Computer Vision

Local interest points

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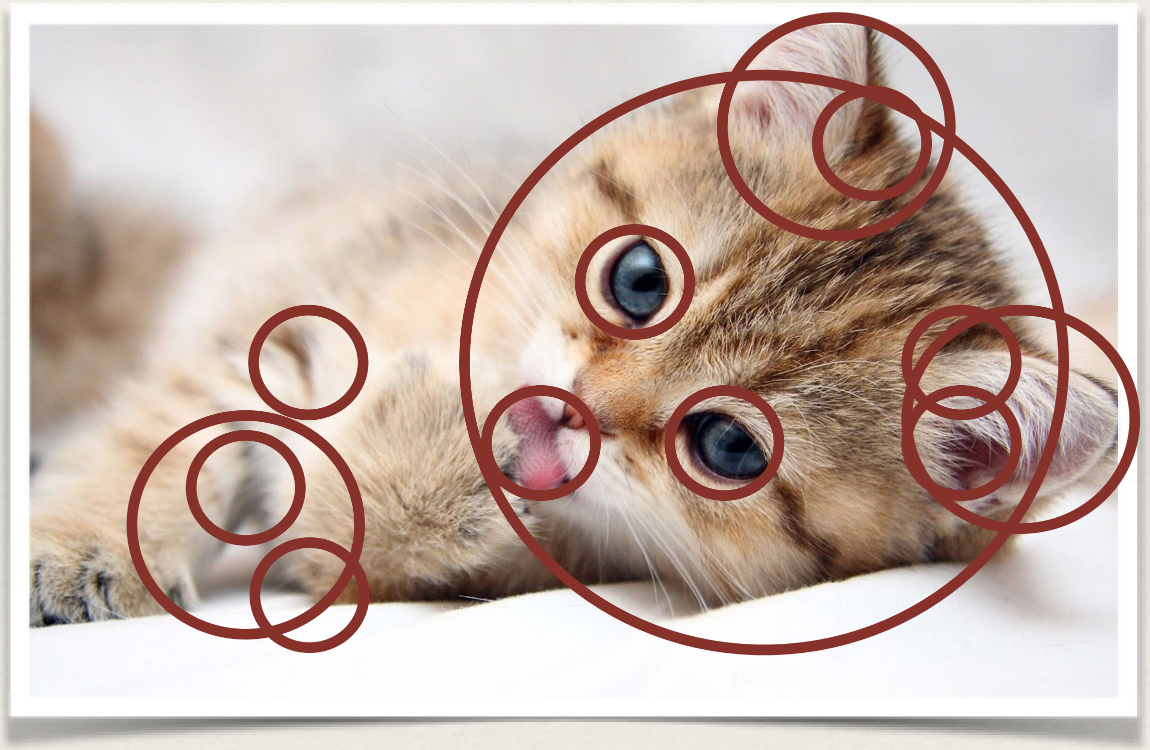
- Finding stable (repeatable) interest points is a key problem in modern computer vision
    - Applications in areas such as tracking, matching, image alignment, making robust features for classification and search, robot navigation, 3D reconstruction, …
    - We'll look at some of these in more detail in future lectures

What makes a good interest point?

- Invariance to brightness change (local changes as well as global ones)
- Sufficient texture variation in the local neighbourhood
  - But not too much!
- Invariance to changes between the angle/position of the scene to the camera

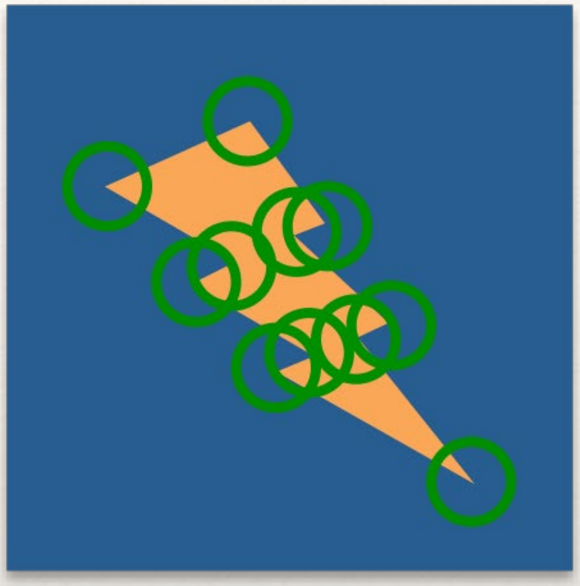


Stable local interest points

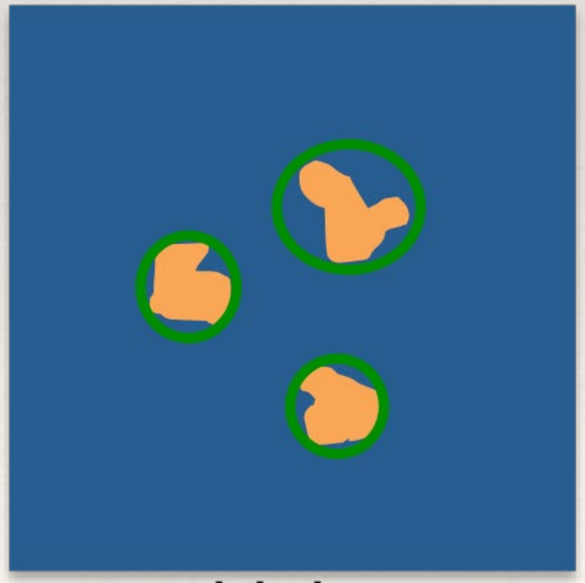
So, how do we find them?

- Lots of different types of interest point types to choose from.
  - We'll focus on two specific types and look in detail at common detection algorithms:
    - Corner detection - *Harris and Stephens*
    - Blob Detection - *Difference-of-Gaussian Extrema*

A blob is a region of an image in which some properties are constant or approximately constant -Wikipedia

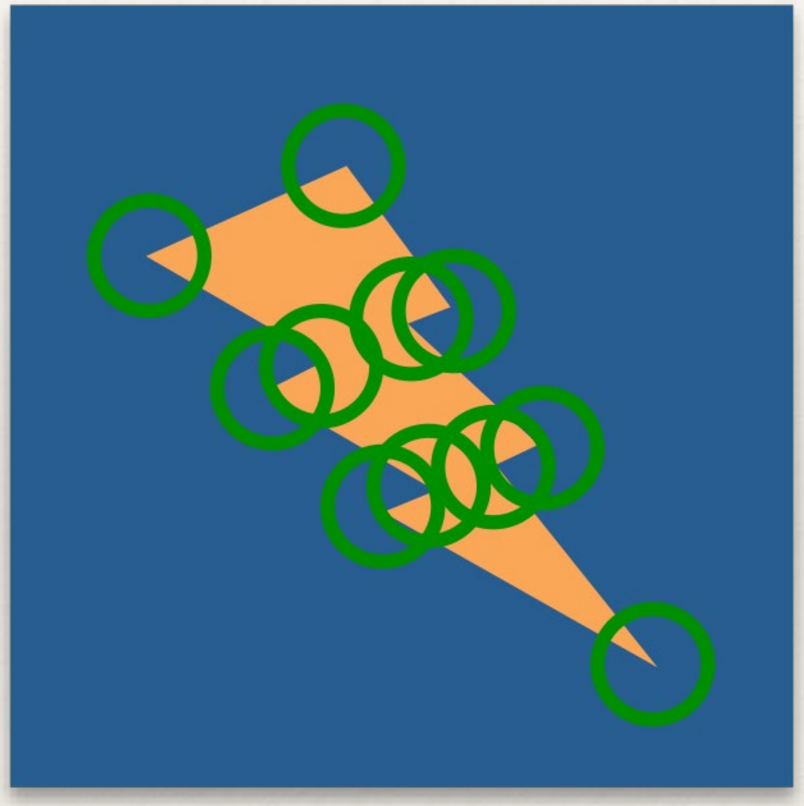


corners



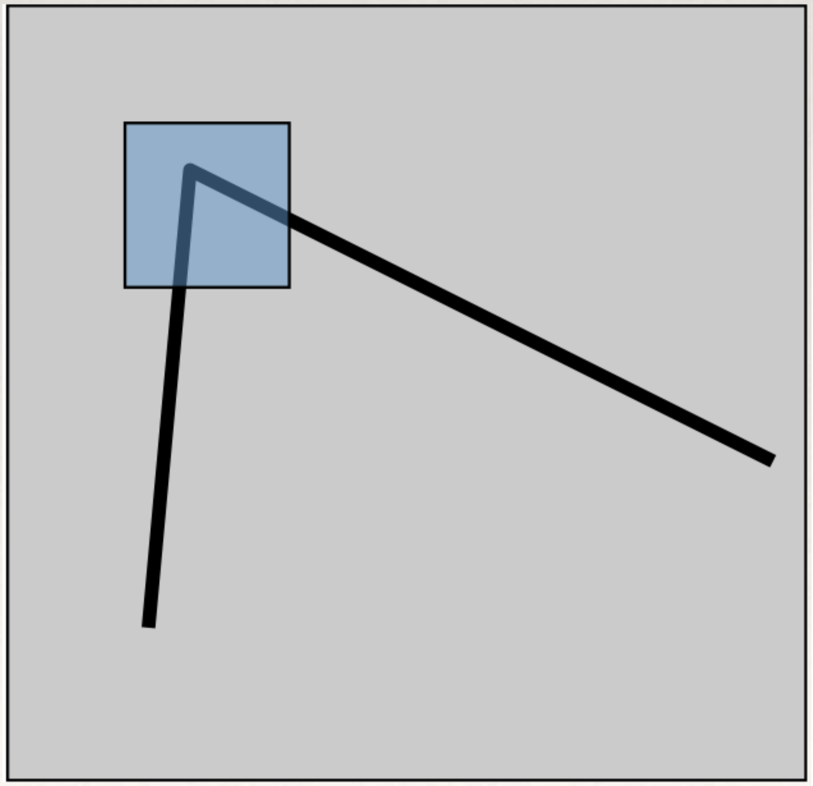
blobs

The Harris and Stephens corner detector

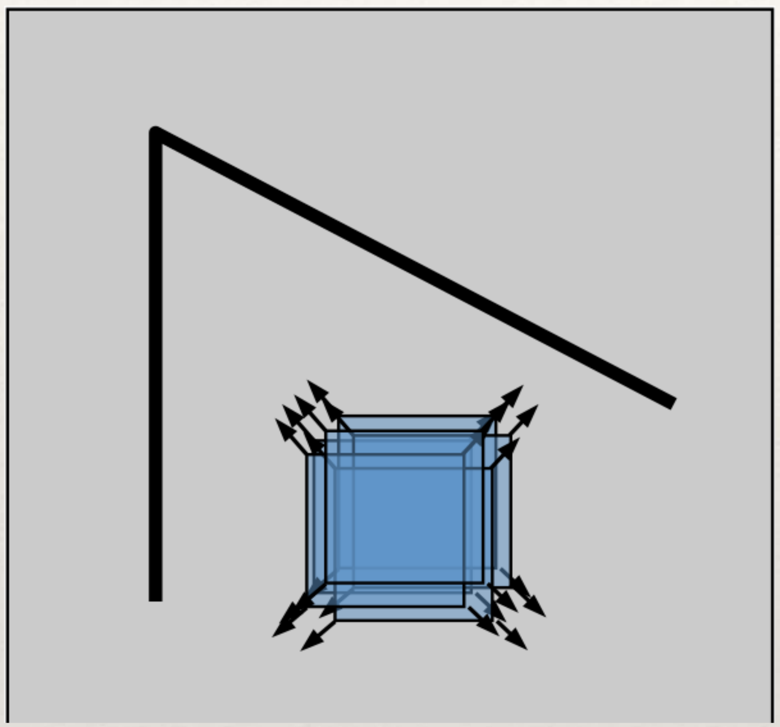


Basic Idea

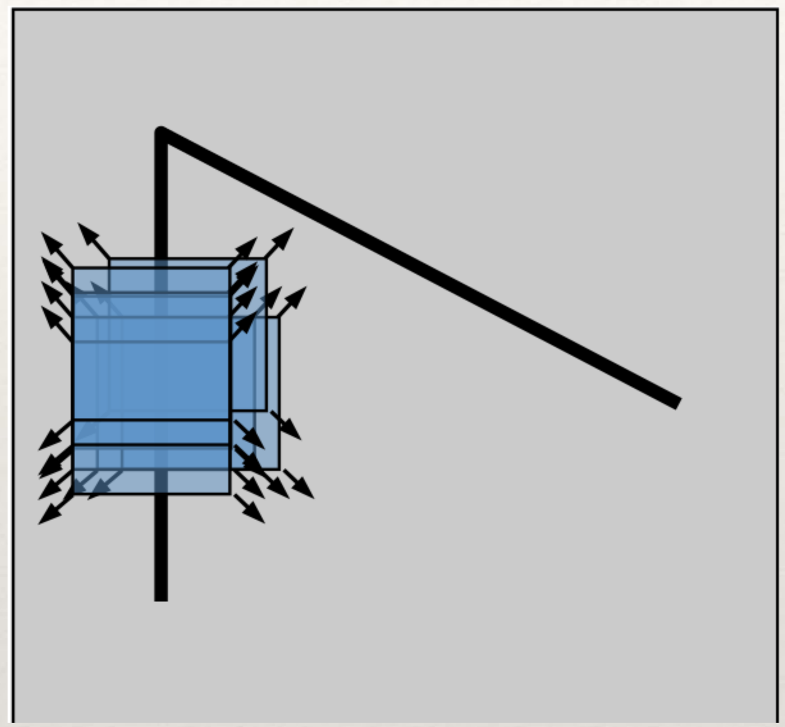
- Search for corners by looking through a small window.
- Shifting that window by a small amount in *any direction* should give a *large change* in intensity



