EME5943
현대제어시스템

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현대제어시스템

Linear systems (2)

Input/output response

- **Convolution equation**

$$\frac{dx}{dt} = Ax + Bu, \qquad y = Cx + Du.$$

Theorem 6.4. *The solution to the linear differential equation* (6.13) *is given by*

$$x(t) = e^{At}x(0) + \int_0^t e^{A(t-\tau)}Bu(\tau)d\tau. \tag{6.14}$$

Definition of convolution

$$(f * g)(t) \stackrel{\text{def}}{=} \int_{-\infty}^{\infty} f(\tau)\,g(t-\tau)\,d\tau = \int_{-\infty}^{\infty} f(t-\tau)\,g(\tau)\,d\tau.$$

Input/output response

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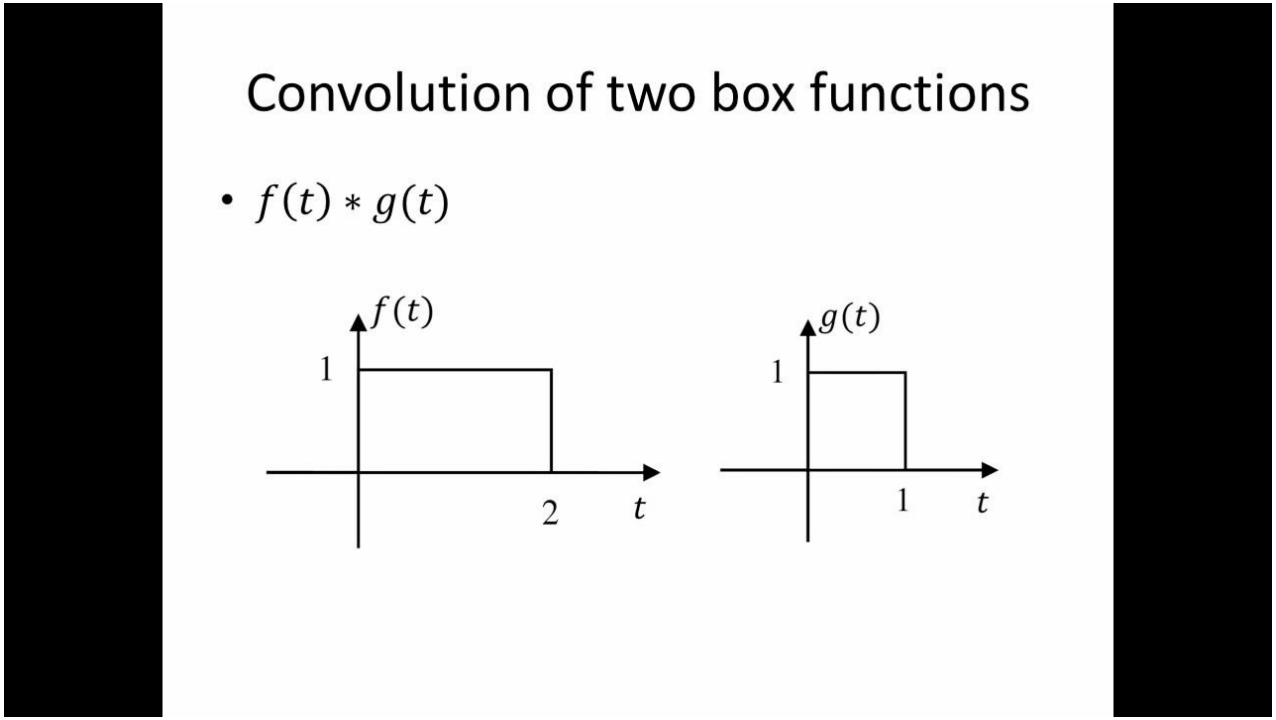


