Lecture 3 Image Sampling

COMP6223 Computer Vision (MSc)

How is an image sampled and what does it imply?

Book
pp
36-56

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Content

1. How does the discrete Fourier transform work, and help?
2. What can go wrong with sampling?

1D Discrete Fourier transfrom

Continuous Fourier:

$$Fp(\xi) = \int_{-\infty}^{\infty} p(t)\, e^{-j2\pi\xi t}\, dt$$

Discrete Fourier calculates
frequency from **data points**

Sampled frequency

$$Fp_u = \frac{1}{N} \sum_{i=0}^{N-1} p_i \; e^{-j\frac{2\pi}{N} iu}$$

N points

Sampled points p_i

1D Discrete Inverse Fourier

Continuous inverse Fourier:

$$p(t) = \int_{-\infty}^{\infty} Fp(\xi)\, e^{j2\pi\xi t}\, d\xi$$

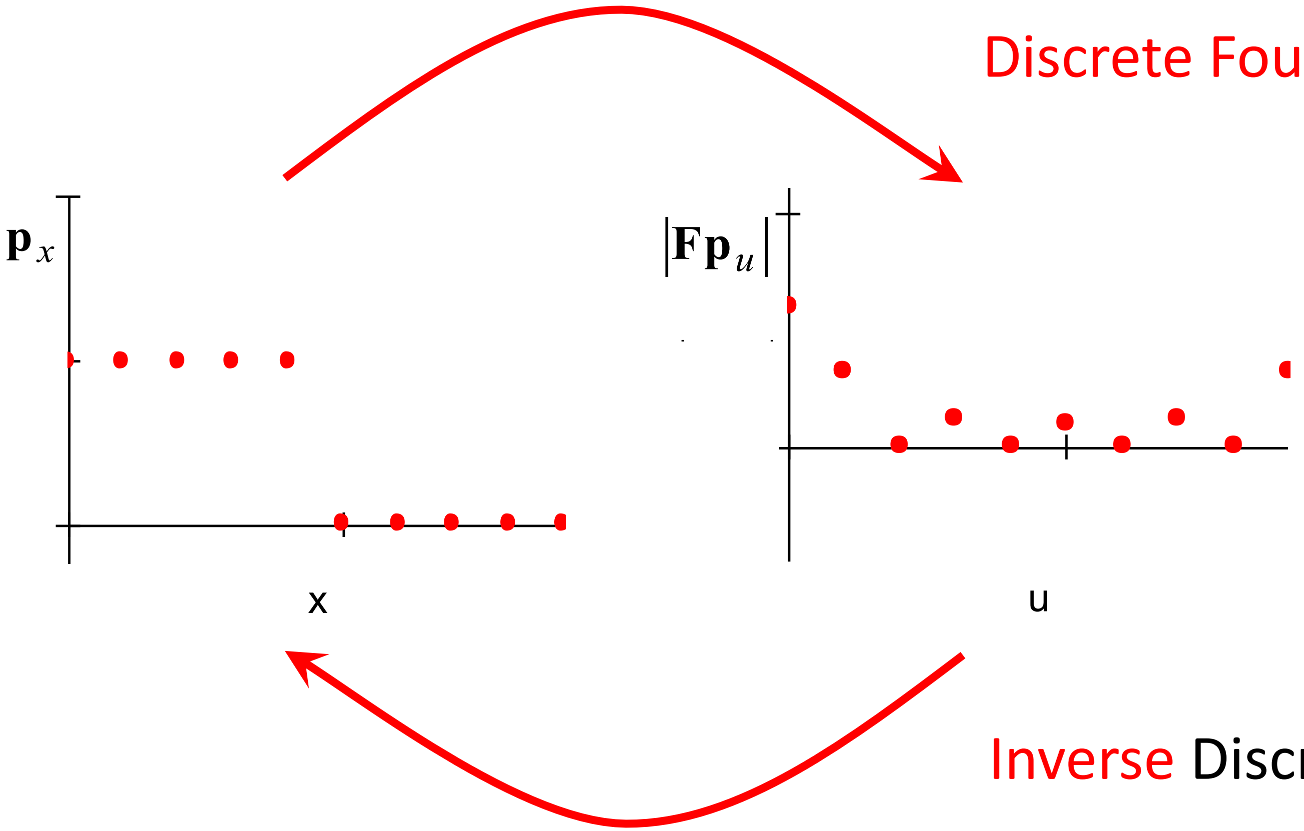
Sampled points p_i

$$p_i = \sum_{u=0}^{N-1} Fp_u\, e^{j\frac{2\pi}{N}iu}$$

Sampled frequency

Transform Pair for Sampled Pulse

Discrete Fourier transform