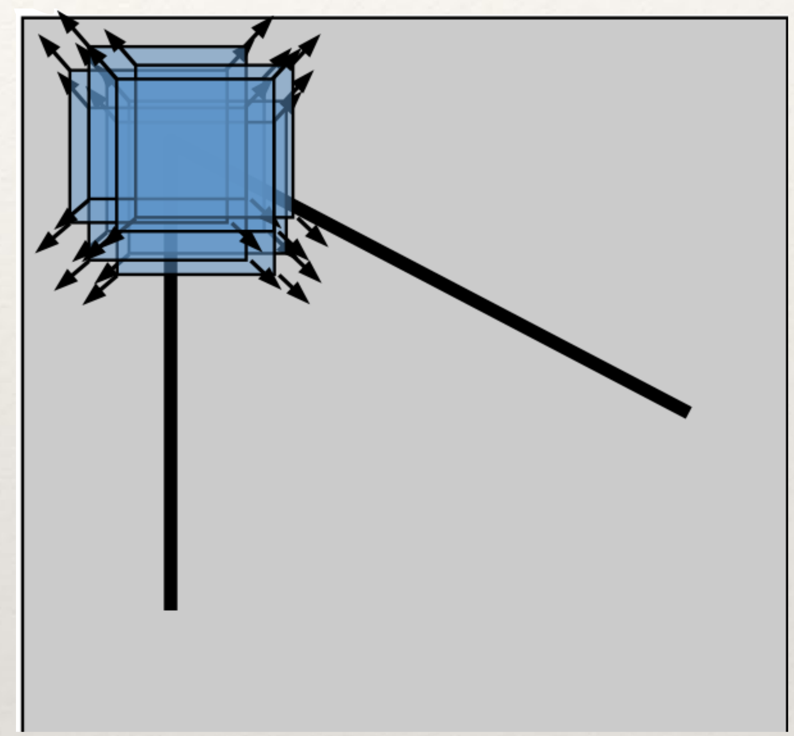
Basic Idea



"flat" region: no change in all directions



"edge": no change along the edge direction



"corner": significant change in all directions

Harris & Stephens: Mathematics

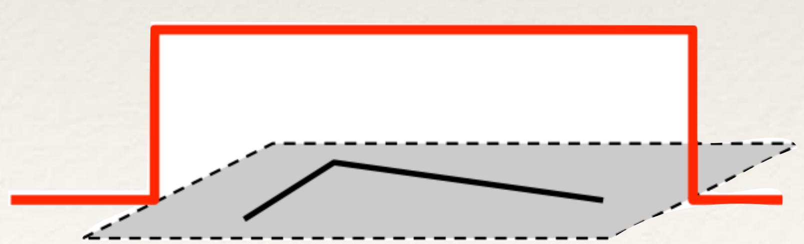
Weighted average change in intensity between a window and a shifted version [by (**Δ**x,**Δ**y)] of that window:

$$E(x,y) = \sum_{W} f(x_i, y_i)[I(x_i, y_i) - I(x_i + \Delta x, y_i + \Delta y)]^2$$

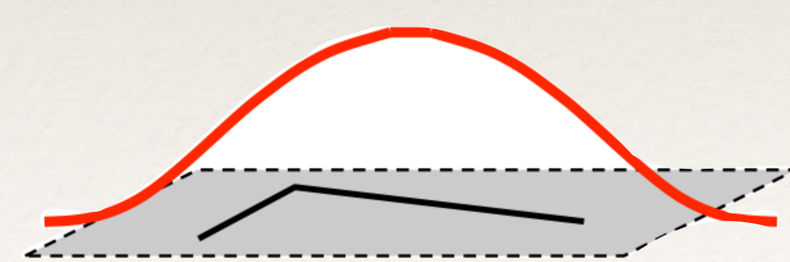
weighting function

intensity in window

intensity in shifted window



flat



Gaussian

Harris & Stephens: Mathematics

- The Taylor expansion allows us to approximate the shifted intensity.
- Taking the first order terms we get this:

$$I(x_i + \Delta x, y_i + \Delta y) \approx I(x_i, y_i) + \begin{bmatrix} I_x(x_i, y_i) & I_y(x_i, y_i) \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix}$$

(partial derivatives of the image)

Harris & Stephens: Mathematics

- Substituting and simplifying gives:

$$
\begin{aligned}
E(x,y) &= \sum_{W} [I(x_i, y_i) - I(x_i + \Delta x, y_i + \Delta y)]^2 \\
&= \sum_{W} \left(I(x_i, y_i) - I(x_i, y_i) - [I_x(x_i, y_i) \quad I_y(x_i, y_i)] \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} \right)^2 \\
&= \sum_{W} \left(-[I_x(x_i, y_i) \quad I_y(x_i, y_i)] \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} \right)^2 \\
&= \sum_{W} \left([I_x(x_i, y_i) \quad I_y(x_i, y_i)] \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} \right)^2 \\
&= [\Delta x \quad \Delta y] \begin{bmatrix} \sum_w \left(I_x(x_i, y_i)\right)^2 & \sum_w I_x(x_i, y_i) I_y(x_i, y_i) \\ \sum_w I_x(x_i, y_i) I_y(x_i, y_i) & \sum_w \left(I_y(x_i, y_i)\right)^2 \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix} \\
&= [\Delta x \quad \Delta y] \mathbf{M} \begin{bmatrix} \Delta x \\ \Delta y \end{bmatrix}
\end{aligned}
$$

Structure Tensor

The **square symmetric** matrix **M** is called the *Structure Tensor* or the *Second Moment matrix*

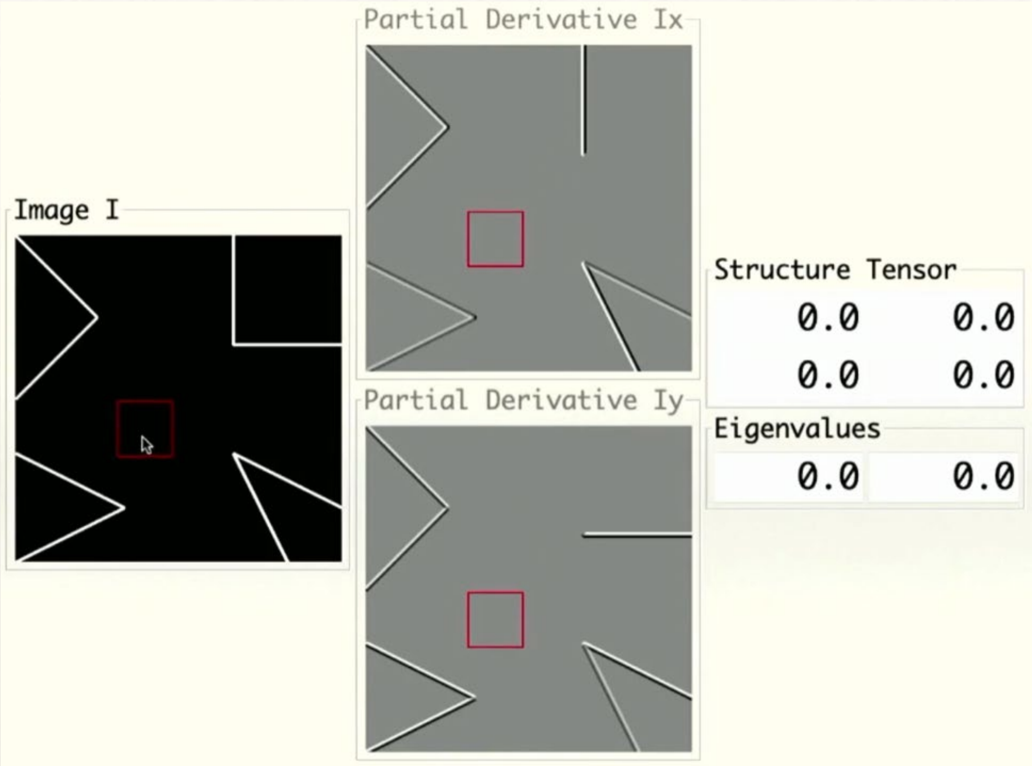
$$
\mathrm{M} = \begin{bmatrix} \sum_{w} \left(I_x(x_i, y_i)\right)^2 & \sum_{w} I_x(x_i, y_i) I_y(x_i, y_i) \\ \sum_{w} I_x(x_i, y_i) I_y(x_i, y_i) & \sum_{w} \left(I_y(x_i, y_i)\right)^2 \end{bmatrix}
$$

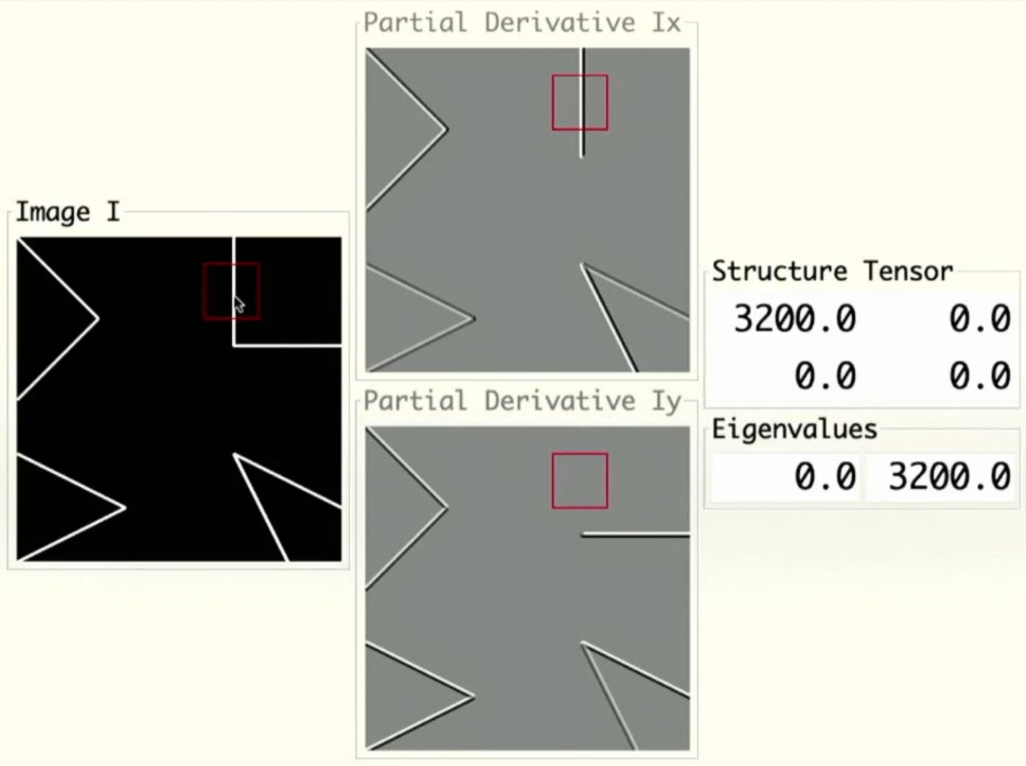
It concisely encodes the how the local shape intensity function of the window changes with small shifts

