SEMESTER 2 EXAMINATION 2016 - 2017

ADVANCED INTELLIGENT AGENTS

DURATION 120 MINS (2 Hours)

This paper contains 10 questions

Answer ALL questions from section A (40 marks)
and THREE of the five questions from section B (60 marks).

This examination provides 75% of the module's marks.

An outline marking scheme is shown in brackets to the right of each question.

Only university approved calculators MAY be used.

A foreign language dictionary is permitted ONLY IF it is a paper version of a direct Word to Word translation dictionary AND it contains no notes, additions or annotations.

16 page examination paper.

Section A

Question A1.

Consider the extensive form game with players 1 and 2 shown in Figure 1.

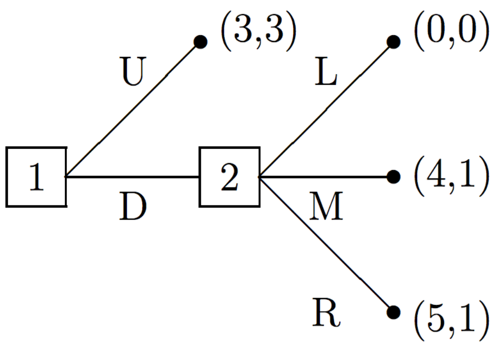

FIGURE 1: Extensive form game.

(a) Write down the normal form representation (i.e., the payoff matrix) of the game. Find all pure strategy Nash equilibria. [5 marks]

(b) One of the pure strategy Nash equilibria is NOT a subgame perfect Nash equilibrium. Which one? Explain your answer. [3 marks]

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Question A2.

Suppose a VCG mechanism is applied to sell two objects $\{A, B\}$ to three buyers $\{1, 2, 3\}$. A buyer can buy none, one, or both of the objects. The valuation function of each buyer depends only on the set of objects assigned to that buyer and is given by:

$$
\begin{array}{llll}
v_1(\emptyset) = 0 & v_1(\{A\}) = 3 & v_1(\{B\}) = 2 & v_1(\{A, B\}) = 15 \\
v_2(\emptyset) = 0 & v_2(\{A\}) = 2 & v_2(\{B\}) = 8 & v_2(\{A, B\}) = 10 \\
v_3(\emptyset) = 0 & v_3(\{A\}) = 10 & v_3(\{B\}) = 3 & v_3(\{A, B\}) = 12
\end{array}
$$

(a) Determine the assignment of objects to buyers and the payments of the buyers, under truthful bidding. [6 marks]

(b) Discuss why buyer 1 might have an objection to the outcome. [2 marks]

VCG

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Question A3.

Consider the cooperative game (N, v) with three agents $N = \{a, b, c\}$, represented as an MC-net as follows:

$$\begin{array}{ll} (a, 8) & (a \wedge b, 7) \\ (b, 5) & (a \wedge c, -5) \\ (c, 3) & (b \wedge c, -3) \\ & (a \wedge b \wedge c, 1) \end{array}$$

(a) Find the characteristic function v of the game. [4 marks]

(b) Verify if the game is motonone. [2 marks]

(c) Verify if the game is super-additive. [2 marks]

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Question A4.

(a) Explain the stochastic multi-armed bandit model. [4 marks]

(b) Give the definition of external regret in the adversarial bandit setting. [2 marks]

(c) What is the difference between the stochastic and adversarial settings of bandit problems? [2 marks]

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Question A5.

(a) Define the UCB index in the stochastic bandit algorithm UCB, and explain how it can be used to choose the next arm. [4 marks]

(b) Which one is the better algorithm, the UCB or the epsilon-greedy? Explain your answer. [4 marks]

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Section B

Question B1.

best response和纳什均衡

(a) Two firms, A and B, compete *a la Cournot*. The (inverse) market demand function is $p = \max\{8 - q_A - q_B, 0\}$, where p is the market price **per unit** and q_i is the quantity produced by firm i for $i \in \{\mathrm{A}, \mathrm{B}\}$.