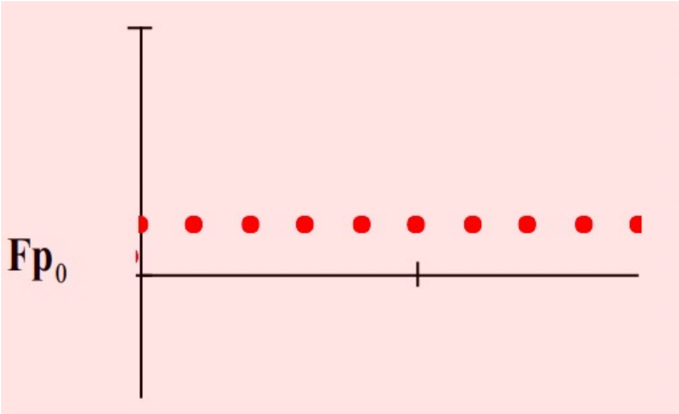


Inverse Discrete Fourier transform

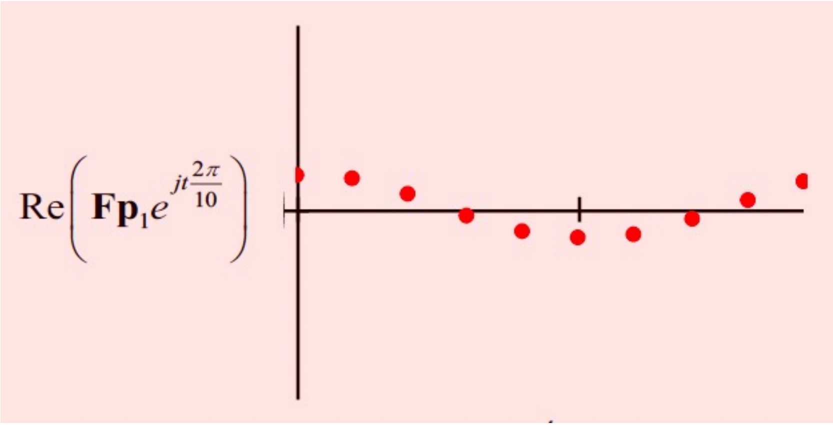
(a) sampled pulse

(b) DFT of sampled pulse

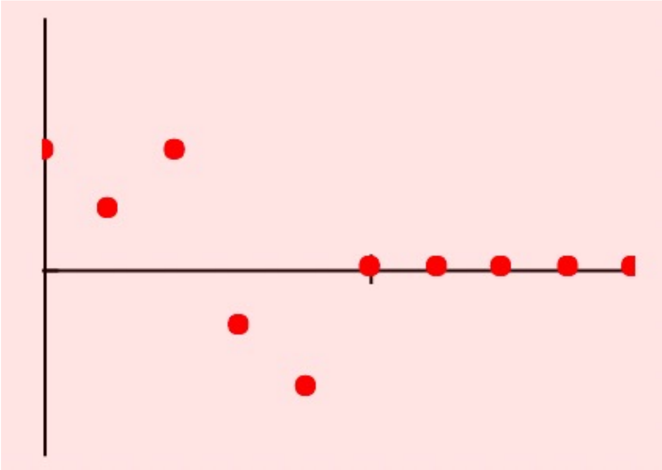
Signal reconstruction from its transform components



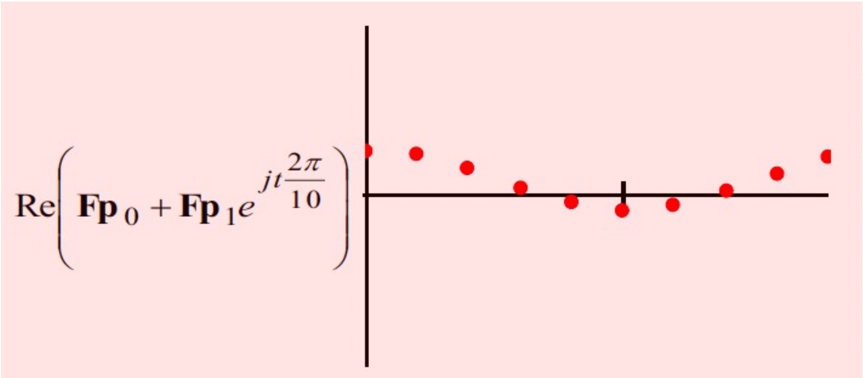
(b) first coefficient Fp_0



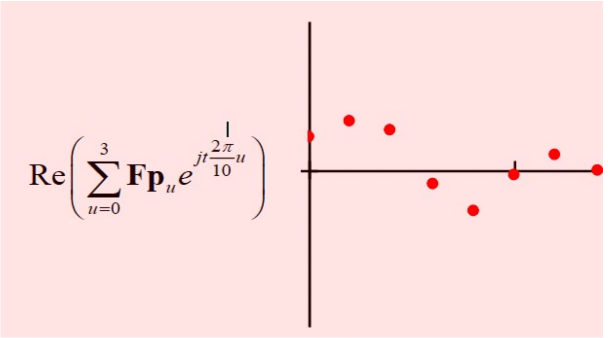
(c) second coefficient Fp_1



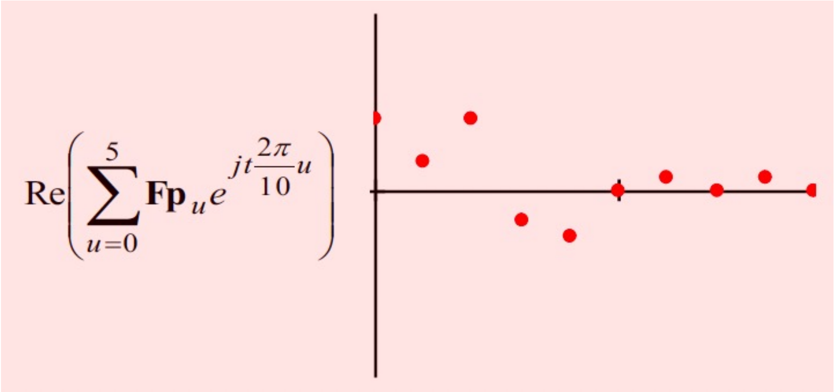
(a) original sampled signal



(d) adding Fp_1 and Fp_0



(e) adding Fp_0, Fp_1 , Fp_2 and Fp_3



(f) adding all six frequency components

$$p_i = \sum_{u=0}^{N-1} Fp_u \, e^{j\frac{2\pi}{N}iu}$$

2D Fourier transform

Two dimensions of space, *x* and *y*

Two dimensions of frequency, *u* and *v*

Forward transform:

$$\mathbf{FP}_{u,v} = \frac{1}{N^2}\sum_{x=0}^{N-1}\sum_{y=0}^{N-1}\mathbf{P}_{x,y}\, e^{-j\left(\frac{2\pi}{N}\right)(ux+vy)}$$

