COMP6207
Algorithmic Game Theory

Lecture 18 Generalised Second Price Auction (GSP)

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Learning Outcomes

- By the end of this session, the students should be able to
  - ***Describe*** possible strategic behaviors in Google's auction model
  - ***Compute*** the allocation and pricing of the VCG auction
  - ***Compare*** the VCG and GSP auction models
  - ***Apply*** the principles of mechanism design on auction models, and ***evaluate*** the solutions.

Search Engine

- Products of Search Engines
  - k slots to sell
  - Each slot can generate a certain number of clicks

- How to match slots to advertisers?
- How to charge advertisers?
- Obj: how to maximize revenue?

- Pay-per-impression
- Pay-per-click
- Pay-per-action

Advertiser/bidder

- Each has a set of interested keywords
- Each has a private value for each of the keyword
- Each has a private daily/weekly/monthly budget
- Use their budget to buy clicks
- Obj: how to maximise utility, subject to budget?

VCG Review
Vickrey–Clarke–Groves

What choice does VCG pick (i.e. who gets A and who gets B), and what is the payment for each agent?

Combinatorial auction setting with two agents

- $v_1(A) = 3$, $v_1(B) = 2$, $v_1(AB) = 6$
- $v_2(A) = 1$, $v_2(B) = 4$, $v_2(AB) = 4$
- The social welfare maximising allocation awards A to agent 1 and B to agent 2, hence achieving social welfare of $v_1(A) + v_2(B) = 7$.
- $p_1 = 4 - 4 = 0$ and $p_2 = 6 - 3 = 3$.

VCG in Sponsored Search

- In the context of the entire keywords-slots matching market, it is a multi-item auction setting.
- In the context of a single keyword, VCG degenerates to a pseudo Vickery auction.
  - Advertisers bid a value on a keyword, not on a slot.
- However, none of the search engines use VCG. Why?
  - Computational efficiency
  - Disadvantages of VCG
  - Difficult to explain to their customers

Generalised First Price auction (GFP)

- In an advertising slot auction with *k* slots
  - The bidders with the highest *k* prices each wins a slot.
  - The *k*-th highest-price bidder gets the *k*-th slot and pay its bidding price.
  - The other bidders lose.

GFP was introduced by Overture (then GoTo, then part of Yahool!) in 1997.

GFP was not good in terms of stability, therefore it was not adopted by Google.

Generalised Second Price auction (GSP)

- In an advertising slot auction with k slots
  - The bidders with the highest k prices each win a slot.
  - The i-th highest-price bidder gets the i-th slot and pay the $(i+1)$-th highest bidding price.
  - The other bidders lose.

- On top of it, Google implements a weighted version of this auction.

Google's Quality Score

- Google computes advertiser *j*'s quality score β_j based on (amongst other things):
  - Historic click frequency
  - Quality of the *landing page*

  Formula kept secret
- Estimation of the click probability

Generalised Second Price (GSP) Auction

- Google's auction:
  - The highest bidder $b_1\beta_1$ gets the first slot; the second-highest bidder $b_2\beta_2$ gets the second slot; etc.
  - The highest bidder pays the second-highest *effective* price, i.e., $p_1\beta_1 = b_2\beta_2 \quad \Leftarrow \quad p_1 = b_2\beta_2/\beta_1$; and so on.
- Pay the smallest amount you could have bid and still retain the same position
- Bidder *j*'s expected utility = $\beta_j (v_j - p_j)$
- Search engine's expected revenue: $\sum_j \beta_j \times p_j$

Exercise

- Find the position and payment for the following example, given that there are 2 slots

	Quality score	Bid
Advertiser 1	0.60	£10
Advertiser 2	0.90	£7
Advertiser 3	0.40	£5

Click-through-rate

- Click-through-rate CTR_{ij}
  - the probability that a user will click on the *i*-th slot when it is showing ad *j*. (estimated by search engines, primarily taking the quality of bidders and the relevance of their ads to queries into account)

