# Deploying Wireless Sensor Networks under Limited Mobility Constraints

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**Abstract**—In this paper, we study the issue of sensor network deployment using limited mobility sensors. By limited mobility, we mean that the maximum distance that sensors are capable of moving to is limited. Given an initial deployment of limited mobility sensors in a field clustered into multiple regions, our deployment problem is to determine a movement plan for the sensors to minimize the variance in number of sensors among the regions and simultaneously minimize the sensor movements. Our methodology to solve this problem is to transfer the nonlinear variance/movement minimization problem into a linear optimization problem through appropriate weight assignments to regions. In this methodology, the regions are assigned weights corresponding to the number of sensors needed. During sensor movements across regions, larger weight regions are given higher priority compared to smaller weight regions, while simultaneously ensuring a minimum number of sensor movements. Following the above methodology, we propose a set of algorithms to our deployment problem. Our first algorithm is the Optimal Maximum Flow-based (*OMF*) centralized algorithm. Here, the optimal movement plan for sensors is obtained based on determining the minimum cost maximum weighted flow to the regions in the network. We then propose the Simple Peak-Pit-based distributed (*SPP*) algorithm that uses local requests and responses for sensor movements. Using extensive simulations, we demonstrate the effectiveness of our algorithms from the perspective of variance minimization, number of sensor movements, and messaging overhead under different initial deployment scenarios.

**Index Terms**—Sensor networks, deployment, limited mobility sensors.

## 1 Introduction

Provisioning sensors with mobility is a topic that has received significant attention recently. The XYZ sensor platform in [1], sensors in DARPA's self-healing minefield program in [2], and the Robomote platform in [3] are recent instances of mobility-enabled sensor models. Basically, the motivation behind this line of activity is the significant advantages that today's sensor networks will be able to leverage from mobility in sensors. For instance, using mobility, sensors can move toward coverage holes to enhance the quality of initial deployment [4], [5], [6], better routes for packets can be found [7], data reliability can be enhanced if sensors move closer to events [8], etc. However, despite the advantages that mobility offers to sensor networks, there is one critical constraint on it that cannot be avoided. Sensors are severely energy constrained and available energy has to be shared for sensing, data processing, transmission, etc. Since mobility also consumes energy, it is very likely that there is a limit on the overall movement distance capability of the sensors. This is especially true in environments when sensors have to work unattended after deployment and where recharging sensors is not always feasible (e.g., hostile zones, battlefields, etc.)

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*Manuscript received 10 Sept. 2005; revised 8 June 2006; accepted 28 Nov. 2006; published online 7 Feb. 2007.*
*For information on obtaining reprints of this article, please send e-mail to: tmc@computer.org, and reference IEEECS Log Number TMC-0270-0905.*
*Digital Object Identifier no. 10.1109/TMC.2007.1032.*

To validate our above claim on mobility limitations, we briefly discuss two recent mobile sensor models implemented in practice. Lymberopoulos and Savvides in [1] have designed a motion-enabled and power-aware sensor node platform. A battery-enabled miniature geared motor actuates sensor motion. The maximum movement distance of sensors in this design is 165 meters. In another development, as part of the self-healing minefield program, DARPA has designed a class of sensors with limited hop-by-hop mobility to detect and repair breaches in battlefields [2]. Mobility here is powered by a fuel-propeller mechanism and the sensors can make up to 100 hops. While the internal mobility semantics may be different in both models, the fact is that the sensor's maximum movement distance is limited.

In this paper, we address an important sensor network deployment problem under mobility constraints on sensors. The sensor network in our problem is a square field that has been clustered into multiple regions. The deployment objective is for each region to have a certain number of sensors (denoted by $\bar{k}$) that are application decided. In this scenario, our problem statement is: Given a deployment of limited mobility sensors in the network, the objective is to determine a sequence of sensor movements in order to minimize the variance in the number of sensors from $\bar{k}$ among all regions in the network and simultaneously minimize the total number of sensor movements.