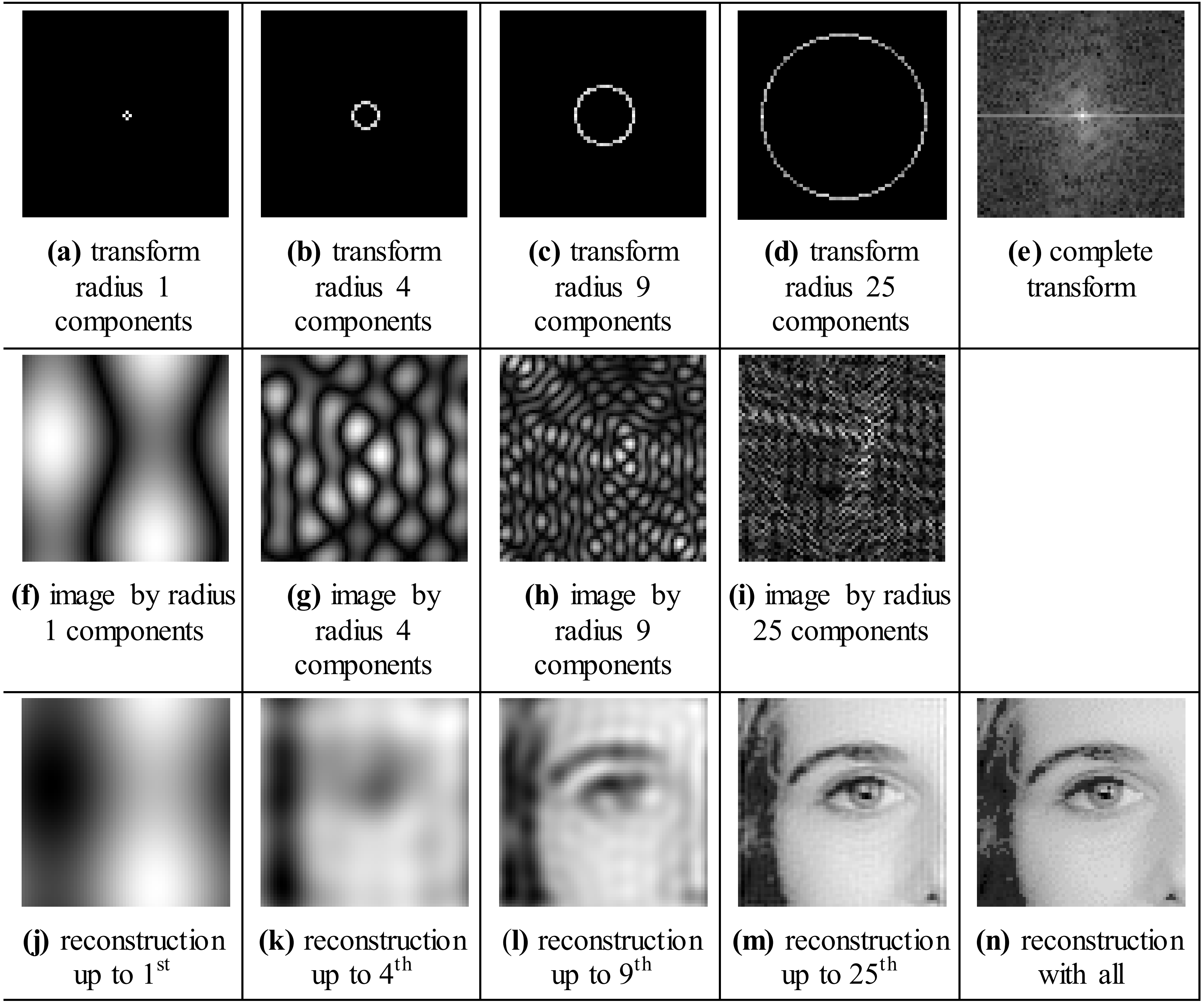
image NxN pixels $\mathbf{P}_{x,y}$

Inverse transform:

$$\mathbf{P}_{x,y} = \sum_{u=0}^{N-1}\sum_{v=0}^{N-1}\mathbf{FP}_{u,v}\, e^{j\left(\frac{2\pi}{N}\right)(ux+vy)}$$

Reconstruction

$$\mathbf{P}_{x,y} = \sum_{u=0}^{N-1}\sum_{v=0}^{N-1} \mathbf{FP}_{u,v} e^{j\left(\frac{2\pi}{N}\right)(ux+vy)}$$



