1. (a) Bidder 1 chooses A

Bidder 2 chooses B

Bidder 3 chooses ϕ

The Maxmium social welfare is $10+8=18$

Bidder 1 payment $14-8=6$

Bidder 2 payment $15-10=5$

Bidder 3 payment $18-18=0$

Explain: Firstly, The Maxmium Social welfare is 18, It doesn't matter which bidder is chosen, The sum of bidder 1 and bidder 2 is less than 18. And other combinations also less than 18.
When bidder 1 doesn't have option A, The maximum combination chosen from the remaining choices is $\{\phi, \phi, AB\}$ The Maxmium Social welfare is 14
When bidder 1 chosen A, The Maxmium Social welfare combination is $\{A, B, \phi\}$ and Social welfare is 18, Now $\sum_{j\neq i} \hat{v}_j(x(\hat{v})) = 8$
Bidder 3 did not affect combinations with the Maxmium Social welfare.

(b). Because each bidder only cares about what good s/he receives. But bidder3 have no choice, so he receive 0.

(c). When bidder 1 didn't participate the game,

Maxmium Social welfare is that bidder 2 chosen ϕ, bidder 3 chosen A and B

Social welfare is 14.

Bidder 2 payment $= 14-14=0$

Bidder 3 payment $= 12-0=12$

(d)

Adding an extra bidder will not decrease the revenue of the Vickery (single-item) auction.

Explain: For the Vickery auction (single-item)

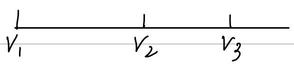
The winner need to pay the second valuation.

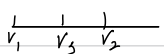
For Vickery auction,

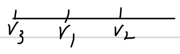
$X(\hat{v}) = \arg\max_{x} \sum_i \hat{v}_i(x)$ Because of single item, so the maximium Social welfare is that highest valuation adding zeros (Because other agent got nothing).

$P_i(\hat{v}) = \sum_{j\neq i} \hat{v}_j(x(\hat{v}_{-i})) - \sum_{j\neq i} \hat{v}_j(x(\hat{v}))$ Means the second highest valuation minus zero.

There are three situations. when adding a new bidder 3 and his valuation is V_3

situation 1. $V_3 < V_2$ Auction will still chooses bidder 1 and pay V_2. Ther revenue will not decrease. 

situation 2. $V_1 > V_3 > V_2$ Because $V_3 > V_2$, Bidder 1 need to pay V_3. The revenue will not decrease. 

situation 3. $V_3 > V_1 > V_2$ Auction will choose bidder 3 and bidder 3 need to pay V_1 Becanse $V_1 > V_2$, so the revenue will increase. 

2. For example:

Bidder 1 reports $V_1' = (0, 5, 6, 14)$

His true valuation $V_1 = (0, 3, 7, 13)$

Bidder 2 reports $V_2 = (0, 3, 9, 15)$

If bidder 1 gives his truthful valuations.

Social welfare: $V_2(AB) = 15$

$t_1 = 0$

$t_2 = 13-0=13$

$P_1 = t_1 - \frac{13}{2} = -6.5$

$P_2 = t_2 - \frac{13}{2} = 6.5$

Bidder 1 utility is $0-(-6.5) = 6.5$

If bidder 1 reports V_1'

Social welfare: $V_2(AB) = 15$

$t_1' = 0$

$t_2' = 14-0=14$

$P_1' = t_1' - \frac{14}{2} = -7$

$P_2' = t_2' - \frac{14}{2} = 7$

Bidder 1 utility is $0-(-7) = 7$

So $7 > 6.5$, that means mechanism is not DS truthful without using or relying on the characterisation of DS truthful mechanisms or any impossibility results.

3.

(a) To compute the cost of each $\sqrt{v_i}$ is $20n$

To compute the cost of whole VCG

$x(\hat{v}) = \arg\max_x \sum_i \hat{v}_i(x)$ Cost: $n! + 2n(n-1)$

$P_i(\hat{v}) = \sum_{j \neq i} \hat{v}_j(x(\hat{v}_{-i})) - \sum_{j \neq i} \hat{v}_j(x(\hat{v}))$ Cost: $(n-1) + 2(n-1))! + [2(n-1)(n-2)] + 2 + n! + 2n(n-1)) + (n-1))$

Because we need to compute every agent payment.

