

# Lecture 9 Finding More Shapes

COMP6223 Computer Vision (MSc)

**How can we go from conic sections to general shapes?**



**Book**  
**pp**  
**215-246**

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Electronics and  
Computer Science**

**UNIVERSITY OF  
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School of Electronics  
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# Content

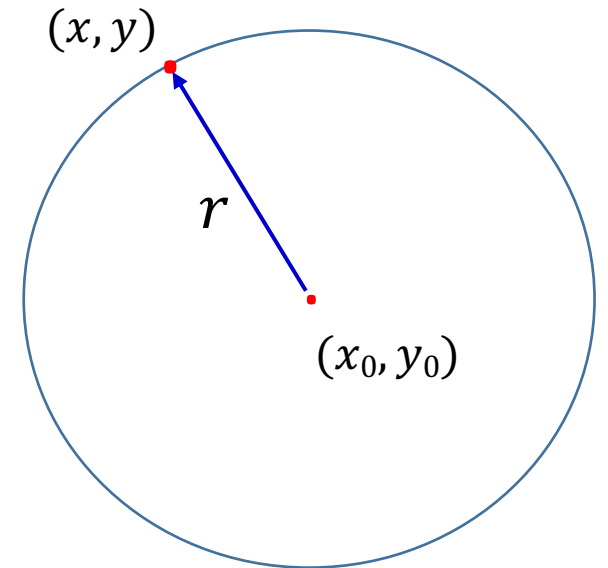
1. What more versions of the Hough transform are possible?
2. What are its limits?
3. Can it be used to detect shapes that are not given by an equation?

# Hough Transform for Circles

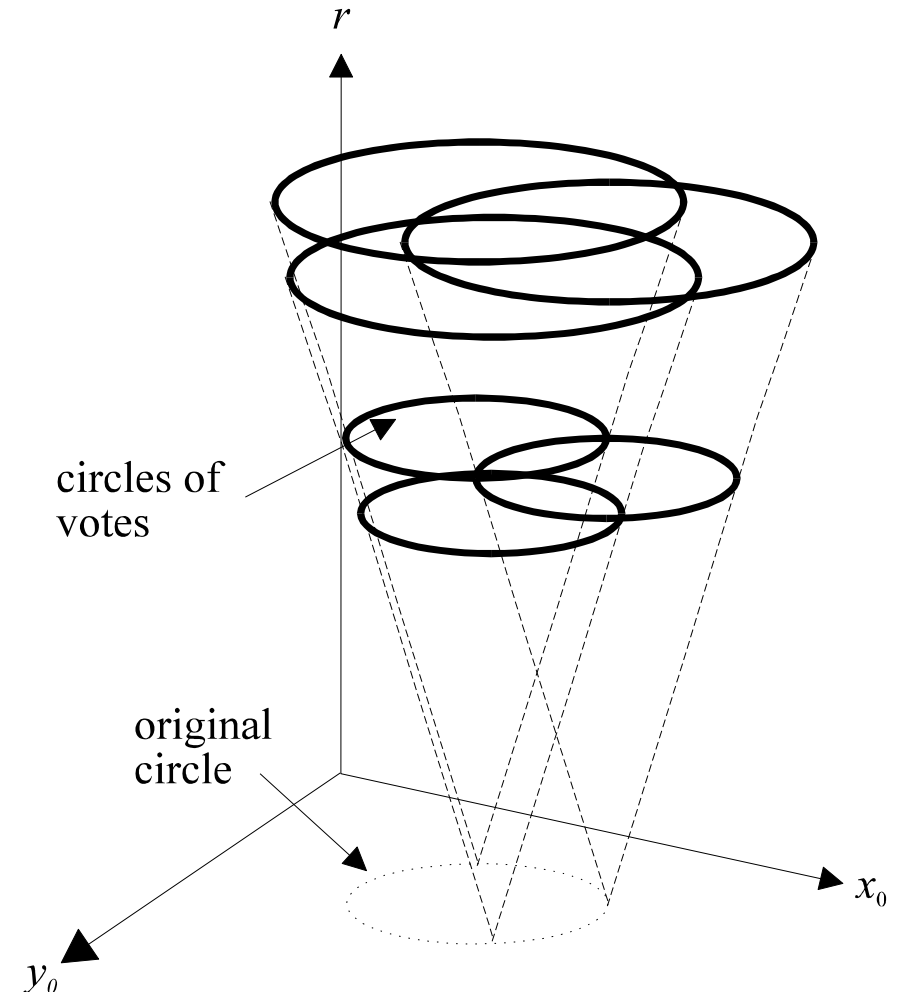
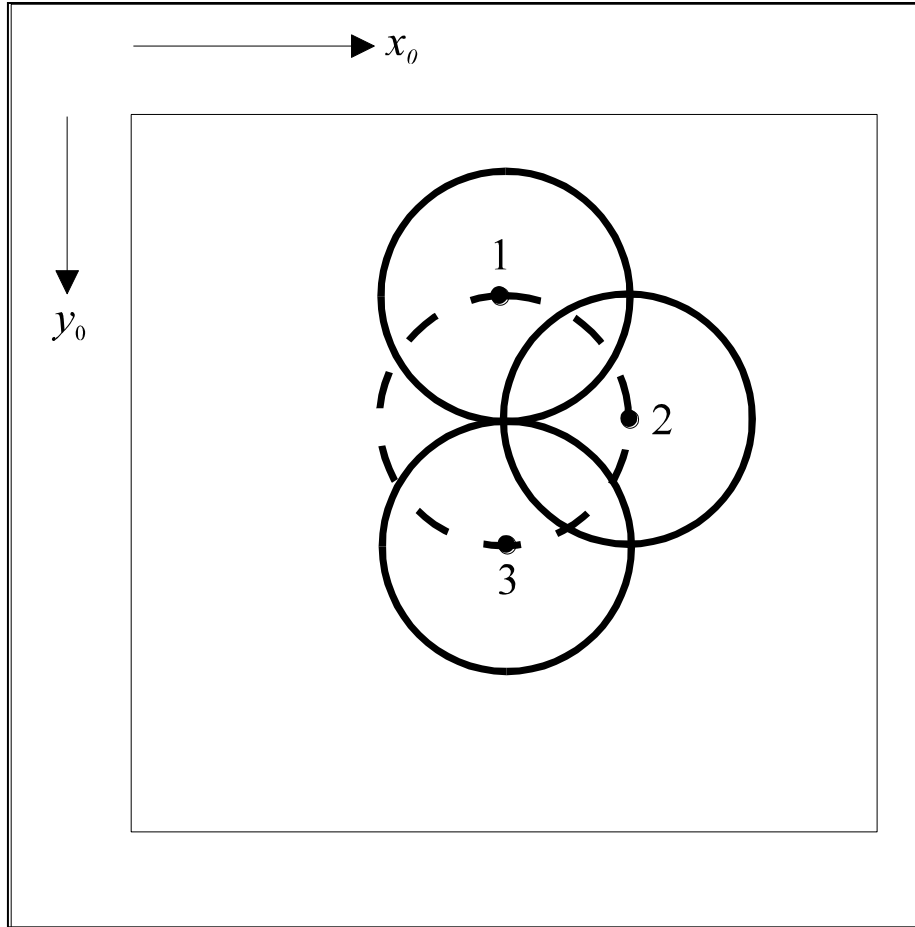
Again, it's **duality**:  $(x - x_0)^2 + (y - y_0)^2 = r^2$