(a) transform radius 1 components

(b) transform radius 4 components

(c) transform radius 9 components

(d) transform radius 25 components

(e) complete transform

(f) image by radius 1 components

(g) image by radius 4 components

(h) image by radius 9 components

(i) image by radius 25 components

(j) reconstruction up to 1^{st}

(k) reconstruction up to 4^{th}

(l) reconstruction up to 9^{th}

(m) reconstruction up to 25^{th}

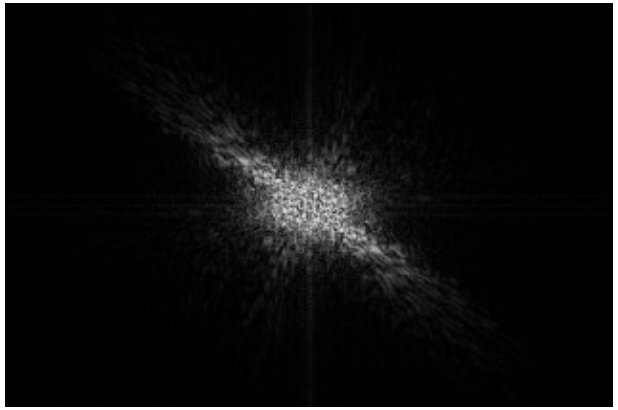
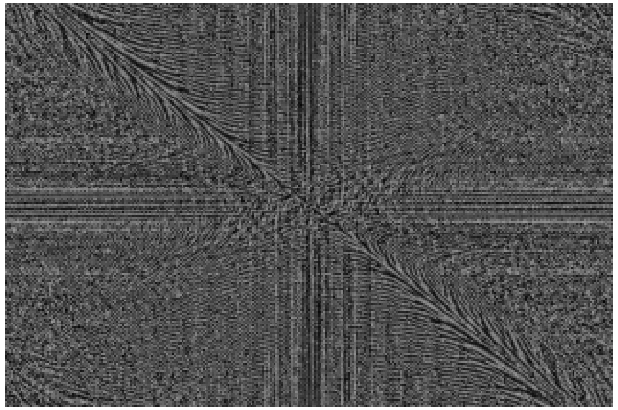
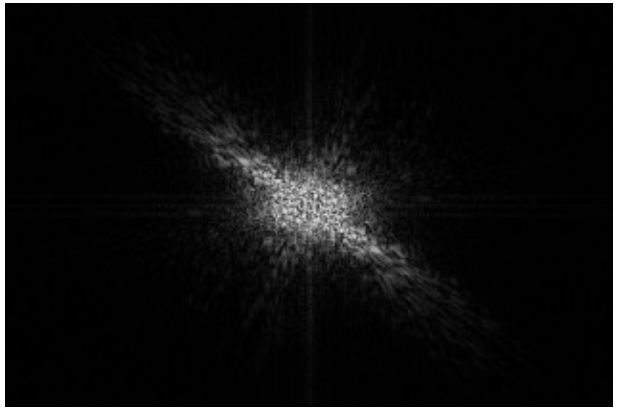
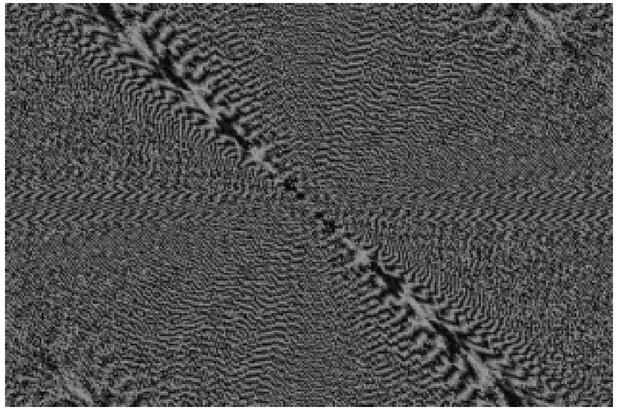
(n) reconstruction with all

Implementation is via (Fast) FFT

```
while L<cols %iterate until log2(cols)-1 levels have been performed
  for j=1:2*L:cols %do all the points in L/2 batches
    for i=1:L %now do L butterflies
      upp(((j+1)/2)+i-1)= Fp(j+i-1)+Fp(j+L+i-1)*exp(-1j*2*pi*(i-1)/(L*2));
      low(((j+1)/2)+i-1)= Fp(j+i-1)-Fp(j+L+i-1)*exp(-1j*2*pi*(i-1)/(L*2));
    end
  end
  for j=1:2*L:cols %copy the components across, to the right places
    for i=1:L
      Fp(j+i-1)=upp(((j+1)/2)+i-1);
      Fp(j+L+i-1)=low(((j+1)/2)+i-1);
    end
  end
L=L*2; %and go and do the next level (up)
end
```

(This is a 1-D FFT)

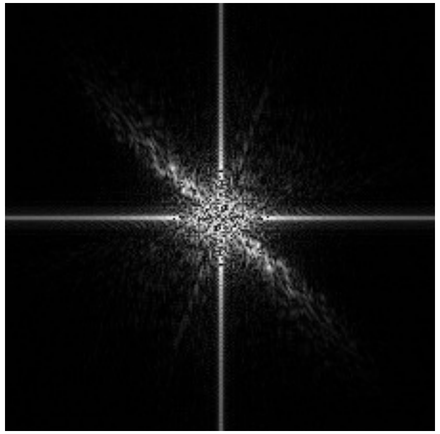
Shift invariance

 **(a)** original image	 **(b)** magnitude of Fourier transform of original image	 **(c)** phase of Fourier transform of original image
 **(d)** shifted image	 **(e)** magnitude of Fourier transform of shifted image	 **(f)** phase of Fourier transform of shifted image

