Lecture 6 Edge Detection

COMP6223 Computer Vision (MSc)

What are edges and how do we find them?

Book
pp
118 -
130

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Content

1. Differentiation/ differencing can be used to find edges of features
2. How can we improve the differencing process?

Edge detection

What is an edge? It's contrast

(a) original image (b) Sobel edge magnitude (c) thresholded magnitude

U2’s Edge can’t detect edges

http://metro.co.uk/2015/05/15/the-edge-falls-off-the-edge-of-the-stage-in-spectacular-style-during-u2s-world-tour-5199503/

First order edge detection

- Vertical edges, **Ex** $$\mathbf{Ex}_{x,y} = \left|\mathbf{P}_{x,y} - \mathbf{P}_{x+1,y}\right|$$
- Horizontal edges, **Ey** $$\mathbf{Ey}_{x,y} = \left|\mathbf{P}_{x,y} - \mathbf{P}_{x,y+1}\right|$$
- Vertical and horizontal edges $$\mathbf{E}_{x,y} = \left|2\times\mathbf{P}_{x,y} - \mathbf{P}_{x+1,y} - \mathbf{P}_{x,y+1}\right|$$

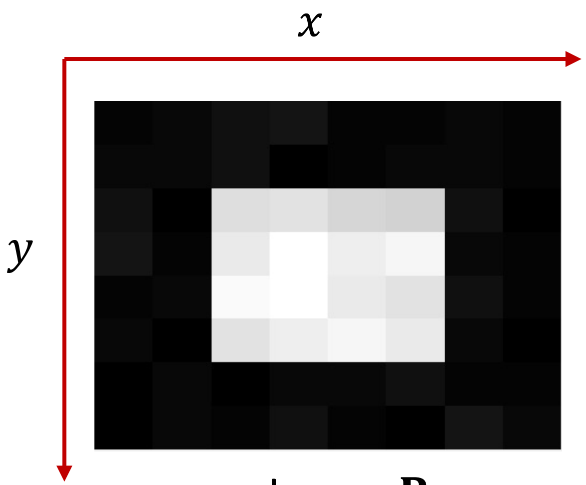
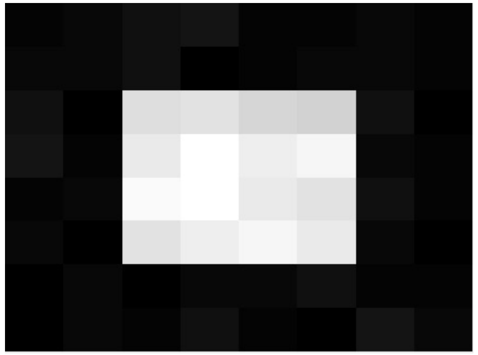


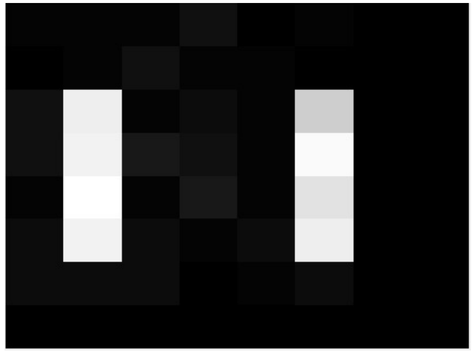
Image **P**

Horizontal differencing

difference



result

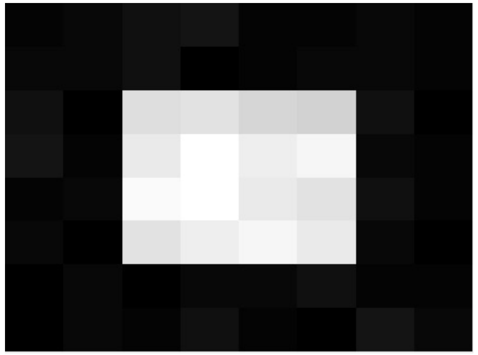


$$\mathbf{Ex}_{x,y} = \left|\mathbf{P}_{x,y} - \mathbf{P}_{x+1,y}\right|$$

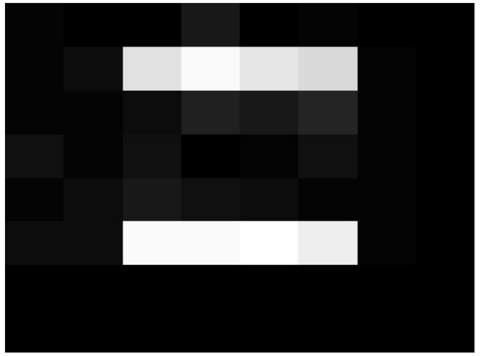
Horizontal differencing detects
vertical edges