http://youtube.com/watch?v=C1N55M1VD2o

Differentiation of convolution

$$\frac{d}{dx}(f * g) = \frac{df}{dx} * g = f * \frac{dg}{dx}$$

Input/output response

$$\frac{dx}{dt} = Ax + Bu$$

$$x(t) = e^{At}x(0) + \int_0^t e^{A(t-\tau)}Bu(\tau)d\tau$$

$$\frac{dx}{dt} = Ae^{At}x(0) + \int_0^t Ae^{A(t-\tau)}Bu(\tau)d\tau + Bu(t)$$

Theorem 6.4. *The solution to the linear differential equation* (6.13) *is given by*

$$x(t) = e^{At}x(0) + \int_0^t e^{A(t-\tau)}Bu(\tau)d\tau. \tag{6.14}$$

$$y(t) = Ce^{At}x(0) + \int_0^t Ce^{A(t-\tau)}Bu(\tau)d\tau + Du(t).$$

Input/output response

- **Coordinate invariance**

$$z = Tx$$

$$\frac{dx}{dt} = Ax + Bu \quad \Rightarrow \quad \begin{aligned} \frac{dz}{dt} &= T(Ax + Bu) = TAT^{-1}z + TBu \\ &=: \tilde{A}z + \tilde{B}u \end{aligned}$$

$$y = Cx + Du = CT^{-1}z + Du =: \tilde{C}z + Du$$

Input/output response

$$\tilde{A} = TAT^{-1}, \qquad \tilde{B} = TB, \qquad \tilde{C} = CT^{-1}$$

$$x(t) = e^{At}x(0) + \int_0^t e^{A(t-\tau)}Bu(\tau)d\tau$$

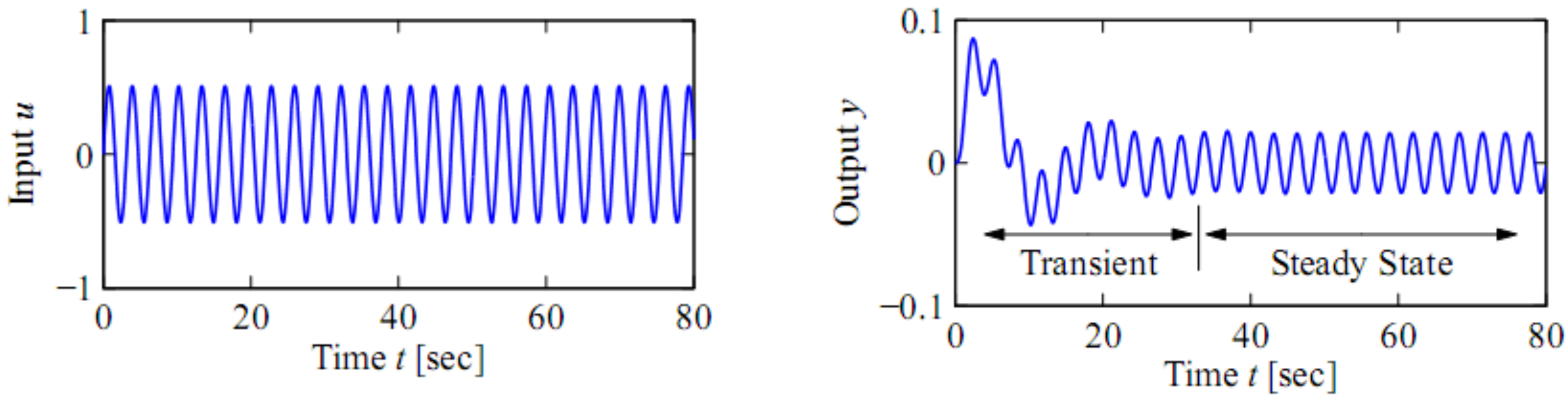
$$\Downarrow$$

$$e^{TST^{-1}} = Te^{S}T^{-1}$$

$$x(t) = T^{-1}z(t) = T^{-1}e^{\tilde{A}t}Tx(0) + T^{-1}\int_0^t e^{\tilde{A}(t-\tau)}\tilde{B}u(\tau)d\tau$$

Input/output response

- **Steady-state response**



- Transient response
  - ✓ Reflecting mismatch between initial condition and steady-state solution

- Steady-state response
  - ✓ Reflecting long-term behavior of system under given input

