Problem 1

Question 1: To prove that current house allocation isn't Pareto optimal, I'd like to find another allocation that is dominating the current one.

$a_1 \to h_2 \to a_2 \to h_2$ (a_2, h_2)

$a_1 \to h_3 \to a_3 \to h_1 \to a_1$ (a_1, h_3) (a_3, h_1)

$a_4 \to h_4 \to a_4$ (a_4, h_4)

∴ a_1: $h_2 > h_3 > h_4 > h_1$ original: a_1 $h_2 > h_3 > h_4 > h_1$

a_2: $h_2 > h_4 > h_3 > h_1$ a_2 $h_2 > h_4 > h_3 > h_1$

a_3: $h_1 > h_4 > h_3 > h_2$ a_3 $h_1 > h_4 > h_3 > h_2$

a_4: $h_1 > h_3 > h_4 > h_2$ a_4 $h_1 > h_3 > h_4 > h_2$

original: $\{(a_1, h_1), (a_2, h_2), (a_3, h_3), (a_4, h_4)\}$

New: $\{(a_1, h_3), (a_2, h_4), (a_3, h_1), (a_4, h_4)\}$

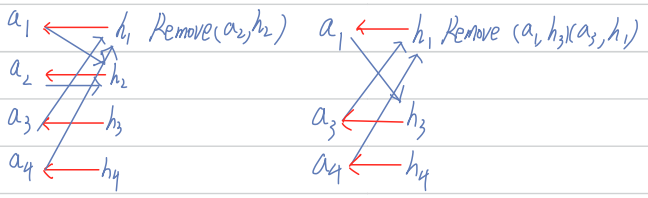
∵ a_1 and a_3 got the better houses, a_2 and a_4 still keep the original allocations.

∴ The new allocation is Pareto-optimal to current allocation

Question 2: $a_1 \to h_2 \to a_2 \to h_2$ (a_2, h_2)

$a_1 \to h_3 \to a_3 \to h_1 \to a_1$ (a_1, h_3) (a_3, h_1)

$a_4 \to h_4 \to a_4$ (a_4, h_4)



a_1: $h_2 > h_3 > h_4 > h_1$

a_2: $h_2 > h_4 > h_3 > h_1$

a_3: $h_1 > h_4 > h_3 > h_2$

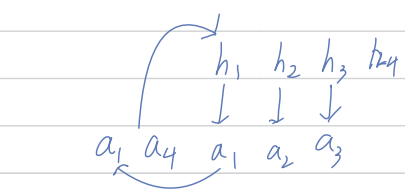
a_4: $h_1 > h_3 > h_4 > h_2$

∴ Follow the TTC mechnism, answer is $\{(a_1, h_3), (a_2, h_4), (a_3, h_1), (a_4, h_4)\}$

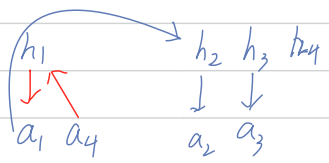
Question 3:

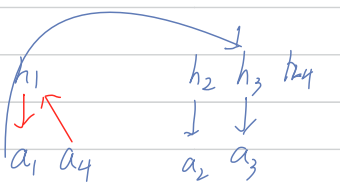
$A_E = \{a_1, a_2, a_3\}$ $A_N = \{a_4\}$ $H_0 = \{h_1, h_2, h_3\}$ $H_N = \{h_4\}$

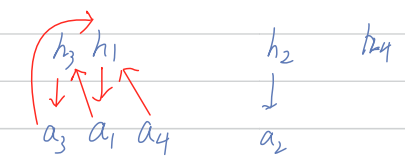
A_E			A_N
a_1	a_2	a_3	a_4
h_2	h_2	h_1	h_1
h_3	h_4	h_4	h_3
h_4	h_5	h_3	h_4
h_1	h_1	h_2	h_2

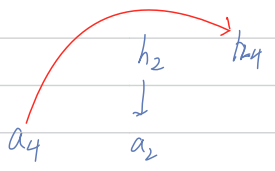


∴ The answer is $\{(a_1, h_3), (a_2, h_2), (a_3, h_4), (a_4, h_4)\}$









Question 4:

a_1 $h_2 > h_3 > h_4 > h_1$ answer is:

