Welcome!

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COMP6207
Algorithmic Game Theory

Lecture 12 One-sided Matching With Initial Ownership

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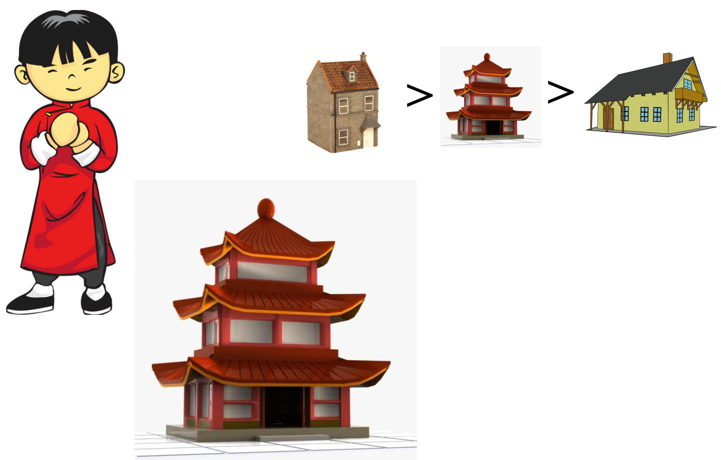
Learning Outcomes

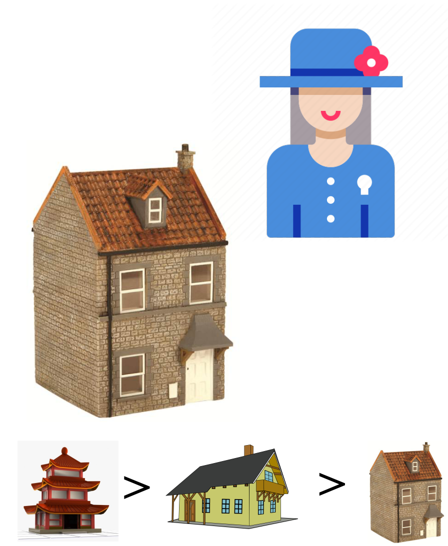
- By the end of this session, the students should be able to
  - ***Describe*** the one-sided matching problem
  - **Understand** the Top Trading Cycle mechanism for the *House Allocation* problem (*HA*)
  - **Understand** Pareto-optimality
  - ***Compute*** the outcome of TTC and YRMH-IGYT

One-sided Matching

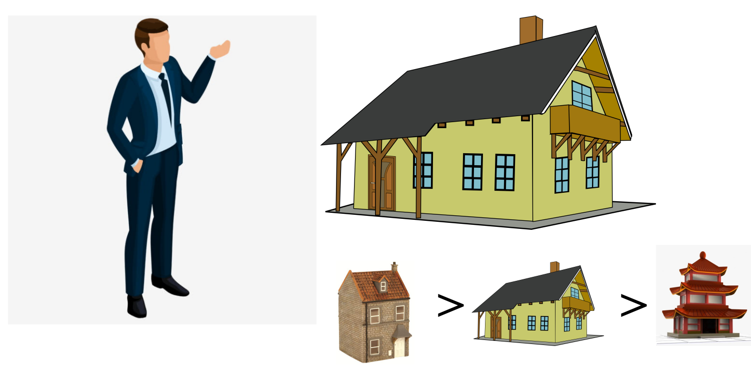
- Bipartite structure
  - Individuals in one group has preferences over the individuals in the other group.
- Examples
  - *House allocation*
    - Each individual owns a house and is interested in improving its utility by swapping with others
  - Other allocation problems
    - No initial ownership

House allocation

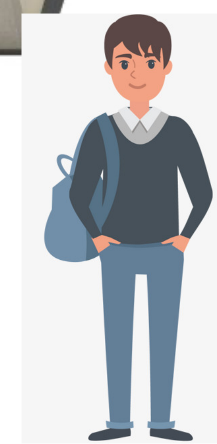




Can we benefit from swapping houses?



Student rooms allocation



Random Serial Dictatorship with Squatting Rights.
Used in undergrad housing in many universities.

1. Each existing tenant decides whether they want to participate in the housing lottery or keep the current house. Those who decide to keep their houses are assigned the current houses. All other houses become available for assignment in later steps.
2. An ordering of agents is decided. The ordering may be uniformly random or may favor some subgroup of agents (for example, seniors over juniors).
3. Serial dictatorship is applied to all available houses and agents (except for existing tenants already assigned their current houses).

What do you think of that mechanism?

Problem:
Existing tenants are not guaranteed to get at least as good a house as their current house: Individually irrational!

Some existing tenants may not want to enter the lottery even if they want to move.

This may result in loss of gains from trade, and the resulting matching may not be Pareto efficient.

Desired properties

- Some good properties we want for the *student rooms allocation* mechanisms and *house allocation* mechanisms.
  – Individual rationality
  – Pareto efficiency
  – Strategy-proofness

- Can we find mechanisms with the above good properties?

House Allocation Problem

- A model for allocation of indivisible goods.
- A set of n agents, each owns a unique house and a strict preference ordering over all n houses.
- The objective is to reallocate the houses among the agents to improve the agents' utility.

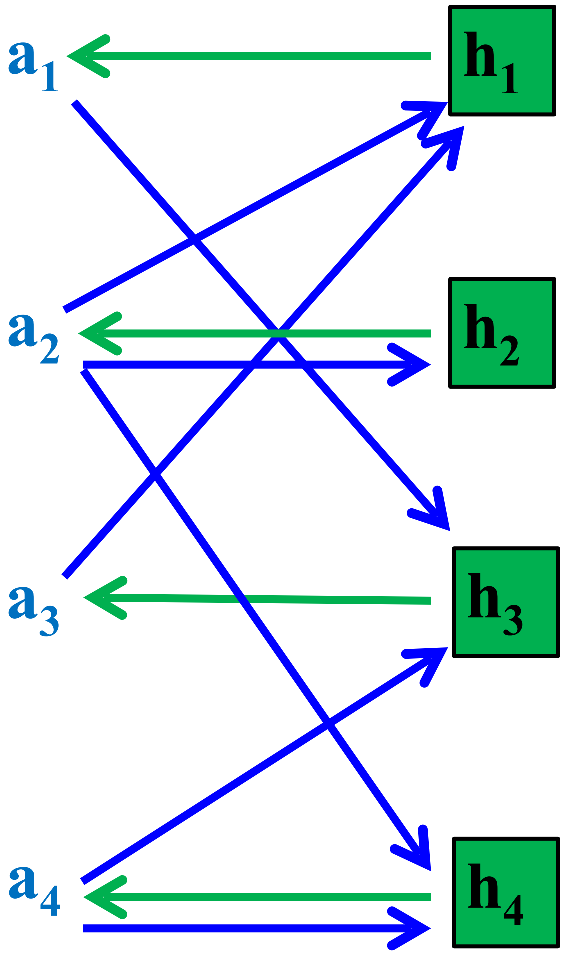
- Top Trading Cycle (TTC) mechanism:
  - Introduced by David Gale and works in rounds.
  - Initially #|agents| = #|houses|, but let's check out this generalized version.

