UNIVERSITY OF SOUTHAMPTON COMP6207W1

SEMESTER 2 EXAMINATION 2015 - 2016

ADVANCED INTELLIGENT AGENTS

DURATION 120 MINS (2 Hours)

This paper contains 10 questions

Answer ALL questions from section A (40 marks)
and THREE of the FIVE questions from section B (60 marks).

An outline marking scheme is shown in brackets to the right of each question.

This examination is worth 72%. The coursework was worth 28%.

University approved calculators MAY be used.

A foreign language dictionary is permitted ONLY IF it is a paper version of a direct 'Word to Word' translation dictionary AND it contains no notes, additions or annotations.

Section A

Question A1.

Consider the 2-player game in Table 1:

主导策略，消除，纳什均衡等

	L	C	R
U	1, 2	3, 5	2, 1
M	0, 4	2, 1	3, 0
D	-1, 1	4, 3	0, 2

TABLE 1: 2-player game.

(a) Does either player have dominated strategies? If so, identify them. [1 marks]

(b) Does either player have a dominant strategy? Why or why not? [1 marks]

(c) Use elimination of dominated strategies (EDS) to solve this game. Show the order in which you are eliminating strategies, and specify whether you are eliminating strictly or weakly dominated strategies. [2 marks]

(d) Is your EDS solution a Nash equilibrium? Why or why not? [1 marks]

(e) Is there a best response strategy which is not rationalisable? Explain why this can or cannot be possible. [1 marks]

(f) Find all rationalisable strategies for both players. [1 marks]

(g) Is there a strong equilibrium in this game? [1 marks]

Question A2.

Two firms, A and B, compete *a la Cournot*. The (inverse) market demand function is $p = \max\{8 - q_A - q_B, 0\}$, where p is the market price **per unit** and q_i is the quantity produced by firm i for $i \in \{\mathrm{A}, \mathrm{B}\}$.

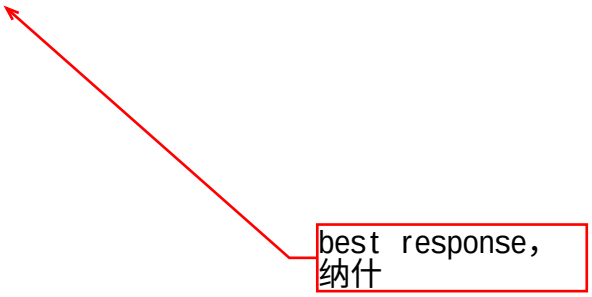
Firm A has a constant cost $c_A = 3$ **per unit** it produced, and firm B has a constant cost $c_B = 5$ **per unit** it produced.

Answer the following questions and show your calculations!

(a) Find the best response functions of firms A and B. [4 marks]

(b) Compute the Nash equilibrium. [2 marks]

(c) Draw the best response functions and find the equilibrium graphically. [2 marks]



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Question A3.

(a) Explain the exploration vs. exploitation dilemma. [2 marks]

(b) Give the definition of regret in the stochastic bandit setting. [2 marks]

(c) What is the definition of Hannan consistency? [2 marks]

(d) Provide the optimal asymptotic lower bound of stochastic bandit algorithms, and explain the meaning of it. [2 marks]

Question A4.

What is the difference between

(a) stochastic and adversarial settings of bandit problems? [2 marks]

(b) epsilon-first and epsilon-greedy? [2 marks]

(c) external and internal regrets of adversarial bandits? [2 marks]

(d) the knowledge of the arm pulling distribution at each time step of the Exp3 and FPL algorithms? [2 marks]

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Question A5.

The school board is voting whether to award the contract for building a new elementary school to Ants (A), Bees (B), Centipedes (C), Dragonflies (D) or Earthworms (E).

There are 12 members of the board, whose preferences over the candidates are distributed as follows:

# voters	Ranking
5 :	$B \succ E \succ A \succ C \succ D$
3 :	$C \succ E \succ A \succ D \succ B$
3 :	$D \succ A \succ E \succ C \succ B$
1 :	$A \succ C \succ E \succ D \succ B$

(a) Find the winner of the election held using each of the following voting schemes. Provide the corresponding winning score or the elimination procedure when relevant.

(i) Plurality vote.

(ii) STV.

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(iii) Maximin.

(iv) Copeland.

[6 marks]

(b) Is there a Condorcet winner or a Condorcet loser in this profile?

