COMP6207
Algorithmic Game Theory

Lecture 19 Social Choice

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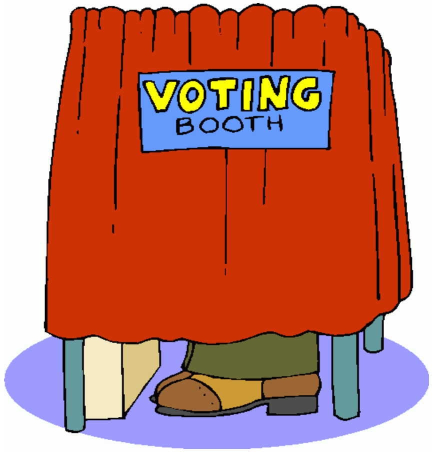
Social Choice

- What is social choice theory?
- How do we define a “good” voting system?
- Voting between two candidates
- Voting among three or more candidates
- Arrow’s Impossibility Theorem
- A view of CS approaches

- **Social choice theory:** how can we measure individual interests and preferences and combine them into one *collective* decision?
  - Finding an outcome that reflects "the will of the people"



- I.e., making decisions based on the preferences of multiple agents
- Largely, but not exclusively, focused on voting

Choosing Between Two Candidates

We do not want

- Football team captain candidates: Bob and Charlie
- Dictatorship
  - Whoever David votes for is declared the winner.
- Imposed rule:
  - No matter how anyone votes, Bob will be named the winner.
  - The winner is decided before the election takes place.

We want

- Some **fairness**
  - Definition?
- **Anonymity:** opposite to Dictatorship
  - A voting system should treat all of the voters equally. So if any two voters swap their ballots, the outcome of the election should remain the same.
- **Neutrality:** opposite to Imposed rule
  - A voting system should treat the candidates the same. So if every voter switched their vote, the outcome would change accordingly.
- **Monotonicity:**
  - It is impossible for a winning candidate to become a losing candidate by gaining votes, and also for a losing candidate to become a winning candidate by losing votes.

Choosing Between Two Candidates

Majority rule: Each voter indicates a preference for one of the two candidates and the candidate with the most votes wins.

- To avoid discussing about ties: assuming odd number of voters

Advantages of Majority Rule

- All voters are treated equally.
  - anonymity
- Both candidates are treated equally.
  - neutrality
- It is **monotone**: If a new election were held and a single voter changed her ballot from the loser of the previous election to the winner, but everyone else voted exactly as before, the outcome of the new election would be the same.

Majority Rule for n=2

- Is there any better voting system?

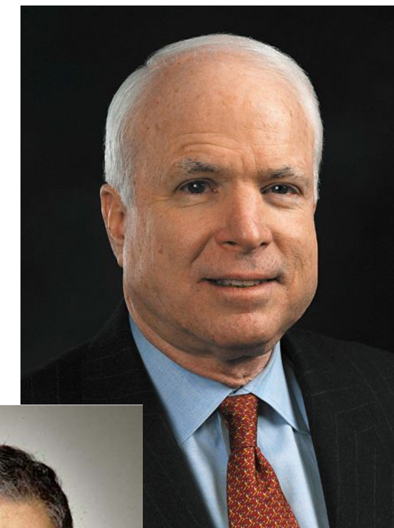
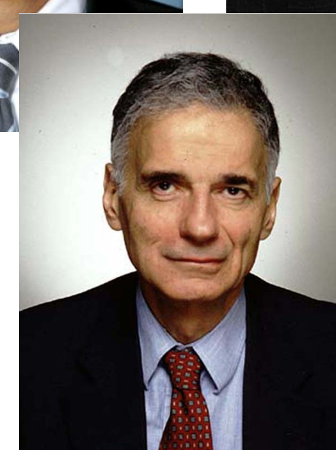
- **May's Theorem (**Kenneth May 1952**):**

Among all possible two-candidate voting systems that never result in a tie, majority rule is the *only* one that is anonymous, neutral, and monotone.

Three or more candidates?

Several different possibilities for voting systems exist:

1. Condorcet's Method
2. Plurality Voting
3. The Borda Count
4. Sequential pair-wise voting
5. The Hare System
6. Approval Voting

- Advantages and disadvantages of each method

1. Condorcet's Method

- A candidate is a winner if he or she would *defeat every other candidate* in an one-on-one contest using majority rule.
  - A generalization of majority rule to multiple candidates, satisfies properties that treats all voters equally, treats both candidates equally, and is monotone.

Any problem?