Top Trading Cycle mechanism (TTC)

In each round:

1. Each agent points to her most preferred house (possibly her own house); each house points back to its owner
2. This creates a directed graph; in this graph, identify cycles
   - Finite: cycle must exist
   - Strict preferences: each agent is in at most one cycle
3. Assign each agent in a cycle to the house she is pointing at and remove her from the mechanism with her assigned house.
4. If all houses are assigned or all agents are assigned or all preference lists are empty, stop.
5. Otherwise, Repeat (i.e. go to step 1)

Example: TTC



$a_1 : h_3 > h_2 > h_1$

$a_2 : h_1 > h_4 > h_2$

$a_3 : h_1 > h_4 > h_3$

$a_4 : h_3 > h_4$

- a_1 is matched to h_3
- a_3 is matched to h_1
- a_4 is matched to h_4
- a_2 is matched to h_2

Example: TTC

$a_1 : h_3 > h_2 > h_1$

$a_2 : h_1 > h_4 > h_2$

$a_3 : h_1 > h_4 > h_3$

$a_4 : h_3 > h_4$

- a_1 is matched to h_3
- a_3 is matched to h_1
- a_4 is matched to h_4
- a_2 is matched to h_2

Quiz

- What matching does TTC return?

$$\mathbf{a_1} : h_2 > h_3 > \mathbf{h_1}$$

$$\mathbf{a_2} : h_1 > \mathbf{h_2}$$

$$\mathbf{a_3} : h_1 > h_4 > \mathbf{h_3}$$

$$\mathbf{a_4} : h_2 > h_1 > h_3 > \mathbf{h_4}$$

Desired Properties

- Reflection: what are the desired properties in this scenario?
  - Truthfulness
  - Pareto optimality
  - What is the matter with the agents' ownership?
    - Individual rationality
    - Core

Incentive in TTC

Theorem.
TTC is dominant-strategy truthful.

Proof sketch.
For any agent matched in round **k** if truthful

- No change in her report can give her a house that was assigned in earlier rounds
  - (this step of the proof needs some more argument)
- No house assigned in a later round will make her better off
- So no benefit in reporting any other preferences but the truth.

Pareto Optimality

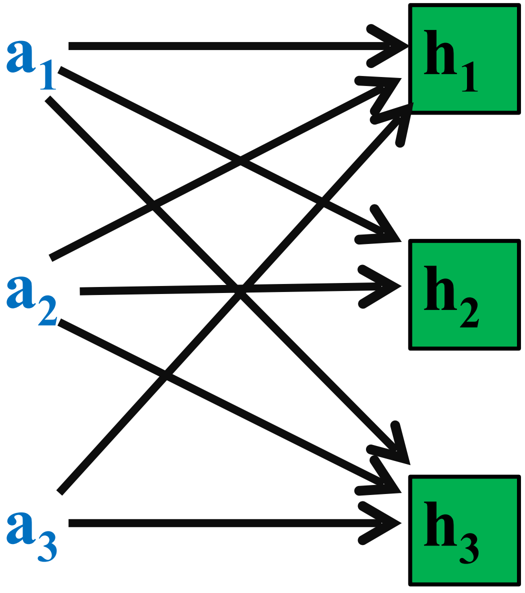
A matching $\mathbf{M_1}$ is *Pareto optimal (**PO**)* if there is no other matching $\mathbf{M_2}$ such that

- some agent prefers $\mathbf{M_2}$ to $\mathbf{M_1}$
- no agent prefers $\mathbf{M_1}$ to $\mathbf{M_2}$

$\mathbf{a_1}$: $h_1 > h_2 > h_3$

$\mathbf{a_2}$: $h_2 > h_1 > h_3$

$\mathbf{a_3}$: $h_1 > h_3$



Pareto optimality is also known as the **Pareto efficiency**

Pareto Optimality

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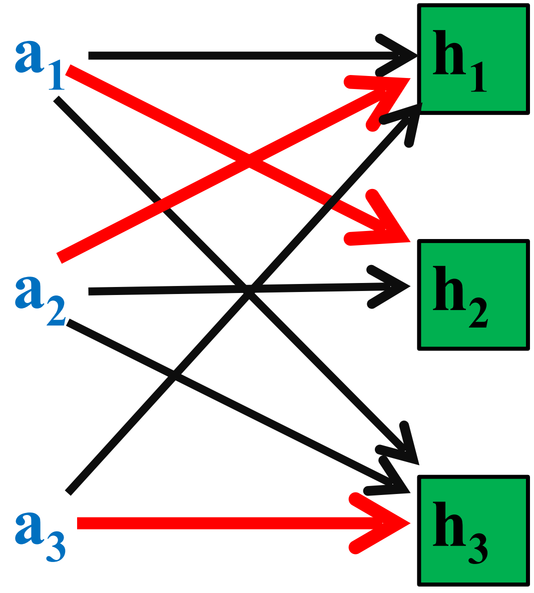
$\mathbf{a_1} : h_1 > h_2 > h_3$