Points:  $x, y$  centre:  $x_0, y_0$  radius:  $r$

“  $x_0, y_0$  “  $x, y$  “  $r$



# Circle Voting and Accumulator Space



# Pseudocode

```
accum=0
```

```
for all x,y
```

```
    if edge(y,x)>threshold
```

```
        for r = min_r, max_r
```

```
            for theta = 0, 2*pi
```

```
                x0=x+r*cos(theta)
```

```
                y0=y+r*sin(theta)
```

```
                accum(y0, x0, r ) PLUS 1
```

```
y0, x0, r= argmax(accum)
```

!look at all points

!check significance

!do values of radius

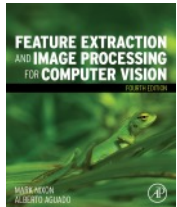
!go around a circle

!generate x

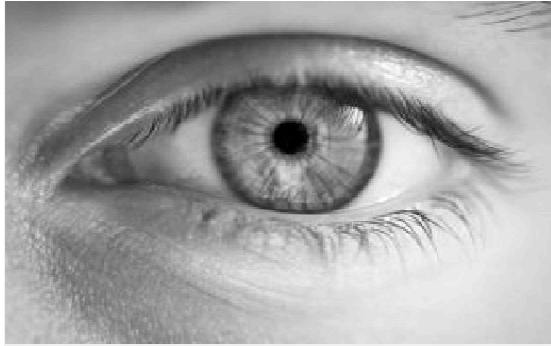
!generate y

!vote in accumulator

!peak gives parameters



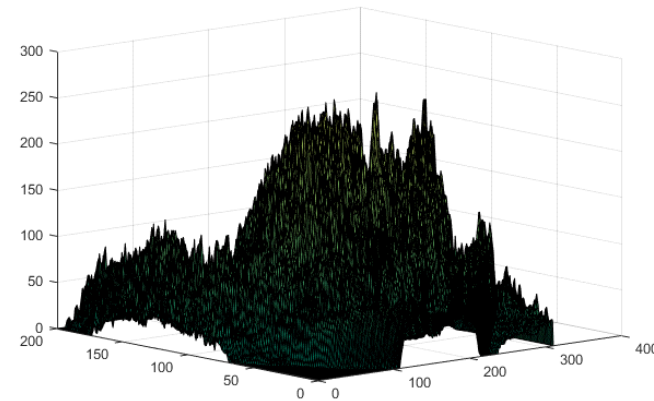
# Applying the HT for circles



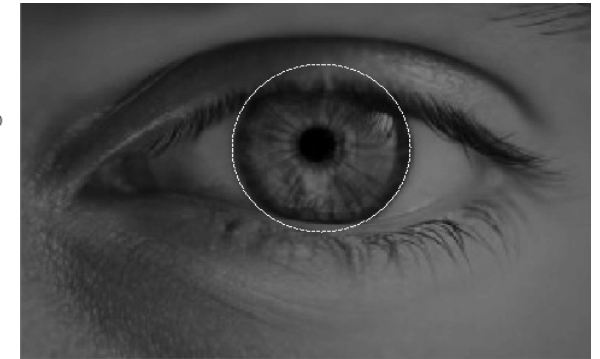
image



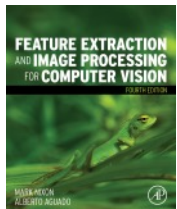
(Sobel) edges



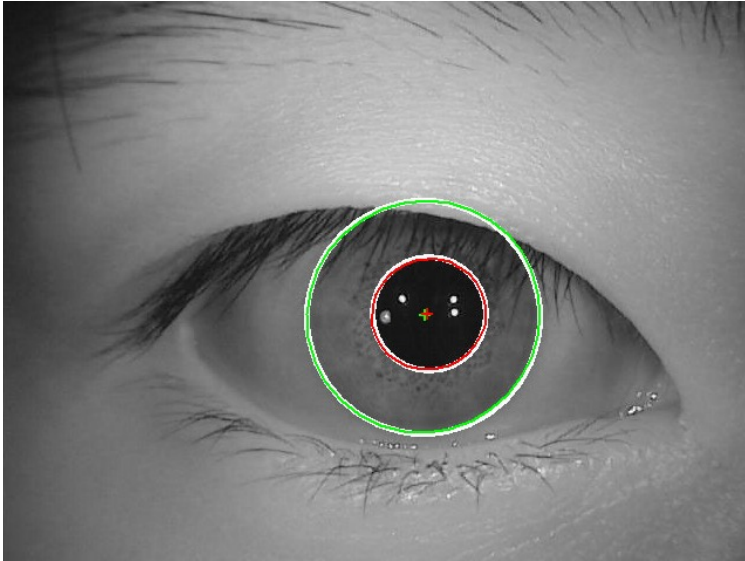
accumulator



small and large circles



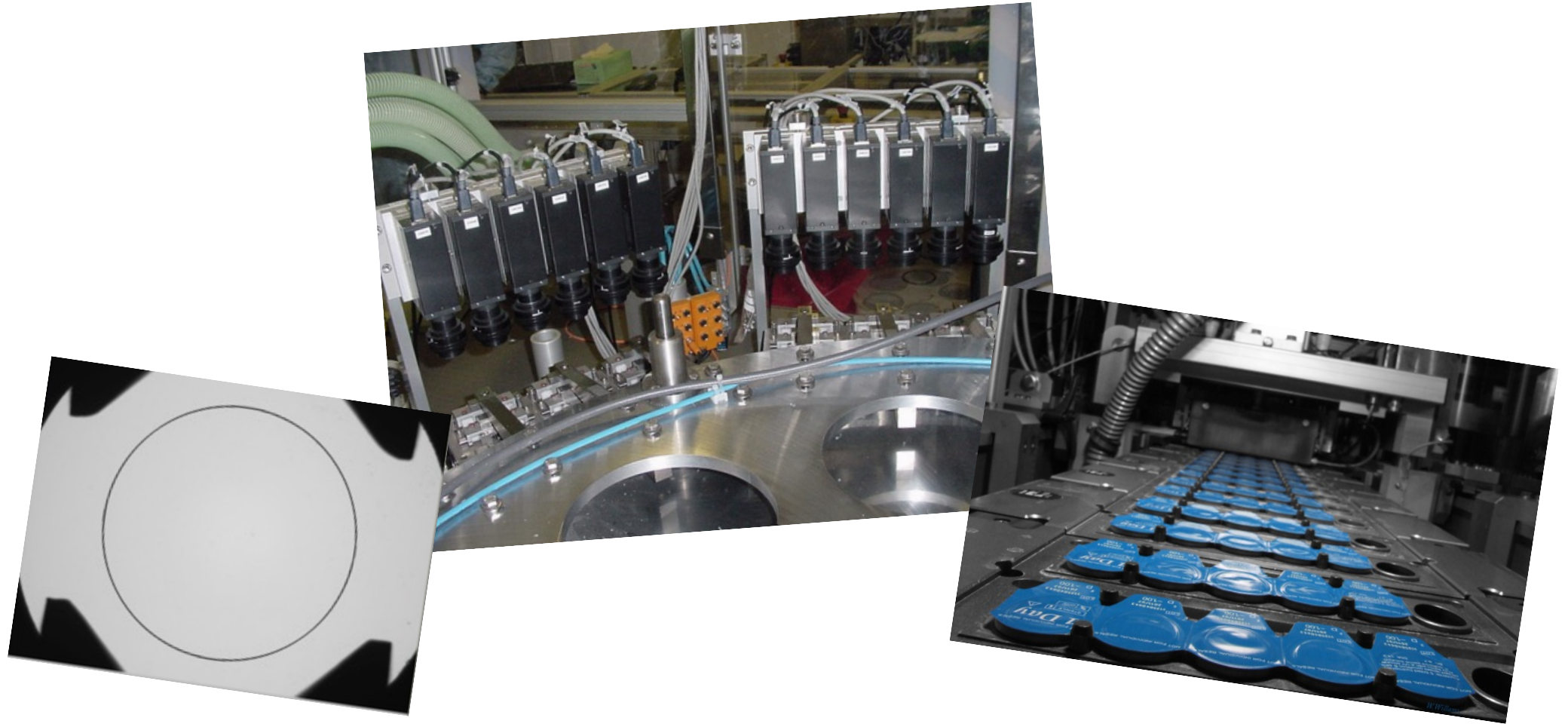
# Integrodifferential operator?



