Properties

Definition (Efficiency)

A quasilinear mechanism is efficient if in equilibrium selects a choice x that maximises $\sum_{i=1}^{n} v_i(x)$.

Definition (Dominant-strategy truthful mechanism)

A direct quasilinear mechanism is truthful if for each agent i and for all $\hat{v}_i$, declaring $\hat{v}_i = v_i$ maximises i's utility.

Definition (Ex post individual rationality)

A mechanism is ex post individually rational if in equilibrium, the utility of each agent is at least 0.

Definition (Budget balance)

A quasilinear mechanism is budget balance if in equilibrium the sum of the payments of the agents is equal to 0.

Definition (Weak Budget balance)

A quasilinear mechanism is budget balance if in equilibrium the sum of the payments of the agents is at least 0.

Vickrey-Clarks-Groves (VCG) mechanism

The VCG mechanism is a direct quasilinear mechanism (χ, p) where

$$\chi(\hat{v}) = \operatorname{argmax}_x \sum_i \hat{v}_i(x)$$

$$p_i(\hat{v}) = \sum_{j \neq i} \hat{v}_j(\chi(\hat{v}_{-i})) - \sum_{j \neq i} \hat{v}_j(\chi(\hat{v}))$$

- Every agent pays his/her social cost.
- VCG is also called pivotal mechanism.
- **Question**: What is u_i in terms of social welfare?

Two Assumptions

Definition (Choice-set monotonicity)

A setting exhibits choice-set monotonicity if $\forall i, X_{-i} \subseteq X$.

- In other words: removing any agent weakly decreases—-that is, never increases—the mechanism's set of possible choices X.

Definition (No negative externalities)

A setting exhibits no negative externalities if $\forall i, \forall x \in X_{-i}, v_i(x) \geq 0$.

- In other words: every agent has zero or positive utility for any choice that can be made without her participation.

Another property

Definition (No single-agent effect)

A setting exhibits no single-agent effect if $\forall i, \forall v_{-i}, \forall x \in \operatorname{argmax}_y \sum_j v_j(y)$ there exists a choice x' that is feasible without i and that has $\sum_{j \neq i} v_j(x') \geq \sum_{j \neq i} v_j(x)$.

- In other words, welfare of agents other than i is weakly increased by dropping i.

Example

Consider a single-item auction. Dropping an agent reduces the amount of competition, making the others better off.