Algorithmic Game Theory
COMP6207

Lecture 7: Revelation Principle

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Recap

Truthfulness in Groves mechanisms

Theorem

Truth-telling is a dominant strategy under Groves mechanisms.

Theorem (Green-Laffont)

Informal statement: in settings where agents may have unrestricted quasilinear utilities, Groves mechanisms are the only mechanisms that are both efficient and dominant-strategy truthful.

VCG and Individual Rationality

Theorem

The VCG mechanism is ex post individual rational when the **choice set monotonicity** *and* **no negative externalities** *properties hold.*

- Choice-set monotonicity implies that removing any agent weakly decreases—that is, never increases—the mechanism's set of possible choices X.
- No negative externalities means that every agent has zero or positive utility for any choice that can be made without her participation.

VCG and Budget Balance

Theorem

The VCG mechanism is weakly budget-balanced when the **no single-agent effect** *property holds.*

- No single-agent effect means that the (total) welfare of agents other than i is weakly increased by dropping i.

Good News

Theorem (Krishna & Perry, 1998)

In any Bayesian game setting in which VCG is ex post individually rational, VCG collects at least as much revenue as any other efficient and ex interim individually-rational mechanism.

- Ex interim individual rationality is a weaker condition than ex post individual rationality
- This result somewhat surprising: does not require dominant strategies, and hence compares VCG to all Bayes-Nash mechanisms.
- A useful corollary: VCG is as budget balanced as any efficient mechanism can be

Bad News

Theorem (Green-Laffont; Hurwics)

No dominant-strategy truthful mechanism is always both efficient and weakly budget balanced, even if agents are restricted to the simple exchange setting.

- *simple exchange* is an environment consisting of buyers and sellers with quasilinear utility functions, all interested in trading a single identical unit of some good.

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- *simple exchange* is an environment consisting of buyers and sellers with quasilinear utility functions, all interested in trading a single identical unit of some good.

Theorem (Myerson-Satterthwaite)

No Bayes-Nash incentive-compatible mechanism is always simultaneously efficient, weakly budget balanced and ex interim individual rational, even if agents are restricted to quasilinear utility functions.

- Class of Bayes-Nash incentive-compatible mechanisms includes the class of dominant-strategy truthful mechanisms
- ex interim IR is weaker than ex post IR

Revelation Principle

Learning Outcomes

By the end of this lecture, you should be able to

- **Describe** the *revelation principle* and **prove** its correctness.
- **Outline** the implications and limitations of *revelation principle*.
- **Outline** the conditions that are necessary and sufficient for a mechanism to be DS truthful (*"direct characterisation theorem"*), and **prove** that these conditions are indeed necessary and sufficient.

Dominant-strategy (DS) truthfulness revisited

So far in mechanism design, we focused on dominant-strategy truthful mechanisms. For good reasons that you pointed out in an earlier session, such as

- as a participant it's easy to figure out how to play
- as a designer it's easy to predict the outcome, assuming players follow their dominant strategies.

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- as a participant it's easy to figure out how to play
- as a designer it's easy to predict the outcome, assuming players follow their dominant strategies.

Notice that when assuming dominant-strategy truthfulness, we in fact making two separate assumptions

1. Every participant in the mechanism has a dominant strategy, no matter what her private valuation is,
2. This dominant strategy is direct revelation, where the participant truthfully reports all of its private information to the mechanism.

DS truthfulness revisited, contd.

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Question:

- Are we imposing unnecessary restriction by asking that the DS is revealing the truth?
- In other words: are there social choice functions that can be implemented in dominant strategies by a mechanism, but cannot be implemented in dominant strategies by a direct truthful mechanism?

Revelation Principle

The revelation principle tells us that given (1), there is no need to relax requirement (2), it comes for free.

In other words, any social choice function that can be implemented (in DS) by any mechanism, can also be implemented by a direct, truthful mechanism.

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Theorem (Revelation Principle)

For every mechanism in which every participant has a dominant strategy (no matter what her private information), there is an equivalent direct dominant-strategy truthful mechanism.

