Discrete Math
Worksheet – Banzhaf!

Name **Key**

1. Consider the weighted voting system [20:14, 8, 6, 4].

a. Write down all the winning coalitions.

$$\underline{14, \; 8_2}$$
$$\underline{14, \; 6_3}$$

$$\underline{14, \; 8_2, \; 6_3, \; 4_4}$$

$$\underline{14, \; 8_2 \; 6_3}$$
$$\underline{14, \; 8_2 \; 4_4}$$
$$\underline{14, \; 6_3 \; 4_4}$$

b. Find the Banzhaf power index for each of the four players.

$B_1 = 6$ $\beta_1 = \frac{6}{10} = 60\%$

$B_2 = 2$ $\beta_2 = \frac{2}{10} = 20\%$

$B_3 = 2$ $\beta_3 = \frac{2}{10} = 20\%$

$\underline{B_4 = 0}$ $\beta_4 = \frac{0}{10} = 0\%$

$T = 10$

c. Is there a player (or players) that would be considered a dictator? A dummy? Having veto power?

Dictator : none

Dummy : P_4 (critical zero times)

Veto : P_1 (critical in every coalition)

2. In each of the following weighted voting systems, determine which players, if any, (i) are dictators; (ii) are dummies; (iii) have veto power.

a. [19: 9, 7, 5, 3, 1]

$$\underline{9, \; 7_2 \; 5_3}$$
$$\underline{9, \; 7_2 \; 3_4}$$
$$\underline{9, \; 7_2 \; 5_3 \; 3_4}$$
$$\underline{9, \; 7_2 \; 3_4 \; 1_5}$$
$$\underline{9, \; 7_2 \; 5_3 \; 3_4 \; 1_5}$$

Dictator : none

Dummy : P_5

Veto : P_1, P_2

b. [17: 13, 5, 2, 1]

$$\underline{13, \; 5_2}$$
$$\underline{13, \; 5_2 \; 2_3}$$
$$\underline{13, \; 5_2 \; 1_4}$$
$$\underline{13, \; 5_2 \; 2_3 \; 1_4}$$

Dictator : none

Dummy : $P_3 \; P_4$

Veto : P_1, P_2

c. [25: 12, 8, 4, 2]

$$\underline{12 \quad 8 \quad 4 \quad 2}$$

Dictator : none

Dummy : none

Veto : $P_1 \; P_2 \; P_3 \; P_4$

FLIP
OVER!!

3. Consider the weighted voting system [9: 5, 5, 4, 2, 1].

a. Write down all the winning coalitions.

$\underline{5_1 5_2}$ $\qquad$ $\underline{5_1 5_2 4_3 2_4}$

$\underline{5_1 4_3}$ $\qquad$ $\underline{5_1 5_2 2_4 1_5}$

$\underline{5_2 4_3}$ $\qquad$ $\underline{5_1 5_2 4_3 1_5}$

$5_1 5_2 4_3$ $\qquad$ $\underline{5_1 4_3 2_4 1_5}$

$\underline{5_1 5_2 2_4}$ $\qquad$ $\underline{5_2 4_3 2_4 1_5}$

$\underline{5_1 5_2 1_5}$ $\qquad$ $5_1 5_2 4_3 2_4 1_5$

$\underline{5_1 4_3 2_4}$

$\underline{5_1 4_3 1_5}$

$\underline{5_2 4_3 2_4}$

$\underline{5_2 4_3 1_5}$

b. Find the Banzhaf power distributions of this weighted voting system.

$B_1 = 8$ $\qquad$ $\beta_1 = \frac{8}{24} = 33.3\%$

$B_2 = 8$ $\qquad$ $\beta_2 = \frac{8}{24} = 33.3\%$

$B_3 = 8$ $\qquad$ $\beta_3 = \frac{8}{24} = 33.3\%$

$B_4 = 0$ $\qquad$ $\beta_4 = \frac{0}{24} = 0\%$

$B_5 = 0$ $\qquad$ $\beta_5 = \frac{0}{24} = 0\%$

$T = 24$