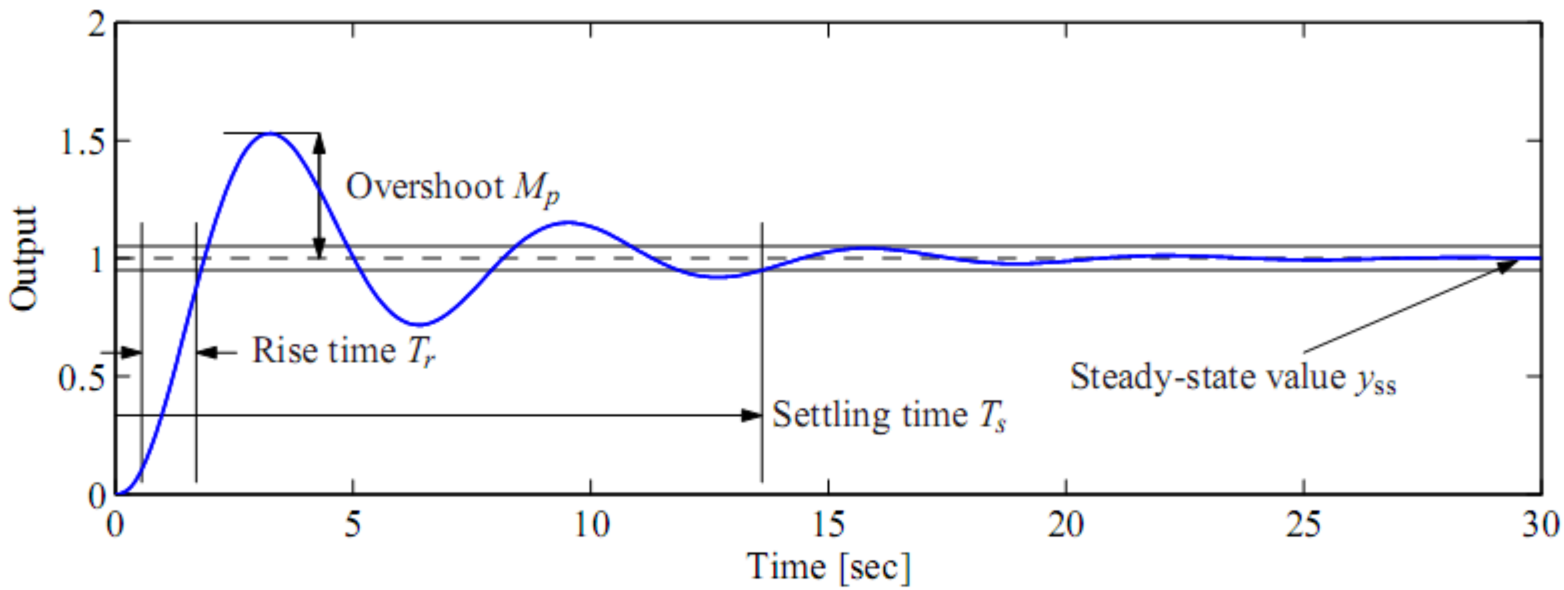
Input/output response

- **Steady-state response**
  - Step response : under unit step

unit step $$u = S(t) = \begin{cases} 0 & t = 0 \\ 1 & t > 0 \end{cases}$$

$$\begin{aligned} y(t) &= Ce^{At}x(0) + \int_0^t Ce^{A(t-\tau)}Bu(\tau)d\tau + Du(t) \\ &= C\int_0^t e^{A(t-\tau)}Bd\tau + D \\ &= C\int_0^t e^{A\sigma}Bd\sigma + D = C\left(A^{-1}e^{A\sigma}B\right)\Big|_{\sigma=0}^{\sigma=t} + D \\ &= CA^{-1}e^{At}B - CA^{-1}B + D \end{aligned}$$

Input/output response



- Steady-state value
- Rise time
  - ✓ Amount of time required for signal to go from 10% to 90% of its final
- Overshoot
  - ✓ Percentage of final value by which signal initially rises above final value
- Settling time
  - ✓ Amount of time required for signal to stay within 2% of its final value for all future