_{\max} \\ u_{\min} & \text{if } e \leq e_{\min}, \end{cases}$$

$$u(t) = k_i \int_0^t e(\tau) d\tau.$$

$$e(t + T_d) \approx e(t) + T_d \frac{de(t)}{dt}$$

$$u(t) = k_p e(t) + k_i \int_0^t e(\tau) d\tau + k_d \frac{de(t)}{dt}$$



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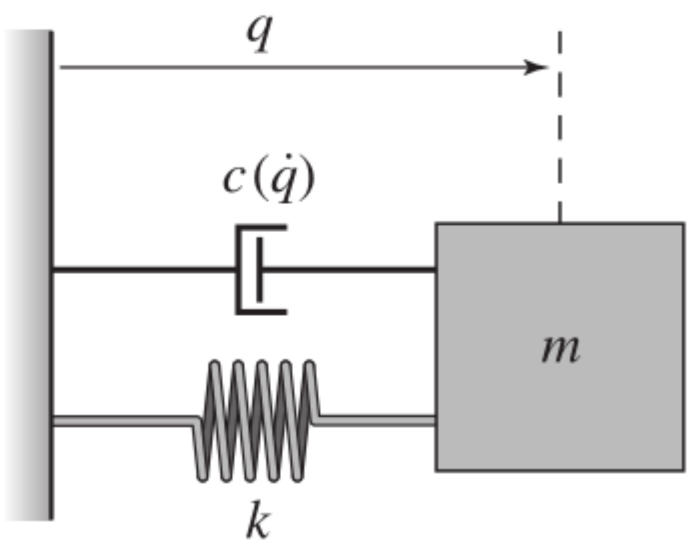
System modeling

Modeling concept

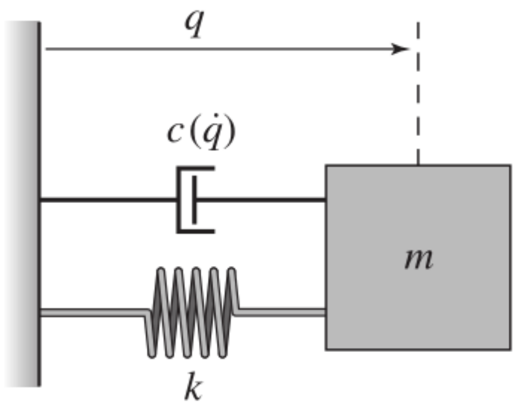
- **Model**
  - A mathematical representation of a system
  - To reason about a system
  - To make predictions about how a system will behave



$$m\ddot{q} + c(\dot{q}) + kq = 0$$

- Ordinary differential equations (ODEs)

Modeling concept



$$m\ddot{q} + c(\dot{q}) + kq = 0$$

- **Input**
  - A system variable that is independently prescribed, or defined by the environment
- **Output**
  - Any system variable of interest
- **State variables**
  - A minimum set of system variable that completely characterizes the motion of a system for the purpose of predicting future motion

Modeling concept

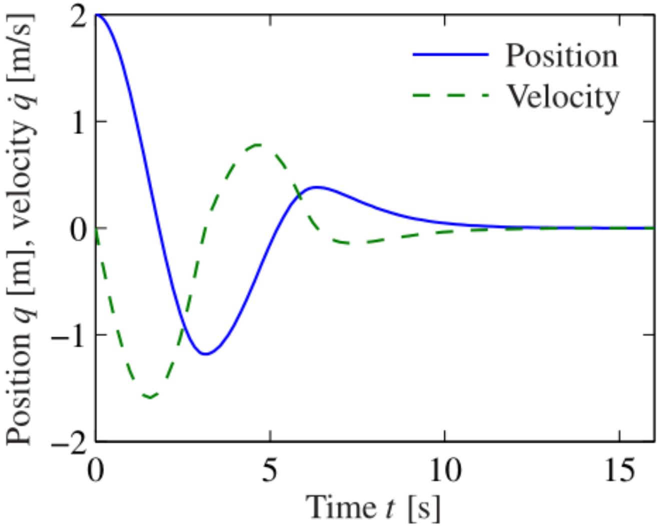
- **State vector**
  - A vector whose elements consists of state variables

- **State equation**
  - The relationship among the change of state, present state, and input

