SEMESTER 2 EXAMINATION 2018 - 2019

ADVANCED INTELLIGENT AGENTS

DURATION 120 MINS (2 Hours)

This paper contains 9 questions

Answer **ALL** parts of the question in **Section A** and **THREE** questions from **Section B**.
Section A carries 40% of the total marks for the exam paper.
Section B carries 60% of the total marks for the exam paper.

An outline marking scheme is shown in brackets to the right of each question.

University approved calculators MAY be used.

A foreign language dictionary is permitted ONLY IF it is a paper version of a direct Word to Word translation dictionary AND it contains no notes, additions or annotations.

11 page examination paper.

TTC topic2

Section A

Question A1.

(a) Explain how the Top Trading Cycle (TTC) mechanism for school choice works.

[5 marks]

(b) Consider an instance of school choice with three students s_1, s_2 and s_3 and three colleges c_1, c_2 and c_3. Assume that each college has capacity one. Let the preferences of the students and colleges be as follows:

$s_1 : c_2 > c_3$ $\quad$ $c_1 : s_2$

$s_2 : c_3 > c_2 > c_1$ $\quad$ $c_2 : s_2 > s_1 > s_3$

$s_3 : c_3 > c_2$ $\quad$ $c_3 : s_1 > s_3 > s_2$

Compute the matching produced by the Boston mechanism. Show the intermediate steps as well as the matching returned.

[5 marks]

Question A2.

(a) For each of the following statements, choose whether it is true or false. You do not need to provide a proof or explanation.

(i) In every instance of the Stable Marriage problem (SM), the man-optimal stable matching is the worst stable matching for the women.

(ii) Every instance of the Hospitals/Residents problem with Couples (HRC) admits a stable matching.

[2 marks]

(b) Consider an instance of the House Allocation problem (HA) with three agents a_1, a_2 and a_3 and three houses h_1, h_2 and h_3. Let the preferences of the agents be as follows:

$a_1 : h_2 > h_1$

$a_2 : h_1 > h_2 > h_3$

$a_3 : h_1 > h_3$

How many distinct Pareto optimal matchings does this instance admit? List all the Pareto optimal matchings.

[4 marks]

(c) Construct an instance of the Stable Marriage problem (SM) in which there is more than one stable matching. Prove that your instance indeed admits more than one stable matching. (Hint: 2 men and 2 women are enough to create such an instance.) [4 marks]

TURN OVER

Question A3.

(a) Briefly describe the revelation principle and its positive implication.

[2 marks]

(b) Give one reason why a mechanism designer may prefer to design and implement an indirect mechanism.

[2 marks]

(c) Under what condition or conditions a quasilinear mechanism (χ, p) is dominant-strategy truthful? Please describe the definition of this condition (or these conditions).

[3 marks]

(d) Briefly explain why bidding truthfully is a dominant-strategy for all agents in the optimal (single-item) auction.

[3 marks]

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Question A4.

(a) Explain the exploration vs. exploitation dilemma.

[3 marks]

(b) Why it is good to have a Hannan consistent bandit algorithm?

[3 marks]

(c) What are the main differences between multi-armed bandit and duelling bandits models?

[4 marks]

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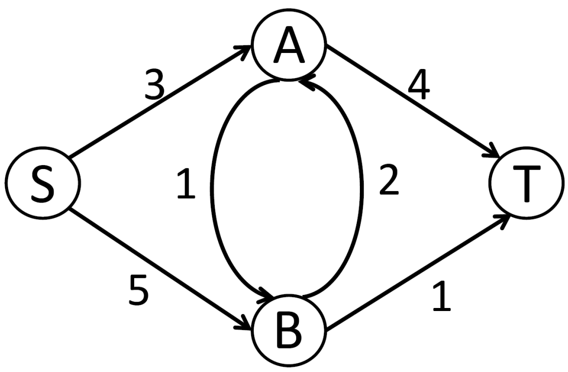
Section B

Question B1.

Consider the transportation network shown below. A mechanism designer wishes to find the lowest-cost route from node S to T in this network. Each edge is owned by a single agent and the number on the edge denotes the cost of transporting along that edge. Note that these numbers are costs, not benefits. So if we choose a path that crosses an edge of cost c, its owner is incurring a cost of c which means her value for this path is $-c$. An agent has a value of 0 if the chosen path does not cross her edge. The cost of each edge is the private information of its owner.



(a) Compute the path selected by VCG.

[2 marks]

(b) Compute the VCG payment of each agent.

[8 marks]

(c) Is VCG weakly budget balanced in this setting? Justify your answer.

[2 marks]

(d) Consider a single-good auction with at least three bidders. Prove or disprove the following statement: “Awarding the good to the highest bidder, at a price equal to the third-highest bid, yields an auction that is dominant-strategy truthful.”