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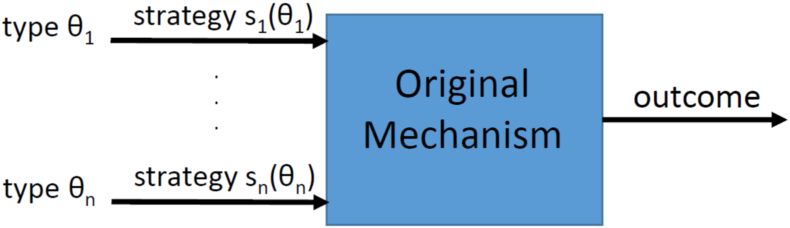

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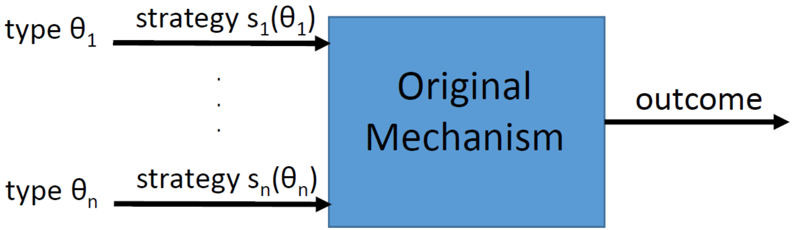
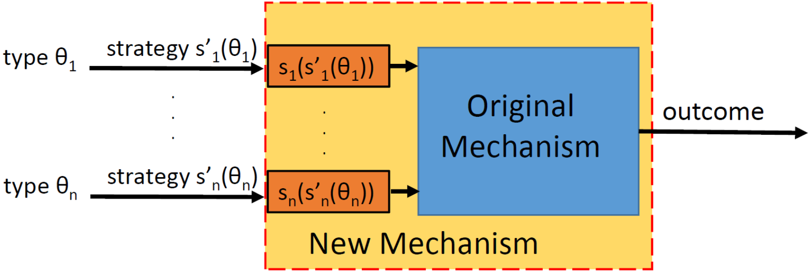


- The revelation principle claims that: for every mechanism in which every participant has a dominant strategy (no matter what its private information), there is an equivalent direct dominant-strategy truthful mechanism.
- Consider an arbitrary, non-truthful mechanism (e.g. may be indirect)
- Recall that a mechanism defines a game, and consider a DS equilibrium $s = (s_1, \ldots, s_n)$

Revelation principle: Proof contd.



- We can construct a new direct mechanism, as shown above.
- This mechanism is truthful by exactly the same argument that s was a DS equilibrium in the original mechanism.
- The agents don't have to lie, because the mechanism already lies for them.
- So, $s_i'(\theta_i) = \theta_i$.

Discussion of the Revelation Principle

- What is the revelation principle good for?
  - recognition that truthfulness is not a restrictive assumption
  - for analysis purposes, we can consider only truthful mechanisms, and be assured that such a mechanism exists
  - recognition that indirect mechanisms can't do (inherently) better than direct mechanisms

Discussion of the Revelation Principle

- What is the revelation principle good for?
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  - for analysis purposes, we can consider only truthful mechanisms, and be assured that such a mechanism exists
  - recognition that indirect mechanisms can't do (inherently) better than direct mechanisms
- What to be careful about: The set of equilibria is not always the same in the original mechanism and revelation mechanism
  - of course, we've shown that the revelation mechanism does have the original equilibrium of interest
  - however, in the case of indirect mechanisms, even if the indirect mechanism had a unique equilibrium, the revelation mechanism can also have new, bad equilibria

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- Full revelation can sometimes place an unreasonable burden on the communication channel.
- Agents' equilibrium strategies might be difficult to compute; in this case the additional burden absorbed by the mechanism might be considerable.

Characterization of DS Truthful Mechanisms

Dominant Strategy Implementation

- So far we have been focused on implementing an efficient (i.e. social-welfare maximising) outcome.
- What other social choice functions can we implement in dominant strategies?
- In some applications, maximising social welfare is a hard computational problem (NP-complete), but there are efficient algorithms that *approximate* the maximum social welfare. Can we implement their corresponding social choice functions?

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- In some settings the mechanism designer has some other objectives that she wants to optimise. E.g. :
  - revenue
  - some kind of fairness
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Note: We still assume quasilinear setting.

Direct Characterisation

Let $X_i(\hat{v}_{-i}) \subseteq X$ denote the set of choices that can be selected by the choice rule χ given the declaration $\hat{v}_{-i}$ by the agents other than i (i.e. the range of $\chi(., \hat{v}_{-i})$).