<https://stackoverflow.com/questions/27058057/comparing-irises-images-with-opencv>



# Contact lenses





# Extensions to conic sections

Ellipse

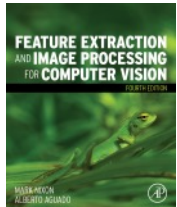
$$\frac{(x - x_0)^2}{a^2} + \frac{(y - y_0)^2}{b^2} = 1$$

described by 4 parameters (i.e.,  $x_0$ ,  $y_0$ ,  $a$ ,  $b$ ).

If each has 100 values, accumulator size:

$$10^2 \times 10^2 \times 10^2 \times 10^2 = 10^8 = 0.1\text{GB}$$

Add rotation, that's 10GB .... Ouch!



Motivates approaches to save memory and improve speed (since result is optimal)

# Speeding it up.....

Use the circular shape as an example.

It's a **3D** accumulator

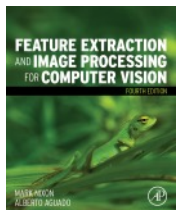
Differentiating  $(x - x_0)^2 + (y - y_0)^2 = r^2 \rightarrow \frac{dy}{dx} = -\frac{(x - x_0)}{(y - y_0)}$

$\left(\frac{dy}{dx}\right)^2 (y - y_0)^2 + (y - y_0)^2 = r^2$

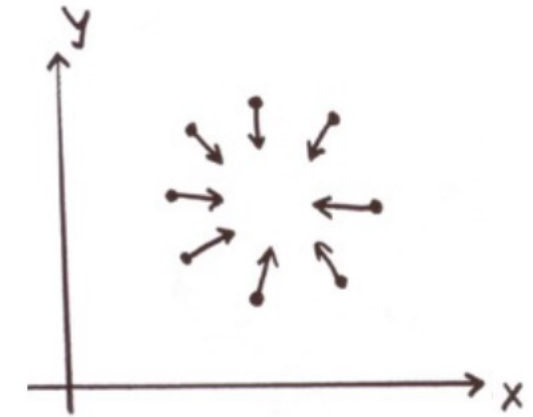
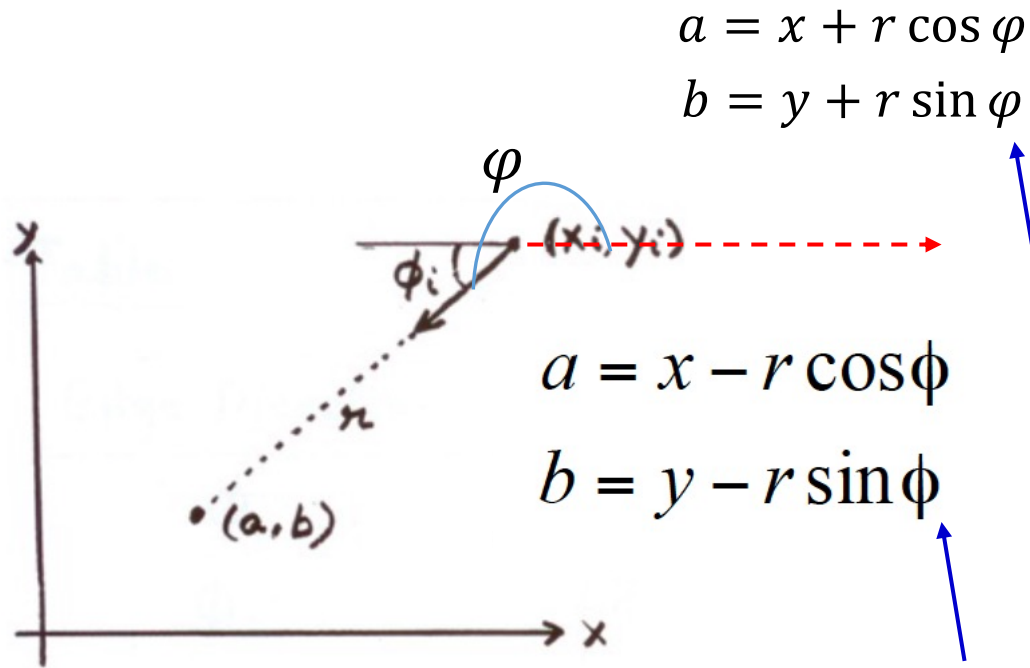
$$y - y_0 = \frac{r}{\sqrt{1 + \left(\frac{dy}{dx}\right)^2}}$$

**2D accumulator**

This is the **edge direction**

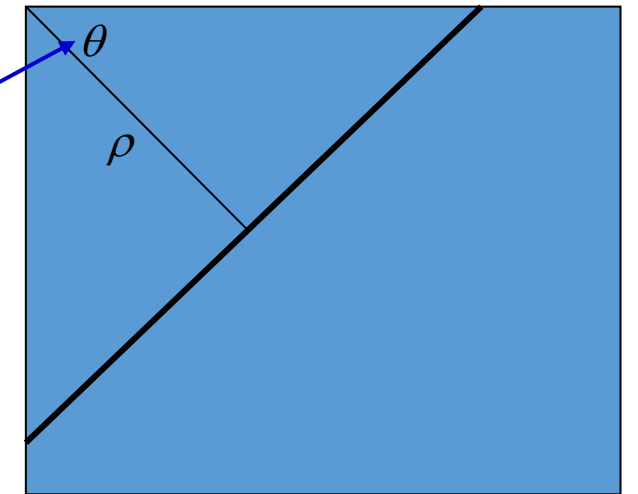


# Fireside



Circle

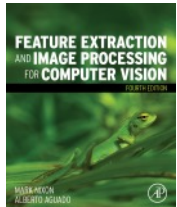
Edge direction can be used to compute  $\phi$ ,  $\varphi$  and  $\theta$  to speed up the Hough transform speed.