$\mathbf{a_2} : h_2 > h_1 > h_3$

$\mathbf{a_3} : h_1 > h_3$

$\mathbf{M_1=\{(a_1,h_2), (a_2,h_1), (a_3,h_3)\}}$



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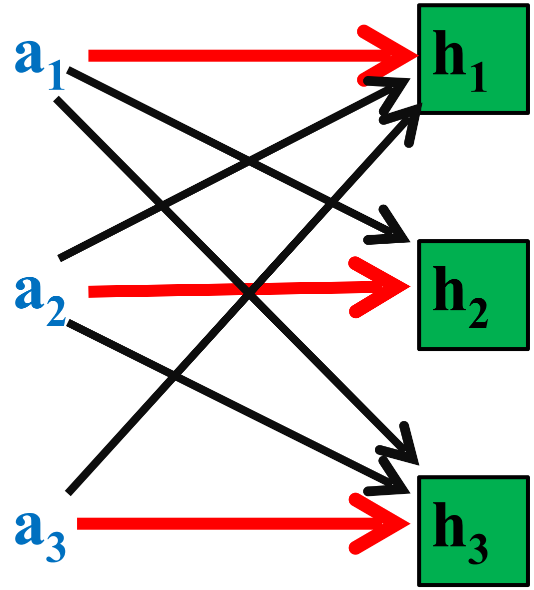
$\mathbf{M_1=\{(a_1,h_2), (a_2,h_1), (a_3,h_3)\}}$

$\mathbf{M_2=\{(a_1,h_1), (a_2,h_2), (a_3,h_3)\}}$



M_2 Pareto dominates M_1

Quiz

Is the **red** matching Pareto optimal?

$\mathbf{a_1}$: $h_2 > \mathbf{h_1}$

$\mathbf{a_2}$: $h_3 > \mathbf{h_4} > h_2$

$\mathbf{a_3}$: $h_4 > \mathbf{h_3}$

$\mathbf{a_4}$: $h_3 > h_4$

Quiz

Is the **red** matching Pareto optimal?

$\mathbf{a_1}$: $\mathbf{h_2} > h_1$

$\mathbf{a_2}$: $h_3 > \mathbf{h_4} > h_2$

$\mathbf{a_3}$: $h_4 > \mathbf{h_3}$

$\mathbf{a_4}$: $h_3 > h_4$

Quiz

Is the **red** matching Pareto optimal?

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$\mathbf{a_2}$: $\mathbf{h_3} > h_4 > h_2$

$\mathbf{a_3}$: $h_4 > h_3$

$\mathbf{a_4}$: $h_3 > \mathbf{h_4}$

Pareto optimality in TTC

Theorem.

TTC allocation is always Pareto optimal.

Proof sketch.

By induction:

- None of the agents matched in round 1 can improve their assignment (they are already getting their most favourite house)
- Fixing the assignment of those agents matched in round 1, none of the agents in round 2 can improve their assignment (they are already getting their most favourite among the remaining houses)
- And so on.

The Core

- The agents own the houses => if a subset of agents can make all of its members better off by exchanging the houses within them, then the TTC allocation would be unstable as these agents would have an incentive to reallocate their houses.
- The allocation returned by TTC is such that no such subset of agents exists.
- Alternatively speaking, the TTC outcome is a core allocation.
- In addition, the TTC allocation is the only such assignment => it returns the unique core allocation.

Student rooms allocation

- A set of existing students $\{a_1, a_{2,} \ldots, a_n\}$, each occupies a room
- A set of newcomers $\{a_{n+1}, a_{n+2,} \ldots, a_{n+m}\}$
- A set of vacant rooms $\{1, 2, \ldots, m\}$

- A mechanism:
  - You request my house, I get your turn (YRMH-IGYT)

You request my house, I get your turn (YRMH-IGYT)

- Fix a priority order of the agents
- Let the agent with the top priority receive her first-choice room, the second agent receive her top choice among the remaining goods and so on, until someone requests the room of an existing tenant.
- If the existing tenant whose room is requested has already received a room, then proceed the assignment to the next agent. *Otherwise, insert the existing tenant at the top of the priority order and proceed with the procedure.*
- If at any step a cycle forms, the cycle is formed by existing tenants (a_1,..,a_k) where a_1 points to the house of agent a_2, who points to the house of a_3, and so on. In such a case assign these houses by letting them exchange, and then proceed with the algorithm.

T. Sonmez, M. U. Unver, House allocation with existing tenants: an equivalence, *Games and Economic Behavior*, 2005.

An example

Let $A_E = \{a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9\}$ be the set of existing tenants

$A_N = \{a_{10}, a_{11}, a_{12}, a_{13}, a_{14}, a_{15}, a_{16}\}$ be the set of newcomers

$H_0 = \{h_1, h_2, h_3, h_4, h_5, h_6, h_7, h_8, h_9\}$ be the set of occupied houses

$H_V = \{h_{10}, h_{11}, h_{12}, h_{13}, h_{14}, h_{15}, h_{16}\}$ be the set of vacant houses

Let the preference profile P be given as

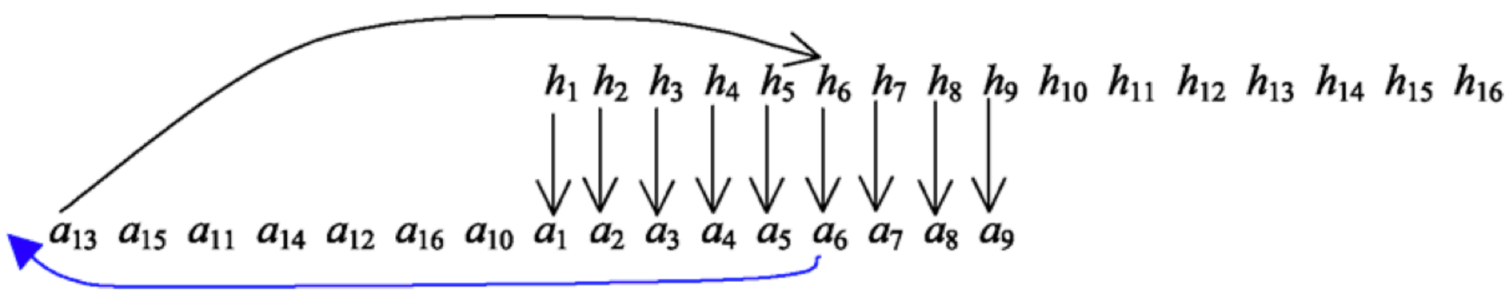
$$
\overbrace{\begin{array}{ccccccccc}
a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 & a_8 & a_9 \\
\hline
h_{15} & h_3 & h_1 & h_2 & h_9 & h_6 & h_6 & h_6 & h_{11} \\
\vdots & h_4 & h_3 & \vdots & \vdots & \vdots & h_7 & h_{12} & \vdots \\
 & \vdots & \vdots & & & & \vdots & \vdots & \\
 & & & & & & & & \\
\end{array}}^{A_E}
\qquad
\overbrace{\begin{array}{ccccccc}
a_{10} & a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} \\
\hline
h_7 & h_2 & h_4 & h_6 & h_8 & h_1 & h_5 \\
h_3 & h_4 & h_{14} & h_{13} & \vdots & \vdots & \vdots \\
h_{12} & h_{16} & \vdots & \vdots & & & \\
h_{10} & \vdots & & & & & \\
\end{array}}^{A_N}
$$