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Theorem

A mechanism is dominant-strategy truthful if and only if it satisfies the following conditions for every agent i and every $\hat{v}_{-i}$:

1. *The payment function $p_i(\hat{v})$ can be written as $p_i(\hat{v}_{-i}, \chi(\hat{v}))$.*

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1. *The payment function $p_i(\hat{v})$ can be written as $p_i(\hat{v}_{-i}, \chi(\hat{v}))$.*
2. $\forall v_i$, $\chi(v_i, \hat{v}_{-i}) \in \text{argmax}_{x \in X_i(\hat{v}_{-i})}(v_i(x) - p_i(\hat{v}_{-i}, x))$.

- An agent's payment can only depend on other agents' declarations and the selected choice
- The mechanism optimises for each agent: taking the other agents' declarations and the payment function into account, from every player's point of view the mechanism selects the most preferable choice.

Proof - if part

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(if part): if the mechanism satisfies the two conditions then it is DS truthful.

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- Denote $x = \chi(v_i, \hat{v}_{-i})$, $x' = \chi(v_i', \hat{v}_{-i})$, $p_x = p_i(\hat{v}_{-i}, x)$, and $p_{x'} = p_i(\hat{v}_{-i}, x')$. (By the first condition the payment functions can be written in this way).

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- The utility of i when telling the truth is $v_i(x) - p_x$.

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- The utility of i when telling the truth is $v_i(x) - p_x$.
- By the second condition, the utility of i when she declares v_i (and choice x is consequently picked) is no less than her utility when declaring v_i' (where choice x') is picked.

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- By the second condition, the utility of i when she declares v_i (and choice x is consequently picked) is no less than her utility when declaring v_i' (where choice x') is picked.
- It is a dominant-strategy for i to declare v_i.

Proof - only if part (first condition)

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First condition:

- Assume, for a contradiction, that the payment function depends (in addition to $\hat{v}_{-i}$ and the choice of the mechanism) also on $\hat{v}_i$.

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- Assume, for a contradiction, that the payment function depends (in addition to $\hat{v}_{-i}$ and the choice of the mechanism) also on $\hat{v}_i$.
- Therefore, for some v_i and v_i' we have that $\chi(v_i, \hat{v}_{-i}) = \chi(v_i', \hat{v}_{-i})$ but $p_i(v_i, \hat{v}_{-i}) > p_i(v_i', \hat{v}_{-i})$.

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- Then a player with type v_i will increase her utility by declaring v_i', a contradiction (to the mechanism being DS truthful)

Proof - only if part (second condition)

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(only if part): if the mechanism is DS truthful then it satisfies the two conditions.

Second condition:

- Assume for a contradiction that for some v_i, $\chi(v_i, \hat{v}_{-i}) \notin \text{argmax}_{x \in X_i(\hat{v}_{-i})}(v_i(x) - p_i(\hat{v}_{-i}, x))$.

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- Fix $x' \in \text{argmax}_{x \in X_i(\hat{v}_{-i})}(v_i(x) - p_i(\hat{v}_{-i}, x))$. Thus, for some v_i' we have that $x' = \chi(v_i', \hat{v}_{-i})$.

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- Therefore a player with type v_i will increase her utility by declaring v_i', a contradiction (to the mechanism being DS truthful)

What was in this lecture

- Recap of VCG's properties and some important well-known theorems
- Revelation principle
- Characterisation of DS truthful mechanisms

Books

- Twenty Lectures on Algorithmic Game Theory, by Tim Roughgarden
- Multiagent Systems: Algorithmic, Game-Theoretic, and Logical Foundations by Yoav Shoham and Kevin Leyton-Brown
  - From now on we will refer to this book as **MAS**
- Algorithmic Game Theory, edited by Noam Nisan, Tim Roughgarden, Eva Tardos,Vijay V. Vazirani
  - From now on we will refer to this book as **AGT**

Further reading/watching

Revelation principle:

- Read MAS chapters 10.2.2
- Read AGT Chapters 9.4.1, 9.4.2, 9.4.3 (note that MAS and AGT sometimes use different notations and definitions for the same concepts)
- Watch Game Theory II - Week 2 (Mechanism Design): video 3

Introduction to DS implementation:

- Read MAS chapter 10.5.1 (we have only covered the first page or so; some of the rest will be covered in the next lecture).
- Read AGT Chapter 9.5.1 and the intro to 9.5 (feel free to check the rest of 9.5 which we will mostly cover in the next lecture).

Acknowledgment

Some of the slides in this lecture were based on the slides by Kevin Leyton-Brown.