Vertical differencing

difference



result

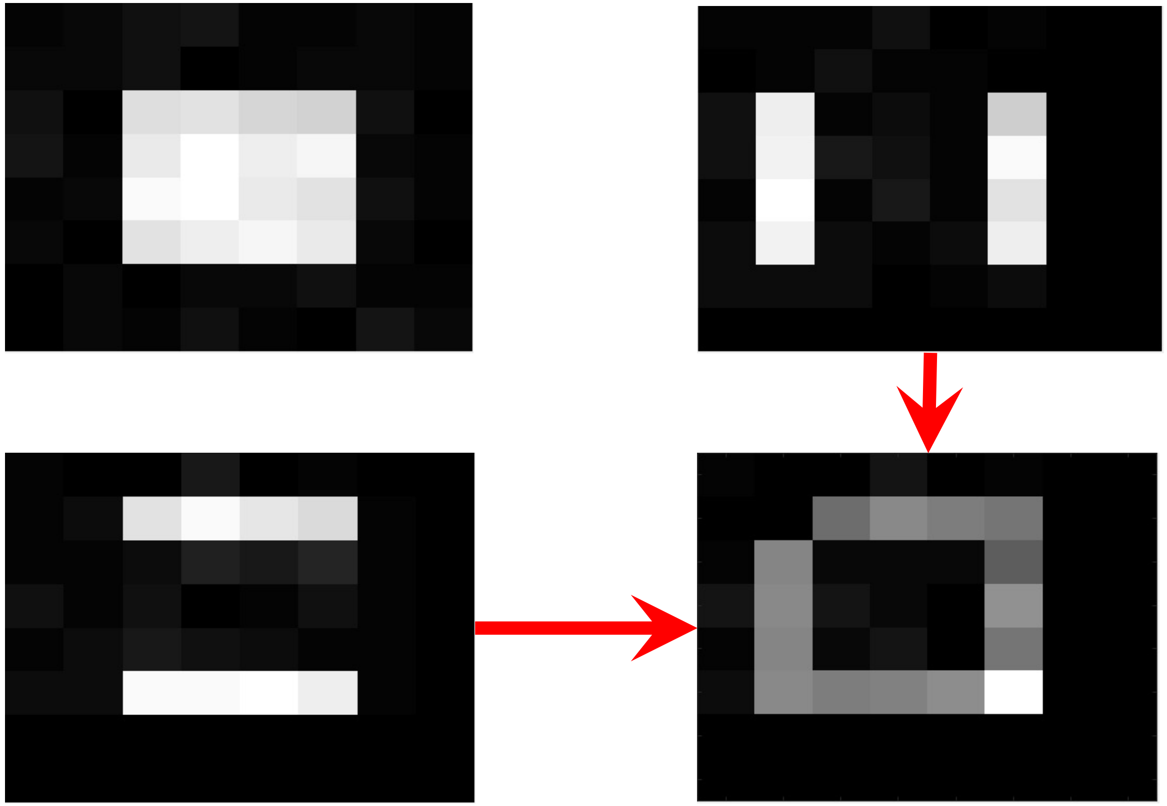


Vertical differencing detects horizontal edges

$$\mathbf{Ey}_{x,y} = \left|\mathbf{P}_{x,y} - \mathbf{P}_{x,y+1}\right|$$

First order edge detection

$$\mathbf{E}_{x,y} = \left|2\times\mathbf{P}_{x,y} - \mathbf{P}_{x+1,y} - \mathbf{P}_{x,y+1}\right|$$



Addition of horizontal and vertical

First order edge detection

Template

2	-1
-1	0

$$\mathbf{E}_{x,y} = \left| 2 \times \mathbf{P}_{x,y} - \mathbf{P}_{x+1,y} - \mathbf{P}_{x,y+1} \right|$$

Code

```
function edge = basic_difference(image)

for x = 1:cols-2 %address all columns except border
  for y = 1:rows-2 %address all rows except border
    edge(y,x)=abs(2*image(y,x)-image(y+1,x)-image(y,x+1)); % Eq. 4.4
  end
end
```

How can we improve it?