So the final cost is $20n$ + Maxmium Social welfare + n payment.

$20n + n! + 2n(n-1) + n[(n-1) + 2(n-1))! + [2(n-1)(n-2)] + 2 + n! + 2n(n-1)) + (n-1))]$

(b) Agent need to report the valuations.

Action: compute the maxmium Social welfare and payments of every agent.

To choose the agent who has highest valuation and paies the second highest payment square of valuation.

Result: The agent who has highest valuation wins the item.

For reducing the caculation of cost. I set that let all winning agents bid as the square of the valuation.

The agents than will report the $\sqrt{v}$ instead of v.

Explain why dominant strategy equilibrium.

In this mechanism, the agent with the highest bid only needs to pay the second highest valuation.

Because it is in each bidder's best interest for each agent to bid for the item's true valuation.

Agents cannot profit by bidding above or below the true valuation,

If Agent submits a bid that is lower than their true valuation, they risk losing the auction to another bidder who values the item higher.

If they submit a bid that is higher than their true valuation, they risk winning the auction and paying more than the item is worth to them.

Total cost: Since the largest valuation has been selected, the second largest valuation is determined. In this case, the valuation of other agents is 0

we can exempt ourselves from caluating the cost of social welfare.

$n! + n[(n-1) + 2(n-1))! + 2 + n! + (n-1)]$

4.

truth-telling

l_1 t_2 t_1 t_3 t_1: l_1 l_2 l_3

l_2 t_1 t_2 t_3 t_2: l_2 l_1 l_3

l_3 t_1 t_2 t_3 t_3: l_1 l_3 l_2

As shown on the left, when t_1 tells truth, it will get match with l_1 and l_2 in the leader oriented and follower oriented. But when t_1 misreports, it will still chooses l_1. So t_1 will benefit from it. It will choose misreporting. At the same time, misreporting will make the matching by the leader do not exist.

So no stable matching mechanism exists for which truth-telling is a dominant strategy for every agent.

misreport.