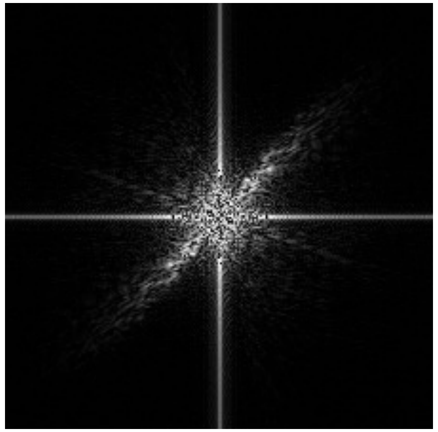
Rotation

(a) original image

(b) rotated image



(c) transform of original image

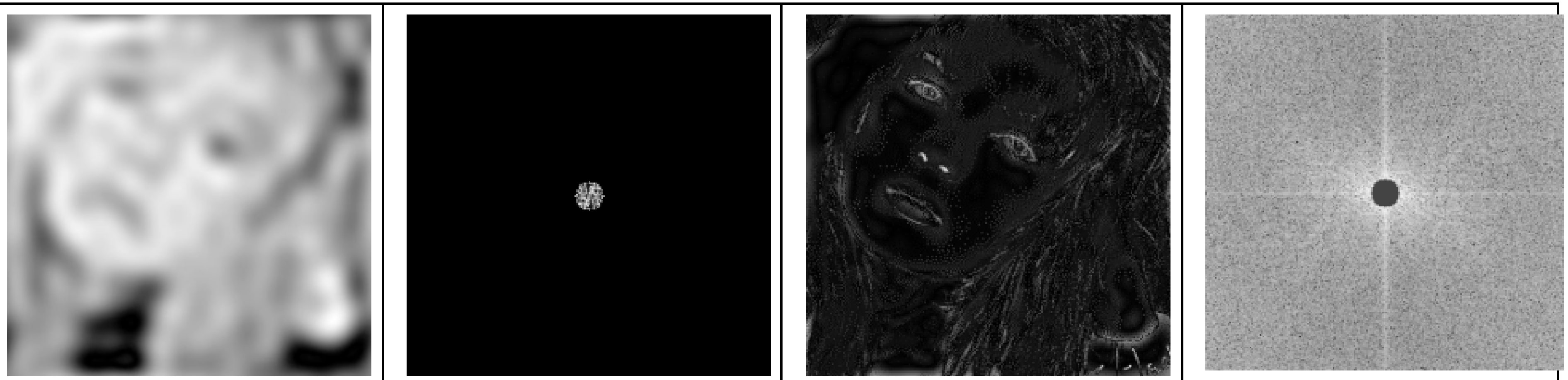


(d) transform of rotated image

$$\mathbf{FP}_{u,v} = \frac{1}{N^2}\sum_{y=0}^{N-1}\sum_{x=0}^{N-1}\mathbf{P}_{x,y}e^{-j\left(\frac{2\pi}{N}\right)(uy+vx)}$$

Filtering

Fourier gives access to frequency components



(a) low-pass filtered image

(b) low-pass filtered transform

(c) high-pass filtered image

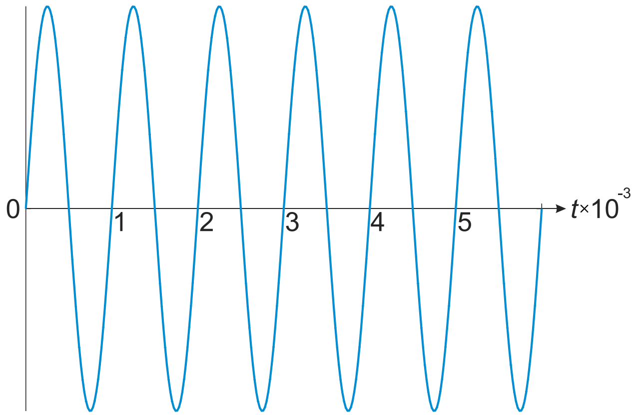
(d) high-pass filtered transform

Applications of 2D FT

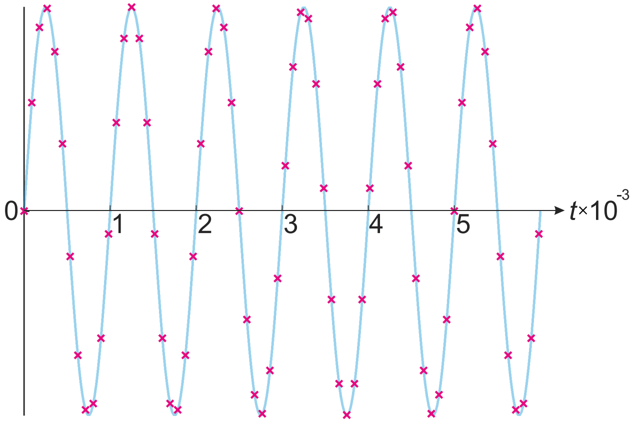
- Understanding and analysis
- Speeding up algorithms
- Representation (invariance)
- Coding
- Recognition/ understanding (e.g. texture)