Firm A has a constant cost $c_A = 3$ **per unit** it produces, and firm B has a constant cost $c_B = 5$ **per unit** it produces.

Answer the following questions and show your calculations.

(i) Find the best response functions of firms A and B.

(ii) Compute the Nash equilibrium.

[7 marks]

(b) Consider the 2-player game in Table 1:

	L	**C**	**R**
U	1, 2	3, 5	2, 1
M	0, 4	2, 1	3, 0
D	-1, 1	4, 3	0, 2

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TABLE 1: 2-player game.

(i) Use elimination of dominated strategies (EDS) to solve this game. Show the order in which you are eliminating strategies, and specify whether you are eliminating strictly or weakly dominated strategies.

(ii) Find all rationalisable strategies for both players.

(iii) Is there a strong equilibrium in this game?

[6 marks]

(c) Consider a security game with a set of three targets $T = \{t_1, t_2, t_3\}$. The attacker (A) can attack any single target, and the defender (D)

TURN OVER

has one resource that can cover any single target. The utilities of the players are as defined in Table 2:

	t_1	t_2	t_3
	covered / uncovered	covered / uncovered	covered / uncovered
D	0 / -1	0 / -1	0 / -1
A	1 / 3	1 / 3	0 / 2

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TABLE 2: Security game.

(i) Write down the normal form representation of the game.

(ii) Compute all (mixed) Nash equilibrium strategies of player D.

(iii) Can you find the minimax and Stackelberg strong equilibrium strategies of player D? Explain!

[7 marks]

Question B2.

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(a) Facebook applies the Groves mechanism with Clarke pivot rule (a.k.a. VCG) for its display advertising. Consider a setting where 2 slots are sold using this mechanism for a visiting user, and there are 3 interested advertisers (agents) A, B, and C competing for these slots. One of the slots is more prominent and therefore more valuable to all agents. Also, two of the advertisers, A and B are direct competitors since they are selling similar products (e.g. Apple vs Microsoft). Therefore, if A's ad is shown in either slot, the value of the remaining slot for B is diminished, and similarly for A if B's ad is shown. Consequently, the utility not only depends on which slot they receive, but also on who receives the other slot (there are so-called allocative externalities). All advertisers prefer to receive both slots compared to one slot.

The valuations are given in Table 3, where v indicates the value without the slot awarded to the direct competitor, and v' the value in case the other slot is given to the direct competitor.

Agent	Slot 1	Slot 2	Slots 1 and 2
A	$v_{A,1} = 2.0, v'_{A,1} = 1.5$	$v_{A,2} = 1.6, v'_{A,2} = 0.9$	$v_{A,1+2} = 2.3$
B	$v_{B,1} = 2.5, v'_{B,1} = 1.6$	$v_{B,2} = 2.0, v'_{B,2} = 1.5$	$v_{B,1+2} = 2.9$
C	$v_{C,1} = 1.0$	$v_{C,2} = 0.7$	$v_{C,1+2} = 1.1$

TABLE 3: Agent Preferences

(i) Given the valuations in Table 3, what is the allocation that maximises the social welfare? How much is the social welfare in this case?

(ii) Compute the transfers for *all* agents when applying the Groves mechanism with the Clarke pivot rule.

(iii) Show whether the resulting transfers are (1) individually rational, (2) weakly budget balanced, (3) strongly budget balanced.

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[12 marks]

(b) A school board is voting whether to award the contract for building a new elementary school to Ants (A), Bees (B), Centipedes (C), Dragonflies (D) or Earthworms (E).

There are 12 members of the board, whose preferences over the candidates are distributed as follows:

$$
\begin{array}{ll}
 & \text{Ranking} \\
\# \text{ voters} & \begin{array}{l}
5: \ B \succ E \succ A \succ C \succ D \\
3: \ C \succ E \succ A \succ D \succ B \\
3: \ D \succ A \succ E \succ C \succ B \\
1: \ A \succ C \succ E \succ D \succ B
\end{array}
\end{array}
$$

(i) Find the winner of the election held using each of the following voting schemes. Provide the corresponding winning score or the elimination procedure when relevant.