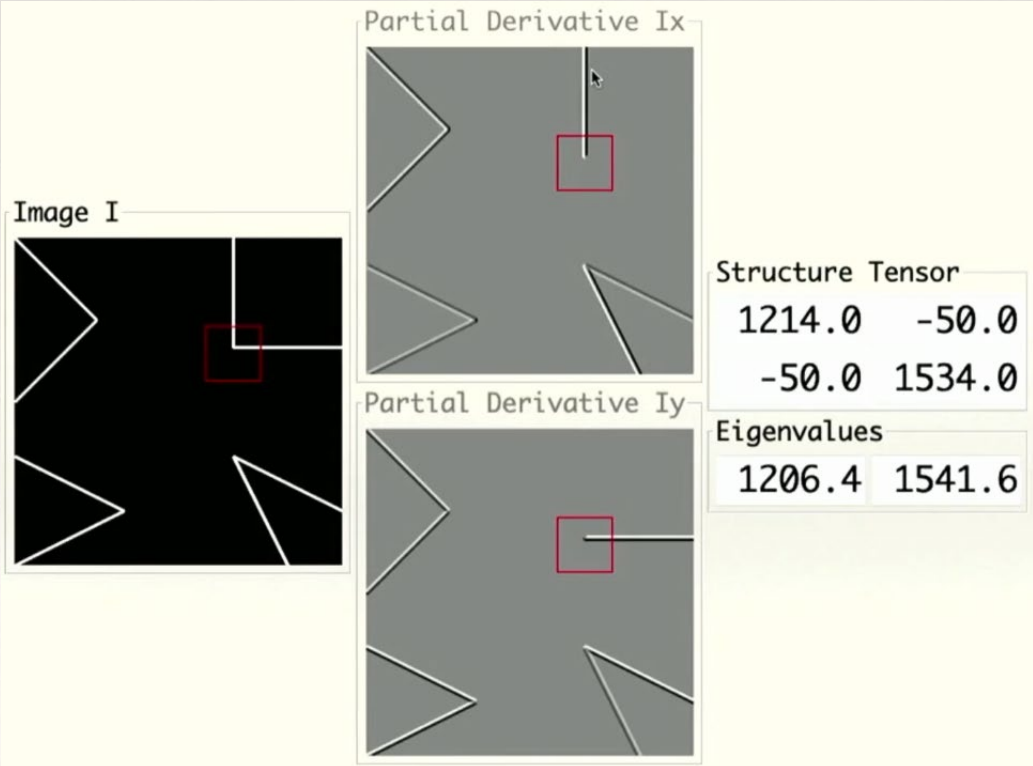
Eigenvalues of the Structure Tensor

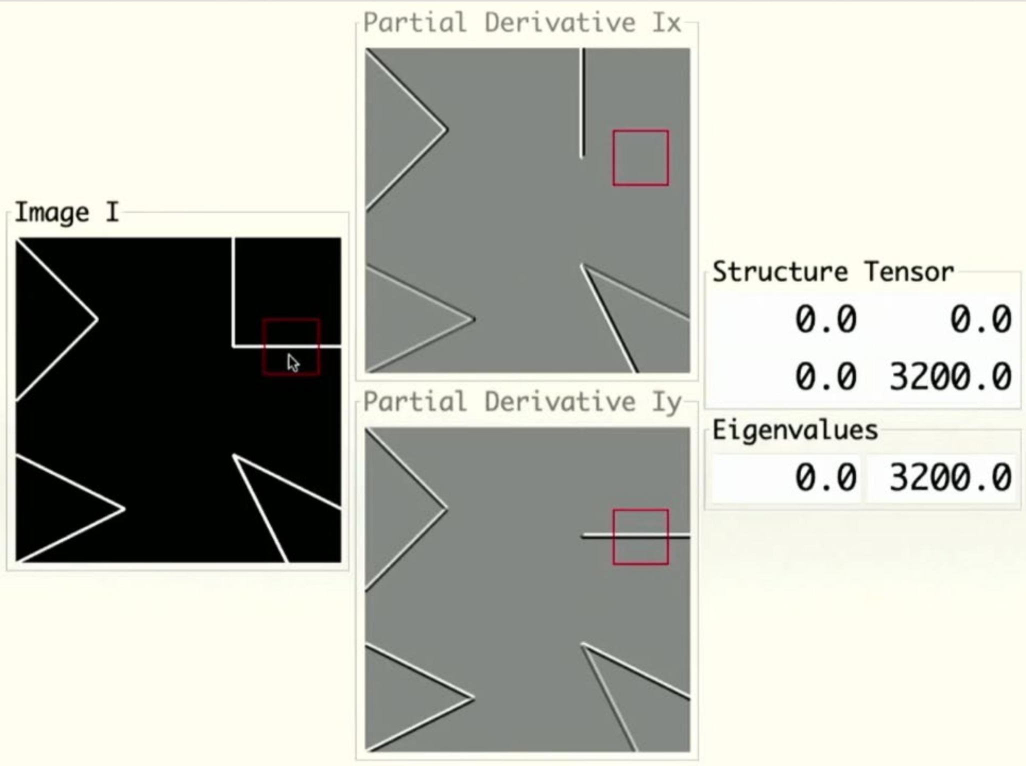
- Think back to Lecture 12 where we looked at covariance matrices…
  - The eigenvalues and vectors tell us the rates of change and their respective directions