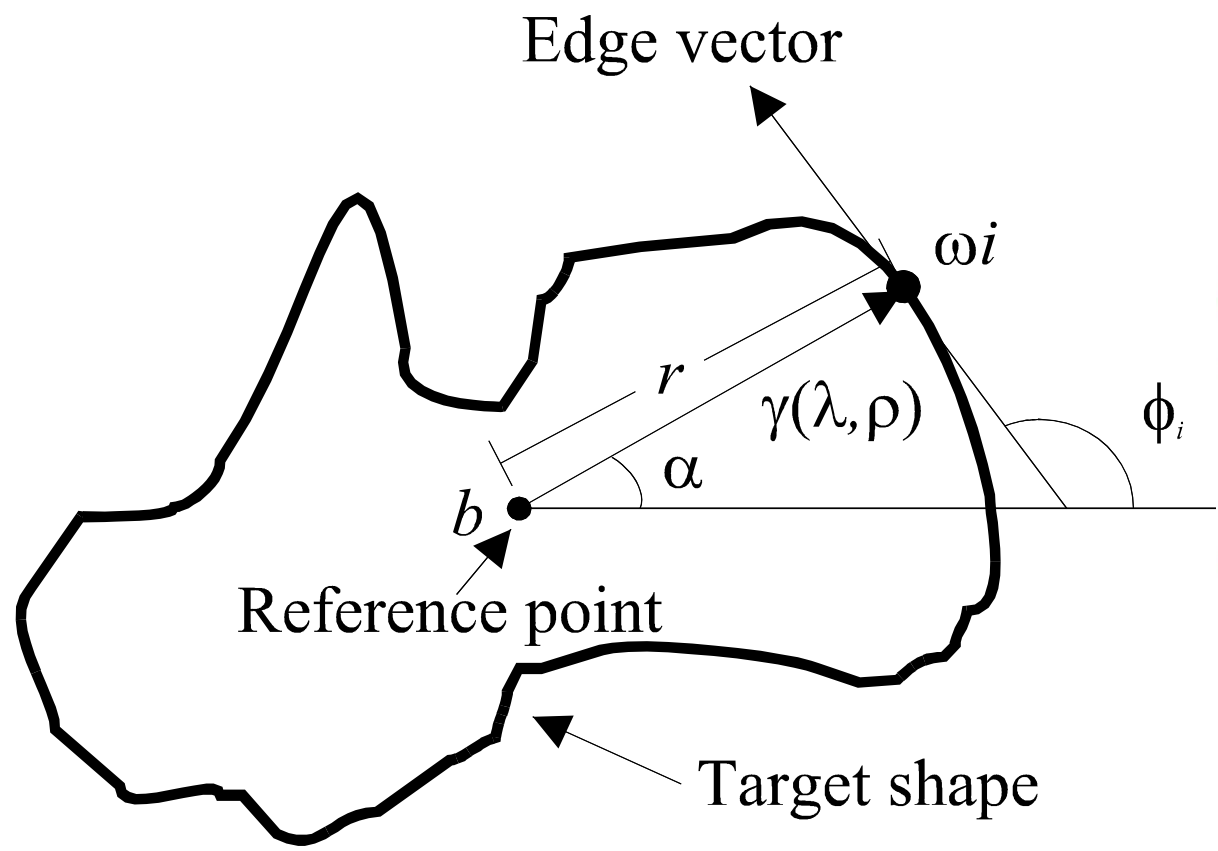
Line  $\rho = x \cos \theta + y \sin \theta$

# Arbitrary Shapes

- Use Generalised Hough transform
- Form (discrete) look-up-table (R-table)
- Vote via look-up-table



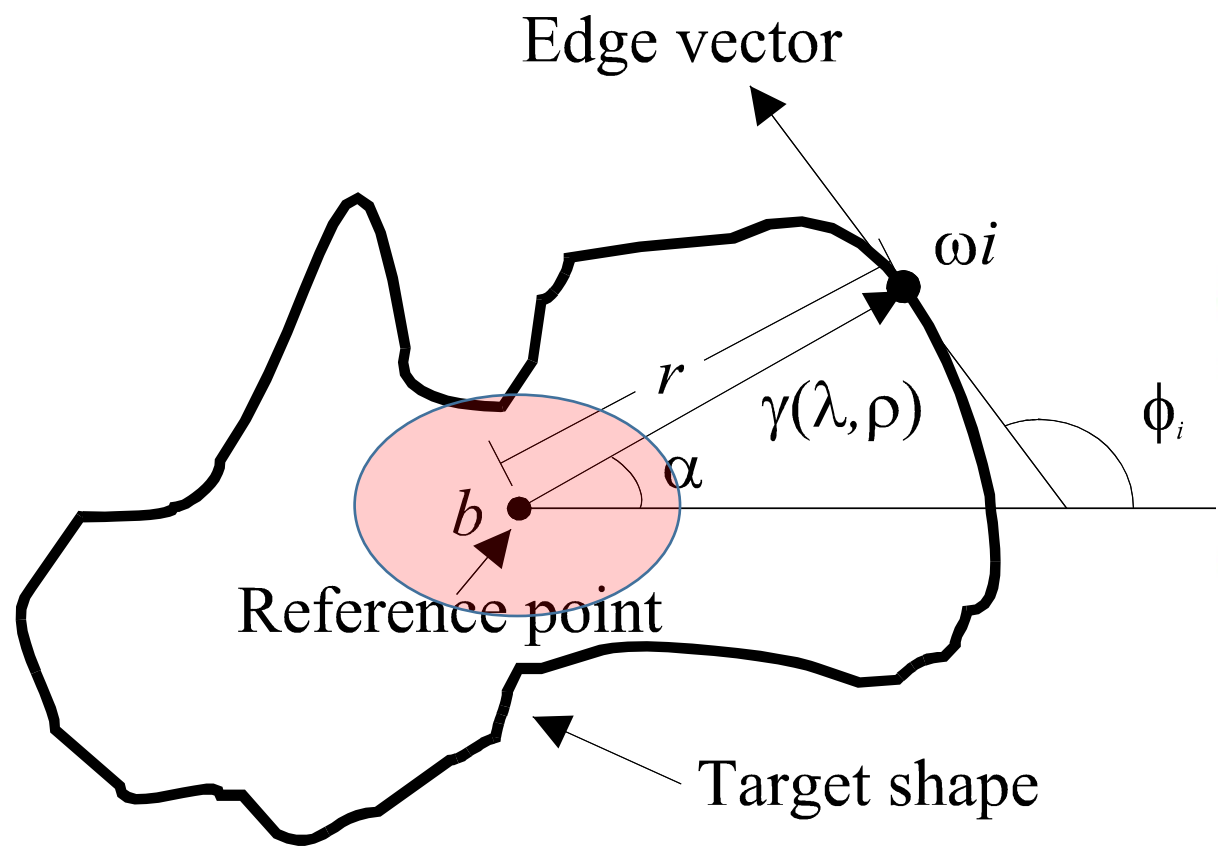
# R-table Construction



$\hat{\phi}_i'$	$\gamma = (r, \alpha)$
0	$(r_0, \alpha_0), (r_1, \alpha_1), (r_2, \alpha_2)$
$\Delta\phi$	$\vdots$
$2\Delta\phi$	$\vdots$
...	...