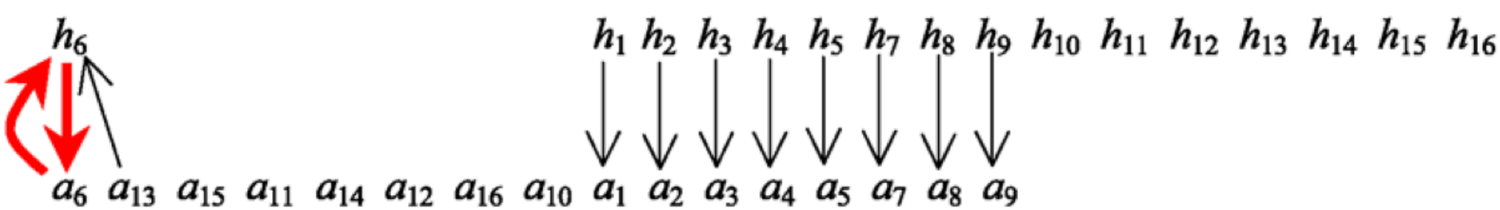
Priority order $>$: $a_{13}, a_{15}, a_{11}, a_{14}, a_{12}, a_{16}, a_{10}, a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9$

Priority order $>$:

$$a_{13}, a_{15}, a_{11}, a_{14}, a_{12}, a_{16}, a_{10}, a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9$$

A_E									A_N						
a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}	a_{11}	a_{12}	a_{13}	a_{14}	a_{15}	a_{16}
h_{15}	h_3	h_1	h_2	h_9	h_6	h_6	h_6	h_{11}	h_7	h_2	h_4	h_6	h_8	h_1	h_5
$\vdots$	h_4	h_3	$\vdots$	$\vdots$	$\vdots$	h_7	h_{12}	$\vdots$	h_3	h_4	h_{14}	h_{13}	$\vdots$	$\vdots$	$\vdots$
	$\vdots$	$\vdots$				$\vdots$	$\vdots$		h_{12}	h_{16}	$\vdots$	$\vdots$			
									h_{10}	$\vdots$					





Priority order $>$:

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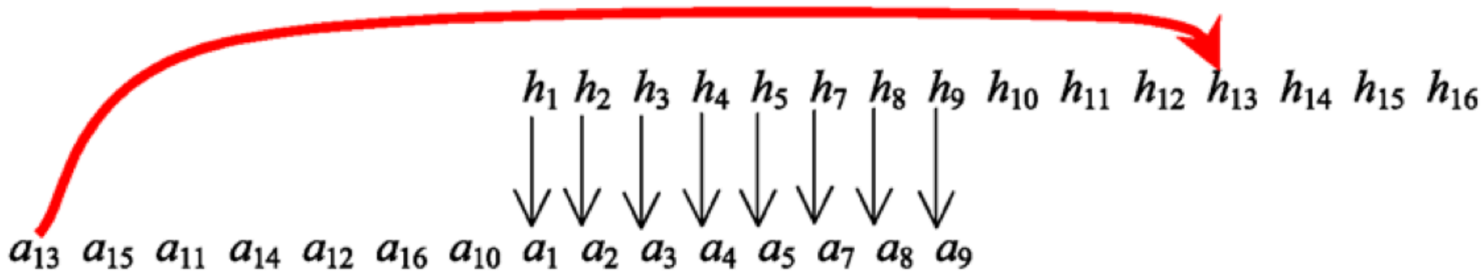
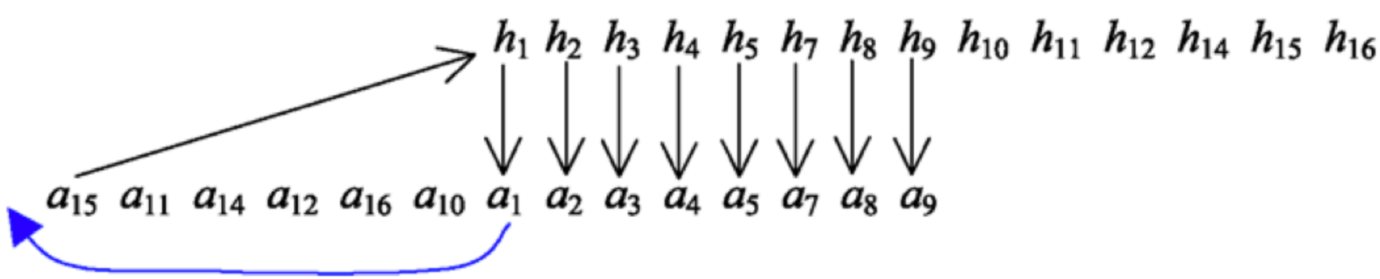
A_E									A_N						
a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}	a_{11}	a_{12}	a_{13}	a_{14}	a_{15}	a_{16}
h_{15}	h_3	h_1	h_2	h_9	h_6	h_6	h_6	h_{11}	h_7	h_2	h_4	h_6	h_8	h_1	h_5
$\vdots$	h_4	h_3	$\vdots$	$\vdots$	$\vdots$	h_7	h_{12}	$\vdots$	h_3	h_4	h_{14}	h_{13}	$\vdots$	$\vdots$	$\vdots$
	$\vdots$	$\vdots$				$\vdots$	$\vdots$		h_{12}	h_{16}	$\vdots$	$\vdots$			
									h_{10}	$\vdots$					