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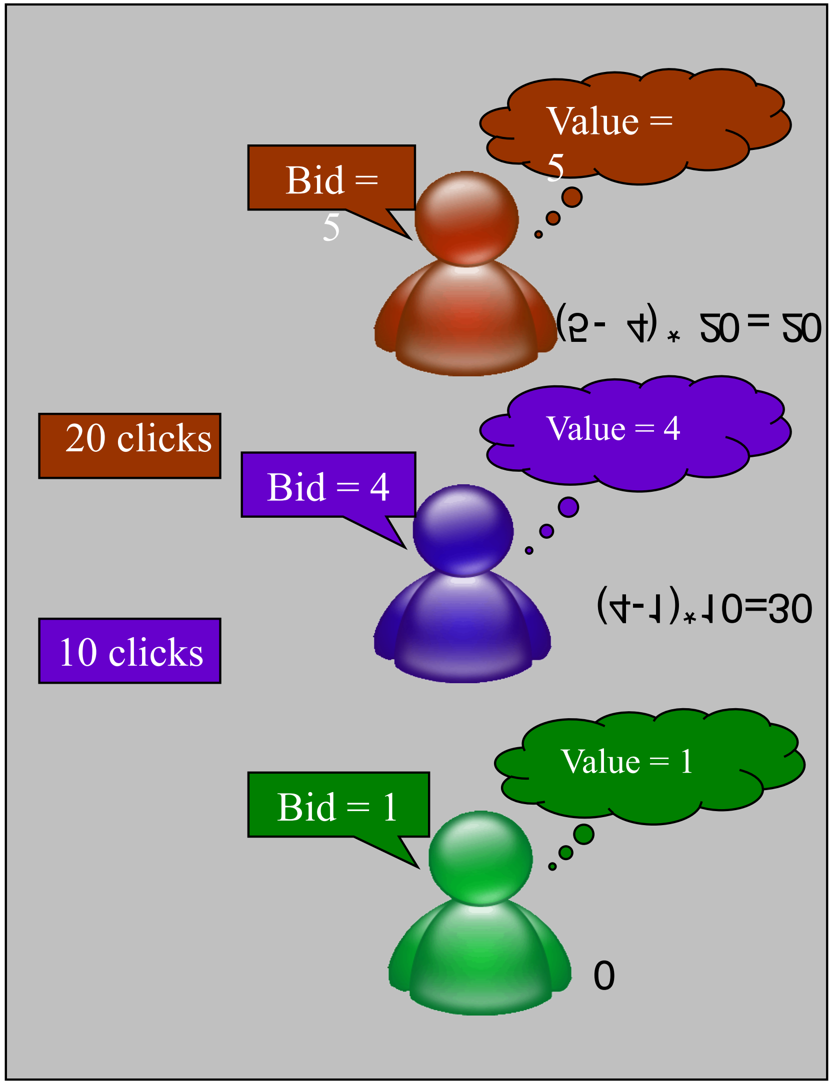
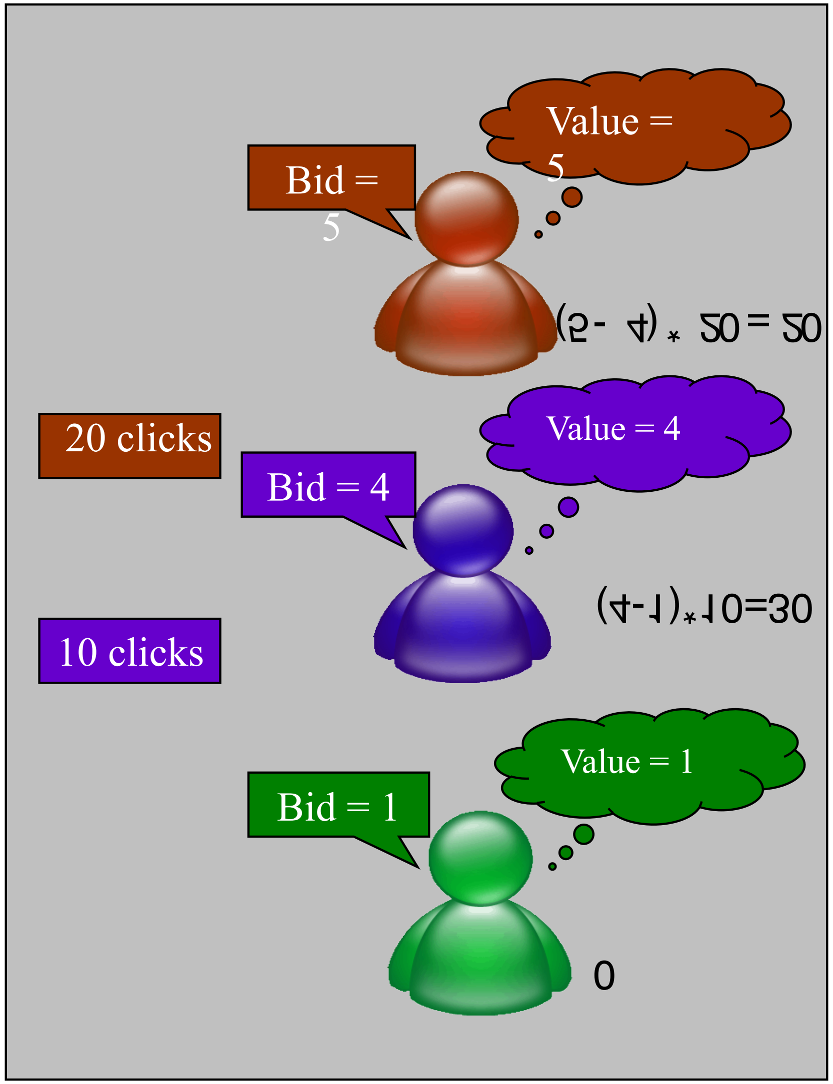
- Separable $CTR_{ij} = \alpha_i \times \beta_j$, where α_i is the quality of slot *i*, and β_j is the quality of bidder j.
  - This is Google's model. Yahoo! sets all beta=1
  - It is assumed that α_i follows the Power Law.
  - Cascade model: click probability α_i depends on whether or not previous ad α_{i-1} was clicked