0.5 0.5

a_2 $h_2 > h_4 > h_3 > h_1$

0.5 0.5

a_3 $h_1 > h_4 > h_3 > h_2$

0.5 0.5

a_4 $h_1 > h_3 > h_4 > h_2$

0.5 0.5

	h_1	h_2	h_3	h_4
a_1	0	0.5	0.5	0
a_2	0	0.5	0	0.5
a_3	0.5	0	0	0.5
a_4	0.5	0	0.5	0

Problem 2 $C_5 > C_4 > C_1 > C_3 > C_2 > C_6$

question 1: $k=2$ C_1, C_2 rejected

$C_1 > C_3$ C_3 rejected

$C_4 > C_1 > C_3 > C_2$ accepted

Pick C_4

question 2: $k=5$ $C_1 C_2 C_3 C_4 C_5$ rejected

C_6 is the last one so accepted

Pick C_6

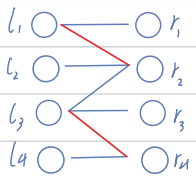
question 3: $k=4$ $C_1 C_2 C_3 C_4$ rejected

$\because C_2 > C_6$ C_6 rejected

$\therefore$ only accept C_5

Problem 3

question 1:

1. 

$\therefore$ 2 will be the maximum, because all nodes are matched and this is the largest possible number of edges.

1 and 3 will be maximal matching

For 1 l_2 just has one match to r_2 But l_1 has matched with r_2

l_4 just has one match to r_4 But l_3 has matched with r_4

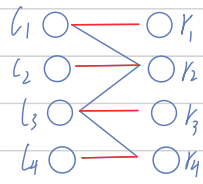
So we can't add more edges.

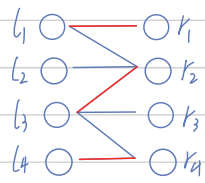
For 3, r_2 has matched to l_3

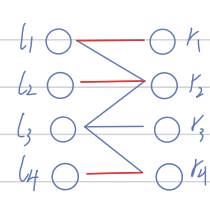
l_2 doesn't have other choice

So both 1 and 3 are maximal.

4 is neither, cause l_3 can match with r_3.

2. 

3. 

4. 

question 2:

	r_1	r_2	r_3	r_4
l_1	1			
l_2		0.5		
l_3		0.5	0.5	
l_4				1

Because of the neighbor relationships

r_1 just only have 1 neighbor l_1, so l_1 will get 1 unit water

r_2 have $l_1 l_2 l_3$ but l_1 is full, so $l_2 l_3$ will get 0.5

l_3 just only have 1 neighbor l_3 and l_3 already has 0.5, so just fill 0.5

For r_4 because its neighbor l_3 is full, so l_4 got 1 unit.

Problem 4

Question 1:

	unanimity	IIA	dictatorship
1	✓	✓	✓
2	✓	×	×
3	×	×	✓
4	×	✓	×

Question 2:

For unanimity

Alice	Bob	Cathy
C, F, I	C	I
	F	F
	I	C

The group preference will follow Bob's preference.
But Alice and Cathy have other preference.
That violated unanimity.

For IIA

Winner

Alice	Bob	Cathy
Chinese	Chinese	Italian
Italian	Italian	Chinese
French	French	French

There is no example to violate the IIA.
Because if someone want choice Chinese which below choice Italian. (These two choices relative)
That agent need to prefer Italian more than Chinese. That violates IIA,
Or agent need to make choices indifferent, let other agent's preference to be group preference. That also violates IIA.
So that function is follow IIA.

winner

Alice	Bob	Cathy
Italian	Chinese	Italian
Chinese	Italian	Chinese
French	French	French

winner

Alice	Bob	Cathy
Italian/Chinese/French	Italian	Italian
	Chinese	Chinese
	French	French

For dictatorship.

Alice	Bob	Cathy
Chinese	Italian	Italian
Italian	Chinese	Chinese
French	French	French

equally	Italian	Italian
	Chinese	Chinese
	French	French

equally	equally	Italian
		Chinese
		French

No matter what the conditions are, there is a dictator in the mechanism to always let the mechanism follow his preference.