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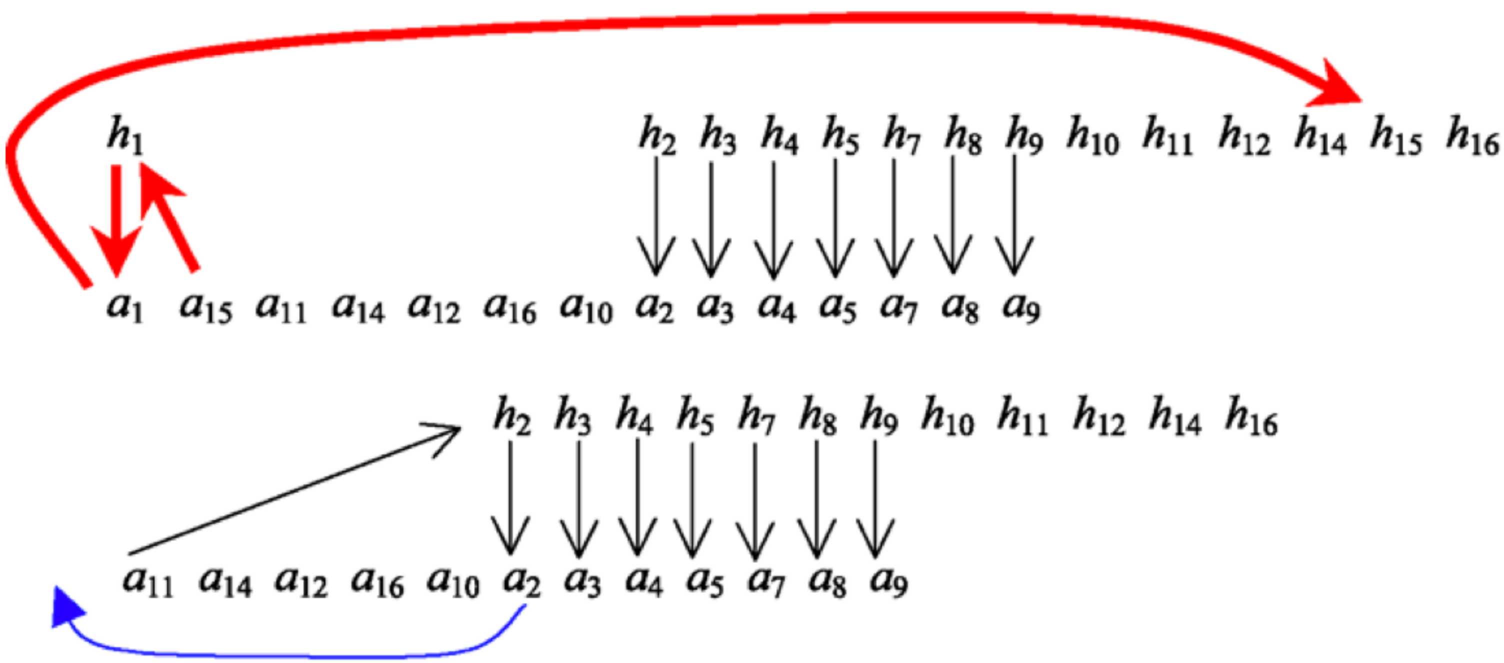
A_E

a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9
h_{15}	h_3	h_1	h_2	h_9	h_6	h_6	h_6	h_{11}
$\vdots$	h_4	h_3	$\vdots$	$\vdots$	$\vdots$	h_7	h_{12}	$\vdots$
	$\vdots$	$\vdots$				$\vdots$	$\vdots$	

A_N

a_{10}	a_{11}	a_{12}	a_{13}	a_{14}	a_{15}	a_{16}
h_7	h_2	h_4	h_6	h_8	h_1	h_5
h_3	h_4	h_{14}	h_{13}	$\vdots$	$\vdots$	$\vdots$
h_{12}	h_{16}	$\vdots$	$\vdots$			
h_{10}	$\vdots$					



Priority order >:

$$a_{13}, a_{15}, a_{11}, a_{14}, a_{12}, a_{16}, a_{10}, a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9$$

A_E									A_N						
a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}	a_{11}	a_{12}	a_{13}	a_{14}	a_{15}	a_{16}
h_{15}	h_3	h_1	h_2	h_9	h_6	h_6	h_6	h_{11}	h_7	h_2	h_4	h_6	h_8	h_1	h_5
$\vdots$	h_4	h_3	$\vdots$	$\vdots$	$\vdots$	h_7	h_{12}	$\vdots$	h_3	h_4	h_{14}	h_{13}	$\vdots$	$\vdots$	$\vdots$
	$\vdots$	$\vdots$				$\vdots$	$\vdots$		h_{12}	h_{16}	$\vdots$	$\vdots$			
									h_{10}	$\vdots$					