	Voter 1	Voter 2	Voter 3
1st	A	B	C
2nd	B	A	A
3rd	C	C	B

- A defeats B (2 to 1), A defeats C (2 to 1), B defeats C (2 to 1)

Therefore, A wins!

Condorcet's Voting Paradox

With three or more candidates, there are elections in which Condorcet's method yields *no* winners!

	Voter 1	Voter 2	Voter 3
1st	A	B	C
2nd	B	C	A
3rd	C	A	B

- A defeats B (2 to 1)
- C defeats A (2 to 1)
- B defeats C (2 to 1)
- No winner!

Manipulation

	Voter 1	Voter 2	Voter 3
1st	A	B	C
2nd	B	A	A
3rd	C	C	B

A wins

	Voter 1	Voter 2	Voter 3
1st	A	B	C
2nd	B	C	A
3rd	C	A	B

No winner

Desirable property: Condorcet Consistency

- Marquis de Condorcet devised a property of good voting rules and new voting system in 18th century
- A **Condorcet winner** is the candidate who always wins in pair-wise elections
- A Condorcet winner does not always exist (**Condorcet paradox**)
- **Condorcet criterion**: If a Condorcet winner exists, it should be selected top in the aggregate ranking
- Rules satisfying this property are called **Condorcet methods** and are said to be **Condorcet consistent**

Marquis de Condorcet

2. Plurality Voting

- Only first place winners are considered
- The candidate with the *most* votes wins
- A natural generalization of majority rule.
- Cons: Fails to satisfy the Condorcet Winner Criterion

1st	A	B	C
2nd	B	A	A
3rd	C	C	B

Condorcet : A wins
Plurality: no winner

- Cons: manipulable: next slide

Manipulation: Example

- 19 voters, 3 candidates (Red, Blue, Green).
  - 9 voters: R > B > G.
  - 8 voters: B > R > G.
  - 2 voters (Alice and Bob): G > B > R.

If I vote for G, then R will get elected, so I'd rather vote for B

If Alice and Bob vote B > G > R, then B is elected

Even one voter can possibly manipulate!

3. The Borda Count

- Assign points to each voter's rankings and then sum these points to arrive at a group's final ranking.
- Each first-place vote is worth n-1 points, each second-place vote is worth n-2 points, and so on down, where n is the number of candidates.
- Candidate with the most point is the winner
  – Who is the winner in the following example?

1st	A	A	A	B	B
2nd	B	B	B	C	C
3rd	C	C	C	A	A

Properties of The Borda Count

- Pros: It satisfies the monotonicity criterion, etc.
- Cons: Does not satisfy the property known as "Independence of irrelevant alternatives".

Independence of Irrelevant Alternatives
The social preferences between candidates A and B depend only on the individual preferences between A and B

See an example next slide.

- Cons: Does not satisfy the Condorcet criterion.
  - Proof by the first part on the next slide

Failure of the IIA

1st	A	A	A	B	B
2nd	B	B	B	C	C
3rd	C	C	C	A	A

Borda scores: A = 6, B = 7, C = 2 so **B** is the winner!

1st	A	A	A	C	C
2nd	B	B	B	B	B
3rd	C	C	C	A	A

Borda scores: A = 6, B = 5, C = 4 so **A** becomes winner

But no one has changed his or her mind about whether B is preferred to A!

4. Sequential Pairwise voting

- Start with an ordered list of the candidates.
- Put the first candidate against the second in a one-on-one contest, according to majority rule
- The winner then moves on to the third candidate in the list, one-on-one.
- Continue this process through the entire list until only one remains at the end.

Example: Sequential Pairwise voting

- Sequentially compare candidates A, B, C, and D

Q1: who is the winner?

A1: D

Q2: is it reasonable? Compare B and D

Observation: It fails to satisfy the following condition

# of Voters	Preference Order
4	A > B > D > C
3	C > A > B > D
3	B > D > C > A

Unanimity

If everyone prefers one candidate A to another candidate C, then C should not be the winner!

Another problem: The winner depends on the order of comparison. e.g., B, D, A, C.

5. The Hare System

- There are different versions but here is the general idea
  - Arrive at a winner by repeatedly deleting candidates that are "least preferred", in the sense of being at the bottom of the ballots.
  - If a single candidate remains after all others have been eliminated, it alone is the winner (otherwise, it is a tie).

