Bayesian Game

A tuple (N, A, Θ, p, u) where

- $N = \{1, \ldots, n\}$ is a finite set of agents
- $A = A_1 \times \ldots \times A_n$, where A_i is the set of actions available to agent i
- $\Theta = \Theta_1 \times \ldots \times \Theta_n$ where Θ_i is the type space of player i
- $p : \Theta \mapsto [0, 1]$ is a common-prior probability distribution on Θ
- $u = (u_1, \ldots, u_n)$, where $u_i : A \times \Theta \mapsto \mathbb{R}$ is the utility function for player i.

Bayesian Game Setting

A tuple (N, O, Θ, p, u)

- $N = \{1, \ldots, n\}$ is a finite set of agents
- O is a set of outcomes
- $\Theta = \Theta_1 \times \ldots \times \Theta_n$ is a set of possible joint type vector
- p is a common-prior probability distribution on Θ
- $u = (u_1, \ldots, u_n)$, where $u_i : O \times \Theta \mapsto \mathbb{R}$ is the utility function for player i.

The key difference with Bayesian Game is that the Bayesian Game Setting does **not include actions** for the agents, and instead defines the utility function over the **set of possible outcomes**.

Mechanism

Definition (Mechanism)

A (deterministic) mechanism (for a Bayesian game setting (N, O, Θ, p, u)) is a pair (A, M), where

- $A = A_1 \times \ldots \times A_n$, where A_i is the set of actions available to agent i, and
- $M : A \mapsto O$ maps each action profile to an outcome.

Footnote: Mechanisms need not to be deterministic (they can be randomised) in which case $M : A \mapsto \Pi(O)$. For now, however, we only focus on deterministic mechanisms.

Mechanisms in the quasilinear settings

In a quasilinear utility setting:

- the set of outcomes is $O = X \times \mathbb{R}^n$ for a finite set of choices X, and
- when outcome $o = (x, (p_1, \ldots, p_n))$ is chosen, the utility of an agent i given joint type θ is $u_i(o, \theta) = u_i(x, \theta) - p_i$.

Definition (Quasilinear mechanism)

A quasilinear mechanism is a triple (A, χ, p), where

- $A = A_1 \times \ldots \times A_n$, where A_i is the set of actions available to agent i,
- Choice function $\chi : A \mapsto X$ maps each action profile to a choice in X, and
- Payment function $p : A \mapsto \mathbb{R}^n$ maps each action profile to a payment for each agent.