Eigenvalues of the Structure Tensor

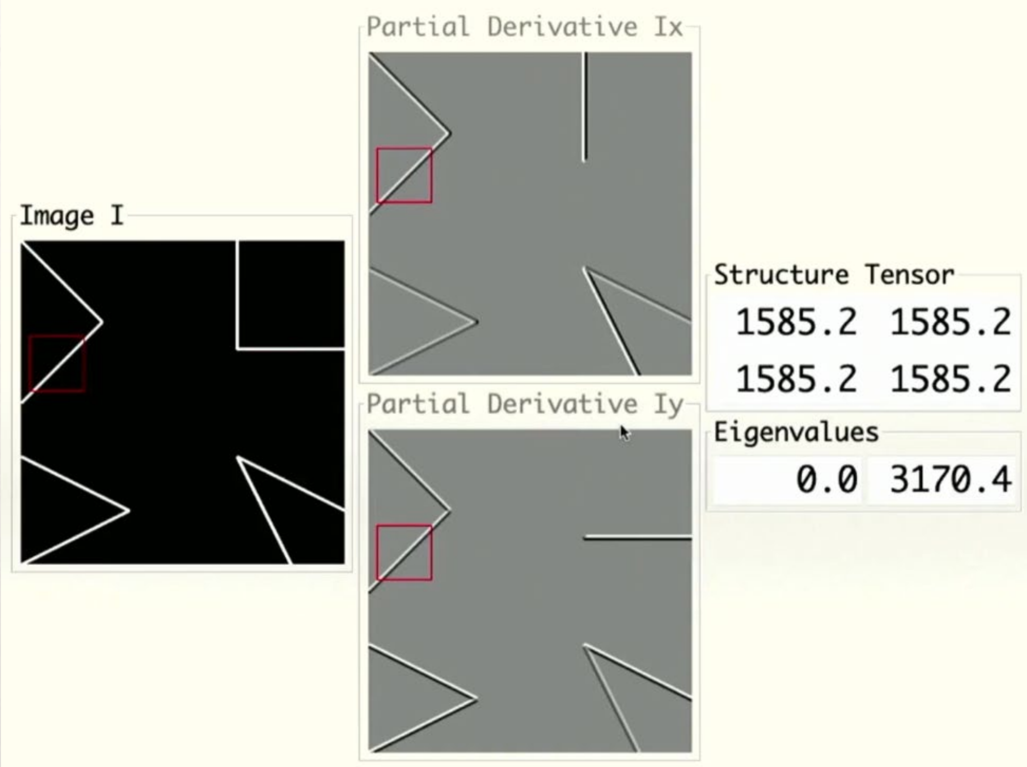


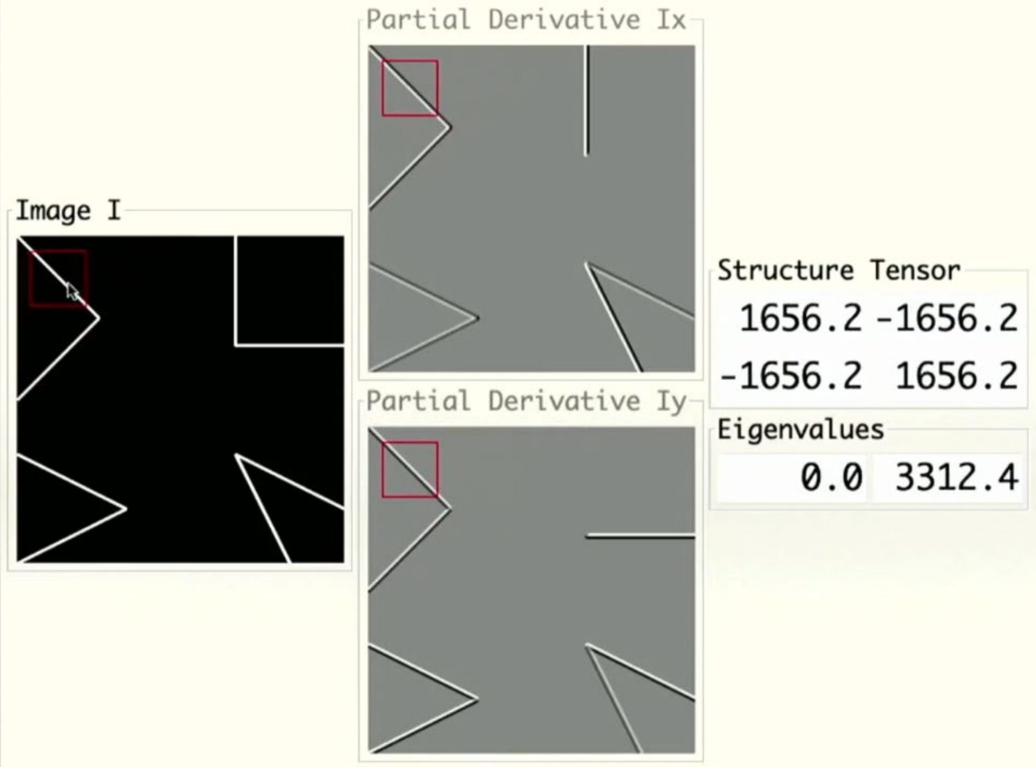


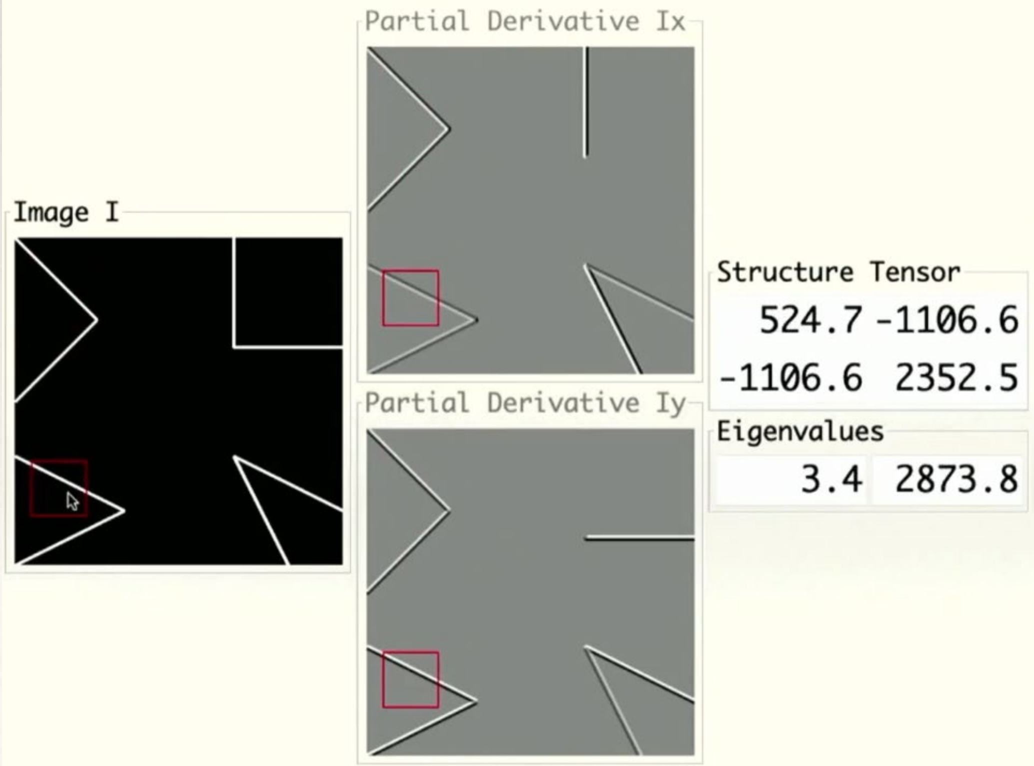


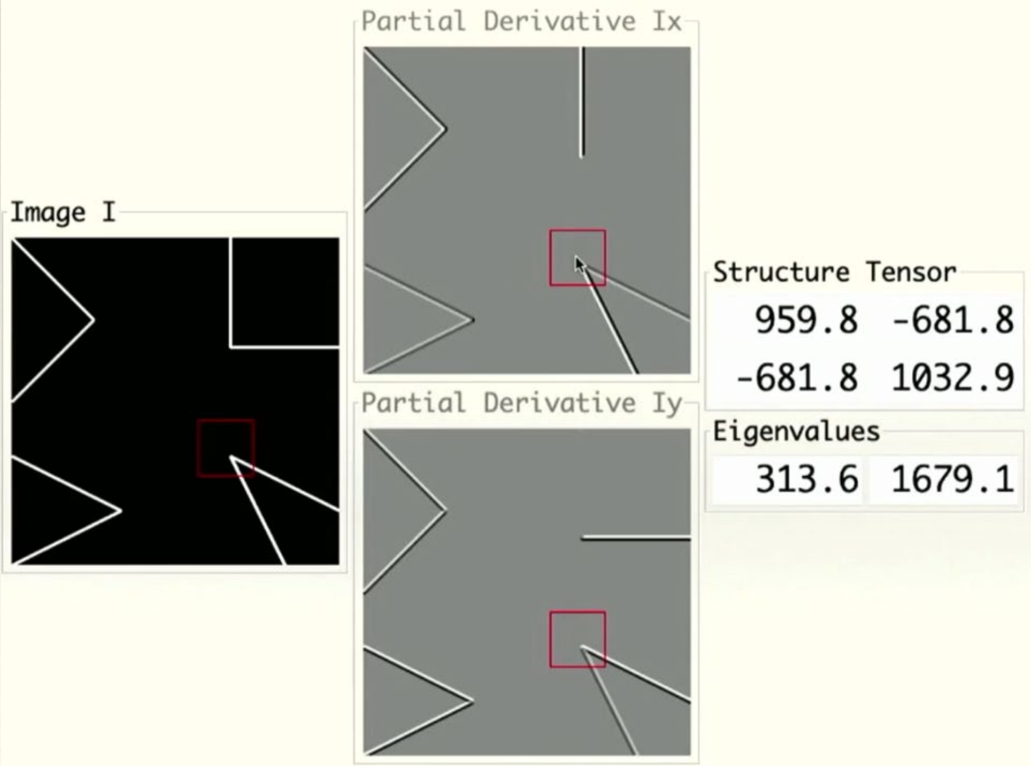


Eigenvalues of the Structure Tensor

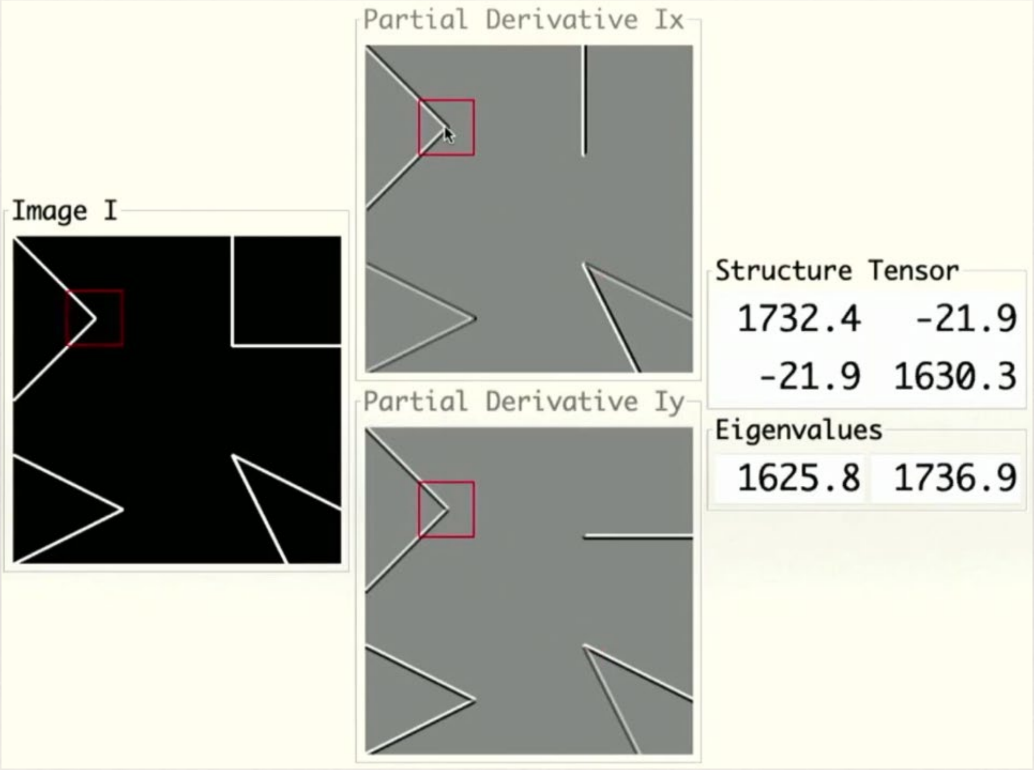


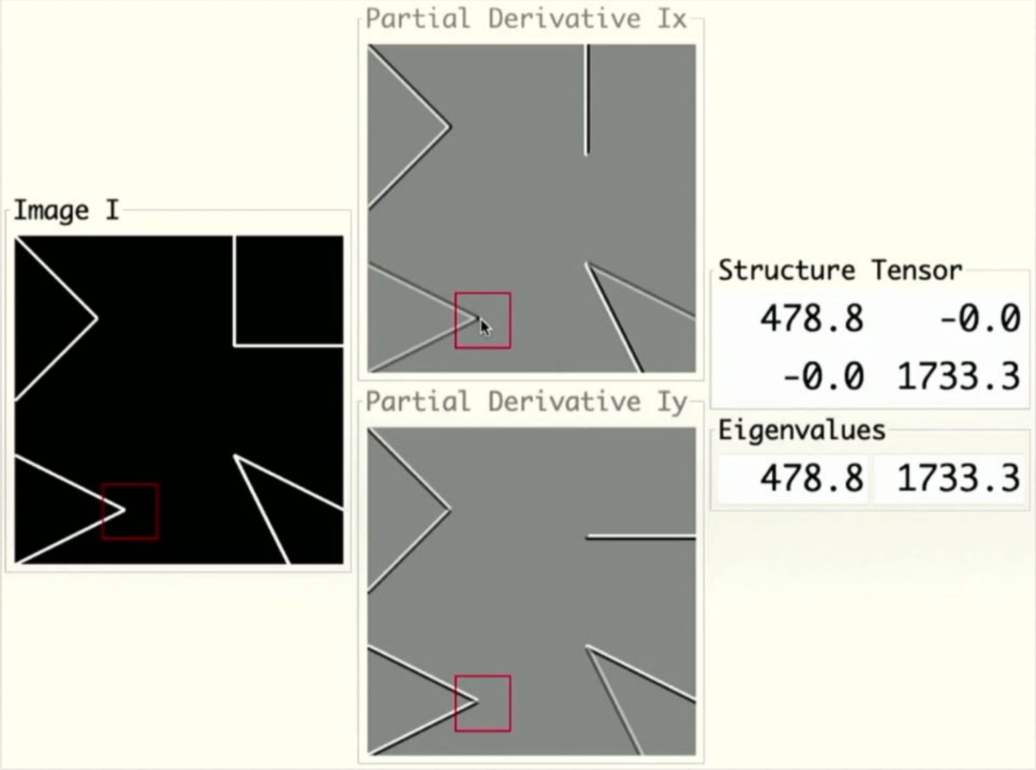


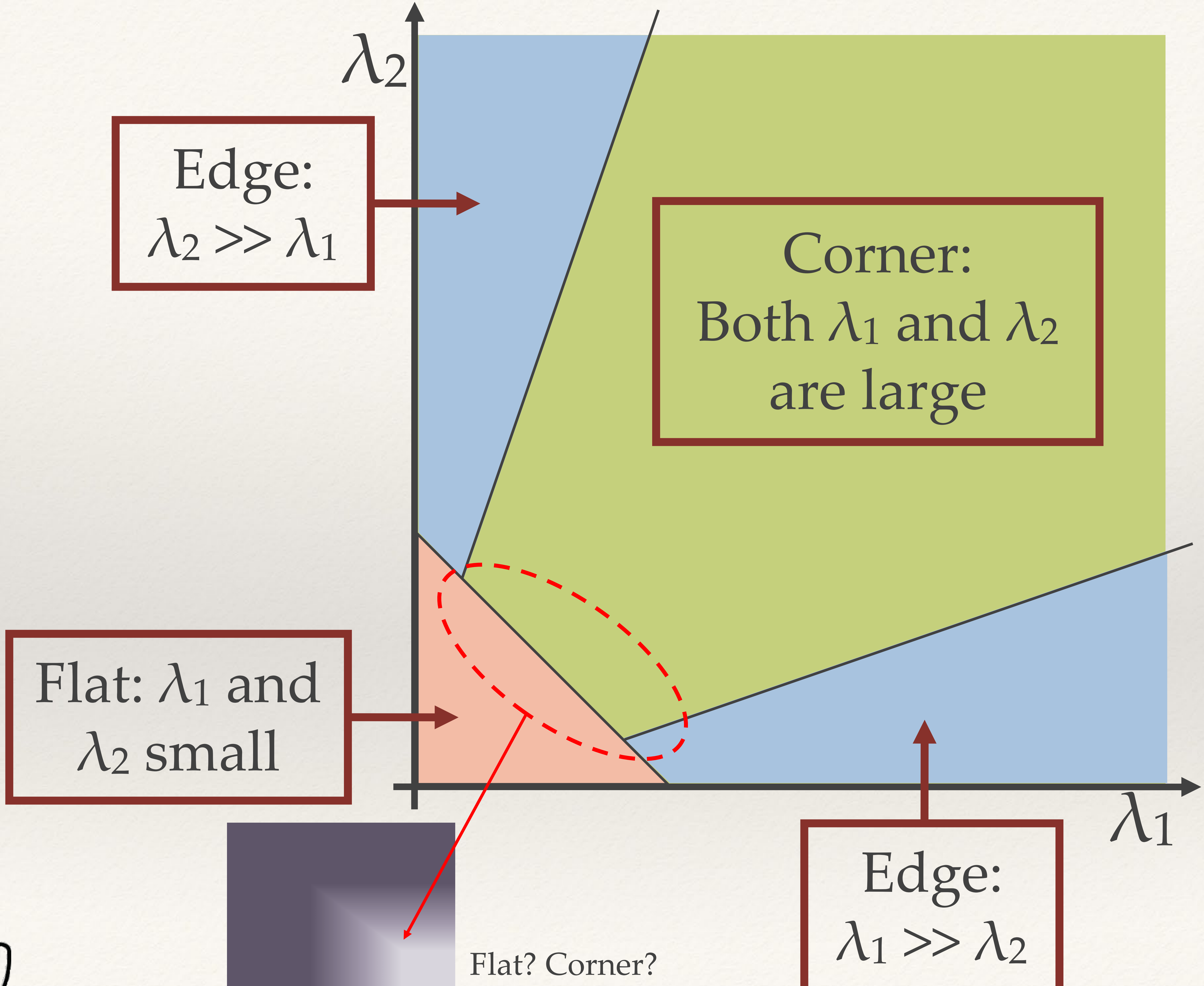




Eigenvalues of the Structure Tensor






λ_2
Edge:
$\lambda_2 >> \lambda_1$
Corner:
Both λ_1 and λ_2
are large
Flat: λ_1 and
λ_2 small
λ_1
Edge:
$\lambda_1 >> \lambda_2$
Flat? Corner?

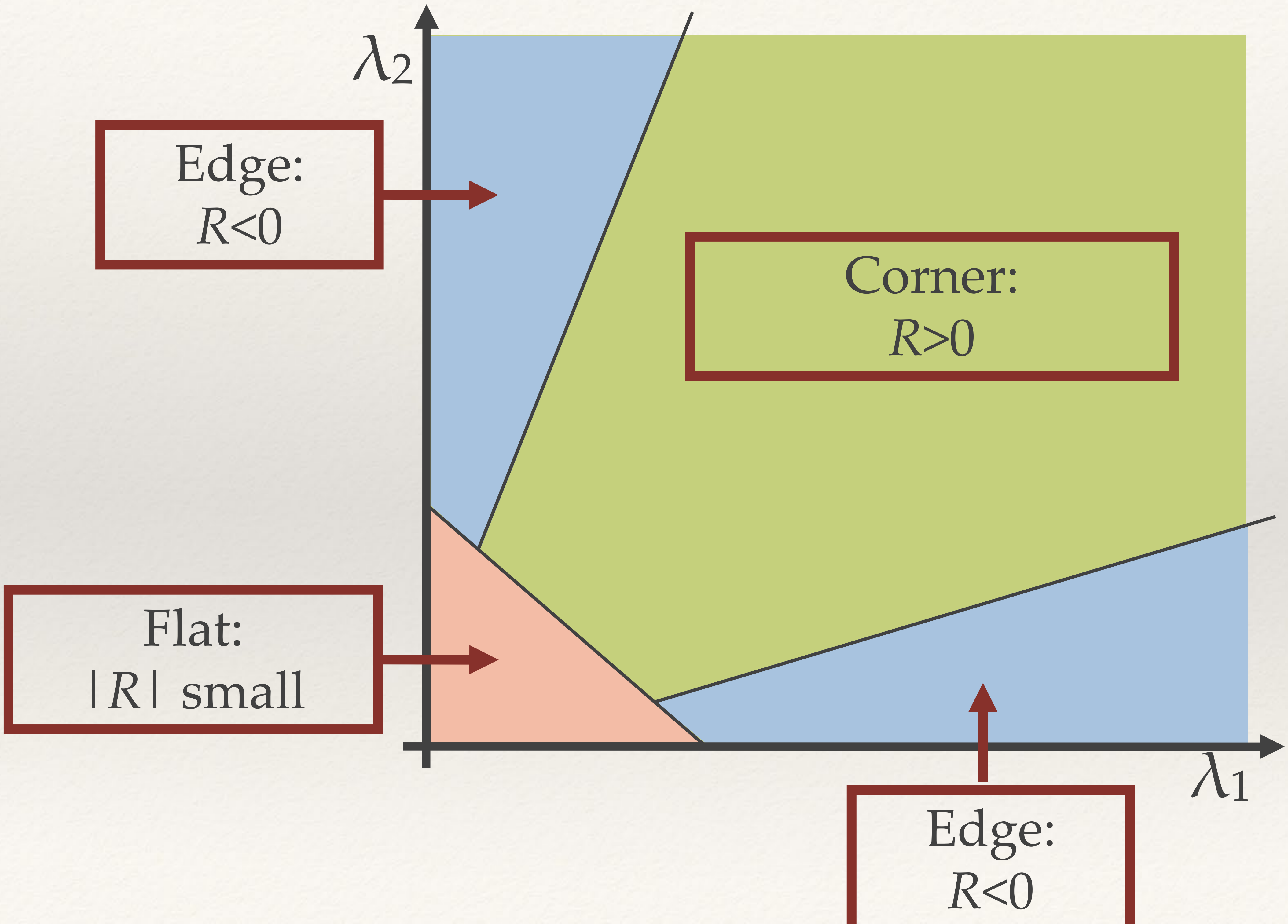
Harris & Stephens Response Function

- Rather than compute the eigenvalues directly, Harris and Stephens defined a corner response function in terms of the determinant and trace of **M**:

$$
\begin{aligned}
&\det(\mathbf{M}) = M_{00}M_{11} - M_{01}M_{10} = M_{00}M_{11} - M_{10}^2 = \lambda_1\lambda_2 \\
&\operatorname{trace}(\mathbf{M}) = M_{00} + M_{11} = \lambda_1 + \lambda_2 \\
&R = \det(\mathbf{M}) - k \operatorname{trace}(\mathbf{M})^2
\end{aligned}
$$

k is a small empirically set constant (usually 0.04 - 0.06)

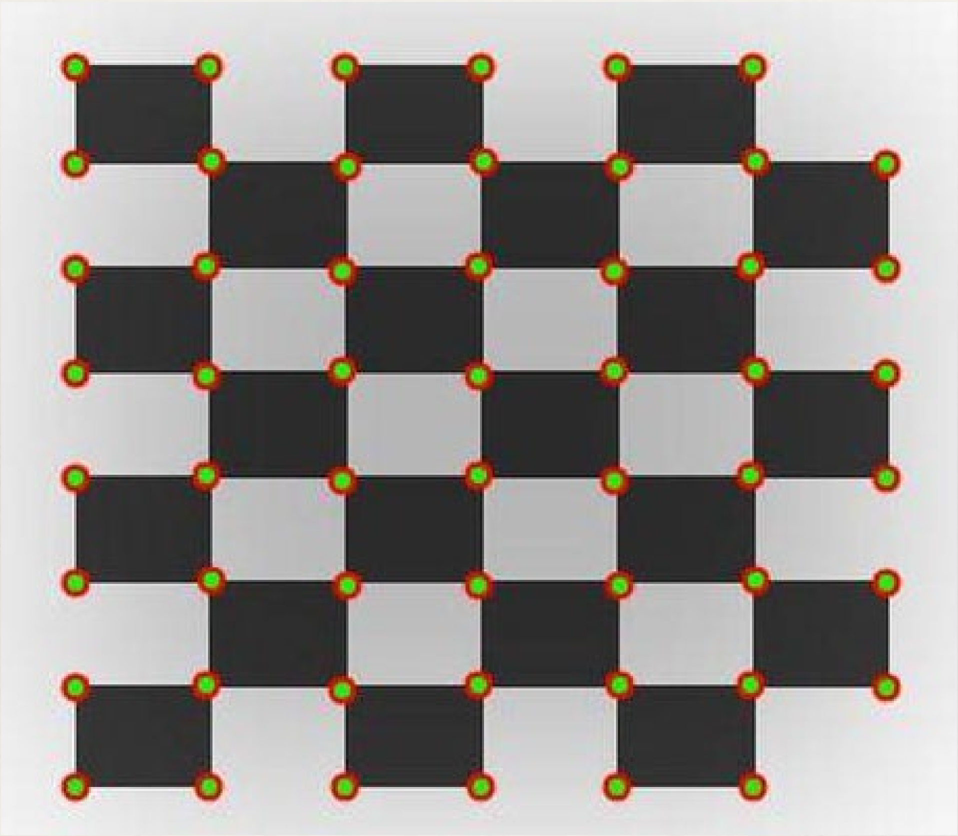
Harris & Stephens Response Function

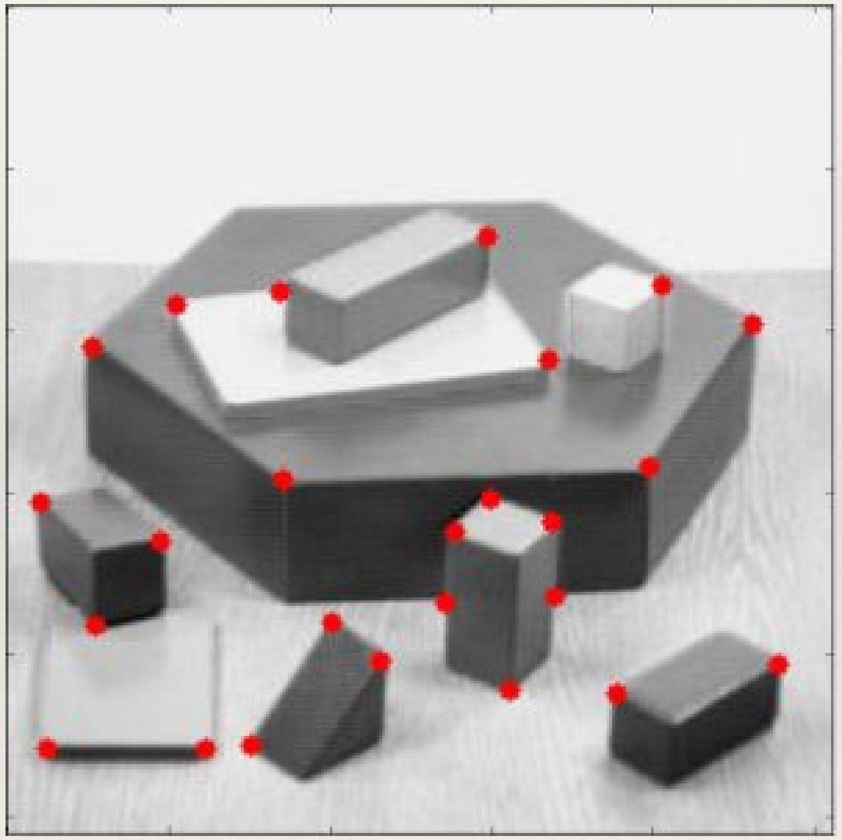


Harris & Stephens Detector

- Simple algorithm:
  - Take all points with the response value above a threshold
  - Then keep only the points that are local maxima (i.e. where the current response is bigger than the 8 neighbouring pixels)

Harris & Stephens Detector





Scale in Computer Vision

The problem of scale

- As an object moves closer to the camera it get larger with more detail… as it moves further away it gets smaller and loses detail…
- If you're using a technique that uses a fixed size processing window (e.g. Harris corners, or indeed anything that involves a fixed kernel) then this is a bit of a problem!

Scale space theory

- Scale space theory is a formal framework for handling the scale problem.
  - Represents the image by a series of increasingly smoothed/blurred images parameterised by a scale parameter t.
  - t represents the amount of smoothing.
  - **Key notion:** Image structures smaller than sqrt(t) have been smoothed away at scale t.

The Gaussian Scale Space

- Many types of scale space are possible (depending on the smoothing function), but only the Gaussian function has the desired properties for image representation.
  - These provable properties are called the "*scale space axioms*".

Gaussian Scale Space

Formally, Gaussian scale space defined as:

$$L(\cdot,\cdot;t) = g(\cdot,\cdot;t) * f(\cdot,\cdot)$$

Note: convolution is over *x, y* for fixed *t*

where t ≥ 0 and,

$$g(x,y;t) = \frac{1}{2\pi t} e^{-(x^2+y^2)/2t}$$

Note: $t = \sigma^2$ = variance of the Gaussian

Gaussian Scale Space

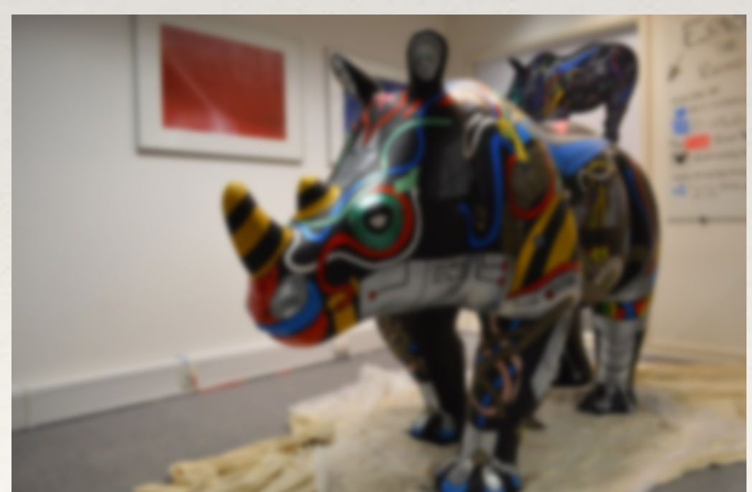
Normally, only a fixed set of values of t are used
- it's common to use integer powers of 2 or $\sqrt{2}$