Taylor series – evaluate $f(t+\Delta t)$

First approximation, original value

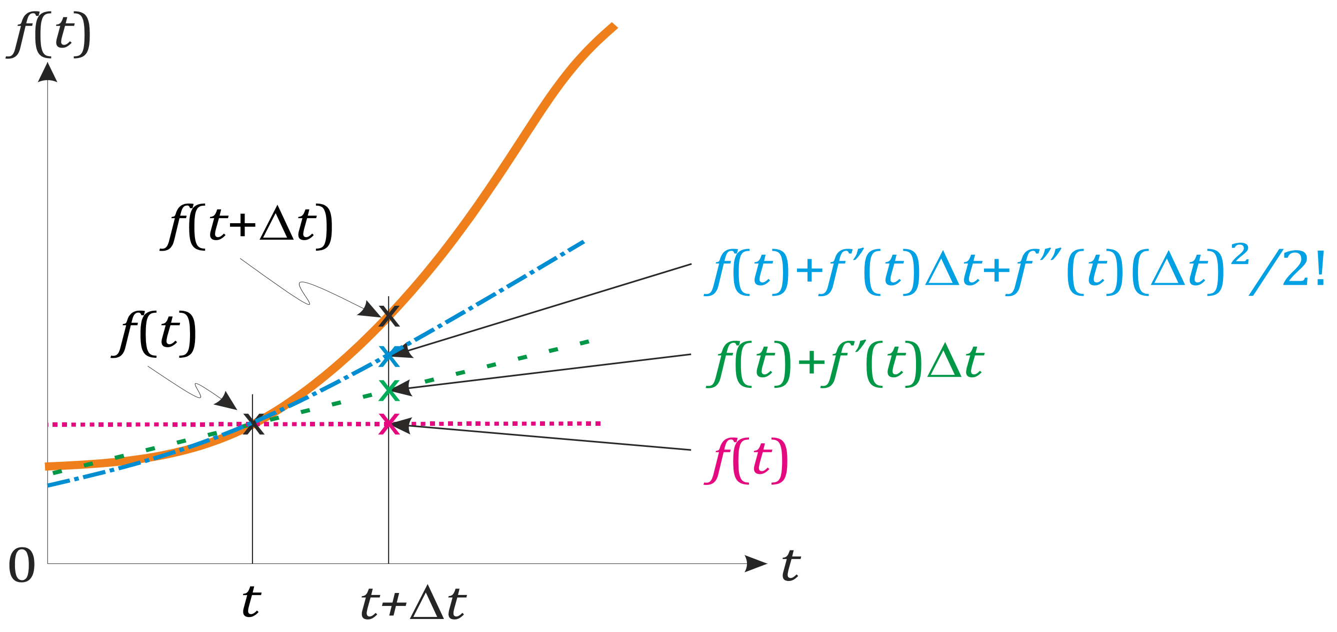
$$f(t+\Delta t) = f(t)$$

Second approximation, add gradient

$$f(t+\Delta t) = f(t) + f'(t)\Delta t$$

Third approximation, add f''

$$f(t+\Delta t) = f(t) + f'(t)\Delta t + \frac{f''(t)}{2!}(\Delta t)^2$$



Taylor series

$$f(t+\Delta t) = f(t) + f'(t)\Delta t + \frac{f''(t)}{2!}(\Delta t)^2 + \frac{f'''(t)}{3!}(\Delta t)^3 + \cdots + \frac{f^n(t)}{n!}(\Delta t)^n$$

Edge detection maths

Taylor expansion for $f(x+\Delta x)$ and $f(x-\Delta x)$:

$$f(x+\Delta x) = f(x) + \Delta x \times f'(x) + \frac{\Delta x^2}{2!} \times f''(x) + O(\Delta x^3) \qquad \textbf{(A)}$$

$$f(x-\Delta x) = f(x) - \Delta x \times f'(x) + \frac{\Delta x^2}{2!} \times f''(x) - O(\Delta x^3) \qquad \textbf{(B)}$$