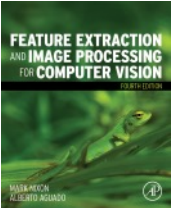


# R-table Construction

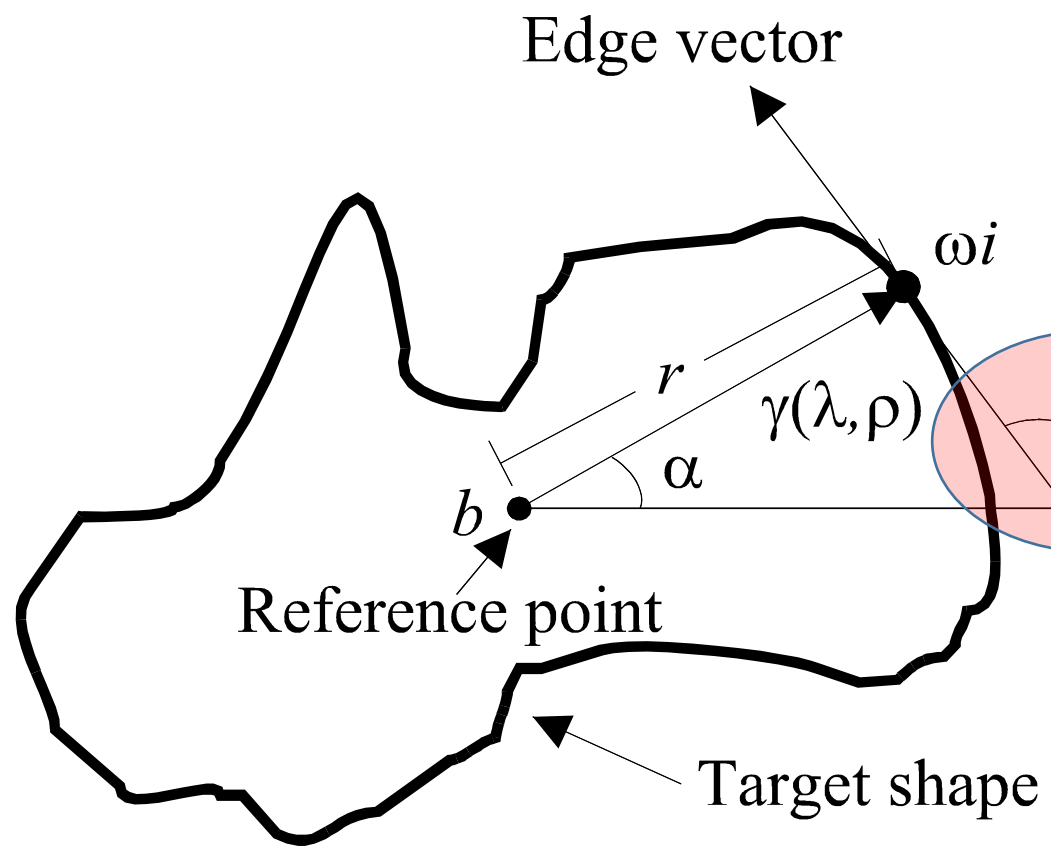


$\hat{\phi}_i$	$\gamma = (r, \alpha)$
0	$(r_0, \alpha_0), (r_1, \alpha_1), (r_2, \alpha_2)$
$\Delta\phi$	$\vdots$
$2\Delta\phi$	$\vdots$
...	...

Need to start somewhere



# R-table Construction



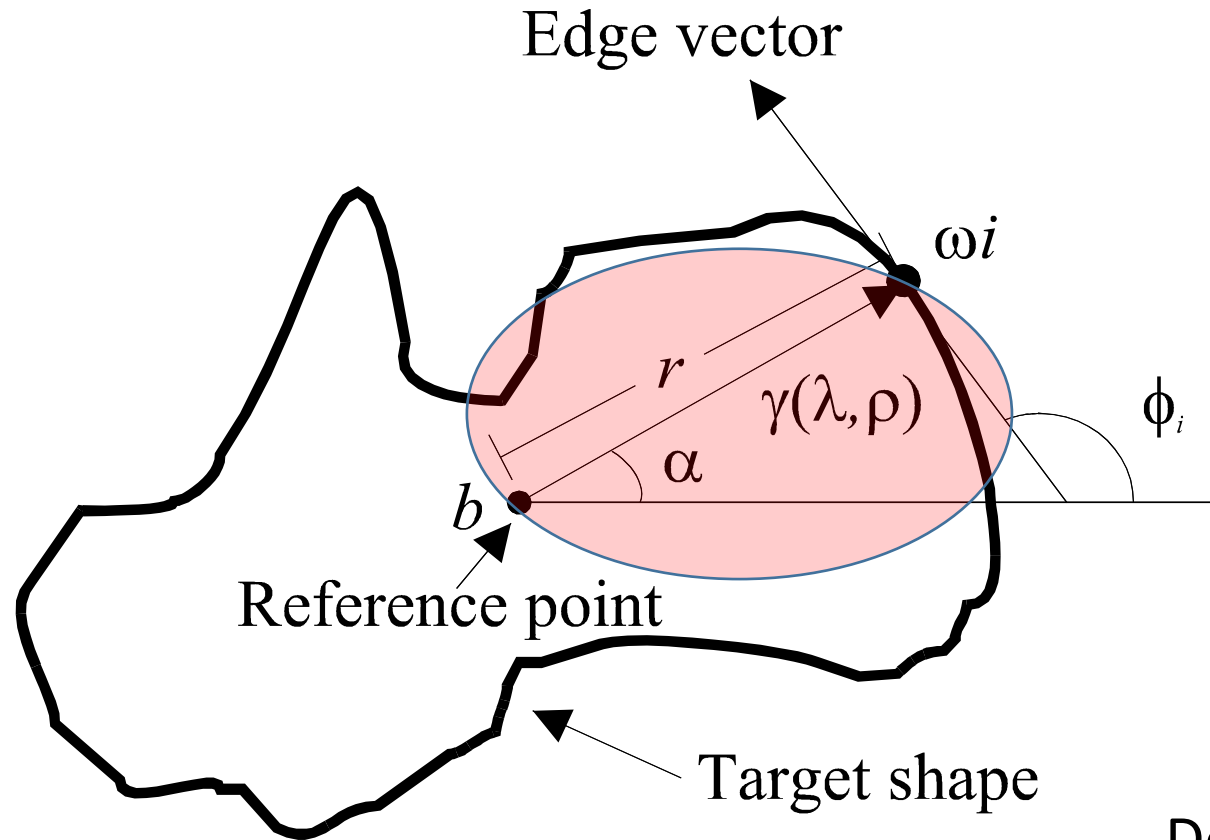
$\hat{\phi}_i'$	$\gamma = (r, \alpha)$
0	$(r_0, \alpha_0), (r_1, \alpha_1), (r_2, \alpha_2)$
$\Delta\phi$	$\vdots$
$2\Delta\phi$	$\vdots$
...	...