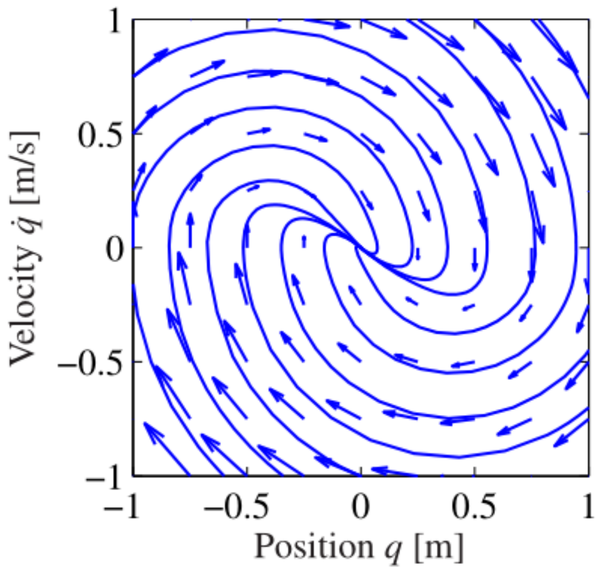
- **State-space model (representation)**
  - A mathematical model of a system using a state equation
  - It determines the system behavior for all time, given
    - ✓ The initial values of state variables
    - ✓ The specification of the inputs to the system for all times

Modeling concept

- **Representations of system behavior**
  - Time plot

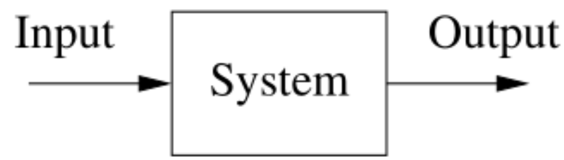


  - Phase portrait



Modeling concept

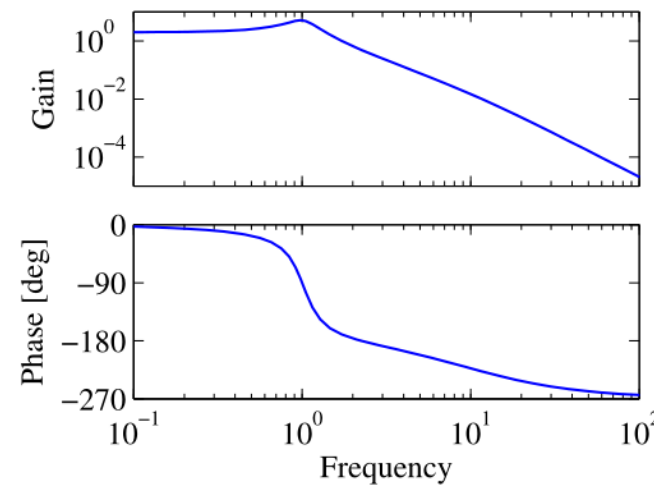
- **Input-output view**
  - Heritage of electrical engineering
  - Useful for
    Linear Time-Invariant (LTI) system



- **Transfer function**
  - Transfer ratio of the Laplace transform of the output to the Laplace transform of the input

$$\frac{Y(s)}{U(s)} = H(s)$$

$$\dot{y}(t) + ky(t) = u(t)$$



State-space model

- **Ordinary differential equations**
  - State space model: a form of differential equation

$$\frac{dx}{dt} = f(x,u) \qquad y = h(x,u)$$

$x \in \mathbb{R}^n$ state vector $\quad u \in \mathbb{R}^p$ input (vector)

$y \in \mathbb{R}^q$ output (vector)

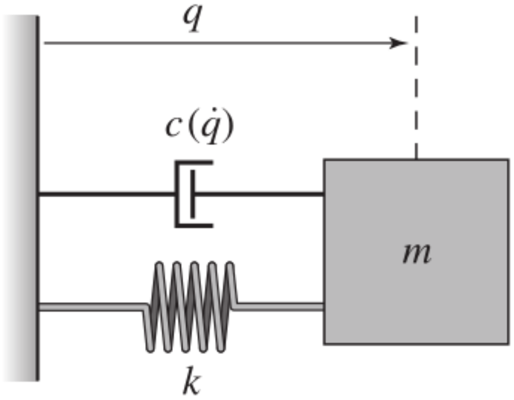
  - Linear state space system

$$\frac{dx}{dt} = Ax + Bu \qquad y = Cx + Du$$

dynamic matrix	control matrix	sensor matrix	direct term

State-space model

- **A simple example**



State-space model

- **Ordinary differential equations**
  - Another form of linear differential equations