**Motivation**. Our deployment problem is representative in many sensor network applications. The approach to cluster sensor networks into regions has been widely adopted in practice [9], [10], [11], [12] as clustering enhances scalability, improves routing and power efficiency, provides better support to higher level functionality, etc. The desired number of sensors $(\bar{k})$ per region can be contingent on one or more factors, including sensing, fault tolerance, resilience to attacks, lifetime, etc. For instance, due to physical limitations on sensors and associated hardware, the sampling rate at which sensors can sense the environment may be limited (e.g., 100 kHz for acoustic sensors [13], and 4,200 Hz for magnetometers [14]). Thus, in applications where the environment needs to be sensed quite frequently, or at all times of operation (e.g., intruder tracking, military surveillance, etc.), multiple sensors per region have to be deployed to meet sensing objectives. Second, there may be obstacles in the regions or external factors (like heat, vibration, etc.) that affect sensing ranges during network operation. Such sensing dynamics can be compensated with multiple sensors per region. Furthermore, fault tolerance and resilience to attacks improve with multiple sensors, lifetime can be prolonged using role rotation among multiple sensors, etc. Hence, multiple sensors per region provide many benefits to sensor networks. However, the desired $\bar{k}$ sensors per region requirement may not always be satisfied. For example, if sensors are randomly deployed (sprayed from a vehicle, airdropped, etc.), the requirement is hard to satisfy. Even if, at initial deployment, all regions have $\bar{k}$ sensors, as time goes on, faults, failures, energy losses, etc., can violate this requirement. In such cases, the limited mobility sensors have to self-adjust their positions to correct such violations.

Recently, some works have appeared where sensor mobility is leveraged to enhance deployment [4], [5], [6], [15]. However, the key shortcoming in such works is that sensor mobility limitations are not explicitly considered. Specifically, it is assumed that, if a sensor wishes to move to a new location, it can do so without any restriction in its movement distance. However, as discussed in the sensor mobility instances above, this may not always be true. Under hard mobility constraints, existing works have limited applicability. For instance, in the well-known virtual force approach [4], [6], [15], sensors exert *virtual forces* among themselves. Two sensors repel (or attract) each other if they are too close (or too far apart). By balancing virtual forces, sensors spread themselves in the field. However, under hard mobility constraints, two sensors may not be able to achieve force balance if the distance required to be traversed is too large. Second, the virtual force approach results in several back and forth movements during force balancing, which, across many iterations, will rapidly deplete sensor mobility capacity. For similar reasons, other works on mobility-assisted deployment also have limited applicability under hard sensor mobility constraints (more discussions on the challenges of limited mobility appear in the next section). In this paper, we address the issue of limited mobility sensor networks deployment.

**Contributions**. In this paper, we design a set of movement algorithms for our deployment problem that can be executed by limited mobility sensors. Our contributions are:

*A Methodology for Translating Our Nonlinear Optimization Problem*. Our first contribution is a weight-based methodology that translates our nonlinear variance objective into a linear one. We propose a weight assignment rule for regions depending on $\bar{k}$ so that, when sensors move, larger weight regions are given higher priority to balance sensor movements among all regions. We then define a new linear objective function called *Score* that captures weighted sensor movements and prove that maximizing the *Score* minimizes the deployment *Variance* and vice versa. The number of sensor movements is minimized by treating each movement as a cost and minimizing the overall costs during *Score* maximization.

*The Optimal Maximum Flow-Based Centralized Algorithm*. Our first algorithm is the Optimal Maximum Flow-based (*OMF*) centralized algorithm. Here, the sensor network at initial deployment is translated into a graph $(G_V)$. Vertices in $G_V$ represent regions and are assigned appropriate weights based on the above methodology. Edges represent movement ability between regions and are assigned corresponding capacities and costs. We first show how the *minimum cost maximum weighted flow plan* in $G_V$ maximizes the *Score* with minimum cost. We then show how this flow plan can be translated as a sensor movement plan that minimizes deployment *Variance* and sensor movements in the network. Note that the *maximum weighted* flow problem is similar to the maximum flow problem except that each target (vertex) has a weight and the objective is to maximize the summation of the flow amount to each target multiplied with the target weight. The maximum flow problem is its special case where the weight of each target is one.

*The Simple Peak-Pit-Based Distributed Algorithm*. We then propose a local, light-weight, and purely distributed Simple Peak-Pit-based (*SPP*) algorithm that is executed by sensors themselves. Regions needing sensors send local requests containing *weights* based on the number of sensors needed. Surplus regions that receive the requests will serve them in a descending order of weights, along with minimizing sensor movements in serving them. As discussed subsequently, path feasibility (to guarantee unbroken chain of movements) is ensured in our algorithms before sensors make real movements.