[4 marks]

(e) A seller is auctioning off k identical copies of an item using the VCG mechanism. Suppose that each bidder wants only one copy of the good (so this is a single-unit demand auction). Compute the revenue generated by VCG. Prove that adding an extra bidder can never decrease the revenue of this auction.

[4 marks]

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Question B2.

Consider a potentially infinite outcome space $O \subset [0,1]$ and a finite set N of n agents. Denote the utility of an agent with type θ_i for outcome o as $u_i(o, \theta_i)$. Constrain the utility functions so that every agent has some unique, most preferred outcome $b(\theta_i) \in O$, and so that $|o' - b(\theta_i)| < |o'' - b(\theta_i)|$ implies that $u_i(o', \theta_i) > u_i(o'', \theta_i)$.

Consider a direct mechanism which asks every agent to declare their most-preferred outcome and then selects the median outcome. If there is an even number of agents, the mechanism chooses the larger of the two middle outcomes (e.g. if we have four agents and they declare 0.1, 0.2, 0.3, 0.4, then the mechanism chooses 0.3).

(a) Prove that truth-telling is a dominant strategy for all agents.

[10 marks]

(b) Prove that if the mechanism designer submits $n-1$ "dummy preferences" of their own with arbitrary values, and then runs the same mechanism on the $2n-1$ preferences, the dominant-strategy truthfulness is preserved.

[5 marks]

(c) As described so far, the mechanism selects the $\lceil \frac{n}{2} \rceil^{th}$-order statistic of the agents' declared preferences. (The k^{th}-order statistic of a set of numbers is the k^{th} largest number in the set.) Explain how to select dummy preferences in such a way that the mechanism selects the k^{th}-order statistic of the agents' declared preferences for any $k \in \{1, ..., n\}$. Of course the dummy preferences must be set in a way that does not depend on the specific declarations made by the agents.

[5 marks]

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Question B3.

Consider a hospitals/residents setting with four residents $r_1, \ldots r_4$ and three hospitals $h_1, \ldots, h_3$. Assume that hospitals h_1 and h_2 each have capacity 1 (i.e. $q_1 = q_2 = 1$) and that the capacity of hospital h_3 is $q_3 = 2$. Let the preferences of the agents be as follows:

$r_1 : h_1 > h_2 > h_3 \qquad h_1 : r_3 > r_2 > r_1$

$r_2 : h_1 > h_3 > h_2 \qquad h_2 : r_2 > r_1 > r_3 > r_4$

$r_3 : h_2 > h_1 \qquad h_3 : r_4 > r_2 > r_1$

$r_4 : h_2 > h_3$

(a) Find the stable matching when residents propose (i.e. the resident-oriented Gale-Shapley is executed). Show the intermediate steps as well as the matching returned.

[10 marks]

(b) How many stable matchings does this instance admit? Justify your answer. (Hint: find the stable matching returned when the hospital-oriented Gale-Shapley is executed.)

[3 marks]

(c) Now consider the Stable Marriage problem. Prove that the woman-oriented Gale-Shapley (i.e. the deferred acceptance algorithm, by Gale-Shapely, in which women are proposing) is not dominant-strategy truthful for men.

[7 marks]

TURN OVER

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Question B4.

In a prediction market, suppose the market maker is using the Logarithmic market scoring rule with $b = 200$, and there are two possible outcomes Y and N for prediction.

(a) How much is the market maker's loss in the worst case?

[5 marks]

(b) Suppose the market is newly open, i.e., 0 share on Y and N. How much does it cost if you want to buy 40 shares on outcome Y?

[10 marks]

(c) Suppose there are 50 shares of Y and 80 shares of N in the market. What is the instantaneous price of Y?

[5 marks]

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Question B5.

(a) Reformulate the following problem as a linear programming problem.

$$\begin{array}{ll} \text{minimize} & 2|x_1| + x_2 \\ \text{subject to} & x_1 + x_2 \geq 3 \end{array}$$

[5 marks]

(b) Reformulate the following problem as a linear programming problem.

$$\begin{array}{ll} \text{maximize} & \min\{x_1, x_2\} \\ \text{subject to} & |2x_1 + x_2| \leq 7 \\ & \frac{3x_1 - x_2}{1 + x_1 + x_2} \leq 0.5 \\ & x_1 \geq 0, x_2 \geq 0 \end{array}$$

[5 marks]

(c) What is the strong duality of linear programming? Given the following Primal problem, write down its Dual problem.

$$\begin{array}{ll} \text{minimize} & x_1 + 2x_2 + 3x_3 \\ \text{subject to} & -x_1 + 3x_2 = 5 \\ & 2x_1 - x_2 + 3x_3 \geq 6 \\ & x_3 \leq 4 \\ & x_1 \geq 0 \\ & x_2 \leq 0 \end{array}$$

[10 marks]

END OF PAPER