Sampling Signals

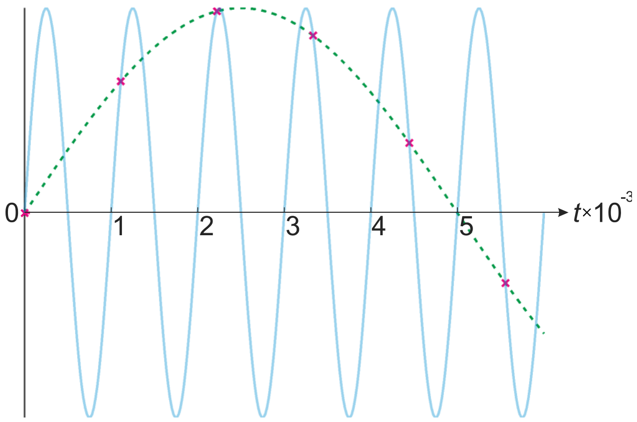
Original continuous signal



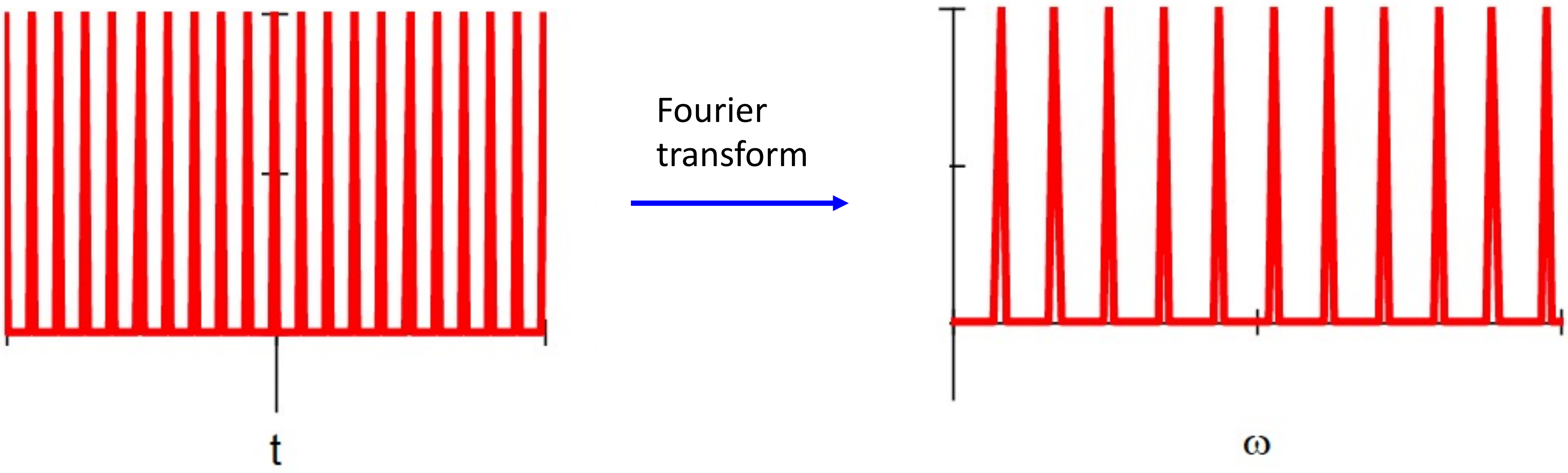
good sampling



bad sampling (aliased)

