Algorithmic Game Theory
COMP6207

Summary on Single-item Auctions

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What is in these slides?

- Definitions of the four single-good auctions covered in Advanced Intelligence (AI) Module: English auction, Dutch auction, First-price sealed-bid auction, and Vickrey auction
- An overview of the relationship between these auctions, in terms of agents' strategies and auctions' revenues.

Recall the assumptions

- Agents' valuations (or types) are private information; i.e. each agent only knows his or her own type.
- Quasilinear utility model.
- Independent private value model
- No collusion between the agents.

English Auction

- One item for sale
- A set of bidders
- Auctioneer starts the bidding at some "reservation price".
- Bidders then shout out ascending prices.
- The auction is terminated once bidders stop shouting.

Rule (Allocation Rule)

The item is allocated to the bidder who shouted the last bid (the highest bid).

Rule (Payment Rule)

The winner is to pay the seller an amount equal to his/her bid.

Dutch Auction

- One item for sale
- A set of bidders
- Auctioneer starts the bidding at a high price.
- The auctioneer lowers the price until someone bids.

Rule (Allocation Rule)

The item is allocated to the bidder who bids.

Rule (Payment Rule)

The winner is to pay the seller an amount equal to his/her bid.

First-price sealed-bid auction

- One item for sale
- A set of bidders
- Bidders are asked to write down their bid on a piece of paper.

Rule (Allocation Rule)

The item is allocated to the bidder with the highest bid.

Rule (Payment Rule)

The winner is to pay the seller an amount equal to his/her bid.

Vickrey auction

- One item for sale
- A set of bidders
- Bidders are asked to write down their bid on a piece of paper.

Rule (Allocation Rule)

The item is allocated to the bidder with the highest bid.

Rule (Payment Rule)

The winner is to pay the seller an amount equal to the second highest bid.

First-price sealed-bid vs. Dutch auction

- In both auctions, if i is the winner she pays her bid b_i
- The amount of available information in both cases is the same (nothing of value is learned until the winner is announced)
- Therefore, the optimal strategies in both Dutch and first-price sealed-bid auctions are identical. These auctions are said to be **strategically equivalent**.

Vickrey auction vs. English auction

- English auction has a more complicated strategy space: a bidder may place several bids throughout the auction, conditioning their new bid on the information revealed (an extensive form game).
- To simplify the representation of a bidder's strategy in English auction, let us interpret the strategy s_i of bidder i as the point at which i should stop bidding (and not bid higher). Note that s_i is not necessarily the same as the last bid placed by bidder i, b_i, but that $b_i \leq s_i$.
- In English auction it is a dominant strategy for bidders to bid up to (and not beyond) their valuation; i.e. setting $s_i = \theta_i$ is a dominant strategy for each bidder i.
- In Vickrey auction it is a dominant strategy to bid truthfully, i.e. to set $b_i = \theta_i$.
- Bidding your true valuation (setting $b_i = \theta_i$) is not exactly the same as bidding up to, and not beyond, your true valuation (setting $s_i = \theta_i$) but it is closely similar. Therefore we say that these two auctions are **weakly strategically equivalent**.

Revenue equivalence

Assume that all bidders are risk-neutral and each has an independent private valuation for the single item, drawn from a common cumulative distribution $F(v)$ that is strictly increasing and atomless on $[L, H]$. Then any auction mechanism in which

- in equilibrium, the item will be allocated to the agent with the highest valuation, and
- any agent with valuation L has an expected utility of zero;

yields the same expected revenue, and hence results in any bidder with valuation v making the same expected payment.

The four single-good auctions we discussed all satisfy the above. Hence they are **revenue equivalent**.