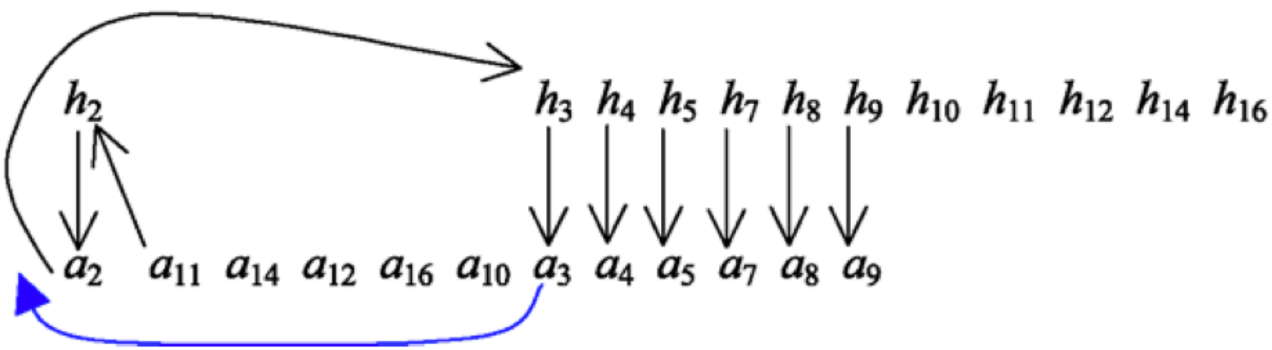
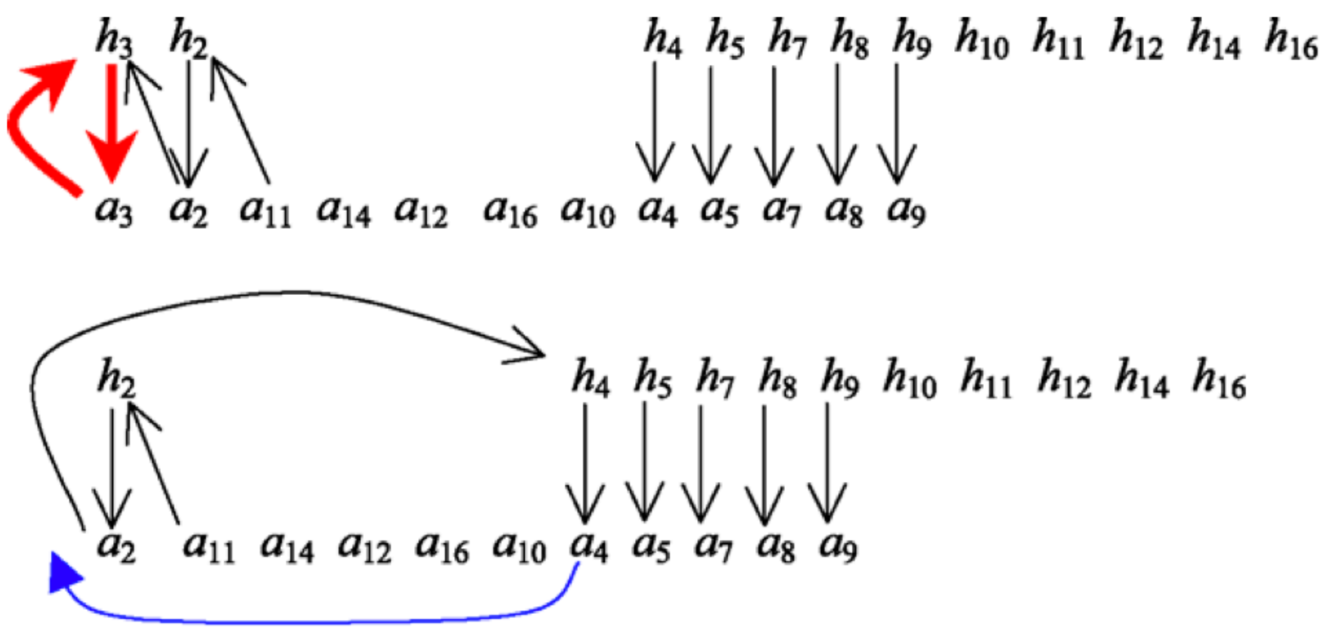


Fig. 1. The sequence of first seven events under the YRMH–IGYT algorithm.

Priority order $>$:

$$a_{13}, a_{15}, a_{11}, a_{14}, a_{12}, a_{16}, a_{10}, a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9$$

A_E									A_N						
a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}	a_{11}	a_{12}	a_{13}	a_{14}	a_{15}	a_{16}
h_{15}	h_3	h_1	h_2	h_9	h_6	h_6	h_6	h_{11}	h_7	h_2	h_4	h_6	h_8	h_1	h_5
$\vdots$	h_4	h_3	$\vdots$	$\vdots$	$\vdots$	h_7	h_{12}	$\vdots$	h_3	h_4	h_{14}	h_{13}	$\vdots$	$\vdots$	$\vdots$
	$\vdots$	$\vdots$				$\vdots$	$\vdots$		h_{12}	h_{16}	$\vdots$	$\vdots$			
									h_{10}	$\vdots$					



Priority order >:

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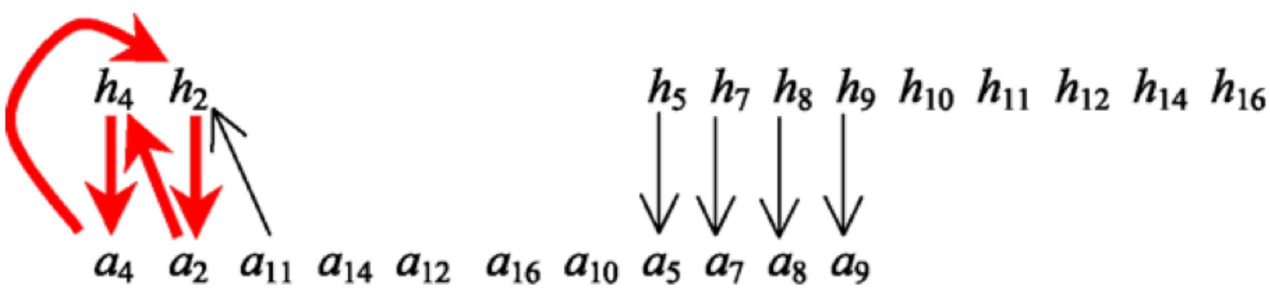
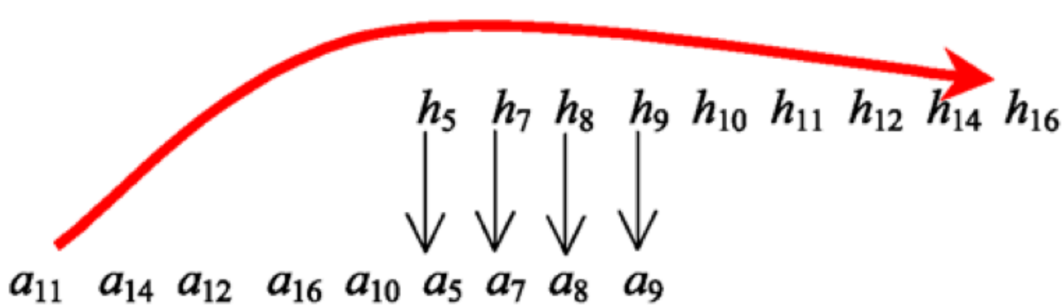
A_E									A_N						
a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}	a_{11}	a_{12}	a_{13}	a_{14}	a_{15}	a_{16}
h_{15}	h_3	h_1	h_2	h_9	h_6	h_6	h_6	h_{11}	h_7	h_2	h_4	h_6	h_8	h_1	h_5
$\vdots$	h_4	h_3	$\vdots$	$\vdots$	$\vdots$	h_7	h_{12}	$\vdots$	h_3	h_4	h_{14}	h_{13}	$\vdots$	$\vdots$	$\vdots$
	$\vdots$	$\vdots$				$\vdots$	$\vdots$		h_{12}	h_{16}	$\vdots$	$\vdots$			
									h_{10}	$\vdots$					