A strategic behaviour in GSP



- Two slots with 20 clicks and 10 clicks
- Three bidders with true values 5, 4, and 1
- If they all bid truthfully:
- Bidder gets 20 clicks and pays 4
  Utility = (5 - 4) * 20 = 20
- Bidder gets 10 clicks and pays 1
  Utility = (4 - 1) * 10 = 30

A strategic behaviour in GSP



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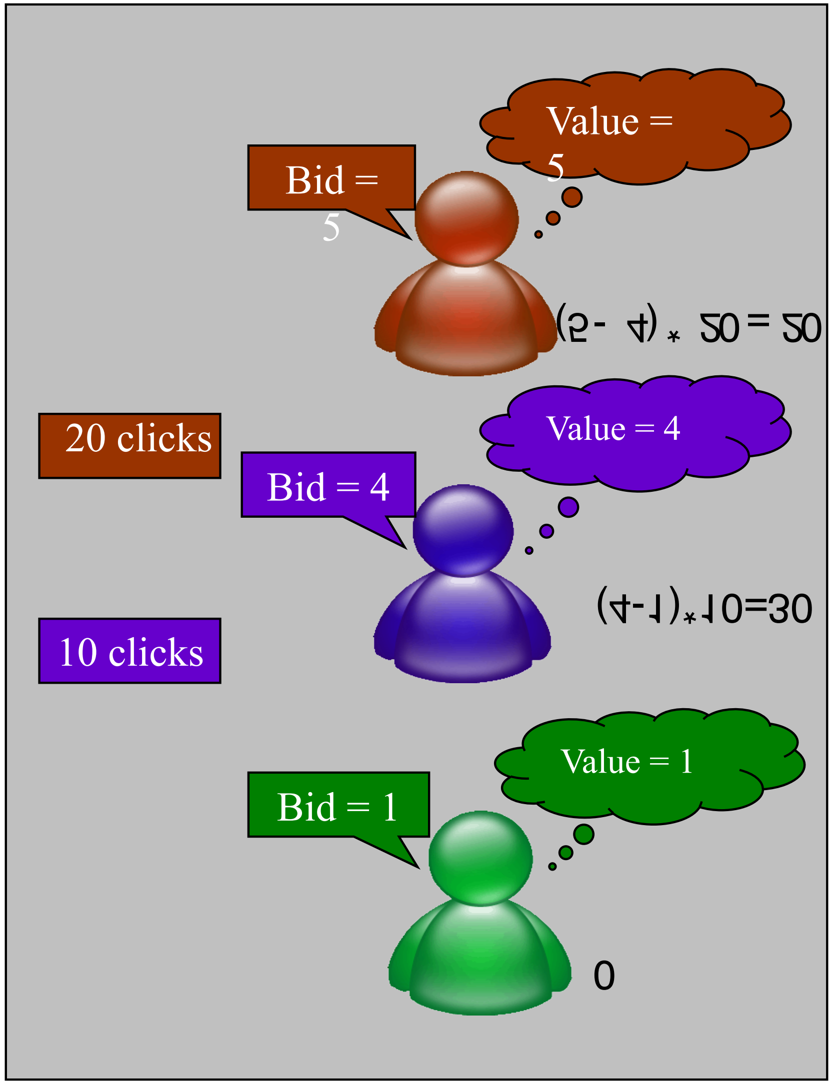
- What if bidder lower its bids?
  - Bid 4.5?
  - Bid 3.5?
  - Bid 2?

A strategic behaviour in GSP



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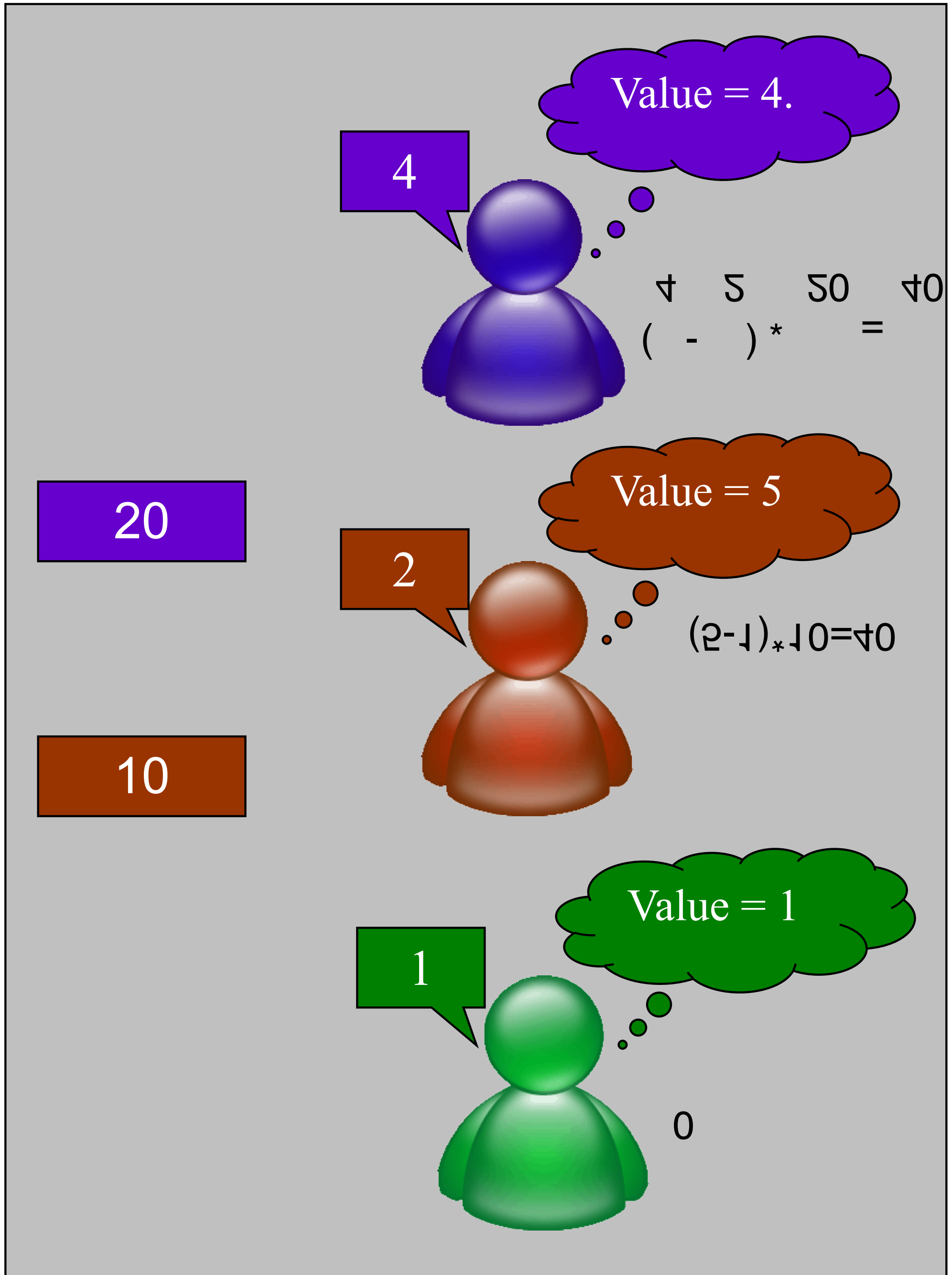
Pure Nash equilibrium of GSP