- Plurality vote. 这个没有
- STV.
- Maximin.
- Copeland.

(ii) Is there a Condorcet winner or a Condorcet loser in this profile?

[8 marks]

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Question B3.

(a) Construct an example of a marriage problem, in which there is more than one stable matching. (You only need two boys and two girls to do this.) [5 marks]

(b) Suppose that the boys all have different favourite girls. How many steps does it take for the Gale-Shapley algorithm to converge? Please explain your answer. [3 marks]

(c) Suppose that the boys have identical preferences. How many steps does it take for the algorithm to converge? Please explain your answer.

[3 marks]

(d) Suppose preferences are given by Tables 4 and 5:

	1	2	3	4	5
Adam	Beth	Amy	Diane	Ellen	Cara
Bill	Diane	Beth	Amy	Cara	Ellen
Carl	Beth	Ellen	Cara	Diane	Amy
Dan	Amy	Diane	Cara	Beth	Ellen
Eric	Beth	Diane	Amy	Ellen	Cara

TABLE 4: Boys' preferences.

	1	2	3	4	5
Amy	Eric	Adam	Bill	Dan	Carl
Beth	Carl	Bill	Dan	Adam	Eric
Cara	Bill	Carl	Dan	Eric	Adam
Diane	Adam	Eric	Dan	Carl	Bill
Ellen	Dan	Bill	Eric	Carl	Adam

TABLE 5: Girls' preferences.

Find a stable matching using the Gale-Shapley algorithm with boys

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making proposals. Find a stable matching using the Gale-Shapley algorithm with girls making proposals. [9 marks]

Question B4.

(a) Consider the cooperative game (N, v) with three agents $N = \{a, b, c\}$ and the following characteristic function:

$$v(\emptyset) = 0 \quad v(\{a\}) = 5 \quad v(\{b\}) = 3 \quad v(\{c\}) = 2$$
$$v(\{a, b\}) = 12 \quad v(\{a, c\}) = 5 \quad v(\{b, c\}) = 4 \quad v(\{a, b, c\}) = 13$$

(i) Find the super-additive cover (N, v^*) of the game.

(ii) Check for the (non)-emptiness of the core. If the core is empty, prove so. Otherwise, find an element in the core and show why it belongs there.

[7 marks]

(b) A research council with the budget of \$360,000 receives applications from three institutions A, B and C for funding their projects, requesting the following amounts:

- A: \$100,000
- B: \$200,000 没学
- C: \$300,000

(i) Compute the characteristic function v of the coalitional game with players $N = \{A, B, C\}$.

(ii) How should the council distribute its budget among the three institutions, according to the Shapley value?

(iii) How should the council distribute its budget among the three institutions, according to the nucleolus?

[13 marks]

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Question B5.

We aim to solve a stochastic multi-armed bandit problem with 3 arms with the UCB algorithm. Each arm, when pulled, provides a random reward between 0 and 1. After 200 pulls, we learnt that the average reward collected from Arms 1, 2, and 3 are 0.4, 0.595, and 0.21, respectively. Also, the number of times we have pulled the arms are: 90 for Arms 1 and 2, and 20 for Arm 3.

(a) What is the probability that Arm 3 will be chosen by the UCB algorithm for pull in the 201^{th} step? Please explain your answer. [6 marks]

(b) Assuming that the real expected reward value of Arm 2 is 0.5, prove that it is more likely that the UCB algorithm will choose Arm 3 at the 202^{th} time step instead of Arm 2. [8 marks]

(c) Using Hoeffding's inequality, estimate the number of times we need to pull Arm 1 to guarantee that the probability of having the expected reward estimation error of Arm 1 to be larger than 0.1 is at most 10%. [6 marks]

END OF PAPER

