

COMP6207 Algorithmic Game Theory 2022/2023

Coursework 2

Due at 11:59pm on 17 May 2023

Problem 1

Agents a_1, a_2, a_3 , and a_4 are currently living in houses h_1, h_2, h_3, h_4 , respectively. Assume that their house preferences are as follows:

$$a_1 : h_2 > h_3 > h_4 > h_1$$

$$a_2 : h_2 > h_4 > h_3 > h_1$$

$$a_3 : h_1 > h_4 > h_3 > h_2$$

$$a_4 : h_1 > h_3 > h_4 > h_2$$

Question 1 [8 marks]. Prove that the current house allocation is not Pareto optimal by showing that it is dominated by another allocation.

Question 2 [8 marks]. Compute the house allocation produced by the Top Trading Cycle mechanism in this situation. Show consecutive steps of the algorithm.

Question 3 [8 marks]. For this question, suppose that agents a_1, a_2 , and a_3 are currently living in houses h_1, h_2, h_3 , respectively. House h_4 is not occupied and agent a_4 is a newcomer who does not have a house assigned yet. Assuming the preferences of the agents are the same as above, what house assignment will be produced by YRMH-IGYT("You request my house, I get your turn") mechanism? Show consecutive steps of the algorithm.

Question 4 [8 marks]. Now, suppose that the houses are not assigned, but the preferences of the agents are the same. What portion of each house will be allocated to which agent using the Simultaneous Eating mechanism? Give the answer by filling in the table below. you do not have to show your computations.

h_1	h_2	h_3	h_4
a_1			
a_2			
a_3			
a_4			

Problem 2

Recall that the Secretary algorithm rejects first k out of n candidates. Suppose that candidates c_1, c_2, c_3, c_4, c_5 , and c_6 arrive in that order. Thus, $n = 6$. Your preferences among candidates are:

$$c_5 > c_4 > c_1 > c_3 > c_2 > c_6.$$

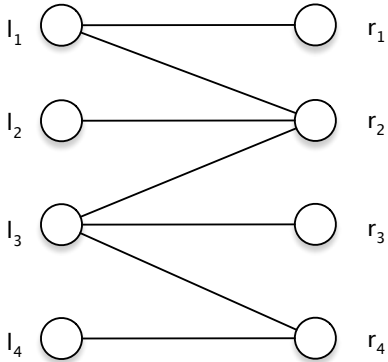
Question 1 [8 marks]. What candidate will be selected if $k = 2$? Explain your answer.

Question 2 [8 marks]. What candidate will be selected if $k = 5$? Explain your answer.

Question 3 [8 marks]. For which values of k the most preferred candidate, c_5 , will be selected? Give all such values. You do not have to explain your answer.

Problem 3

Consider the bipartite graph depicted below.



Question 1 [10 marks]. Which of the following four matchings are *maximum*, which are *maximal* but not *maximum*, and which are neither? You do not have to explain your answer.

1. $(l_1, r_2), (l_3, r_4),$
2. $(l_1, r_1), (l_2, r_2), (l_3, r_3), (l_4, r_4),$
3. $(l_1, r_1), (l_3, r_2), (l_4, r_4),$
4. $(l_1, r_1), (l_2, r_2), (l_4, r_4),$

Question 2 [10 marks]. Suppose that left nodes are “buckets” and right nodes are the sources of water. Each bucket and each source has the same capacity of one unit. What is the distribution of water from different sources between the buckets is produced by the Water Level algorithm? Give the answer by filling in the table below. You do not have to show your computations.

	r_1	r_2	r_3	r_4
l_1				
l_2				
l_3				
l_4				

Problem 4

Alice, Bob, and Cathy decided to go to a restaurant to celebrate the end of the Algorithmic Game Theory module. They consider three possible restaurants: Chinese, French, and Italian. Being good Game Theory students, they first decided to agree on the preference aggregation mechanism. Here are four social choice functions (mechanisms) that they consider:

1. First, see if Alice ranked equally all three options. If she did not, then take Alice's preferences as the group preferences. If yes, see if Bob ranked equally all three options. If he did not, then take Bob's preferences as the group preferences. If yes, take Cathy's preferences as the group preferences.
2. If Alice's and Bob's preferences agree, take them as the group preferences. Otherwise, take Cathy's preferences as the group preferences.
3. If Alice ranked Chinese higher than French, then the group preferences are:

Chinese

French, Italian

If Alice ranked French higher than Chinese, then the group preferences are:

French

Chinese, Italian

Otherwise, the group preferences are:

French, Chinese, Italian

4. First, ignore French option and see if the majority prefers Chinese over Italian. In this case, the group preferences are:

Chinese

Italian

French

If the majority prefers Italian over Chinese then the group preferences are:

Italian

Chinese

French

If there is no majority (for example, one prefers Chinese, one prefers Italian, and the last ranks Chinese and Italian equally), then the group preferences are:

Chinese, Italian

French

Question 1 [12 marks].

Which of these social choice functions satisfy the unanimity condition, which satisfy the IIA condition, and which are dictatorships? Copy the table below and fill it in with Yes/No answers. You do not have to explain your answers.

	unanimity	IIA	dictatorship
1			
2			
3			
4			

You will get 3 marks for each correct row and 0 marks for each incorrect row.

Question 2 [12 marks].

Explain your answers for the FIRST row **only** of Question 1. Each of the three explanations will get up to 4 marks.