$$\frac{d^n y}{dt^n} + a_1 \frac{d^{n-1} y}{dt^{n-1}} + \cdots + a_n y = u$$

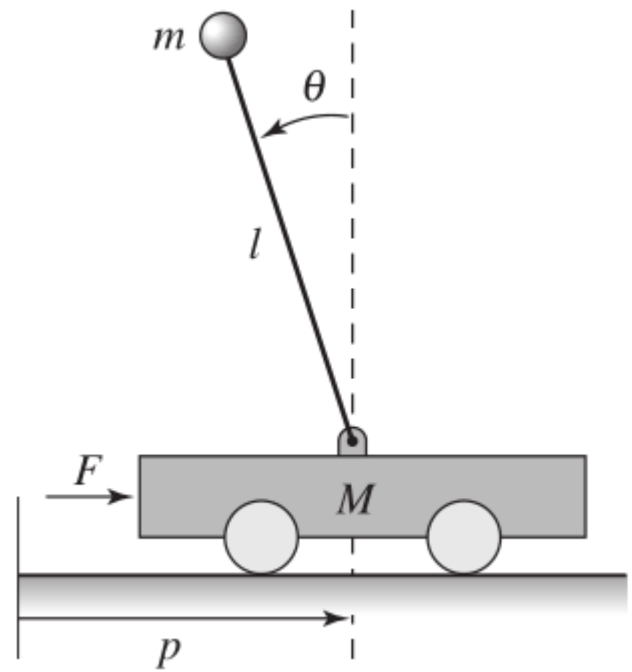
$$x = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_{n-1} \\ x_n \end{pmatrix} = \begin{pmatrix} d^{n-1}y/dt^{n-1} \\ d^{n-2}y/dt^{n-2} \\ \vdots \\ dy/dt \\ y \end{pmatrix}$$

$$\frac{d}{dt}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} -a_1 & -a_2 & \dots & -a_{n-1} & -a_n \\ 1 & 0 & \dots & 0 & 0 \\ 0 & 1 & & 0 & 0 \\ \vdots & & \ddots & & \vdots \\ 0 & 0 & & 1 & 0 \end{pmatrix} x + \begin{pmatrix} 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix} u$$

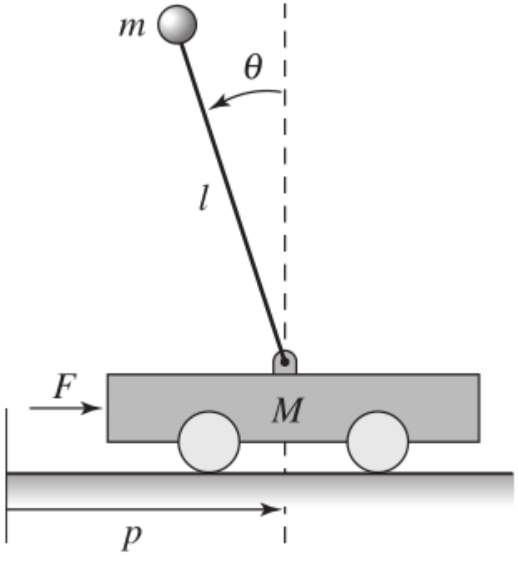
$$y = x_n$$

State-space model

- **Ordinary differential equations**
  - Example: Balance systems



State-space model



Dynamics of system

$$\begin{pmatrix} (M+m) & -ml\cos\theta \\ -ml\cos\theta & (J+ml^2) \end{pmatrix} \begin{pmatrix} \ddot{p} \\ \ddot{\theta} \end{pmatrix} + \begin{pmatrix} c\dot{p} + ml\sin\theta\,\dot{\theta}^2 \\ \gamma\dot{\theta} - mgl\sin\theta \end{pmatrix} = \begin{pmatrix} F \\ 0 \end{pmatrix}$$

$$M_t = M + m \qquad J_t = J + ml^2$$
$$c_\theta = \cos\theta \qquad s_\theta = \sin\theta$$

$$\frac{d}{dt}\begin{pmatrix} p \\ \theta \\ \dot{p} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} \dot{p} \\ \dot{\theta} \\ \dfrac{-mls_\theta\dot{\theta}^2 + mg(ml^2/J_t)s_\theta c_\theta - c\dot{p} - (\gamma/J_t)mlc_\theta\dot{\theta} + u}{M_t - m(ml^2/J_t)c_\theta^2} \\ \dfrac{-ml^2 s_\theta c_\theta\dot{\theta}^2 + M_t gls_\theta - clc_\theta\dot{p} - \gamma(M_t/m)\dot{\theta} + lc_\theta u}{J_t(M_t/m) - m(lc_\theta)^2} \end{pmatrix} \qquad y = \begin{pmatrix} p \\ \theta \end{pmatrix}$$