Measure **edge direction**





# R-table Construction

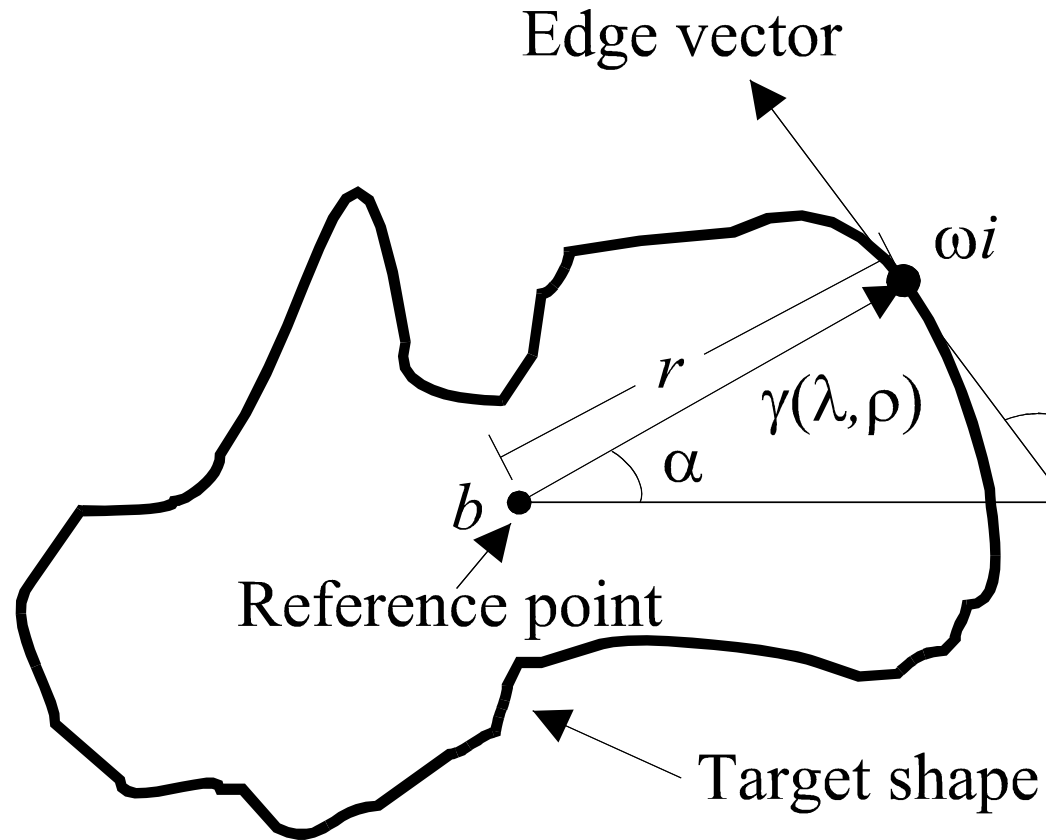


$\hat{\phi}_i$	$\gamma = (r, \alpha)$
0	$(r_0, \alpha_0), (r_1, \alpha_1), (r_2, \alpha_2)$
$\Delta\phi$	$\vdots$
$2\Delta\phi$	$\vdots$
...	...

Determine **length** and **direction** to reference point



# R-table Construction



$\hat{\phi}_i$	$\gamma = (r, \alpha)$
0	$(r_0, \alpha_0), (r_1, \alpha_1), (r_2, \alpha_2)$
$\Delta\phi$	$\vdots$
$2\Delta\phi$	$\vdots$
...	...

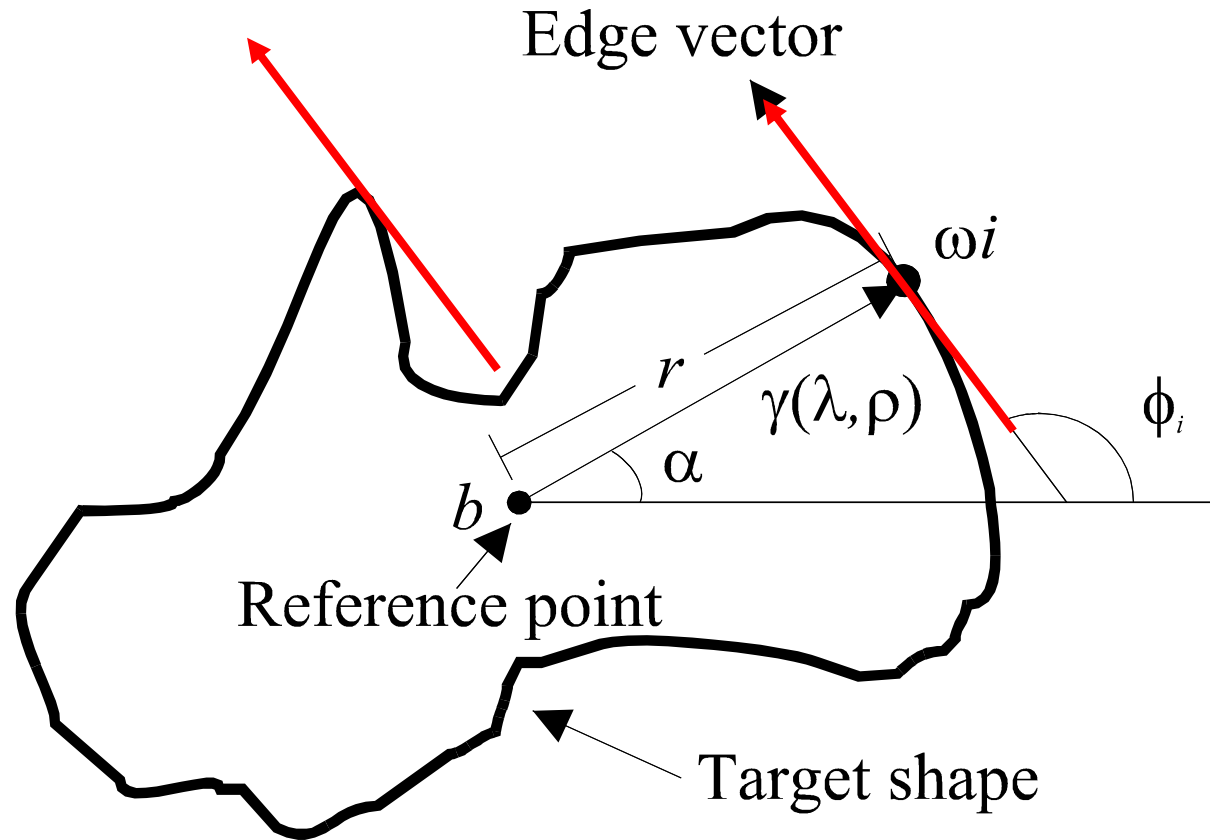
Store **length** and **direction** indexed  
by **edge direction**



# R-table Construction

$$x_c = x_i - r \cos(\alpha)$$

$$y_c = y_i - r \sin(\alpha)$$



$\hat{\phi}_i'$	$\gamma = (r, \alpha)$
0	$(r_0, \alpha_0), (r_1, \alpha_1), (r_2, \alpha_2)$
$\Delta\phi$	$\vdots$
$2\Delta\phi$	$\vdots$
...	...