(A) – (B) :

$$f'(x) = \frac{f(x+\Delta x) - f(x)}{\Delta x} - O(\Delta x)$$

$$\mathbf{Exx}_{x,y} = \left|\mathbf{P}_{x,y} - \mathbf{P}_{x-1,y}\right|$$

1	-1

$$f'(x) = \frac{f(x+\Delta x) - f(x-\Delta x)}{2\Delta x} - O(\Delta x^2)$$

If $\Delta x < 1$, this error is clearly smaller

$$\mathbf{Exx}_{x,y} = \left|\mathbf{P}_{x+1,y} - \mathbf{P}_{x-1,y}\right|$$

1	0	-1

Templates for improved first order difference

1	0	-1

(a) Mx

1
0
-1

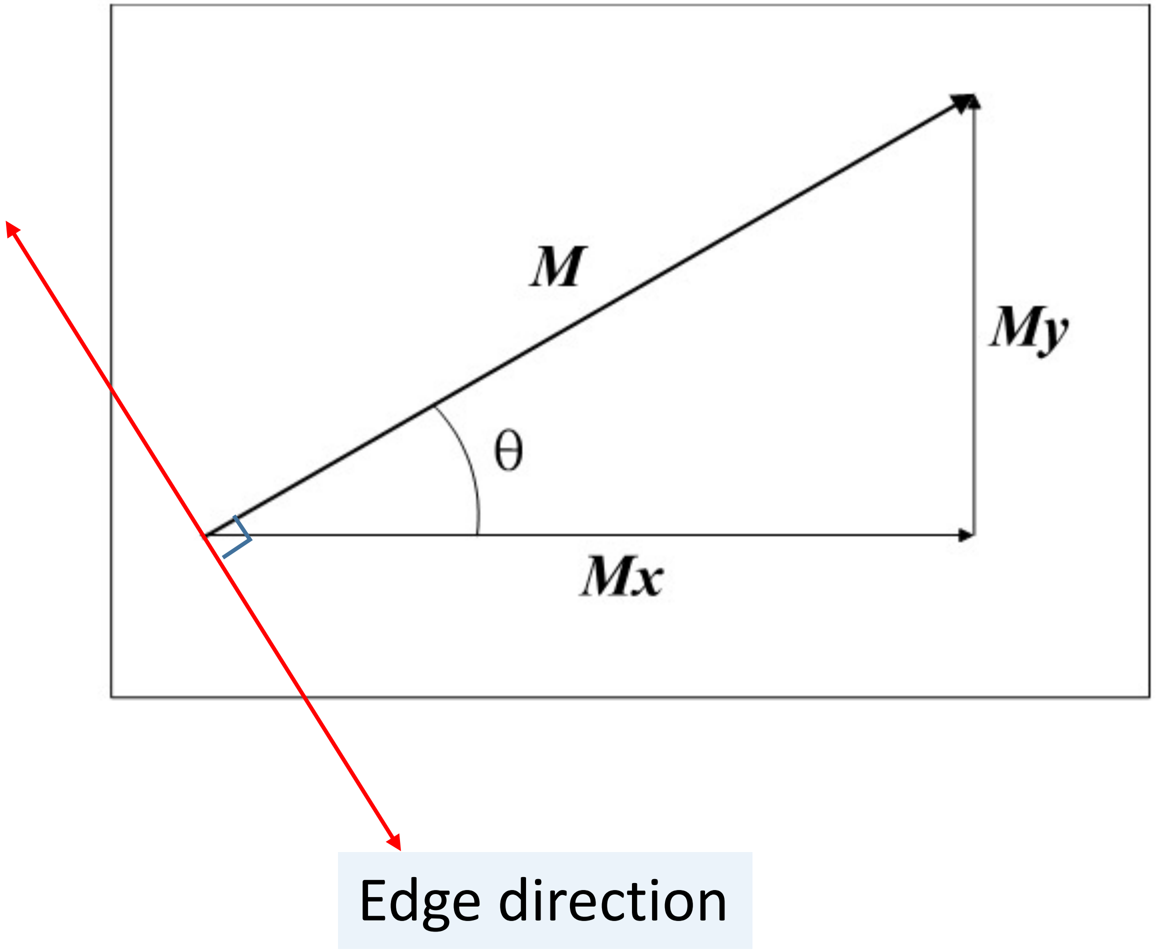
(b) My

Edge Detection in Vector Format

Vectors have magnitude (strength)
