COMP6203 Intelligent Agents 2022/2023

Exercises on Voting

Exercise 1: Celebration lunch

To celebrate the end of COVID-19 pandemic, members of the Intelligent Agents and Complexity (AIC) research group are organising a lunch get-together. To simplify the logistics, they decide to order take-away from a restaurant. They need to decide between 6 cuisine: Italian (I), Thai (T), Japanese (J), British (B), American (A), and Mauritian (M). Assume AIC has 50 members (this number is fictional), and that the preference orderings of these 50 members are as follows:

$$
\begin{aligned}
10 \text{ members} &: I \succ T \succ J \succ B \succ A \succ M \\
5 \text{ members} &: B \succ A \succ M \succ T \succ J \succ I \\
8 \text{ members} &: T \succ J \succ M \succ I \succ B \succ A \\
12 \text{ members} &: J \succ T \succ I \succ M \succ A \succ B \\
9 \text{ members} &: M \succ J \succ T \succ I \succ B \succ A \\
6 \text{ members} &: A \succ B \succ I \succ M \succ T \succ J
\end{aligned}
$$

Answer the following questions:

1. Assuming lexicographic tie breaking rule, order the cuisines according to the aggregated ranking produced by the Copeland method.
2. Assuming lexicographic tie breaking rule, order the cuisines according to the aggregated ranking produced by the Borda count voting method.
3. Is there a Condorcet winner? Argue for your answer.

Exercise 2: Manipulation

Take the setting of the previous exercise, but this time assume that there are only 6 members $\{1, 2, 3, 4, 5, 6\}$ in AIC and each preference ordering is provided by one member. That is:

$$
\begin{aligned}
1 &: I \succ_1 T \succ_1 J \succ_1 B \succ_1 A \succ_1 M \\
2 &: B \succ_2 A \succ_2 M \succ_2 T \succ_2 J \succ_2 I \\
3 &: T \succ_3 J \succ_3 M \succ_3 I \succ_3 B \succ_3 A \\
4 &: J \succ_4 T \succ_4 I \succ_4 M \succ_4 A \succ_4 B \\
5 &: M \succ_5 J \succ_5 T \succ_5 I \succ_5 B \succ_5 A \\
6 &: A \succ_6 B \succ_6 I \succ_6 M \succ_6 T \succ_6 J
\end{aligned}
$$

Assume that we are using Copeland method with lexicographic tie breaking rule.

4. Which of members, if any, does not see his or her top cuisine ranked first by Copeland and is able to manipulate the voting by declaring a different preference ordering (i.e. declaring a different preference ordering that results in Copeland ranking his or her cuisine first)?

50
10

1. IVS T 16/34 T
IVS J 16/34 J
IVS B 39/11 I
IVS A 39/11 I
IVS M 28/22 I
TVS J 29/21 T
TVS B 39/11 T
TVS A 39/11 T
TVS M 30/20 T
JVS B 39/11 J
JVS A 39/11 J
JVS M 30/20 J
BVS A 32/18 B
BVS M 21/29 M
AVS M 21/29 M

T: 5
J: 4-1=3
I: 3-2=1
B: 1-4=-3
M: 2-3=-1
A: 0-5=-5

T>J>I>M>B>A

T>J>I>M>B>A

I: 6×10+1×5+3×8+4×12+3×9+4×6=188
T: 5×10+3×5+6×8+5×12+4×9+2×6=221
J: 4×10+2×5+5×8+6×12+5×9+1×6=213
B: 3×10+6×5+2×8+1×12+2×9+5×6=136
A: 2×10+5×5+1×8+2×12+9×1+6×6=142
M: 1×10+4×5+4×8+3×12+6×9+6×3=170

I I J B A M

Comparison	Result	win
i vs T	16/34	T
i vs J	16/34	J
i vs B	39/11	I
i vs A	39/11	I
i vs M	28/22	I
T vs J	29/21	T
T vs B	39/11	T
T vs A	39/11	T
T vs M	30/20	T
J vs B	39/11	J
J vs A	39/11	J
J vs M	30/20	J
B vs A	32/18	B
B vs M	21/29	M
A vs M	21/29	M