$t=0$

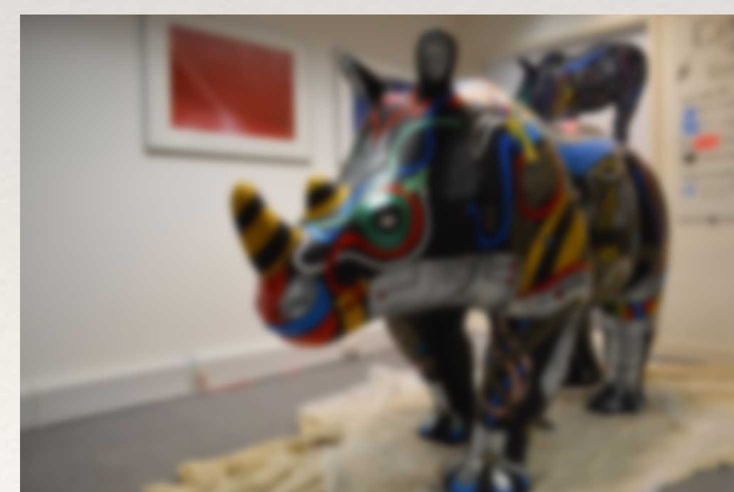
$t=1$

$t=2$

$t=4$



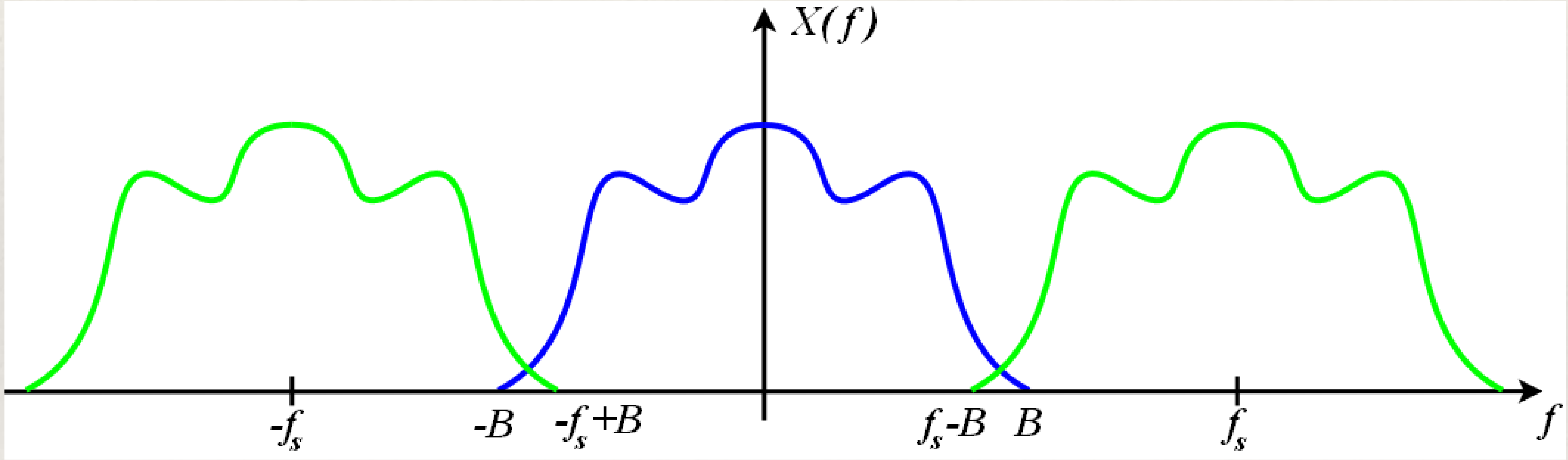
$t=16$



$t=32$

Nyquist-Shannon Sampling theorem

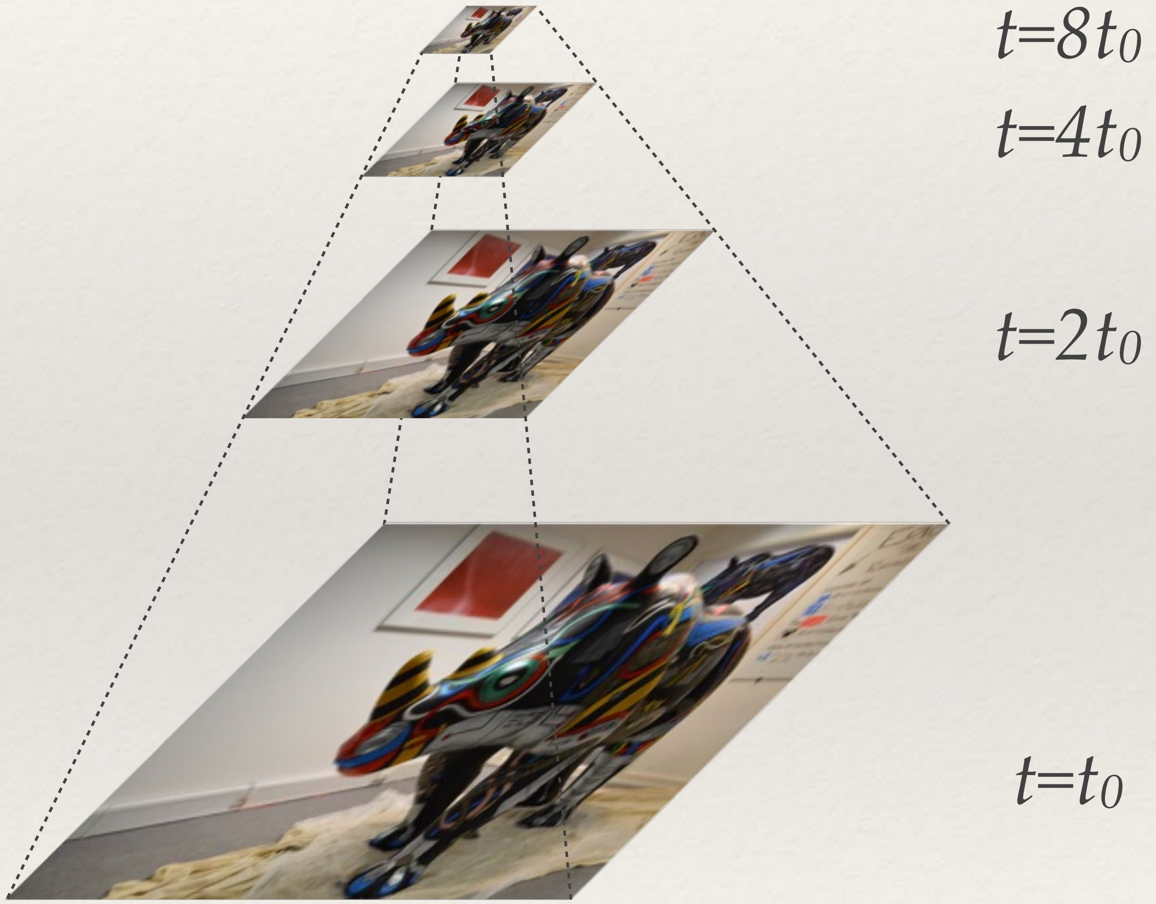
If a function x(t) contains no frequencies higher than B hertz, it is completely determined by giving its ordinates at a series of points spaced 1/(2B) seconds apart.



…so, ***if you filter the signal with a low-pass filter that halves the frequency content, you can also half the sampling rate without loss of information…***

Gaussian Pyramid

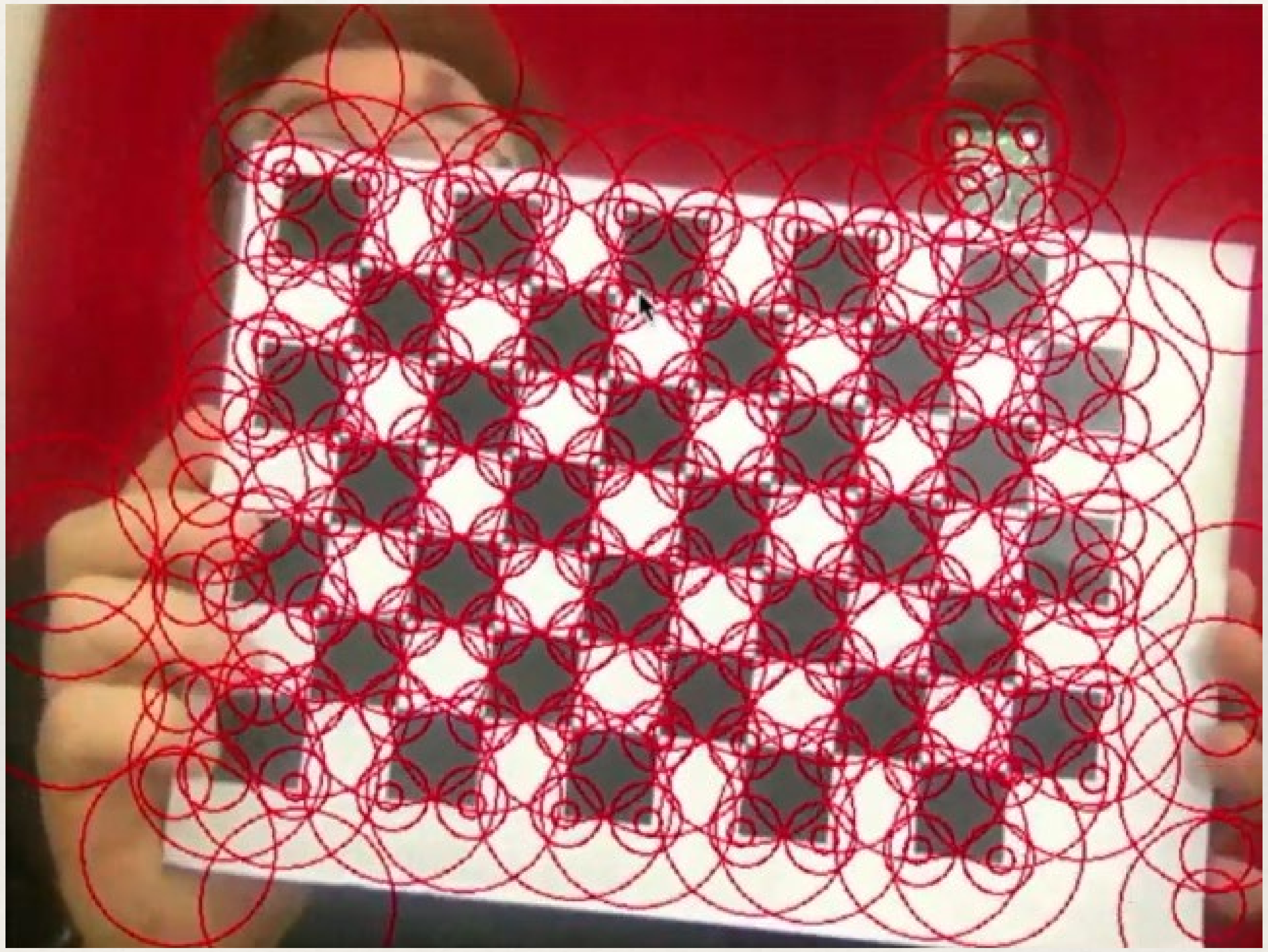
- Every time you double t in scale space, you can half the image size without loss of information!
  - Leads to a much more efficient representation
    - faster processing
    - less memory

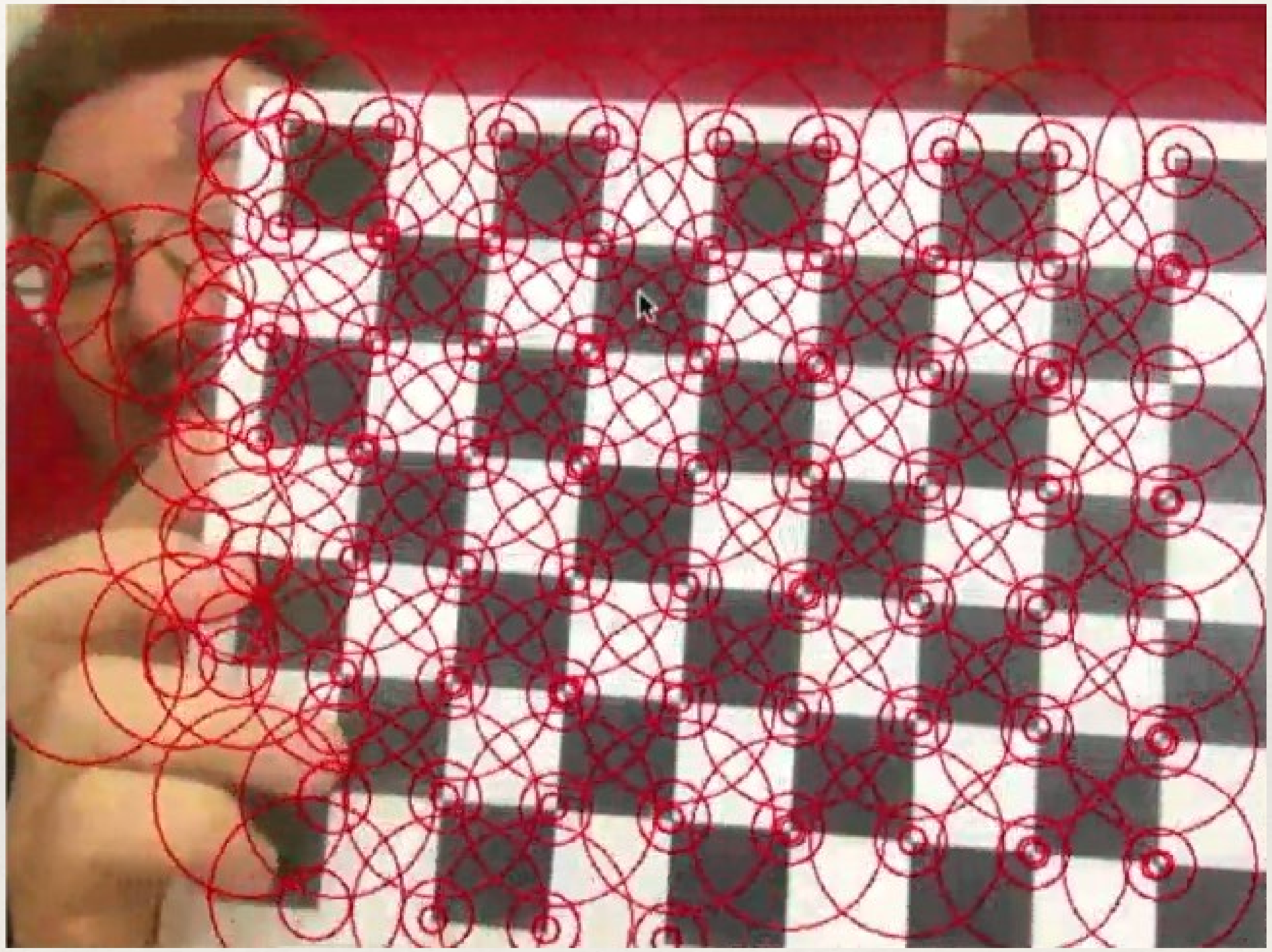


Multi-scale Harris & Stephens

- Extending the Harris and Stephens detector to work across scales is easy…
    - We define a Gaussian scale space with a fixed set of scales and compute the corner response function at every pixel of each scale and keep only those with a response above a certain threshold.