- Pure Nash equilibrium:
  - a bid profile $(b_1,\ldots,b_k,\ldots,b_m)$ s.t., for any bidder k
    - $(v_k - b_{k+1})*\alpha_k >= (v_k - b_j)*\alpha_j$ for any $j<k$;
    - $(v_k - b_{k+1})*\alpha_k >= (v_k - b_{j+1})*\alpha_j$ for any $j>k$.

Is this a Pure Nash Equilibrium?



- BLUE gets 20 clicks
  - Pays 2 each
  - Profit = (4-2)*20 = 40
- RED gets 10 clicks
  - Pays 1 each
  - Profit = (5-1)*10 = 40
- GREEN gets 0 clicks
  - Pay nothing
  - Profit = 0

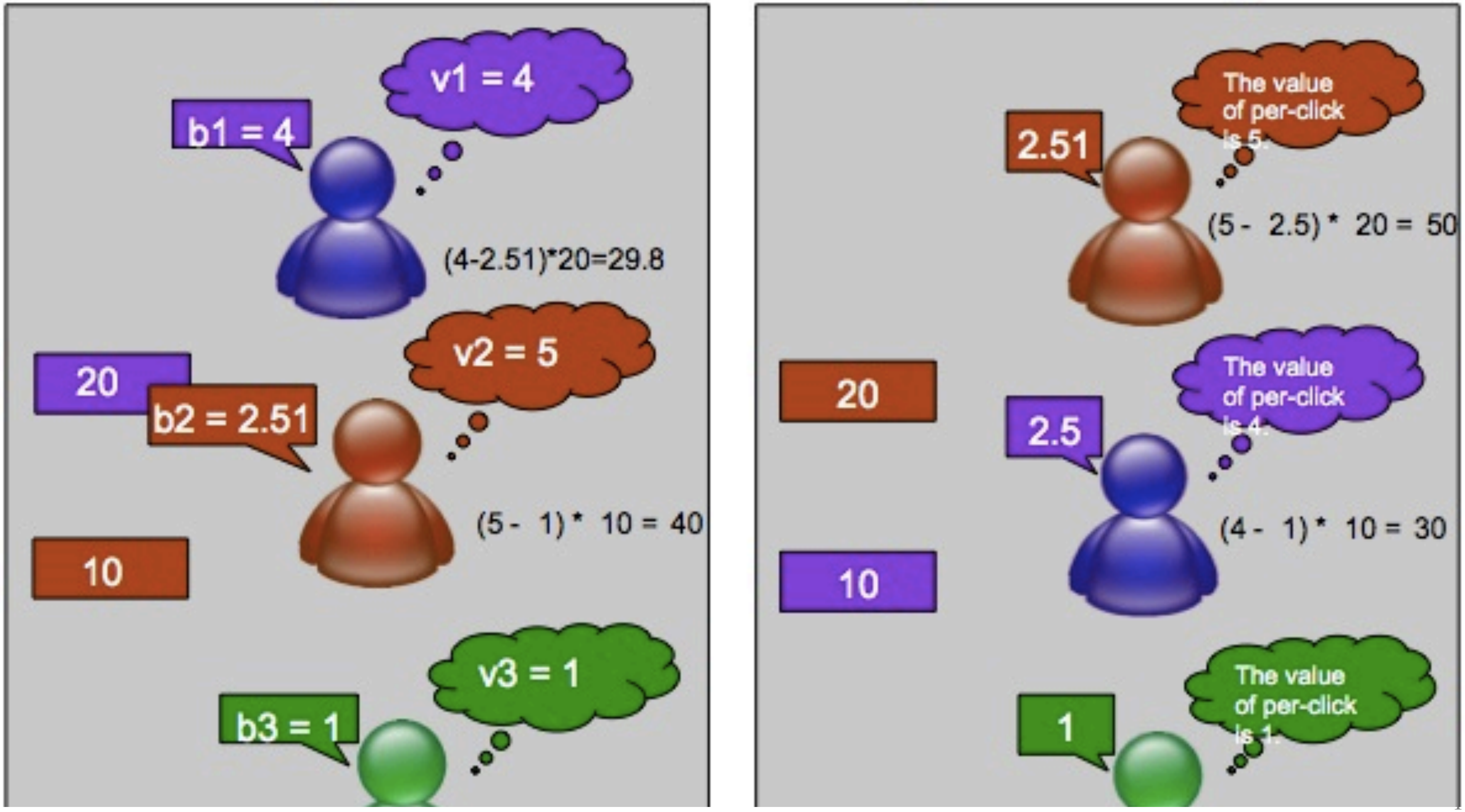
- Try out different strategies

Automated bidding

- Third-Party bid proxy
- Web analytics data warehouse
- Bid management system
- Biding optimizer

Forward-looking behavior

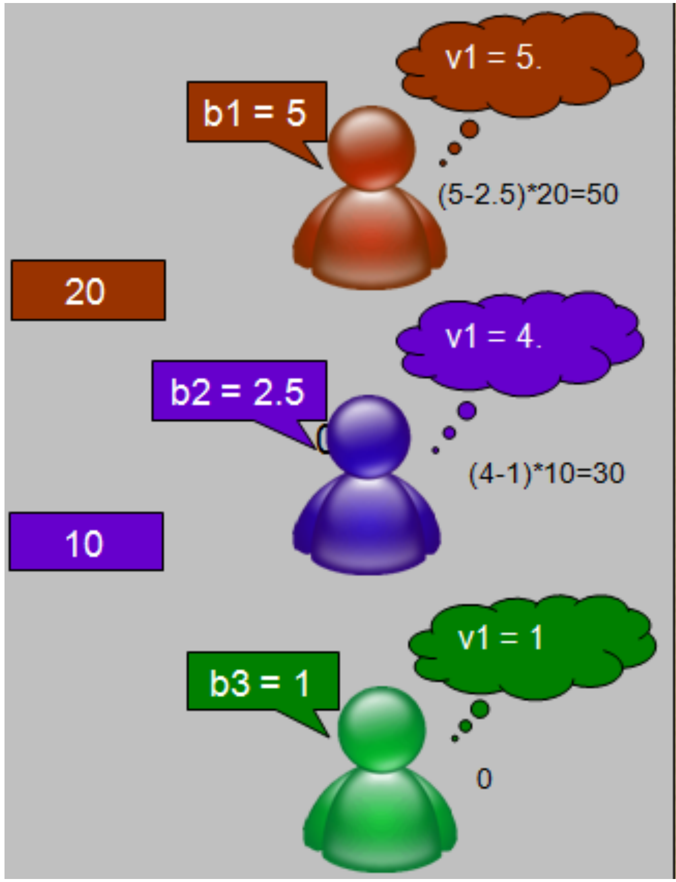
When bidders are farsighted, Nash equilibrium may not be stable.