and direction

Magnitude: $M = \sqrt{M_x^2 + M_y^2}$

Direction: $\theta = \tan^{-1}\left(\frac{M_y}{M_x}\right)$



Templates for 3×3 Prewitt operator

Average improved horizontal and vertical operators

1	0	-1
1	0	-1
1	0	-1

(a) Mx

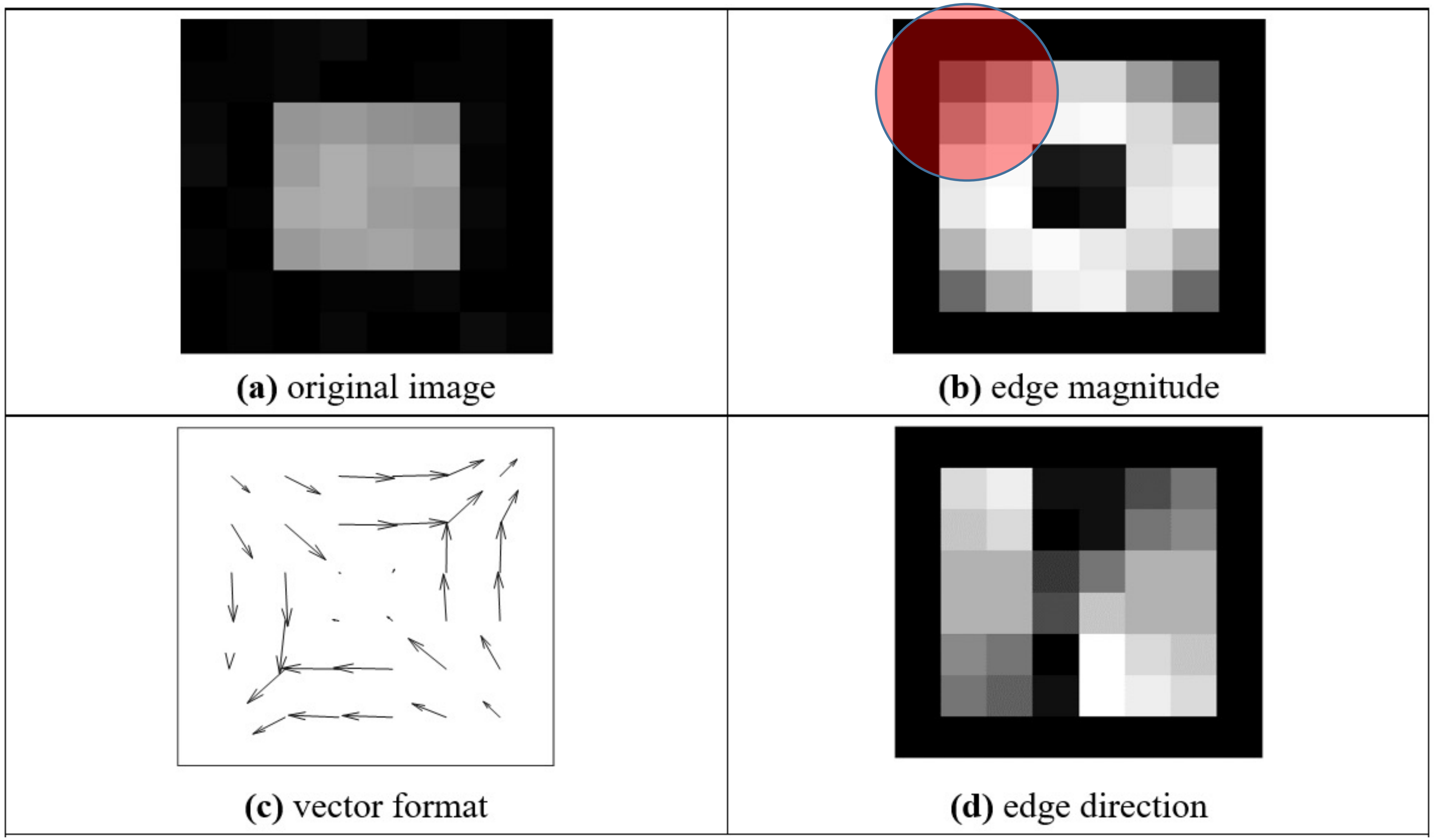
1	1	1
0	0	0
-1	-1	-1

(b) My

Edge magnitude and direction calculated for **centre** point

Applying the Prewitt Operator

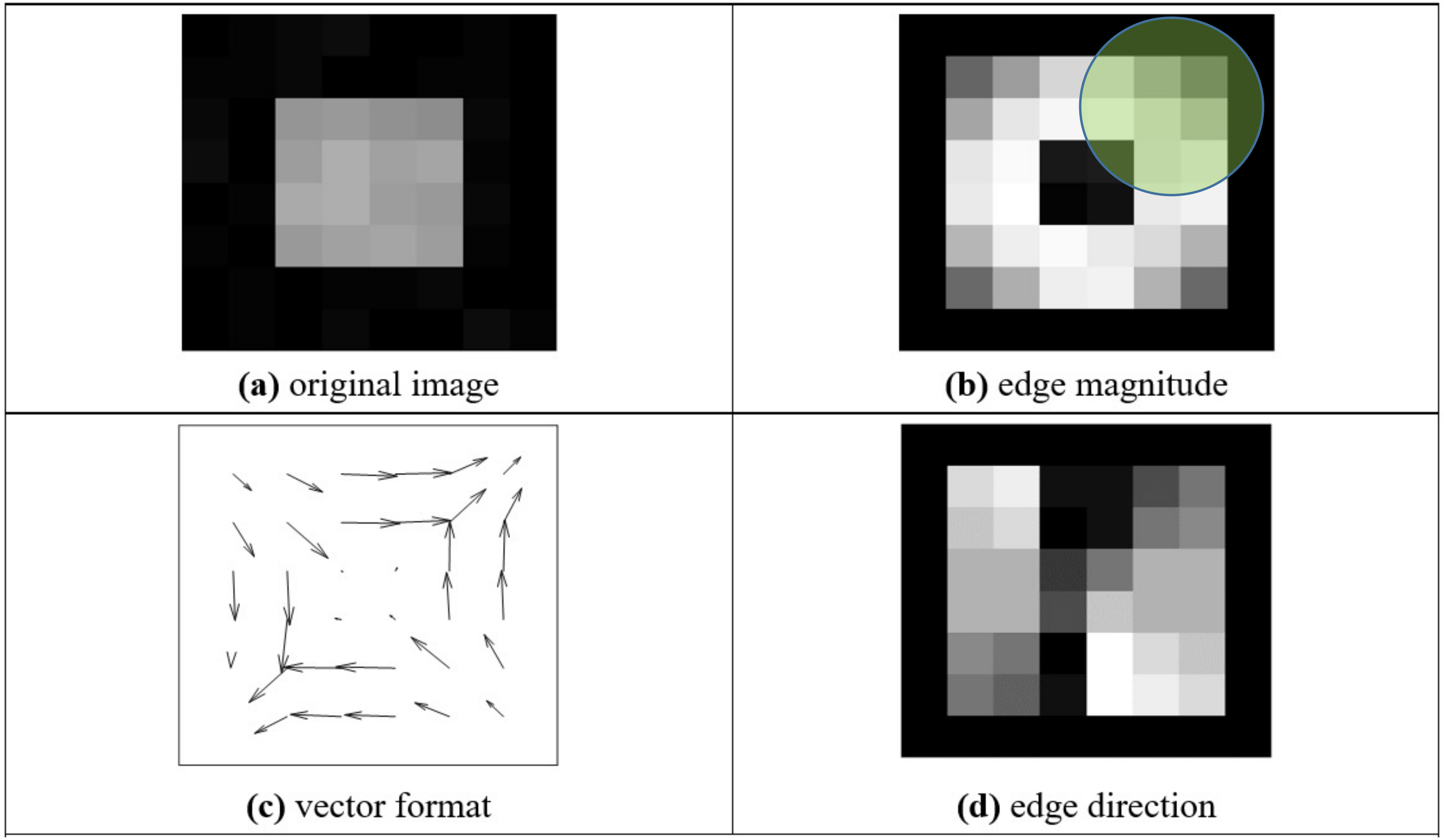
No missing points



(a) original image (b) edge magnitude (c) vector format (d) edge direction