*Theoretical Analysis and Performance Evaluations*. We conduct a detailed theoretical analysis and performance evaluations of our algorithms. We formally prove the optimality of our *OMF* algorithm in minimizing variance and the number of sensor movements and derive its complexity. We then conduct extensive simulations to evaluate the performance of our algorithms. For comprehensiveness, we also simulate the well-known Virtual Force algorithm [4]. In general, the *OMF* algorithm (being optimal) achieves the best variance and sensor movement minimization. However, under certain scenarios (small $\bar{k}$, uniform initial deployment), performance of the *SPP* algorithm is close to the *OMF* algorithm. We also study communication overhead in our algorithms. We observe that the overhead in the *SPP* algorithm is generally lower. However, when initial deployment is highly concentrated, the overhead in the *OMF* algorithm is quite close to the *SPP* algorithm, while being smaller in some cases. Finally, we observe that all our algorithms have better performance than the virtual force algorithm with less overhead. As pointed out before, this is due to many back and forth sensor movements in the virtual force algorithm resulting in rapid expiration of sensor mobility capacity.

Our paper is organized as follows: In Section 2, we formally define our deployment problem and detail the methodology of our proposed algorithms in Section 3. In Section 4, we present our *OMF* algorithm and prove its optimality. In Section 5, we present our *SPP* algorithm and its features. We present some discussions in Section 6 and performance evaluations in Section 7. Related work is presented in Section 8 and we conclude our paper in Section 9.

## 2 Our Sensor Network Deployment Problem

### 2.1 Problem Definition

Our sensor network is a square field of size $Q$. It is clustered into two-dimensional square regions, where each region is of size $R$. The number of regions is denoted as $S$ ($S = (\frac{Q}{R})^2$). We denote the number of sensors in region $i$ at the time of initial deployment as $n_i$. The deployment objective is for each region to have a certain number of sensors, denoted by $\bar{k}$. At the time of initial deployment, not all regions will have $\bar{k}$ sensors. The sensors deployed are limitedly mobile. If a sensor moves from one region to any of its adjacent neighboring regions, we consider that as *one* hop made by the sensor. We denote $H$ as the maximum number of such hops a sensor is capable of. In this context, our problem statement is: Given a sensor network with $S$ regions, each of size $R$, and an initial deployment of $N$ limited mobility sensors, we want to determine a sequence of sensor movements so that 1) at the conclusion of movements, the variance in the number of sensors from $\bar{k}$ among all the regions in the network with less than $\bar{k}$ sensors is minimized, and 2) the overall number of hops of the limited mobility sensors is also minimized. Denoting $k_i$ as the number of sensors in a region $i$ at the conclusion of sensor movements, the variance $Var$ is

$$Var = \frac{1}{S}\sum_{i=1}^{S}\left(\bar{k} - min(k_i, \bar{k})\right)^2. \quad (1)$$

Denoting $h_i$ as the number of hops made by sensor $i$ (where $h_i \leq H$) and denoting $N$ as the number of sensors initially deployed, the overall number of sensor movement hops is

$$M = \sum_{i=1}^{N} h_i. \quad (2)$$

Our problem is to simultaneously minimize two objectives, namely, $Var$ (a nonlinear function) and $M$.

*Problem Features*. Our problem is general since we place no restriction on $\bar{k}$. If $\bar{k} = 1$, then the requirement is *one* sensor per region. To enhance reliability, $\bar{k}$ can be set larger than 1. Also, it is not necessary that $\bar{k}$ is the same for all regions. In nonuniform environments, some regions may need more sensors than others, meaning that $\bar{k}$ is different for different regions. The *Variance* definition still holds, except that $\bar{k}$ in (1) becomes $\bar{k}_i$ for region $i$. Our problem is also not contingent on the number of mobile sensors. It holds even when only a part of the sensors in the network are mobile.[1]

An important feature of our problem is that we do not minimize the variance in number of sensors among *all* regions from $\bar{k}$. We minimize it among only the regions that have *less* than $\bar{k}$ sensors at final deployment, which is captured by the term $min(k_i, \bar{k})$ in (1). In many cases, sensors are *overdeployed*. When the deployment objective is only $\bar{k}$ sensors per region, the nature of our problem will not let extra sensors move when the requirement of at least $\bar{k}$ sensors among all regions has been met. This is to preserve the mobility of sensors in such cases. Eventually, when some sensors fail (due to faults, power losses, etc.), the deficiency in the $\bar{k}$ requirement can be met by the spare sensors whose limited mobility was initially preserved, effectively complementing the motivations for overdeployment.