T: 5
J: 3
I: 1
B: -3
M: -1
A: -5

T > J > I > M > B

I: 6 + 1 + 3 + 4 + 3 + 4 = 105 (12, 5, 9, 12, 7, 6)

T > J > I > M > B > A

T: 5 + 3 + 6 + 5 + 4 + 2 = 221

J: 4 + 2 + 5 + 6 + 5 + 1 = 213

B: 3 + 6 + 2 + 1 + 2 + 5 = 126

A: 2 + 5 + 1 + 2 + 1 + 6 = 122

M: 1 + 4 + 2 + 3 + 6 + 3 = 170

Yes T: 5

Comparison

i vs T	2/4	T
i vs J	2/4	J
i vs B	4/2	i
i vs A	4/2	i
i vs M	3/3	
T vs J	5/1	T
T vs B	4/2	T
T vs A	4/2	T
T vs M	3/3	T
J vs B	4/2	J
J vs A	4/2	J
J vs M	3/3	
B vs A	4/2	B
B vs M	3/3	
A vs M	3/3	

T: 4
J: 3-1=2
i: 2-2=0
B: 1-3=-2
A: 0-4=-4
M: 0

J: 比 I > T

i vs T → 3/3 i

7. Pareto Efficiency

Exercise 3: Desirable properties

In the lecture we discussed some desirable properties for social welfare and social choice functions. In this question we want to examine the three voting protocols we saw in the lecture with respect to these properties.

5. Of the three properties of *weak Pareto efficiency*, *monotonicity* and *nondictatorship*, which properties does the Plurality voting method satisfy? Argue for your answer.

6. Of the three properties of *Pareto efficiency*, *independence of irrelevant alternatives* and *nondictatorship*, which properties does the Borda count voting protocol satisfy? Argue for your answer.

7. Of the three properties of *Pareto efficiency*, *independence of irrelevant alternatives* and *nondictatorship*, which properties does the Copeland method satisfy? Argue for your answer.

Note: To show that a voting method V does not satisfy a particular property X, it is enough to show that for a particular example, V fails to satisfy X.

0.8 0.7 0.56 1.5 1.5 − 0.56 = 0.94

$\frac{0.8 - 0.56}{1.5 - 0.56}$ $\frac{0.24}{0.94}$

$\frac{24}{94}$ $\frac{12}{47}$

I vs T 16/34 T
I vs J 16/34 J
I vs B 39/11 I
I vs A 39/11 I
I vs M 28/22 I
T vs J 29/21 T
T vs B 39/11 T
T vs A 39/11 T
T vs M 30/20 T
J vs B 39/11 J
J vs A 39/11 J
J vs M 30/20 J
B vs A 24/26 B
B vs M 21/29 M
A vs M 21/29 M

T: 8
I: 3-2=1
J: 4-1=3
M: 2-3=-1
B: 1-4=-3
A: 0-5=-5

T ≥ J > I > M > B > A

Twin 5 for pair-wise

I: 60+5+24+48+27+24=182
T: 50+15+48+60+36+12=221
J: 40+10+40+72+45+6=213
M: 10+20+32+36+54+18=170
B: 30+30+16+12+18+30=136
A: 20+25+8+24+9+36=122

T > J > I > M > B > A

I vs T 2/4 T
I vs J 2/4 J
I vs B 4/2 I
I vs A 4/2 I
I vs M 3/3 =
T vs J 4/2 T
T vs B 4/2 T
T vs A 4/2 T
T vs M 3/3 =
J vs B 4/2 J
J vs A 4/2 J
J vs M 3/3 =
B vs A 4/2 B
B vs M 3/3 =
A vs M 3/3 =

I: 2-2=0

J = 3-1=2

T: 4

M: 0

A: -4

B = 1-3=-2

T > J > I > M > B > A

I vs T 2/4 T
I vs J 2/4 J
I vs B 4/2 I
I vs A 4/2 I
I vs M 3/3 =
T vs J 4/2 T
T vs B 4/2 T
T vs A 4/2 T

T vs M 3/3 =
J vs B 4/2 J
J vs A 4/2 J
J vs M 3/3 =
B vs A 4/2 B
B vs M 3/3 =
A vs M 3/3 =

I: 2-2=0
T: 4
J: 3-1=2
M: 0
A: 0-4=-4
B 1-3=-2