$$\frac{dx}{dt} = f(x,u) \qquad\qquad y = h(x,u)$$

State-space model

$$\frac{d}{dt}\begin{pmatrix} p \\ \theta \\ \dot{p} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} \dot{p} \\ \dot{\theta} \\ \dfrac{-mls_\theta\dot{\theta}^2 + mg(ml^2/J_t)s_\theta c_\theta - c\dot{p} - (\gamma/J_t)mlc_\theta\dot{\theta} + u}{M_t - m(ml^2/J_t)c_\theta^2} \\ \dfrac{-ml^2 s_\theta c_\theta\dot{\theta}^2 + M_t gls_\theta - clc_\theta\dot{p} - \gamma(M_t/m)\dot{\theta} + lc_\theta u}{J_t(M_t/m) - m(lc_\theta)^2} \end{pmatrix}$$

$$\frac{d}{dt}\begin{pmatrix} p \\ \theta \\ \dot{p} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & m^2l^2g/\mu & -cJ_t/\mu & -\gamma lm/\mu \\ 0 & M_tmgl/\mu & -clm/\mu & -\gamma M_t/\mu \end{pmatrix}\begin{pmatrix} p \\ \theta \\ \dot{p} \\ \dot{\theta} \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ J_t/\mu \\ lm/\mu \end{pmatrix} u$$

$$y = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} x$$

$$\frac{dx}{dt} = Ax + Bu \qquad y = Cx + Du$$

State-space model

▪ Difference equations

- Not continuously in time, but discrete instants of time

  → Discrete-time system

$$\frac{dx}{dt} = f(x,u) \quad \Rightarrow \quad x[k+1] = f(x[k],u[k])$$

$$y = h(x,u) \quad \Rightarrow \quad y[k] = h(x[k],u[k])$$

difference equation

Linear cases

$$x[k+1] = Ax[k] + Bu[k] \qquad y[k] = Cx[k] + Du[k]$$

State-space model

$$x[k+1] = Ax[k] + Bu[k] \qquad y[k] = Cx[k] + Du[k]$$

Solution ?

$$x[k] = A^k x[0] + \sum_{j=0}^{k-1} A^{k-j-1} Bu[j],$$

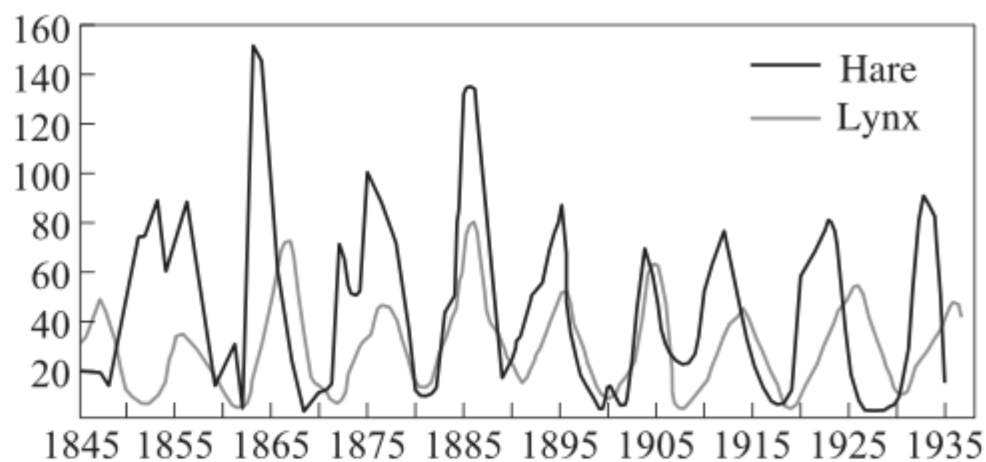
$$y[k] = CA^k x[0] + \sum_{j=0}^{k-1} CA^{k-j-1} Bu[j] + Du[k],$$

$$k > 0.$$

State-space model

- **Difference equations**
  - Example: Predator-prey
    - ✓ Ecological system with two species, one of which feeds on the other

Canadian lynx *vs* Snowshoe hare



A discrete data

State-space model

Model as a difference equation
by keeping track of rate of births and deaths of each species

$$H[k+1] = H[k] + b_r(u)H[k] - aL[k]H[k]$$
$$L[k+1] = L[k] + cL[k]H[k] - d_f L[k],$$

H population of hares

L population of lynxes

$b_r(u)$ hare birth rate

d_f lynx mortality rate

with many simplifying assumptions….

State-space model

$$H[k+1] = H[k] + b_r(u)H[k] - aL[k]H[k]$$
$$L[k+1] = L[k] + cL[k]H[k] - d_f L[k],$$