1. In the following, we discuss solutions for the basic problem first. Extensions to other problems are discussed later.

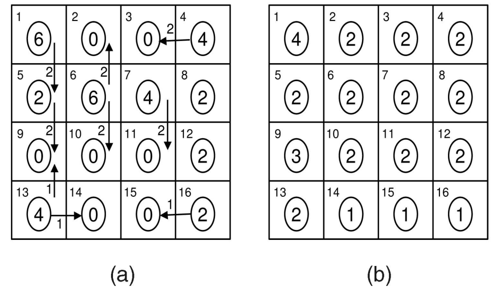

Fig. 1. (a) An instance of initial deployment and an intuitive movement plan to minimize variance and (b) the resulting deployment.

*Assumptions*. We make the following assumptions: We assume that $min\{\frac{S_{sen}}{\sqrt{2}}, \frac{S_{tr}}{\sqrt{5}}\} \geq R$, where $R$ is the region size and $S_{sen}$ and $S_{tr}$ are a sensor's sensing and transmission ranges, respectively. If each region has $\bar{k}$ sensors at final deployment, then $\frac{S_{sen}}{\sqrt{2}} \geq R$ means every point in each region is *covered* by $\bar{k}$ sensors, and $\frac{S_{tr}}{\sqrt{5}} \geq R$ means a sensor in any region can communicate with $\bar{k}$ sensors in each of its four adjacent regions. We also assume sensors are homogeneous in sensing and transmission ranges, and they are unaffected during network operation as in [4], [5], [6], [15]. We assume a free space radio propagation model, where there exists a clear line of sight path between two communicating sensors in the network. We assume that each sensor knows which region it resides in. To do so, sensors can be provisioned with GPS devices or methods in [16] can be used, where sensor location are determined using sensors themselves as landmarks. For simplicity, we first assume that the regions to which a sensor can move to are regions in its adjacent left, right, top, and bottom directions only (denoted as neighboring regions). After discussing this case, the general case where a sensor can move in any arbitrary direction is discussed next. Also, we first assume that the network is not partitioned. The issue of partitions is discussed later.

### 2.2 An Example of Our Problem and Challenges

We illustrate our problem further with an example. Consider an instance of initial deployment in the network, shown in Fig. 1a. The number of inside circles denotes the number of sensors in that region. The number in the upper left corner denotes the corresponding region ID. Let the maximum number of hops $H = 1$ and $\bar{k} = 2$. There are 32 sensors initially deployed. At time of initial deployment, regions 2, 3, 9, 10, 11, 14, and 15 have less than $\bar{k}$ sensors. An intuitive way to minimize the variance from $\bar{k}$ is to let neighboring regions locally synchronize for movement.

Using local information exchanges, it is likely that the sensors move according to the sequence shown in Fig. 1a. The arrows indicate direction of movement and the numbers beside arrows indicate number of sensors moved from that region.

Let us denote regions that have at least *one* sensor at initial deployment as *source regions* (or *sources*) and denote regions that do not have any sensor at initial deployment as *holes*. Region 4 is a source and moves sensors to region 3, since region 3 is close to it and needs sensors. With local information exchange, region 7 will not move sensors to region 3; rather, it will move sensors to region 11 after synchronizing with regions 4 and 6. Similarly, since region 13 has four sensors and since regions 9 and 14 do not have any sensor, a sensor moves from region 13 to fill regions 9 and 14. But, since region 5 has two sensors and it receives two sensors from region 1, two sensors move from region 5 to region 9. Other regions also follow the same intuition and synchronization to move sensors. The final deployment is shown in Fig. 1b. Note that regions 14, 15, and 16 have only one sensor. In fact, with this movement plan, minimum variance (equal to 0) cannot be achieved. Consider region 14. The only way region 14 can get a sensor is from regions 13, 10, or 15. However, regions 10 and 15 initially did not have any sensor. Thus, no sensor can move to region 14 via regions 10 and 15 since $H = 1$. Similarly, no sensor can move to region 13 via region 9. Besides, region 13 has no extra sensor now. Consequently, all paths to region 14 are *blocked* in this movement plan. A pertinent question to raise at this point is whether there exists an optimal movement plan that can make the variance 0. If so, what is the plan, or more importantly, what are the challenges that need to be addressed in this movement plan? We discuss both issues below.