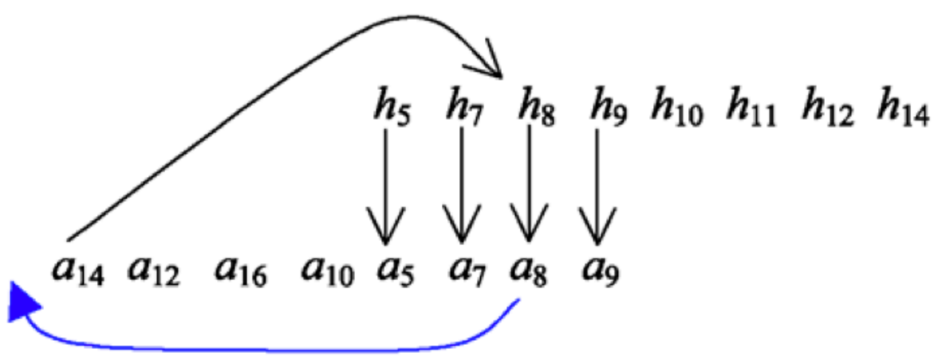


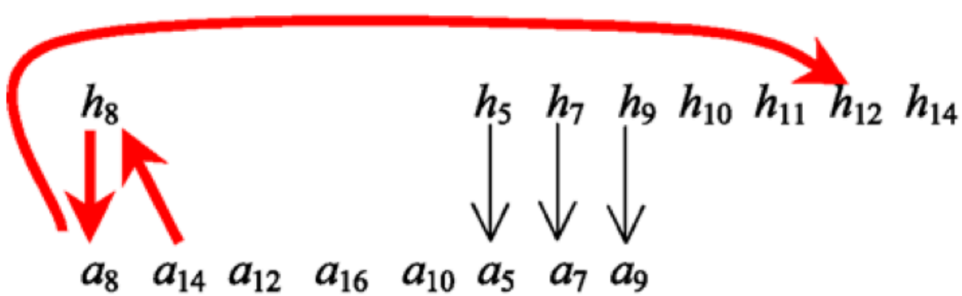


Priority order >:

$$a_{13}, a_{15}, a_{11}, a_{14}, a_{12}, a_{16}, a_{10}, a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9$$

A_E									A_N						
a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}	a_{11}	a_{12}	a_{13}	a_{14}	a_{15}	a_{16}
h_{15}	h_3	h_1	h_2	h_9	h_6	h_6	h_6	h_{11}	h_7	h_2	h_4	h_6	h_8	h_1	h_5
$\vdots$	h_4	h_3	$\vdots$	$\vdots$	$\vdots$	h_7	h_{12}	$\vdots$	h_3	h_4	h_{14}	h_{13}	$\vdots$	$\vdots$	$\vdots$
	$\vdots$	$\vdots$				$\vdots$	$\vdots$		h_{12}	h_{16}	$\vdots$	$\vdots$			
									h_{10}	$\vdots$					





Priority order >:

$$a_{13}, a_{15}, a_{11}, a_{14}, a_{12}, a_{16}, a_{10}, a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9$$

A_E									A_N						
a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}	a_{11}	a_{12}	a_{13}	a_{14}	a_{15}	a_{16}
h_{15}	h_3	h_1	h_2	h_9	h_6	h_6	h_6	h_{11}	h_7	h_2	h_4	h_6	h_8	h_1	h_5
$\vdots$	h_4	h_3	$\vdots$	$\vdots$	$\vdots$	h_7	h_{12}	$\vdots$	h_3	h_4	h_{14}	h_{13}	$\vdots$	$\vdots$	$\vdots$
	$\vdots$	$\vdots$				$\vdots$	$\vdots$		h_{12}	h_{16}	$\vdots$	$\vdots$			
									h_{10}	$\vdots$					

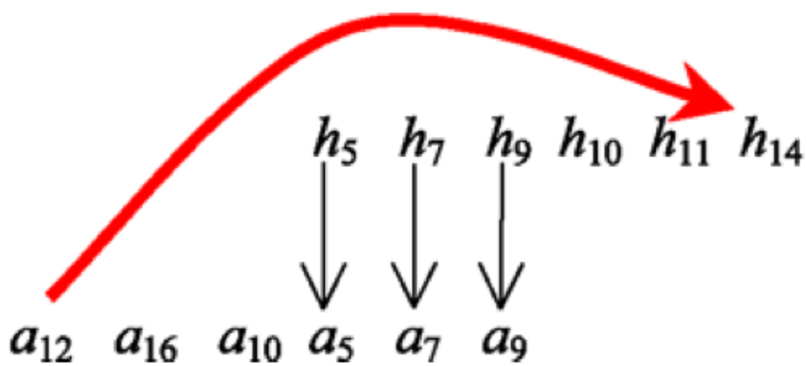
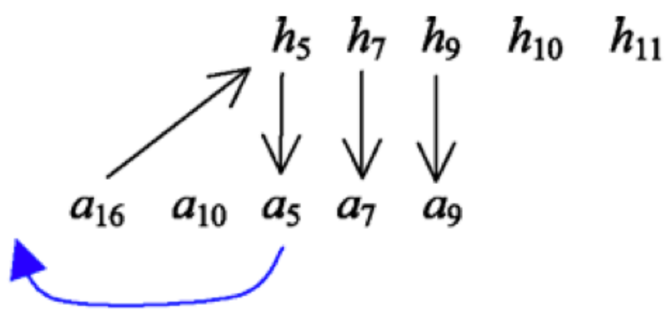


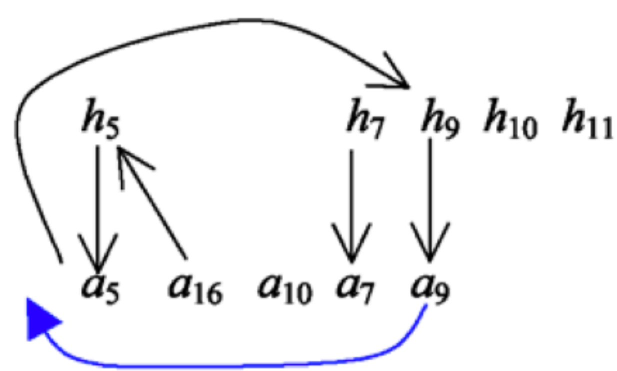
Fig. 2. The sequence of second seven events under the YRMH–IGYT algorithm.