Applying the Prewitt Operator

Blurred edges



(a) original image

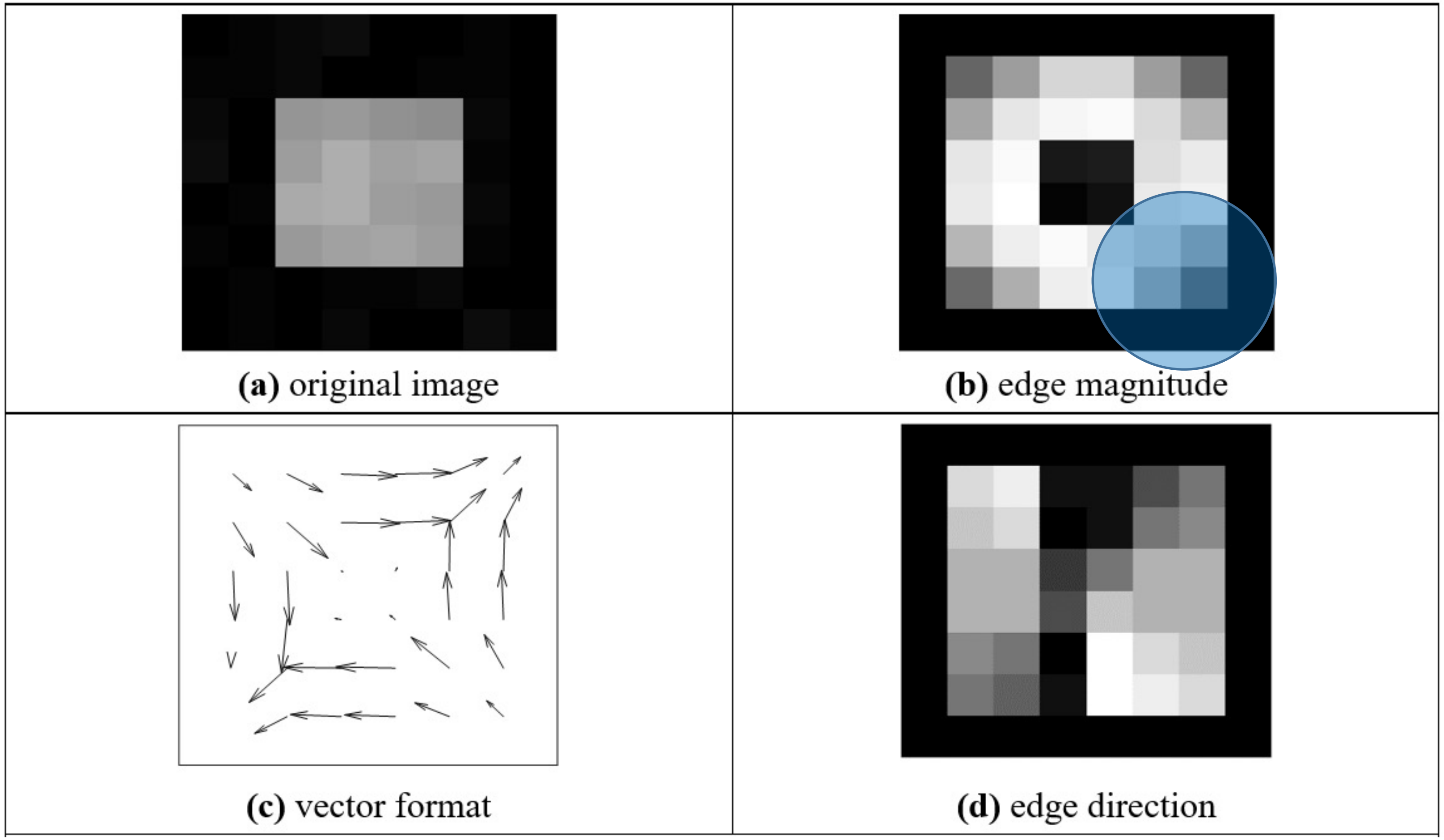
(b) edge magnitude

(c) vector format

(d) edge direction

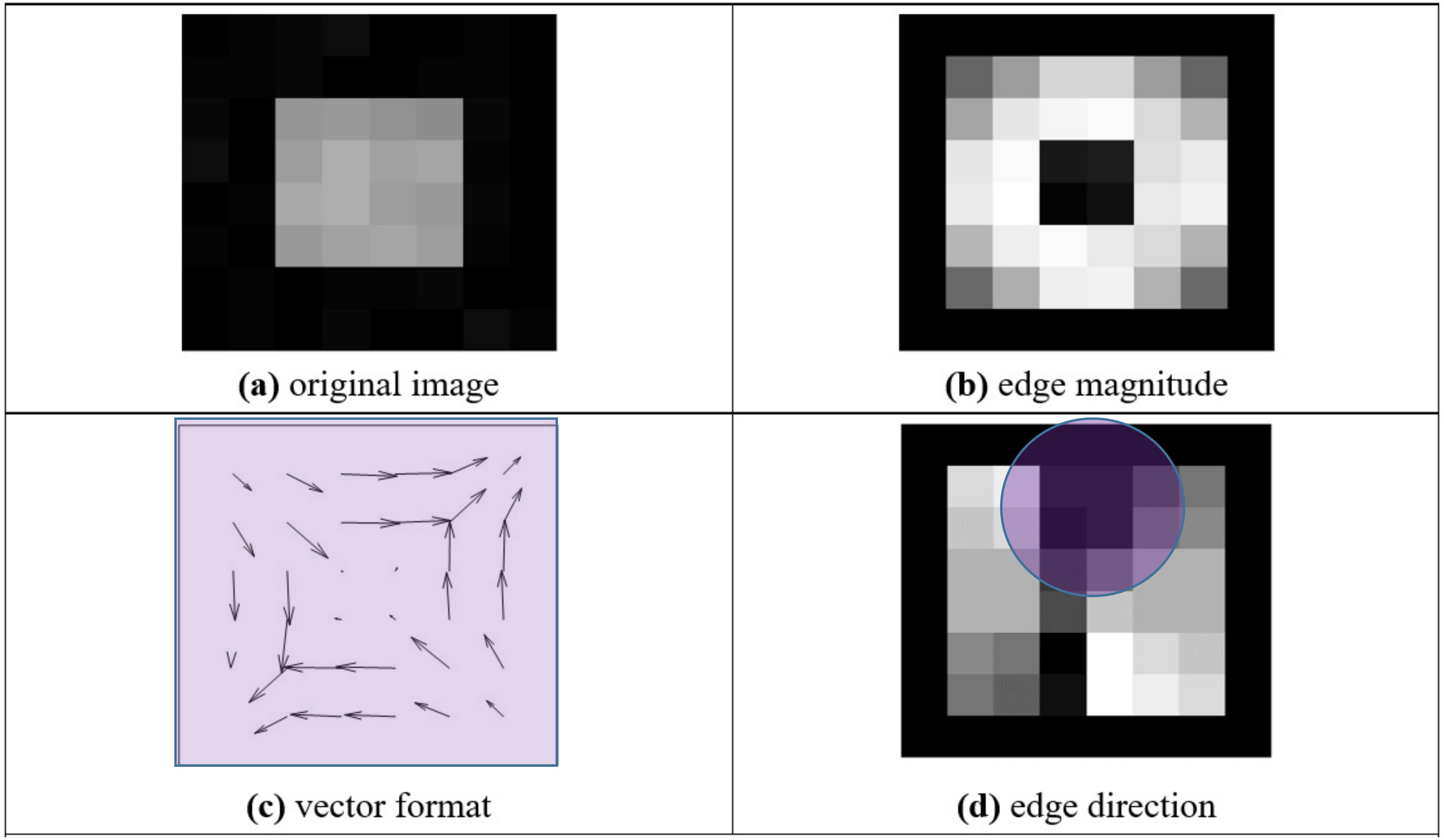
Applying the Prewitt Operator

No double points



Applying the Prewitt Operator

So use vectors



(a) original image (b) edge magnitude (c) vector format (d) edge direction

Displaying directions as an image communicates nothing

Templates for Sobel operator

Sobel is **most popular basic operator**

Double the centre coefficients of Prewitt