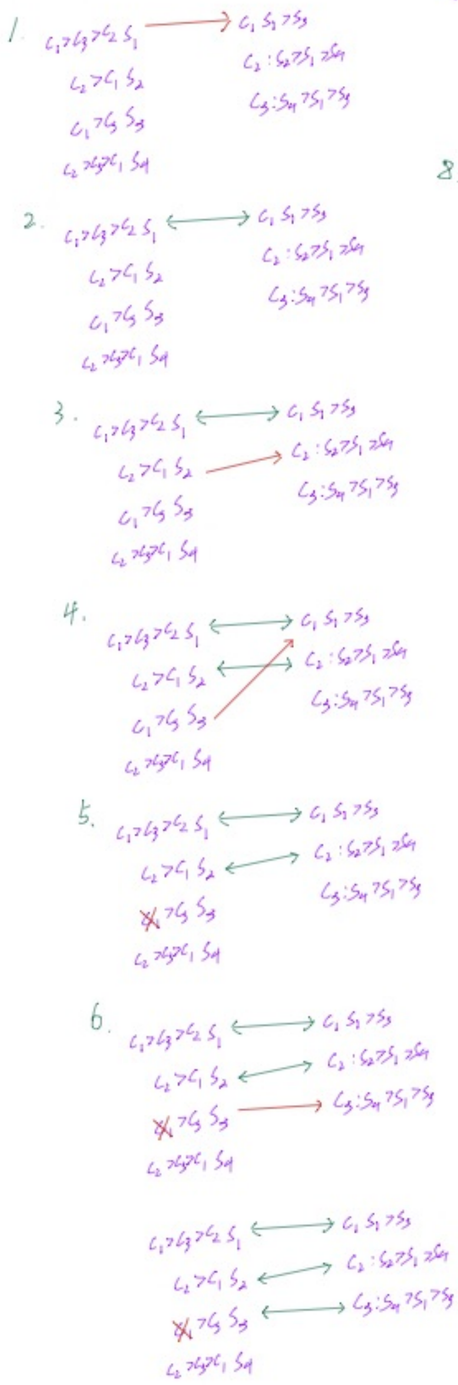
l_1 t_2 t_1 t_3 t_1 l_1 l_3 l_2

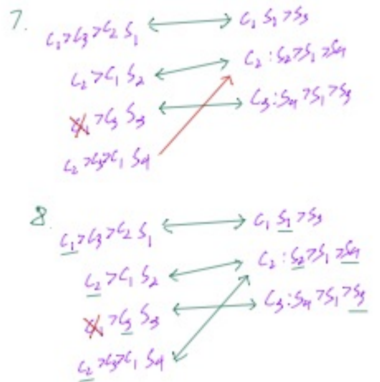
l_2 t_1 t_2 t_3 t_2 l_2 l_1 l_3

l_3 t_1 t_2 t_3 t_3 l_1 l_3 l_2

5.

(a)

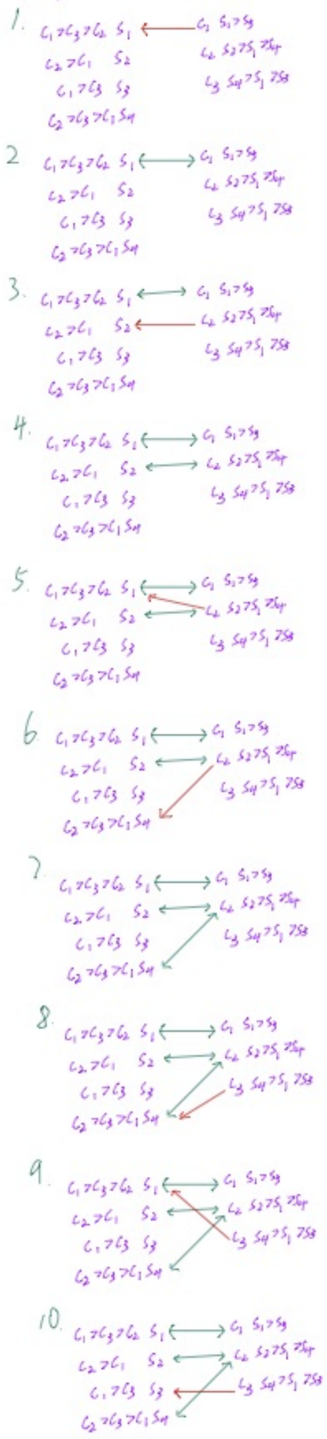


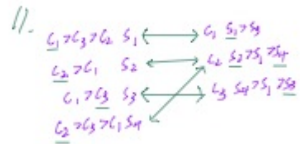


Final answer:

$(S_1, C_1)\ (S_2 C_2)\ (S_3 C_3)\ (S_4 C_2)$

(b)





Final answer:

$(S_1 C_1)\ (S_2 C_2)\ (S_3 C_3)\ (S_4 C_2)$

6.

(a)

$$A : \overline{B} > E > \underline{D} > \cancel{F} > \cancel{G} > \cancel{H} > \cancel{C}$$

$$B : \overline{C} > F > \underline{A} > \cancel{G} > \cancel{H} > \cancel{E} > \cancel{D}$$

$$C : \overline{D} > G > \underline{B} > \cancel{H} > \cancel{E} > \cancel{F} > \cancel{A}$$

$$D : \overline{A} > H > \underline{C} > \cancel{E} > \cancel{F} > \cancel{G} > \cancel{B}$$

$$E : \overline{F} > A > \underline{H} > \cancel{B} > \cancel{C} > \cancel{D} > \cancel{G}$$

$$F : \overline{G} > B > \underline{E} > \cancel{C} > \cancel{D} > \cancel{A} > \cancel{H}$$

$$G : \overline{H} > C > \underline{F} > \cancel{D} > \cancel{A} > \cancel{B} > \cancel{E}$$

$$H : \overline{E} > D > \underline{G} > \cancel{A} > \cancel{B} > \cancel{C} > \cancel{F}$$

So phase-1 table is:

A	B E D
B	C F A
C	D G B
D	A H C
E	F A H
F	G B E
G	H C F
H	E D G

(b) phase-2:

exposed rotation:

1. A H C F A
   E D G B

2. A H C F A
   D G B E

Then we get

A	D
B	C
C	B
D	A
E	F
F	E
G	H
H	G

So we got a stable matching.