Multi-scale Harris & Stephens Corner Detector

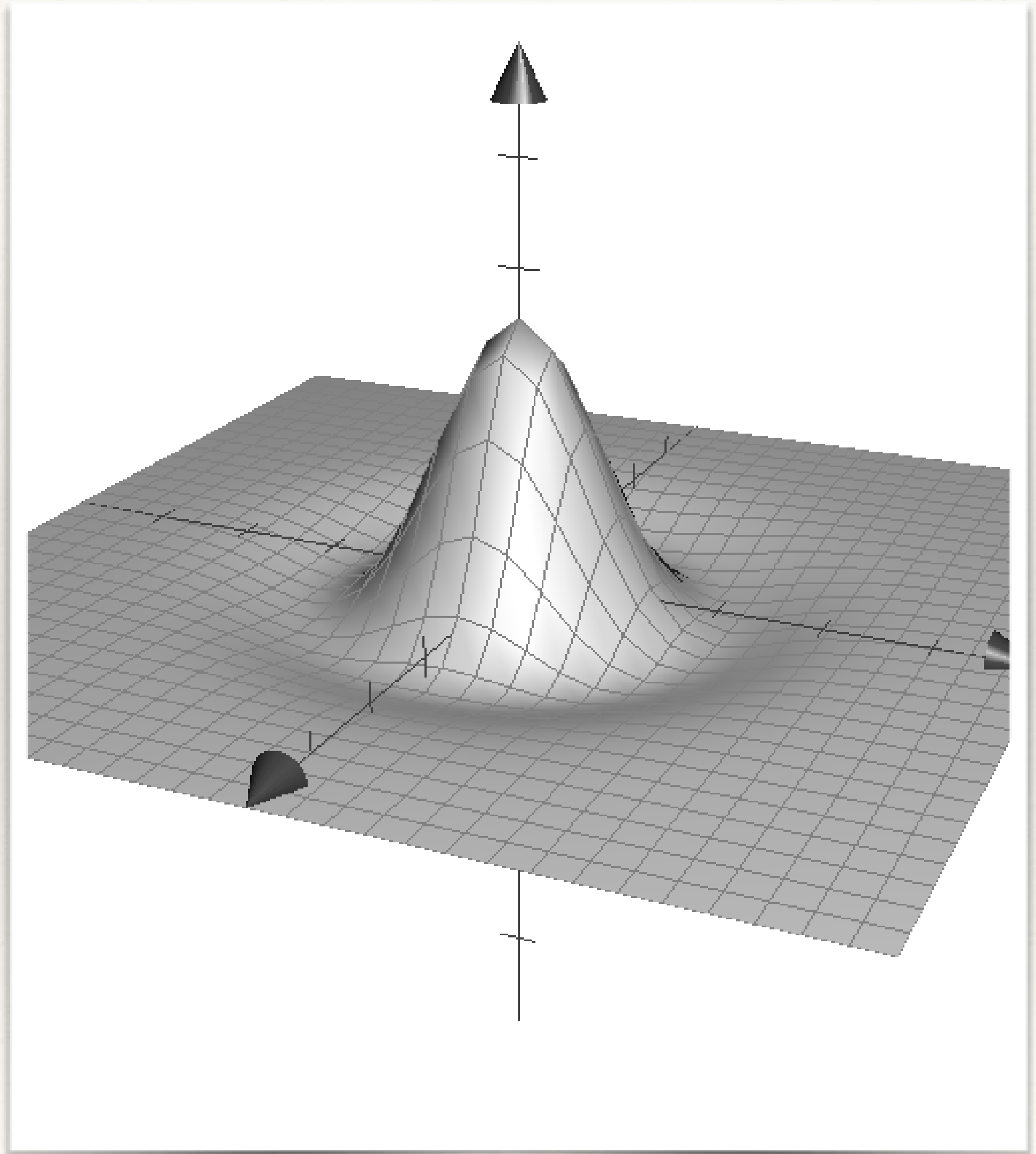




Blob Detection

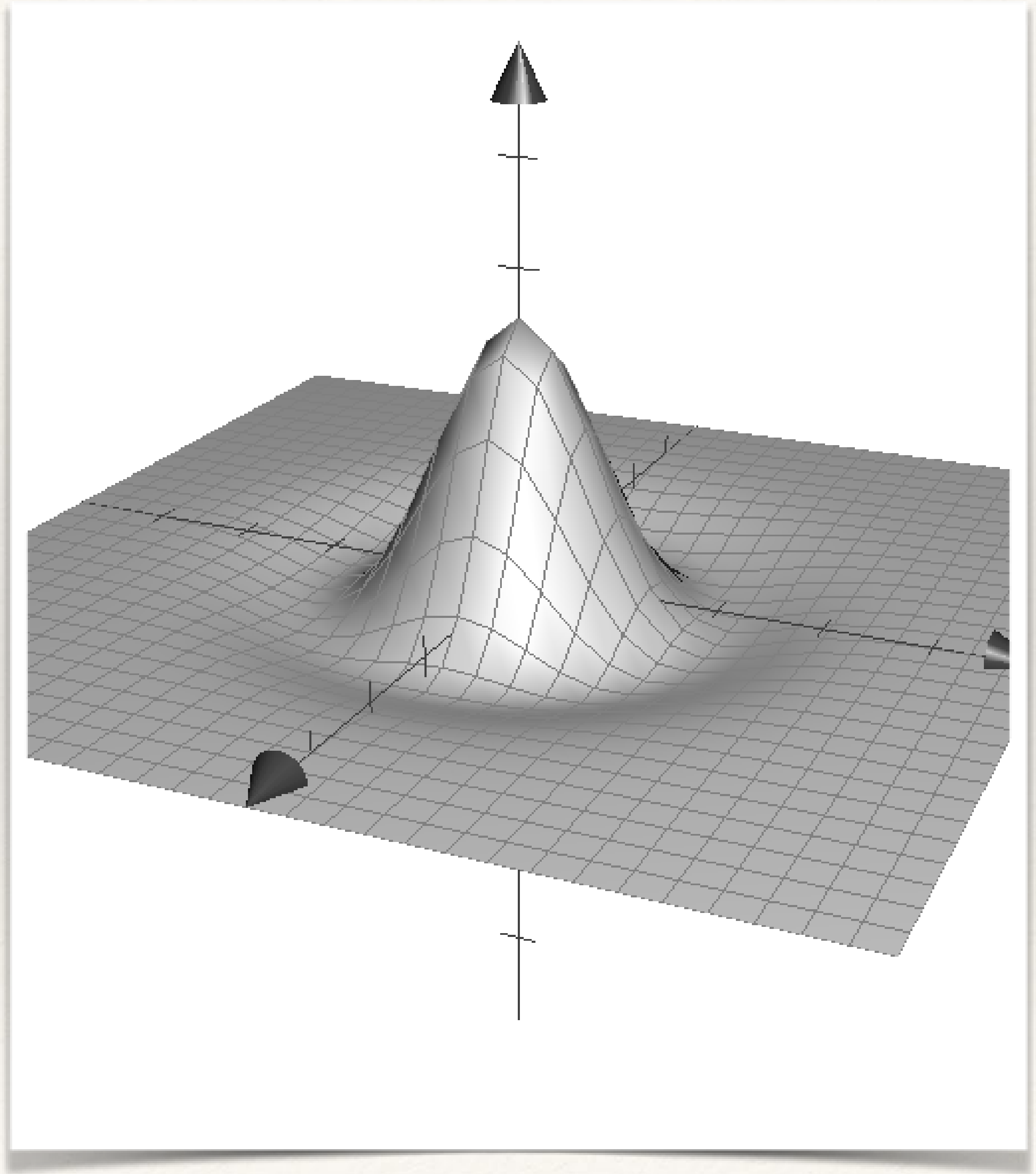
Recap: Laplacian of Gaussian

- Recall that the LoG is the second derivative of a Gaussian
  - Used in the Marr-Hildreth edge detector
    - Zero crossings of LoG convolution



Laplacian of Gaussian

- Instead of finding zero crossings, find local minima or maxima => blob detector!

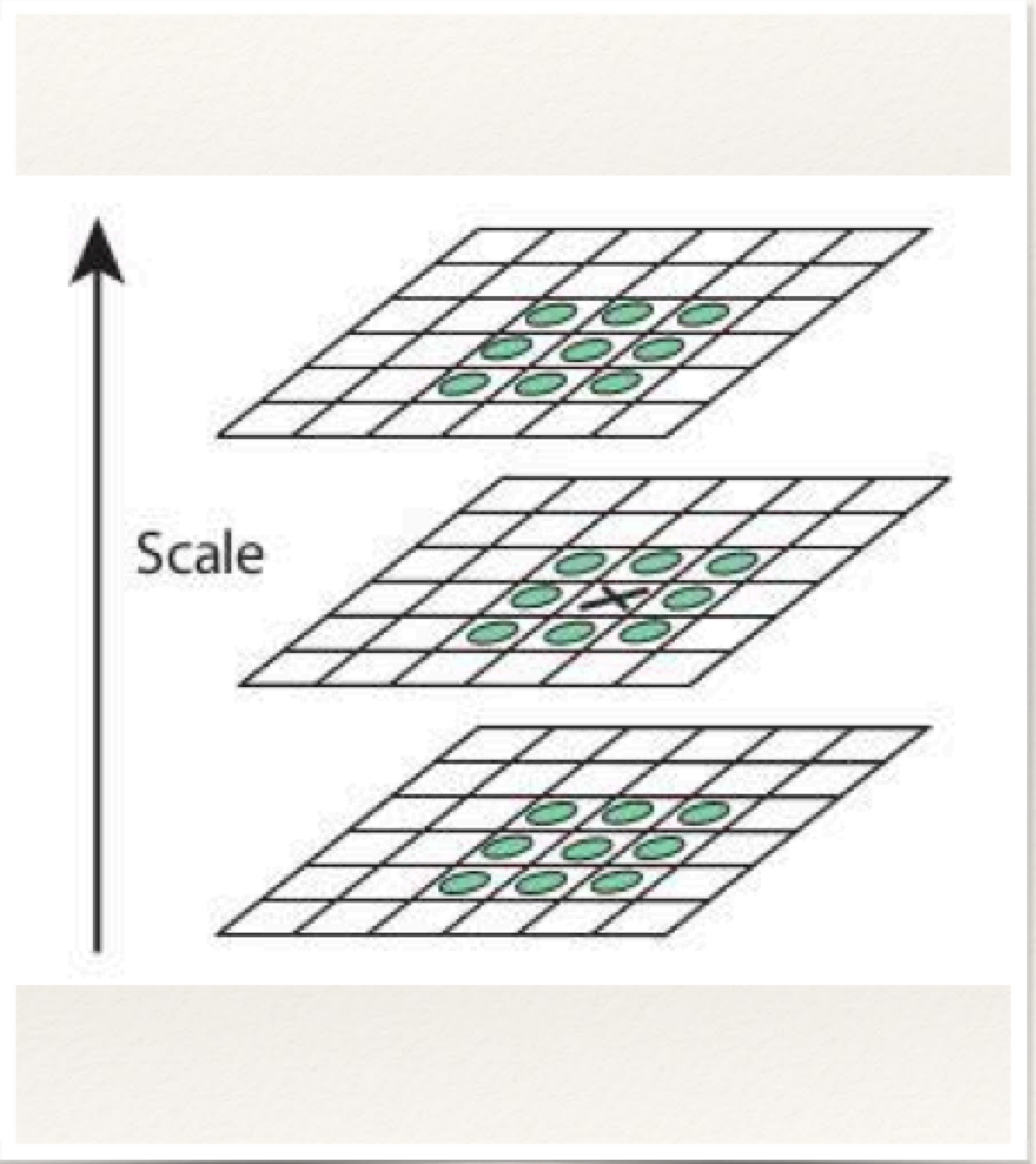


Scale space LoG

- Normalised scale space LoG defined as:

$$\nabla^2_{norm} L(x, y; t) = t(L_{xx} + L_{yy})$$

- By finding extrema of this function in scale space, you can find *blobs* at their representative scale (~sqrt(2t))
  - Just need to look at the neighbouring pixels!



