COMP6203 Intelligent Agents 2022/2023

Exercises on Auctions

Read me: On Envy-Freeness

In the negotiation lectures you have seen the definition of envy-freeness which says "no agent prefers the resources allocated to other agents".

The notion of envy-freeness is not restricted to negotiation settings. In general, envy-freeness means that no agent prefers to receive the outcome received by another agent. In auctions, this means that no agent prefers to receive the allocation and payment of another agent; i.e. they don't prefer to receive the allocation of another agent and pay their payments.

Think of it this way: in the negotiation setting, an agent receives (a share) of one or several resources. In a single-item auction setting, an agent does not just receive an item/resource, s/he also receives a price tag for it.

Exercise 1: Full information game

Consider the following independent private value auction setting for a single item. There are 2 agents, April (A) and Ben (B), wanting to buy the item being auctioned. Assume that the agents' valuations are at least 0 pounds and at most 5 pounds and everyone (both agents and the auctioneer) knows this; therefore agents can only place bids between 0 and 5 pounds. Furthermore, assume that agents can only place bids in full pounds (so this is a discrete bid setting).

Agent A values the item at $\theta_A = 2$ pounds, and B values the item at $\theta_B = 4$ pounds. Assume a complete information setting (i.e. the agents know each others' valuations). Assume that the second-price sealed-bid auction (Vickrey) is used to allocate the item, and that ties are broken in lexicographic order.

1. Compute the utility of the agents when they both bid truthfully.

2. Is the resulting outcome (from truthful bidding) envy-free? Explain.

3. Identify all pure Nash equilibria of the strategic-form game induced by this auction.

4. Identify all pure strategy profiles that result in an envy-free outcome.

5. Does the set of pure Nash equilibria change if a different (deterministic) tie-breaking rule is used?

6. Does the set of envy-free outcomes change if a different (deterministic) tie-breaking rule is used?

7. What can you say about the set of all pure Nash equilibria of this game if the agents' valuations are between 0 and 50 (i.e., can you describe and characterise this set succinctly without identifying each member one by one)?

Exercise 2: Optimal bids

Consider the following independent private value auction setting for a single item. There are n agents wanting to buy the item being auctioned. Assume that agents can only place bid in full pounds (e.g. they can bid 15 pounds bot not 25.5 pounds). Assume that agent i values the item at 7 pounds and that she has beliefs about the bid distribution. Let $F(b)$ denote the probability that all other bids are less than b pounds, and suppose that agent i has the following information:

$$F(2) = 0, F(3) = 0.1, F(4) = 0.25, F(5) = 0.35, F(6) = 0.40, F(7) = 0.65,$$

$$F(8) = 0.85, F(9) = 0.95, F(10) = 1$$

8. Suppose that we are running a first-price sealed-bid auction and that ties are broken in favour of the opponents of i. Compute all optimal bids for bidder i and her expected utility if she places an optimal bid.

9. Suppose that we are running a second-price sealed-bid (a.k.a. Vickrey) and that ties are broken in favour of the opponents of i. Compute all optimal bids for bidder i and her expected utility if she places an optimal bid..