Priority order >:

$$a_{13}, a_{15}, a_{11}, a_{14}, a_{12}, a_{16}, a_{10}, a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9$$

A_E									A_N						
a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}	a_{11}	a_{12}	a_{13}	a_{14}	a_{15}	a_{16}
h_{15}	h_3	h_1	h_2	h_9	h_6	h_6	h_6	h_{11}	h_7	h_2	h_4	h_6	h_8	h_1	h_5
$\vdots$	h_4	h_3	$\vdots$	$\vdots$	$\vdots$	h_7	h_{12}	$\vdots$	h_3	h_4	h_{14}	h_{13}	$\vdots$	$\vdots$	$\vdots$
	$\vdots$	$\vdots$				$\vdots$	$\vdots$		h_{12}	h_{16}	$\vdots$	$\vdots$			
									h_{10}	$\vdots$					

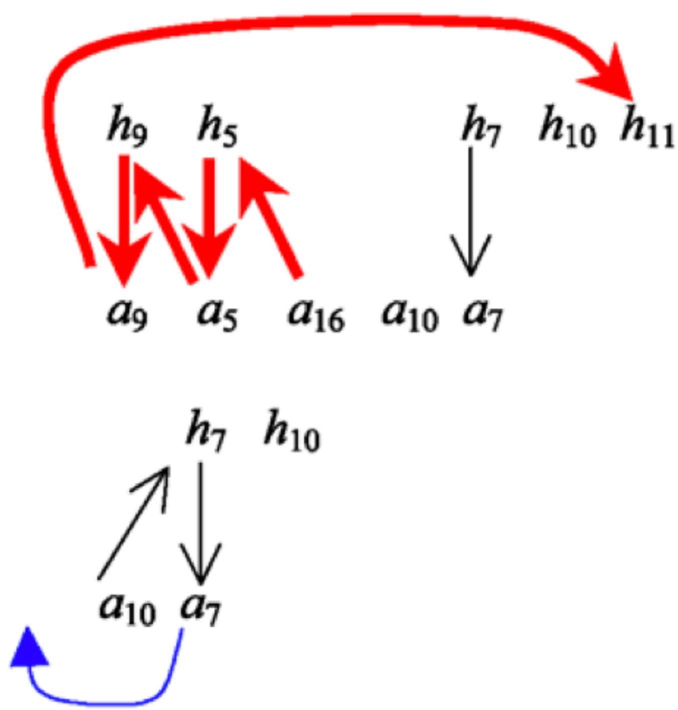




Priority order $>$:

$$a_{13}, a_{15}, a_{11}, a_{14}, a_{12}, a_{16}, a_{10}, a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9$$

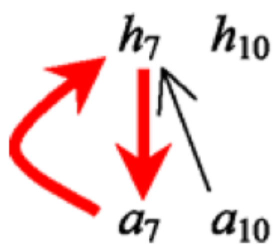
A_E									A_N						
a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}	a_{11}	a_{12}	a_{13}	a_{14}	a_{15}	a_{16}
h_{15}	h_3	h_1	h_2	h_9	h_6	h_6	h_6	h_{11}	h_7	h_2	h_4	h_6	h_8	h_1	h_5
$\vdots$	h_4	h_3	$\vdots$	$\vdots$	$\vdots$	h_7	h_{12}	$\vdots$	h_3	h_4	h_{14}	h_{13}	$\vdots$	$\vdots$	$\vdots$
	$\vdots$	$\vdots$				$\vdots$	$\vdots$		h_{12}	h_{16}	$\vdots$	$\vdots$			
									h_{10}	$\vdots$					



Priority order >:

$$a_{13}, a_{15}, a_{11}, a_{14}, a_{12}, a_{16}, a_{10}, a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9$$

A_E									A_N						
a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}	a_{11}	a_{12}	a_{13}	a_{14}	a_{15}	a_{16}
h_{15}	h_3	h_1	h_2	h_9	h_6	h_6	h_6	h_{11}	h_7	h_2	h_4	h_6	h_8	h_1	h_5
$\vdots$	h_4	h_3	$\vdots$	$\vdots$	$\vdots$	h_7	h_{12}	$\vdots$	h_3	h_4	h_{14}	h_{13}	$\vdots$	$\vdots$	$\vdots$
	$\vdots$	$\vdots$				$\vdots$	$\vdots$		h_{12}	h_{16}	$\vdots$	$\vdots$			
									h_{10}	$\vdots$					



Priority order $>$:

$$a_{13}, a_{15}, a_{11}, a_{14}, a_{12}, a_{16}, a_{10}, a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9$$

A_E									A_N						
a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9	a_{10}	a_{11}	a_{12}	a_{13}	a_{14}	a_{15}	a_{16}
h_{15}	h_3	h_1	h_2	h_9	h_6	h_6	h_6	h_{11}	h_7	h_2	h_4	h_6	h_8	h_1	h_5
$\vdots$	h_4	h_3	$\vdots$	$\vdots$	$\vdots$	h_7	h_{12}	$\vdots$	h_3	h_4	h_{14}	h_{13}	$\vdots$	$\vdots$	$\vdots$
	$\vdots$	$\vdots$				$\vdots$	$\vdots$		h_{12}	h_{16}	$\vdots$	$\vdots$			
									h_{10}	$\vdots$					

The outcome of the algorithm is

$$\begin{bmatrix} a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 & a_8 & a_9 & a_{10} & a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} \\ h_{15} & h_4 & h_3 & h_2 & h_9 & h_6 & h_7 & h_{12} & h_{11} & h_{10} & h_{16} & h_{14} & h_{13} & h_8 & h_1 & h_5 \end{bmatrix}.$$

Properties

Theorem (Theorem 3 in Abdulkadiroğlu and Sönmez (1999)): For a given ordering f, the YRMH-IGYT algorithm yields the same outcome as the top trading cycles algorithm.

We can think of YRMH-IGYT as a variant of Galei's TTC in which all vacant houses (and houses whose initial owners are already assigned houses) point to the highest priority agents rather than the owners of the houses. So we sometimes call the mechanism TTC as well.

The YRMH-IGYT mechanism is Pareto efficient, strategyproof, and makes no existing tenant worse off.

Summary

- One-sided matching problem
  - When agents own items, i.e., so-called House Allocation problem
    - Top Trading Cycle mechanism (TTC)

- Extra reading
  - House allocation
    - Chapter 10, Section 3, AGT book

Preview

- When agents do not own items
  - Probabilistic Serial mechanism (PS)
  - Random Priority (RP)