Scale space DoG

- In practice it's computationally expensive to build a LoG scale space.
- But, the following approximation can be made:

$$\nabla^2_{norm} L(x, y; t) \approx \frac{t}{\Delta t}\left(L(x, y; t + \Delta t) - L(x, y; t - \Delta t)\right)$$

- This is called a Difference-of-Gaussians (*DoG*)
  - Implies that the LoG scale space can be built from subtracting adjacent scales of a Gaussian scale space

Difference of Gaussian Response

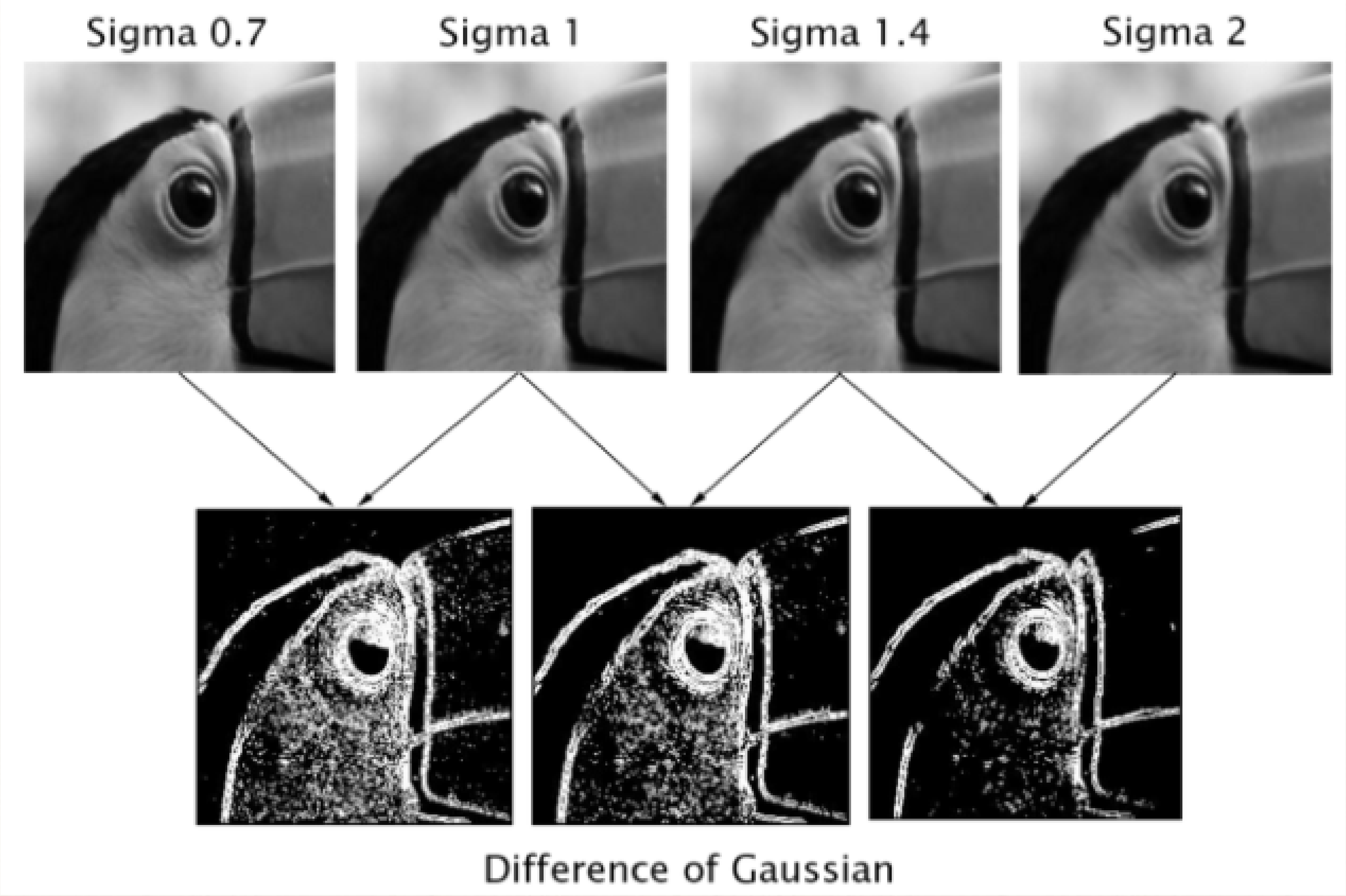
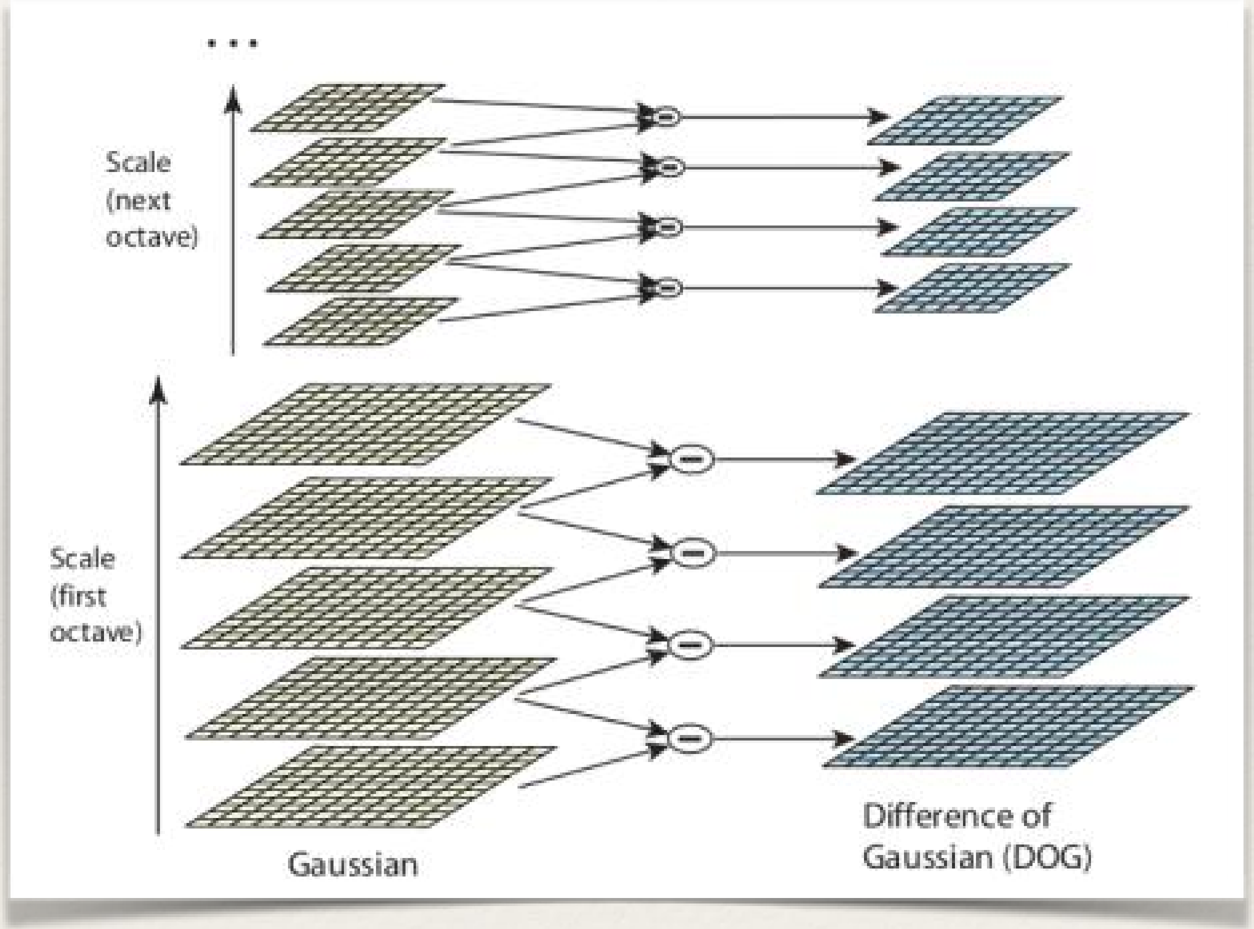


Image from https://medium.com/

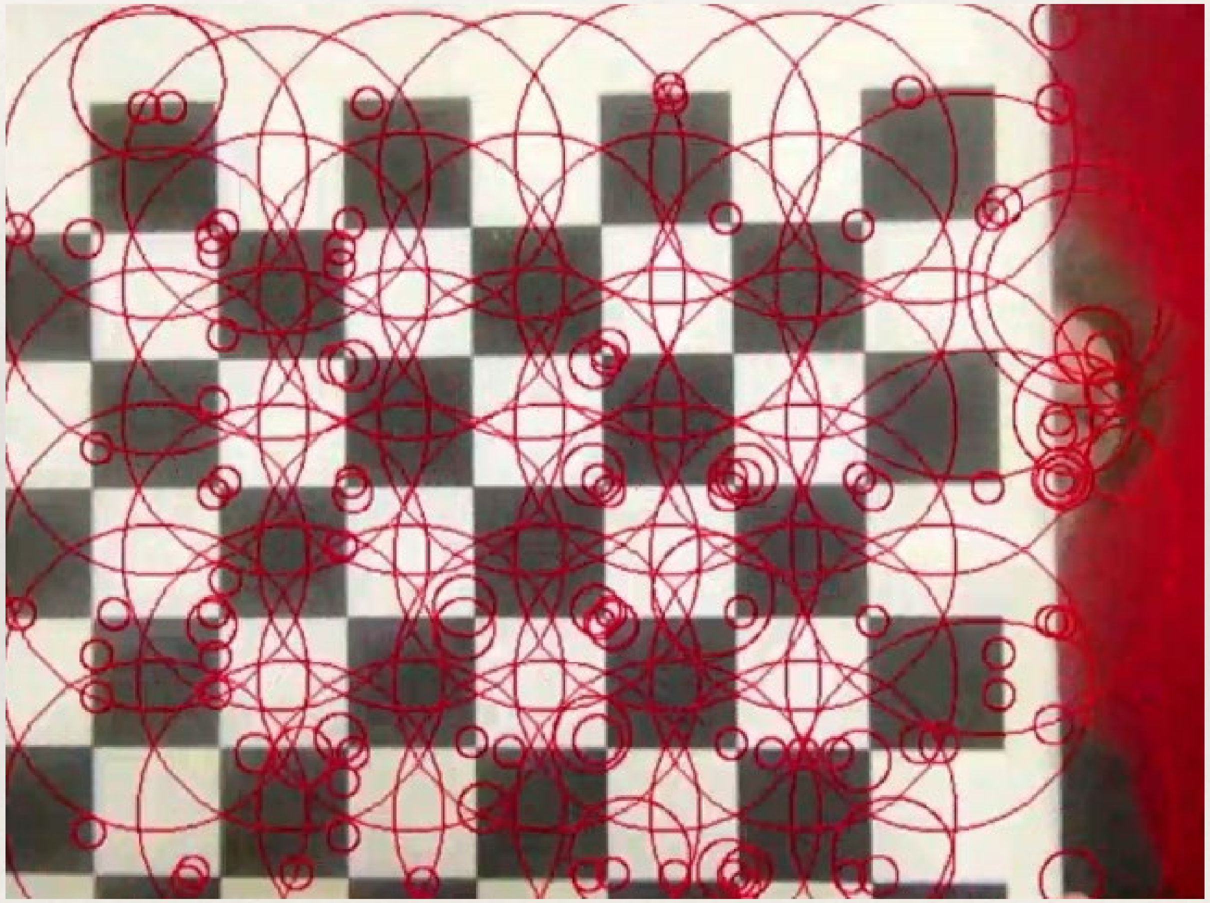
Very useful property: *if a blob is detected at* $(x_0, y_0; t_0)$ *in an image, then under a scaling of that image by a factor* s*, the same blob would be detected at* $(sx_0, sy_0; s^2t_0)$ *in the scaled image.*

DoG Pyramid

- Of course, for efficiency you can also build a DoG pyramid
  - an *oversampled* pyramid as there are multiple images between a doubling of scale.
  - Images between a doubling of scale are an *octave.*



Multi-scale Difference of Gaussian Blob detector



Summary

- Interest points have loads of applications in computer vision.
  - They need to be robustly detected, and invariant to rotation, lighting change, etc.
- We've looked at two types: corners and blobs
  - Harris & Stephens is a common corner detector
  - Finding extrema in a multi scale DoG pyramid provides a robust blob detector
- Scale space theory allows us to find features (corners and blobs) of different sizes

Further reading

- Harris & Stephens
  - Original paper by C. Harris and M. Stephens (1988) http://www.bmva.org/bmvc/1988/avc-88-023.pdf
  - Material also partially covered in Mark's book pp. 159-165
- Scale space http://en.wikipedia.org/wiki/Scale_space_representation
- Blob detection
  - Wikipedia http://en.wikipedia.org/wiki/Blob_detection#The_Laplacian_of_Gaussian
  - https://medium.com/@vad710/cv-for-busy-devs-improving-features-df20c3aa5887#_=_
  - Lowe's seminal SIFT paper also has a good description of finding blobs using the DoG in section 3: http://www.cs.ubc.ca/~lowe/papers/ijcv04.pdf

Exercises

- **Practical exercises**
  - Harris Corner Detection:
    - org.openimaj.image.feature.local.interest.HarrisIPD class in OpenIMAJ
    - https://opencv24-python-tutorials.readthedocs.io/en/latest/py_tutorials/py_feature2d/py_features_harris/py_features_harris.html
  - DoG
    - Dog is a part of SIFT in OpenCV
      https://opencv24-python-tutorials.readthedocs.io/en/latest/py_tutorials/py_feature2d/py_sift_intro/py_sift_intro.html
    - Chapter 5 of the OpenIMAJ tutorial covers finding DoG blobs

* *Acknowledgements:* Based on earlier Computer Vision lecture slides by Dr. Jon Hare.