COMP6203 Intelligent Agents 2022/2023

Solutions to Exercises on Preference Elicitation

Jane, a detective novelist, is planning to stay in a hotel in Dartmoor for few weeks, in order to focus on finishing up her last novel. Her editor is paying for the cost of the stay, for up to £120 per night. She has shortlisted top 10 hotels in BookingDotCom that cost at most £120 per night. For the simplicity of presentation, we will refer to these hotels as A, B, ..., and J. Jane is planning to make a decision based on 4 criteria: price (with range $\pounds[80, 120]$) denoted as criterion 1, star rating (with range $[0, 5]$) denoted as criterion 2, average reviewers' score (with range $[0, 10]$) denoted as criterion 3, and distance from the centre of a nearby town (with range $miles[0, 10]$ denoted as criterion 4. Her evaluation of the hotels on each criterion i (i.e. functions $g_i(o)$) is presented in the following table. Her value (aka utility) function is additive: $U(o) = \sum_{i=1}^{n} u_i(g_i(o))$ for outcome (that is, hotel) o where n is the number of criteria. The marginal value functions, u_i's, are all non-decreasing. (Since the editor pays, she prefers staying in a more expensive hotel, as they usually offer more perks; she also prefers to stay in a quiet place and hence away from town centers.)

Hotels	Price (Criterion 1)	Star rating (Criterion 2)	Average review score (Criterion 3)	Distance to town centre (Criterion 4)
A	115	4	9	3
B	110	5	9	3
C	95	4	8	2
D	100	5	8.5	2.5
E	85	3	9.5	1
F	95	4	8	5
G	115	5	7.5	7
H	120	4	8.5	0.5
I	80	3	9	4
J	95	3	7	6

Out of these 10 hotels, Jane has been to 4 before and knows her preference ordering over them:

$$J \succ I \succ E \sim C$$

Consider the UTA^{GMS} method from the lecture.

1. Write down a linear program to check whether there exists a compatible value function.
2. How many constraints does your LP have (not including non-negativity constraints)?
3. Write down a linear program to check whether A is necessarily weakly preferred to B.
4. Write down a linear program to check whether A is possibly weakly preferred to B.

Solution.

1. There exists a compatible value function if and only the following program returns an optimal solution greater than zero (i.e. $\epsilon > 0$).

The top 3 constraints are there to ensure that $U(J) > U(I)$, $U(I) > U(E)$, and $U(E) = U(C)$, and thus the preference ordering over the four reference outcomes J, I, E, C is preserved.

$$
\begin{aligned}
\textbf{Maximize } & \epsilon \\
\textbf{subject to } & u_1(95) + u_2(3) + u_3(7) + u_4(6) \geq u_1(80) + u_2(3) + u_3(9) + u_4(4) + \epsilon \\
& u_1(80) + u_2(3) + u_3(9) + u_4(4) \geq u_1(85) + u_2(3) + u_3(9.5) + u_4(1) + \epsilon \\
& u_1(85) + u_2(3) + u_3(9.5) + u_4(1) = u_1(95) + u_2(4) + u_3(8) + u_4(2) \\
& \\
& u_1(95) - u_1(85) \geq 0, u_1(85) - u_1(80) \geq 0 \\
& u_1(80) \geq 0, u_1(95) \leq u_1(120) \\
& \\
& u_2(4) - u_2(3) \geq 0 \\
& u_2(3) \geq 0, u_2(4) \leq u_2(5) \\
& \\
& u_3(9.5) - u_3(9) \geq 0, u_3(9) - u_3(8) \geq 0, u_3(8) - u_3(7) \geq 0 \\
& u_3(7) \geq 0, u_3(9.5) \leq u_3(10) \\
& \\
& u_4(6) - u_4(4) \geq 0, u_4(4) - u_4(2) \geq 0, u_4(2) - u_4(1) \geq 0 \\
& u_4(1) \geq 0, u_4(6) \leq u_4(10) \\
& \\
& u_1(80) = 0, u_2(0) = 0, u_3(0) = 0, u_4(0) = 0 \\
& u_1(120) + u_2(5) + u_3(10) + u_4(10) = 1 \\
& \\
& u_i(o_i) \geq 0, \quad \forall o \in \{J, I, E, C\}, \forall i \in \{1, 2, 3, 4\}
\end{aligned}
$$

2. $25 = 3 + 4 + 3 + 5 + 5 + 4 + 1$

3. A is necessarily weakly preferred to B if the optimal solution to the following linear program is nonnegative.

$$
\begin{aligned}
\textbf{Minimize} \quad & (u_1(115) + u_2(4) + u_3(9) + u_4(3)) - (u_1(110) + u_2(5) + u_3(9) + u_4(3)) \\
\textbf{subject to} \quad & u_1(95) + u_2(3) + u_3(7) + u_4(6) \geq u_1(80) + u_2(3) + u_3(9) + u_4(4) + \epsilon \\
& u_1(80) + u_2(3) + u_3(9) + u_4(4) \geq u_1(85) + u_2(3) + u_3(9.5) + u_4(1) + \epsilon \\
& u_1(85) + u_2(3) + u_3(9.5) + u_4(1) = u_1(95) + u_2(4) + u_3(8) + u_4(2) \\
& \\
& u_1(115) - u_1(110) \geq 0, u_1(110) - u_1(95) \geq 0, u_1(95) - u_1(85) \geq 0, u_1(85) - u_1(80) \geq 0 \\
& u_1(80) \geq 0, u_1(115) \leq u_1(120) \\
& \\
& u_2(5) - u_2(4) \geq 0, u_2(4) - u_2(3) \geq 0 \\
& u_2(3) \geq 0, u_2(4) \leq u_2(5) \\
& \\
& u_3(9.5) - u_3(9) \geq 0, u_3(9) - u_3(8) \geq 0, u_3(8) - u_3(7) \geq 0 \\
& u_3(7) \geq 0, u_3(9.5) \leq u_3(10) \\
& \\
& u_4(6) - u_4(4) \geq 0, u_4(4) - u_4(3) \geq 0, u_4(3) - u_4(2) \geq 0, u_4(2) - u_4(1) \geq 0 \\
& u_4(1) \geq 0, u_4(6) \leq u_4(10) \\
& \\
& u_1(80) = 0, u_2(0) = 0, u_3(0) = 0, u_4(0) = 0 \\
& u_1(120) + u_2(5) + u_3(10) + u_4(10) = 1 \\
& \\
& u_i(o_i) \geq 0, \quad \forall o \in \{A, B, J, I, E, C\}, \forall i \in \{1, 2, 3, 4\}
\end{aligned}
$$

Comparing this LP with the LP of Exercise 2, you will notice that, in addition to a different objective function, there are a few new constraints. The constraints in Green font are new. The constraint in blue font looks as if it's a new constraint, but it has actually appeared before for a different reason. As we don't need redundant constraints, we can safely ignore it.

4. The linear program for checking weather A is possibly weakly preferred to B is exactly the same as shown for part (c) except that the objective function is to Maximize:

$$\textbf{Maximize} \quad (u_1(115) + u_2(4) + u_3(9) + u_4(3)) - (u_1(110) + u_2(5) + u_3(9) + u_4(3))$$

And A is possibly weakly preferred to B if the optimal solution to this minimisation linear program is nonnegative.