[2 marks]

Section B

Question B1.

(a) Consider a security game with a set of three targets $T = \{t_1, t_2, t_3\}$. The attacker (A) can attack any single target, and the defender (D) has one resource that can cover any single target. The utilities of the players are as defined in Table 2:

	t_1	t_2	t_3
	covered / uncovered	covered / uncovered	covered / uncovered
D	0 / -1	0 / -1	0 / -1
A	1 / 3	1 / 3	0 / 2

TABLE 2: Security game.

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(i) Write down the normal form representation of the game.

(ii) Compute all (mixed) Nash equilibrium strategies of player D.

(iii) Can you find the minimax and Stackelberg strong equilibrium strategies of player D? Explain!

[8 marks]

(b) Consider a congestion game $G = \langle N, R, (S_i)_{i \in N}, (u_r)_{r \in R} \rangle$ where:

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- $N = \{1, 2\}$ - set of two agents
- $R = \{r_1, r_2\}$ - set of two resources
- $\forall i \in N$, $S_i = \{r_1, r_2\}$ - set of strategies for each agent (an agent can select either resource r_1 or resource r_2 but not both – that is, G is a *resource selection* game)
- $\forall r \in R$, $u_r(k)$ is the utility to each user of resource r if the total number of its users is k.

Write down the payoff matrix for this game. Prove that a pure Nash equilibrium must exist for any assignment of utility functions $u_r(k)$.

[7 marks]

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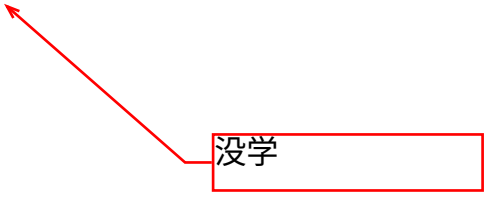
(c) Let G be a resource selection game with agents $N = \{1, \ldots, n\}$, resources $R = \{r_1, \ldots, r_m\}$, strategy sets $S_i \subseteq R$ for $i \in N$ and utility functions u_r for $r \in R$. Show that if the utility functions u_r are *monotonically decreasing*, then every pure strategy Nash equilbrium of G is a strong equilibrium.

Hint: Start by showing that an improving move by any coalition (if existed) would not change the congestion vector. [5 marks]



Question B2.

(a) In the extensive form game with players 1 and 2 shown in Figure 1, one of the Nash equilibria is NOT a subgame perfect Nash equilibrium. Which one? Show your work!

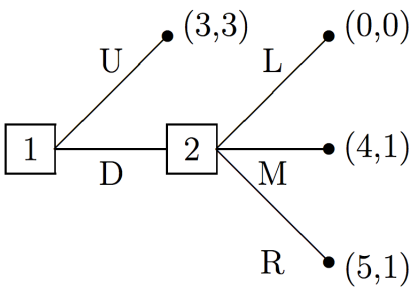


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FIGURE 1: Extensive form game.

[5 marks]

(b) Consider the cooperative game (N, v) with three agents $N = \{a, b, c\}$, represented as an MC-net as follows:

$$\begin{array}{ll} (a, 5) & (a \wedge b, 4) \\ (b, 3) & (a \wedge c, -2) \\ (c, 2) & (b \wedge c, -1) \\ & (a \wedge b \wedge c, 2) \end{array}$$

(i) Find the characteristic function of the game.

(ii) Verify if the game is motonone.

(iii) Verify if the game is super-additive.

(iv) Compute the Shapley value for all the agents.

(v) Is the Shapley payoff distribution in the core of the game? Why?

[15 marks]

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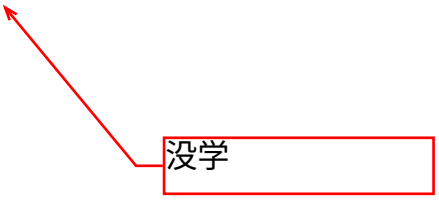
Question B3.

We aim to solve a stochastic multi-armed bandit problem with 3 arms with the UCB algorithm. Each arm, when pulled, provides a random reward between 0 and 1. After 200 pulls, we learnt that the average reward collected from Arms 1, 2, and 3 are 0.4, 0.6, and 0.3, respectively. Also, the number of times we have pulled the arms are: 90 for Arms 1 and 2, and 20 for Arm 3.