Edge direction is **not** a unique  
description  
Gives **noise** in accumulator



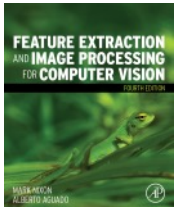
# Procedure for GHT

## Preparation

1. Determine centre of template shape
2. Form R-table from template shape

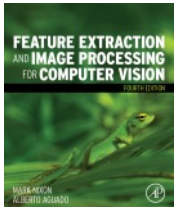
## Application

1. Use R-table to vote for points in the real image  
For edge points  $>$  threshold  
Get edge direction(x,y)  
For all R-table entries with direction(x,y)  
Vote in accumulator (@distance, @direction)
2. Argmax(accumulator) gives centre co-ordinates of shape



# Arbitrary Shapes


- Use Generalised Hough transform
- Form (discrete) look-up-table (R-table)
- Vote via look-up-table
- Scale? scale R-table voting
- Orientation? Rotate R-table voting
- Inherent problems with discretisation




# R-table Construction

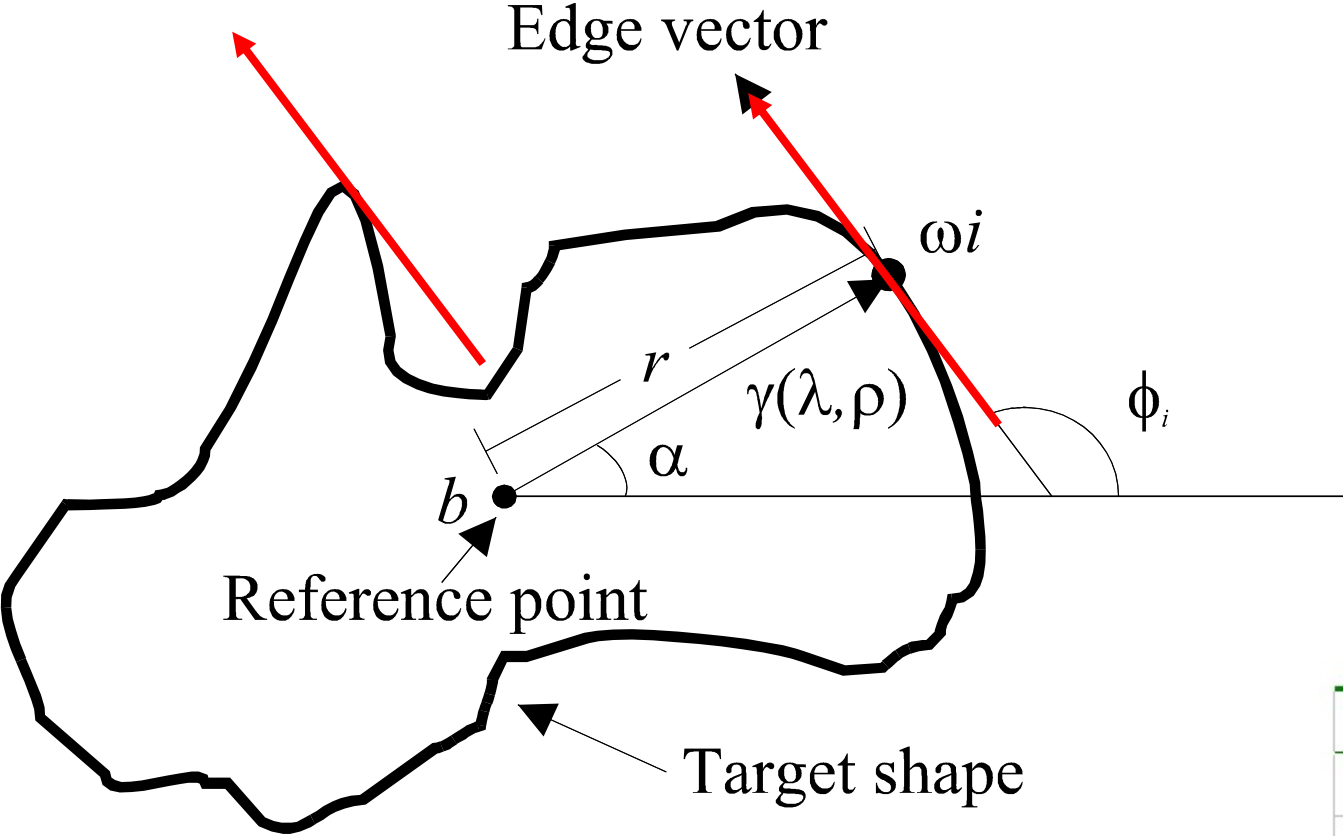
$$x_c = x_i - rS \cos(\alpha + \theta)$$

$$y_c = y_i - rS \sin(\alpha + \theta)$$

  
Scale

  
Orientation

**Important:** for the case of rotation, before checking with the R-table, the edge direction of the edge point needs to minus the rotation  $\theta$

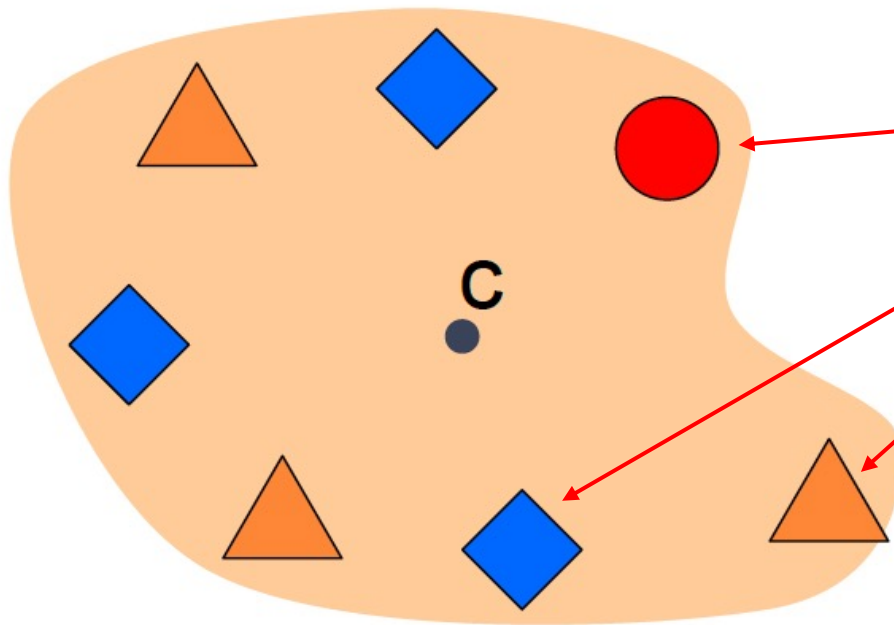


$\hat{\phi}'_i$	$\gamma = (r, \alpha)$
0	$(r_0, \alpha_0), (r_1, \alpha_1), (r_2, \alpha_2)$
$\Delta\phi$	$\vdots$
$2\Delta\phi$	$\vdots$
...	...