- Here is one version:

An Example

- In this example, A has 5 first-place votes, B has 5 first-place votes, and C has 4 first-place votes, so C is eliminated
- Now A has 5 first-place votes, and B has 9, so A is eliminated
- B is the only candidate left, so B is the winner

# Voters	Preference Order
5	A > B > ~~C~~
4	~~C~~ > B > A
3	B > ~~C~~ > A
2	B > A > ~~C~~

# Voters	Preference Order
5	A > B
4	B > A
3	B > A
2	B > A

Another Example

- This time, A has 5 first-place votes, and B and C are tied with 4, so B and C are both eliminated at the same time

# Voters	Preference Order
5	A > B > C
4	C > B > A
3	B > C > A
1	B > A > C

- This leaves only A to win the election

Cons

- Now let's modify the profile from the previous example, so that the 1 voter with preference B > A > C now has preference A > B > C
- Notice that this change moves the winner A higher on that voter's ballot

# Voters	Preference
6	A > ~~B~~ > C
4	C > ~~B~~ > A
3	~~B~~ > C > A

# Voters	Preference
6	~~A~~ > C
4	C > ~~A~~
3	C > ~~A~~

C wins!

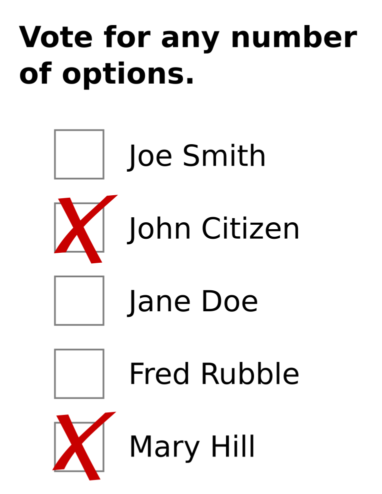
Why is this a problem?

- A was the winner of the original election, and one of the voters changed his ballot to move A higher, causing A to lose

- This shows that the Hare system is not **monotone**

6. Approval Voting

- One alternative possibility is approval voting: Instead of a preference list ballot, each voter is allowed to give one vote to as many of the candidates as he or she finds **acceptable.**
  - No limit is set on the number of candidates for whom an individual can vote.
  - Violate "one person, one vote"
  - The winner under approval voting is the candidate who receives the largest number of approval votes.
- Application: often used to elect new members to the National Academy of Science, etc.



Sounds good. Yet, if I knew my favorite candidate is not going to win, and someone I dislike is going to win, … , maybe approve some other candidates.

- Similar to the manipulation in Plurality Voting.

A summary of voting systems

Voting System	Cons
Condorcet's Method	Voting cycle
Plurality Voting	Fails to satisfy the Condorcet Criterion, manipulable
Borda Count	Fails to satisfy the Independence of Irrelevant Alternatives (IIA) property
Sequential Pairwise Voting	Fails to satisfy Pareto efficiency (*unanimity*)
Hare System	Fails to satisfy monotonicity
Approval Voting	Manipulable

Can we do better?

- Is it possible to find a voting system for three or more candidates as "ideal" as majority rule for two candidates?

Arrow's Impossibility Theorem

With three or more candidates and any number of voters, there does not exist - and there will never exist - a voting system that satisfies:

1. If every voter prefers alternative A over alternative B, then the group prefers A over B. -- *unanimity*
2. If every voter's preference between A and B remains unchanged, then the group's preference between A and B will also remain unchanged (even if voters' preferences between other pairs like A and C, B and C, or D and E change). -- *independence of irrelevant alternatives*
3. No single voter possesses the power to always determine the group's preference. -- *non-dictatorship*

Kenneth Arrow (1021-2017)

Nobel Memorial Prize in Economic Sciences, 1972

Many other impossibility results

- Theorem (Gibbard-Satterthwaite, 1971): every non-dictatorial aggregation rule with ≥ 3 candidates is manipulable.
- All what they say is: no perfect mechanism!
- Why are there so many mechanisms?
- Q: How Do We Get Around These Impossibility Result?

Computer Science approach

- If we cannot make manipulation impossible…
- Then we can try to make it hard!
- Approach: design universal method to turn any voting protocol into a hard-to-manipulate one.

However…

- These hardness results are for worst-case analysis.
- Average-case hardness seems hard to achieve.
- Open question: What is the maximum fraction of manipulators we can tolerate? How can we achieve that?

Further reading

- Chapter 9 of the AGT book.

  Vijay Vazirani, Noam Nisan, Tim Roughgarden, Éva Tardos, *Algorithmic Game Theory*, Cambridge University Press, 2007, ISBN: 9780521872829.