1	0	-1
2	0	-2
1	0	-1

(a) Mx

1	2	1
0	0	0
-1	-2	-1

(b) My

WHY?

Applying Sobel operator

(a) original image (b) Sobel edge magnitude (c) thresholded magnitude

Generalising Sobel - use Pascal’s triangle

1. Averaging

Window size										
2				1		1				
3			1		2		1			Sobel 3×3
4		1		3		3		1		
5	1		4		6		4		1	Sobel 5×5

2. Differencing

Window size										
2				1		-1				
3			1		0		-1			Sobel 3×3
4		1		1		-1		-1		
5	1		2		0		-2		-1	Sobel 5×5

Generalised Sobel

$$\begin{pmatrix} 1 \\ 4 \\ 6 \\ 4 \\ 1 \end{pmatrix} \times \begin{pmatrix} 1 & 2 & 0 & -2 & -1 \end{pmatrix}$$

Generated by: `averaging`^{T} `*(differencing)`

```
>> s=Sobel_templates(5)

s(:,:,1) =

     1     2     0    -2    -1
     4     8     0    -8    -4
     6    12     0   -12    -6
     4     8     0    -8    -4
     1     2     0    -2    -1
```

Main points so far

1. Differencing detects contrast and thus edges
2. Can improve the differencing process (by maths!!)
3. Sobel is a good general purpose operator

We shall go to more sophisticated methods,

coming up next…

Filters for edge detection