# Further example

Template

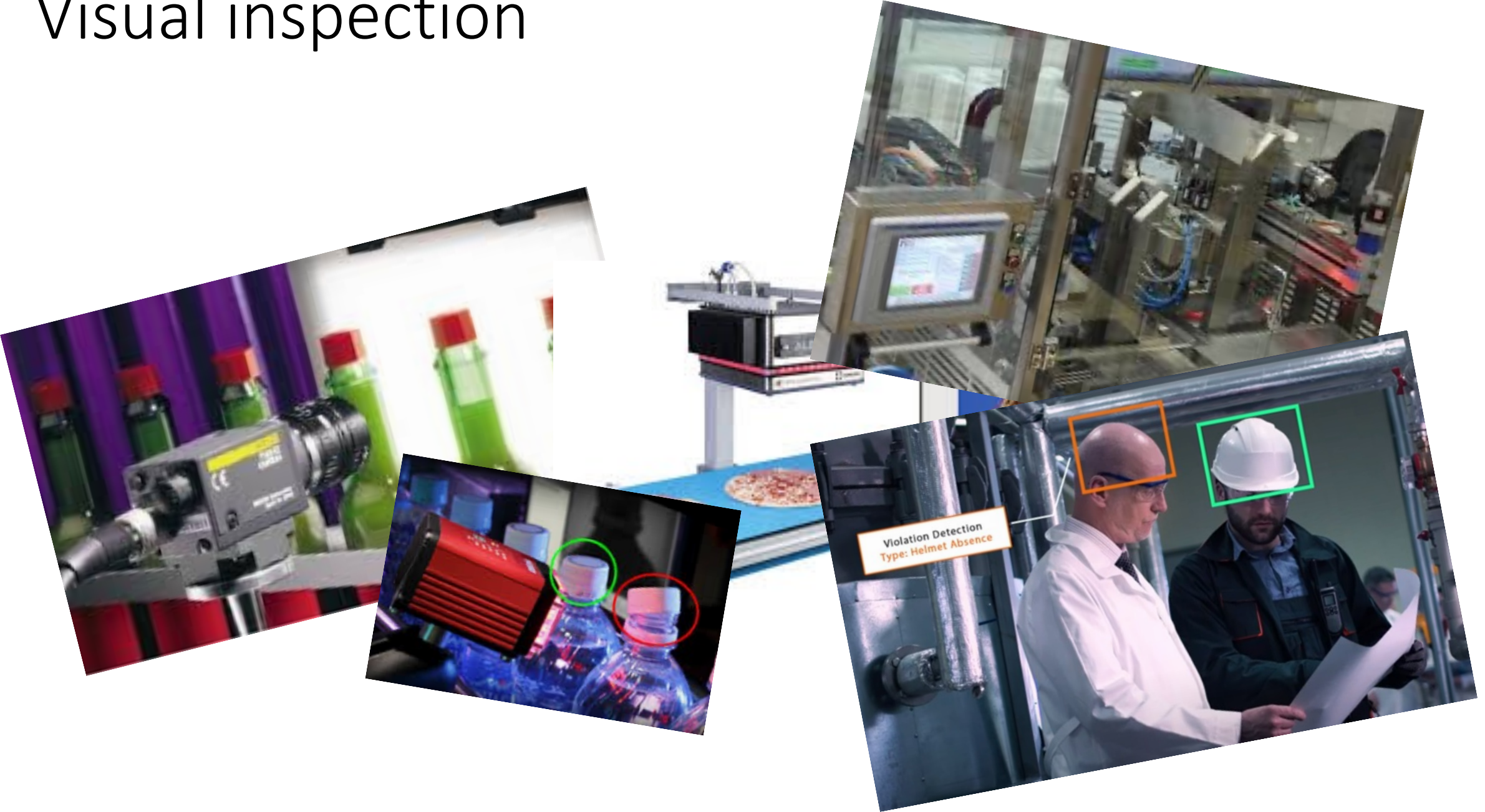


Firstly, generate an R-table for **objects**

Then, use the R-table to vote for points



# Visual inspection

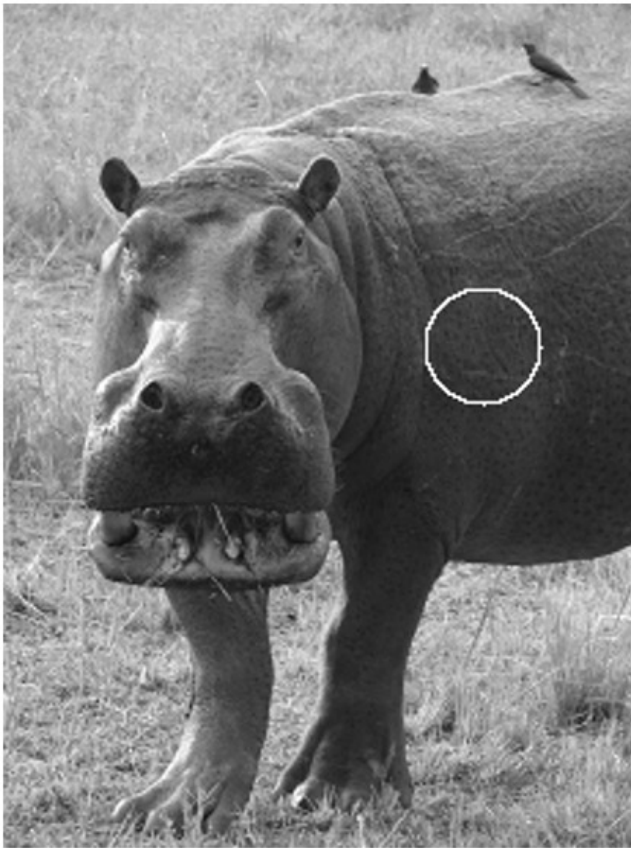


# Active Contours

- For **unknown** arbitrary shapes: extract by **evolution**
- Elastic band analogy
- Balloon analogy
- Discrete vs. continuous



# Geometric active contours



(a) initialisation



(b) result

**Extraction by a Level-Set Based Approach**

# Main points so far

1. **conic sections** become more complex and take more time
2. can use **Generalised Hough Transform** for complex shapes
3. **shape detection** IS computer vision. Many more approaches

Let's see how computer vision can work