(a) Which arm will be chosen by the UCB algorithm to pull on the 101^{th} step? [6 marks]

(b) Let μ_1 and μ_2 denote the true and unknown expected rewards of Arms 1 and 2, respectively. Given the current observations and Hoeffding's inequality, prove that the probability of having $\mu_1 > \mu_2$ is at most 33%. [7 marks]

(c) Suppose that Arm 1 is suboptimal (i.e., not the arm with the highest expected reward), and Δ_1, the sub-optimality gap for Arm 1 is 0.2. Using Markov's inequality, prove that the probability of Arm 1 being the most pulled one at the end of the $20,000^{th}$ pull is lower than 30%. [7 marks]



Question B4.

(a) (i) Describe the revenue equivalence theorem and its implications.

(ii) Discuss why, despite the revenue equivalence theorem, truthful auctions are preferred.

[7 marks]

(b) Facebook applies the Groves mechanism with Clarke pivot rule (a.k.a. VCG) for its display advertising. Consider a setting where 2 slots are sold using this mechanism for a visiting user, and there are 3 interested advertisers (agents) A, B, and C competing for these slots. One of the slots is more prominent and therefore more valuable to all agents. Also, two of the advertisers, A and B are direct competitors since they are selling similar products (e.g. Apple vs Microsoft). Therefore, if A's ad is shown in either slot, the value of the remaining slot for B is diminished, and similar for A if B's ad is shown. Consequently, the utility not only depends on which slot they receive, but also on who receives the other slot (there are so-called allocative externalities). All advertisers prefer to receive both slots compared to one slot.

The valuations are given in Table 3, where v indicates the value without the slot awarded to the direct competitor, and v' the value in case the other slot is given to the direct competitor.

Agent	Slot 1	Slot 2	Slots 1 and 2
A	$v_{A,1} = 2.0, v'_{A,1} = 1.5$	$v_{A,2} = 1.6, v'_{A,2} = 0.9$	$v_{A,1+2} = 2.3$
B	$v_{B,1} = 2.5, v'_{B,1} = 1.6$	$v_{B,2} = 2.0, v'_{B,2} = 1.5$	$v_{B,1+2} = 2.9$
C	$v_{C,1} = 1.0$	$v_{C,2} = 0.7$	$v_{C,1+2} = 1.1$

gsp

TABLE 3: Agent Preferences

(i) Given the valuations in Table 3, what is the allocation that maximises the social welfare? How much is the social welfare in this case?

TURN OVER

(ii) Compute the transfers for *all* agents when applying the Groves mechanism with the Clarke pivot rule.

(iii) Show whether the resulting transfers are (1) individually rational, (2) weakly budget balanced, (3) strongly budget balanced.

[13 marks]

Question B5.

(a) Use the following voting profile with 7 voters to prove that no positional scoring rule with a strictly descending scoring vector will satisfy the Condorcet principle:

# voters	Ranking
3 :	$A \succ B \succ C$
2 :	$B \succ C \succ A$
1 :	$B \succ A \succ C$
1 :	$C \succ A \succ B$

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[5 marks]

(b) Consider the following matching problem with 5 boys and 5 girls, whose preferences are given by Tables 4 and 5:

	1	2	3	4	5
Alan	Yana	Zoe	Wendy	Vera	Xara
Bob	Wendy	Yana	Zoe	Xara	Vera
Chris	Yana	Vera	Xara	Wendy	Zoe
Dean	Zoe	Wendy	Xara	Yana	Vera
Evan	Yana	Wendy	Zoe	Vera	Xara

TABLE 4: Boys' preferences.

	1	2	3	4	5
Zoe	Evan	Alan	Bob	Dean	Chris
Yana	Chris	Bob	Dean	Alan	Evan
Xara	Bob	Chris	Dean	Evan	Alan
Wendy	Alan	Evan	Dean	Chris	Bob
Vera	Dean	Bob	Evan	Chris	Alan

TABLE 5: Girls' preferences.

(i) Find a stable matching using the Gale-Shapley algorithm with boys making proposals.

(ii) Find a stable matching using the Gale-Shapley algorithm with girls making proposals.

[8 marks]

TURN OVER

(c) Suppose that all the boys have identical preferences. How many steps will it take for the Gale-Shapley algorithm with boys making proposals to converge? Why? [4 marks]

(d) Suppose that all the boys have different favourite girls. How many steps will it take for the algorithm with boys making proposals to converge? Why?

[3 marks]

END OF PAPER