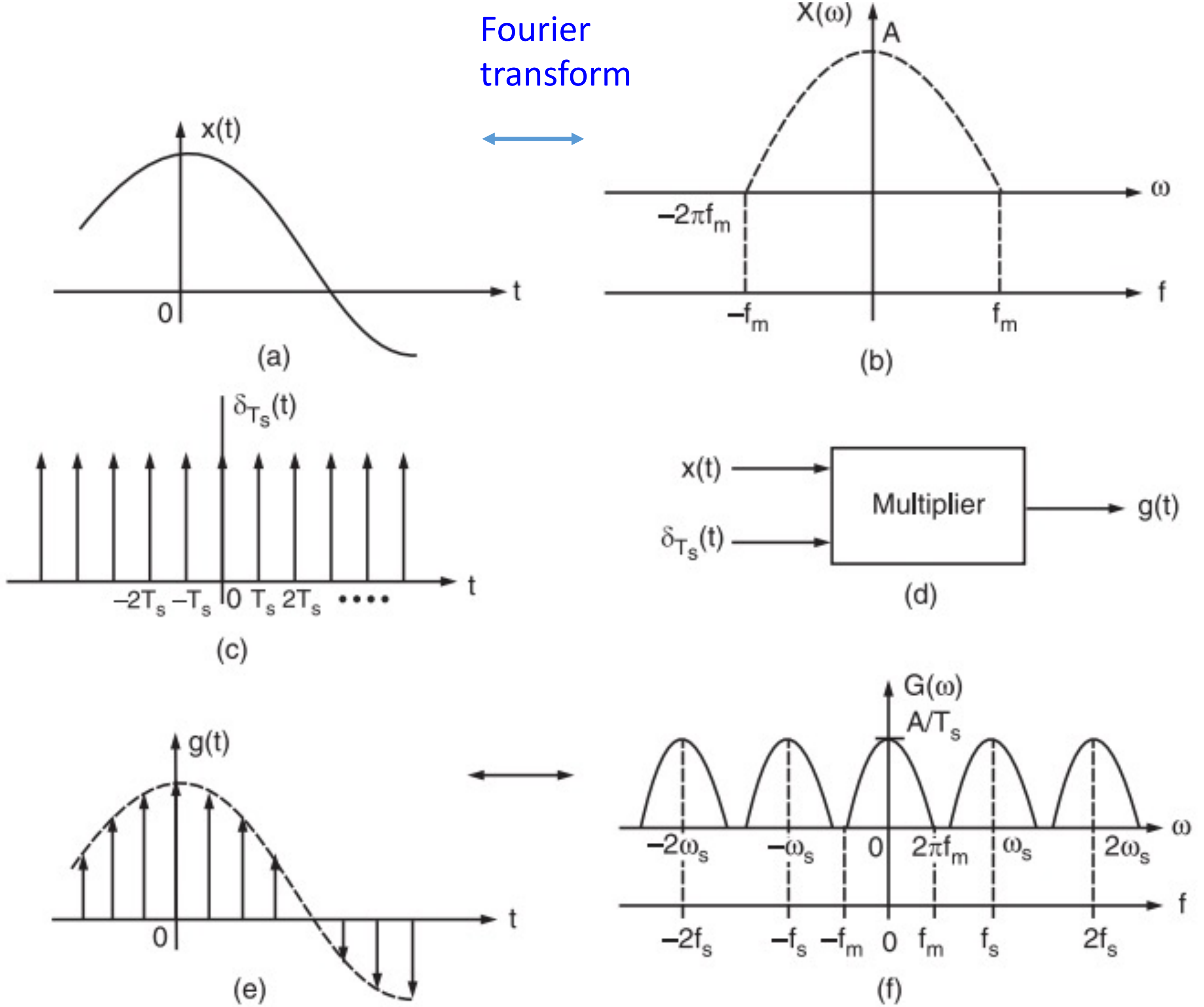


Sampling function



Sampling function in time domain

Sampling function in frequency domain - fft



Fourier transform Property:

$$\mathcal{F}(x\delta) = \mathcal{F}(x) * \mathcal{F}(\delta)$$

Convolution; wait until next next lecture

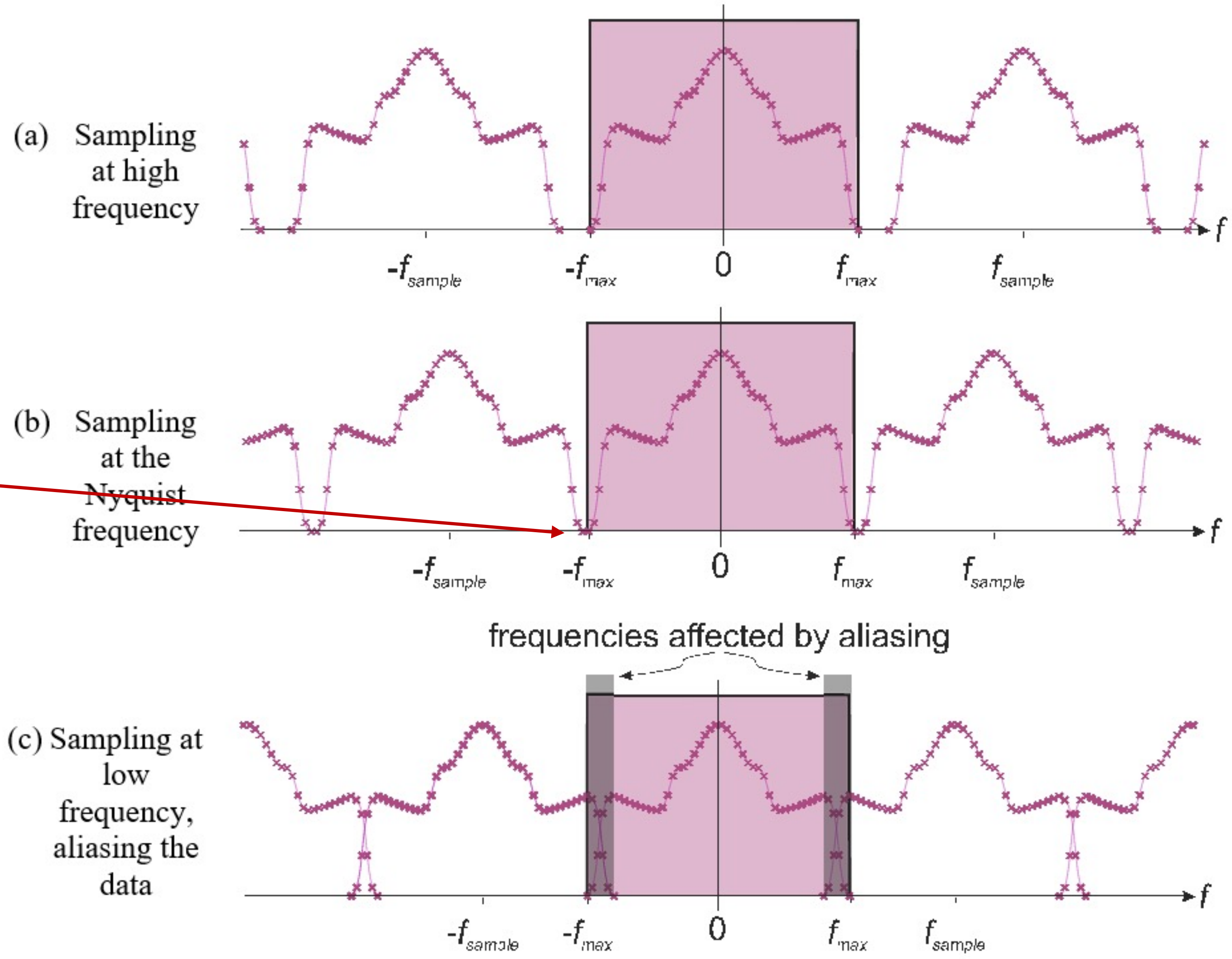
https://electronicspost.com/sampling-theorem/