Locally envy-free equilibrium

- *Envy-freeness*: No body wants to exchange its allocation with anyone else's.
- *Locally envy-free equilibrium*
  - an equilibrium such that no bidder would like to exchange its bid with the bidder immediately above it.
  - For any k: $\alpha_k*(v_k - b_{k+1}) >= \alpha_{k-1}*(v_k - b_k)$.

A locally envy free equilibrium



An Envy Free Solution

- RED gets 20 clicks
  - Pays 2 each
  - Profit = 50
- BLUE gets 10 clicks
  - Pays 1 each
  - Profit = 30

An Envy Free Solution

- If RED and BLUE exchange bids
- BLUE gets 20 clicks
  - Pays 2.5 each
  - Profit = 30
  - which is no more than 30, his original profit.
- RED gets 10 clicks
  - Pays 1 each
  - Profit = 40
  - which is less than 50, his original profit.

GSP vs. VCG

- GSP is not as good as VCG in theory, but it survives due to
  - Simplicity: simpler to explain to advertisers
  - Large market effect: no easy sense of truthfulness in millions of parallel auctions
  - Re-engineering cost: ad systems is much more complicated than an auction
  - Ambiguous revenue effect: short term change from GSP to VCG will hurt revenue, while long term change is uncertain (depending on advertisers' responses, etc)
  - In VCG, it is time-consuming to compute the allocation and pricing
  - Not easy to explain VCG to advertisers

VCG vs. GSP

- Why Google do not use VCG?
  - Less revenue!

- Theorem: $\text{Revenue}_{\text{GSP}} \geq \text{Revenue}_{\text{VCG}}$
  - Equality holds when GSP obtains locally envy-free equilibrium.

Google Online Marketing Challenge

- The Google Online Marketing Challenge is a unique opportunity for students to experience and create online marketing campaigns using Google AdWords and Google+. Over 100,000 students and professors from almost 100 countries have participated in the past 8 years.
- With a $250 AdWords advertising budget provided by Google, students develop and run an online advertising campaign for a business or non-profit organization over a three week period. The teams that develop and communicate the most successful campaigns win awesome prizes, including trips to Google offices. Students also have the opportunity to participate in the optional Google+ Social Media Marketing category by creating and managing a Google+ Page for their clients over a five week period.
- The Challenge is open to student teams of three to six members from undergraduate or graduate programs, regardless of their major. All students must register under a verified faculty member, lecturer or instructor currently employed by an accredited higher education institute.
- http://www.google.com/onlinechallenge/

Google AdSense

- Make Money Online Through Website
- A free, simple way to earn money by placing ads on your website
- https://www.google.co.uk/intl/en/adsense/start/how-it-works/

Review

- Sponsored search auctions
  - Revenue maximization
  - Strategic behaviors
- Extra reading

Sponsored search auctions: an overview of research with emphasis on game theoretic aspects

Patrick Maillé, Evangelos Markakis, Maurizio Naldi, George D. Stamoulis & Bruno Tuffin

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