In the frequency domain

Spectra repeat

If sampling is just right, spectra just touch

Minimum sampling frequency = 2 × max



Sampling process in the frequency domain

Sampling theory

Nyquist's sampling theorem

In order to be able to reconstruct a signal from its samples we must sample at minimum at twice the maximum frequency in the original signal

E.g.: speech 6kHz, sample at 12 kHz

Video bandwidth (CCIR) is 5MHz

Sampling at 10MHz gave 576×576 images

Guideline: "two pixels for every pixel of interest"

Aliasing in Sampled Imagery

(a) high resolution

(c) low resolution – aliased

Correct and Incorrect Apparent Wheel Motion

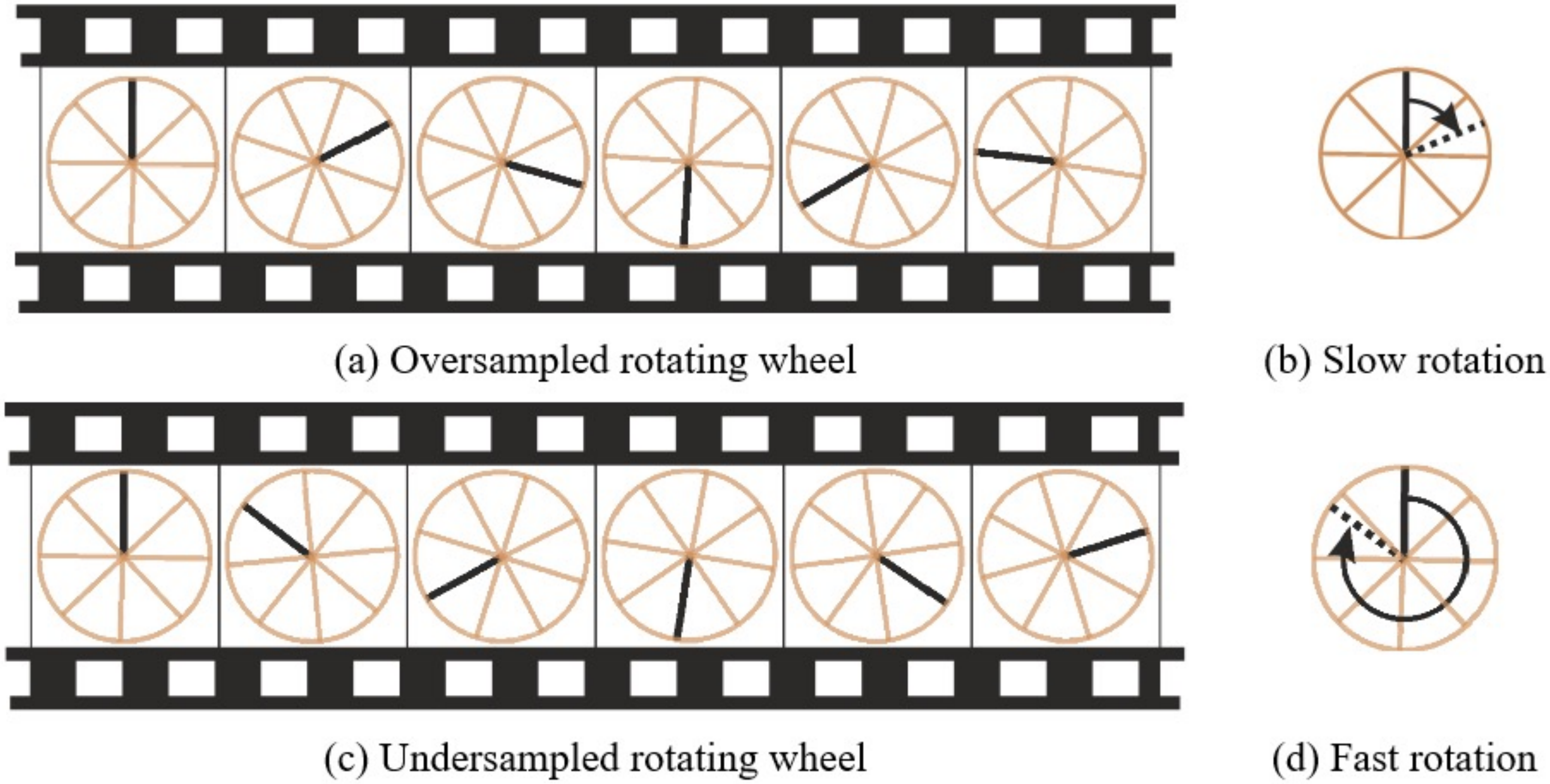


Figure 4.5 Correct and incorrect apparent wheel motion

https://www.youtube.com/watch?v=e1EqXE06xr8

Main points so far

1) Need to sample at a high enough frequency

2) Aliasing corrupts image information

3) Discrete Fourier allows analysis and understanding

4) Fourier has many properties and advantages

…. but it's complex.

We'll move on to processing images …

More sampling theories

Compressed sensing

Many signals are sparse…

Regularisation