There are two key challenges to our problem. The first challenge is due to our objective of simultaneously minimizing variance and the number of sensor movement hops. Consider the movement plan in Fig. 1a. Region 6, which has six sensors in it, wishes to fill regions 2 and 10. The intuition is because both regions are empty and region 6 is close to them. But, this plan, which attempts to minimize hops, cannot minimize variance. There is thus a conflict that may be present in minimizing variance and the number of hops using local information. For optimum deployment, region 6 should move sensors to regions 10 and 15 (in Fig. 2a). The path to region 15 may appear long, but it is the one that makes the global variance 0, shown in Figs. 2a and 2b.

The second challenge is, due to limited mobility, if a sensor in one region wishes to move to some far away region, then, depending on $H$, there must be mobile sensors in one or more intermediate regions (like a chain) in the corresponding path (if $H = 1$, then all intermediate regions in the path need to have a mobile sensor). If there is no mobile sensor after a sensor has traveled $H$ hops to a particular region, no sensor can move beyond that region, resulting in blocked paths. For instance, in Fig. 1b, although region 1 still has extra mobile sensors, all paths from region 1 to region 14 are blocked. The challenge is in determining such optimal chains for sensor movements.

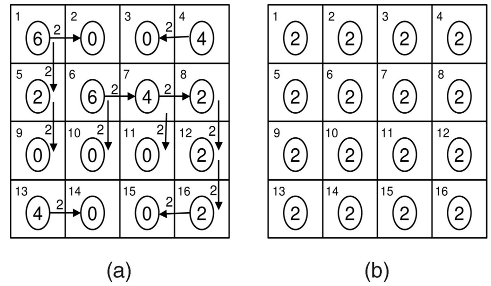

Fig. 2. (a) An instance of initial deployment and an optimal movement plan and (b) the resulting deployment.

Such a path may traverse many intermediate regions, as shown in Fig. 2a, where the path from region 6 to region 15 traverses five regions and a chain of movements is possible in each intermediate region. Determining such a chain of movements for optimal variance and sensor movement hops is not trivial. If sensors make purely local decisions, then optimality cannot be achieved. Also, it is preferable for sensors to make a movement plan (which sensors should move and where) prior to their movement in order to avoid erroneous movements and compensating for such errors later on.

## 3 Methodology of Our Algorithms

Our sensor network deployment problem has two objectives: 1) minimizing variance and 2) minimizing the overall number of movement hops of the limited mobility sensors. In the following, we discuss our methodology to achieve both objectives: Consider any two regions $i$ and $j$ in a sensor network. Let the number of sensors in region $i$ be less than that of region $j$, both of which are less than $\bar{k}$. If one sensor is available to move to one of these two regions, the contribution to global variance minimization in the network is larger if the sensor moves to region $i$ than if it moves to region $j$. Our methodology to capture this notion of priority is by *weight* assignment to regions. When sensors move, larger weight regions are given priority compared to smaller weight regions with the objective of global variance minimization. In the above example, region $i$ will have a larger weight than region $j$ to prioritize sensor movements to region $i$. We discuss our methodology in further detail below.

The overall variance is minimized (equal to 0) when each region has at least $\bar{k}$ sensors. Thus, for each region $i$ in the sensor network, we first create $\bar{k}$ *virtual sinks* (or simply *sinks*) in order to allocate a position (*virtually*) for each of the $\bar{k}$ sensors that are needed in each region. Let each sink in region $i$ be denoted by $s_i^1, s_i^2, s_i^3, \ldots, s_i^{\bar{k}}$. For each sink $s_i^1, s_i^2, s_i^3, \ldots, s_i^{\bar{k}}$, we assign weights to them denoted by $w_i^1, w_i^2, w_i^3, \ldots, w_i^{\bar{k}}$ respectively to prioritize movements toward larger weight sinks. The weights are

$$w_i^j = 2 * j - 1 \; (1 \leq j \leq \bar{k}). \tag{3}$$

Note that sink $s_i^m$ has more weight than $s_i^n$ if $m > n$. Also, $w_i^m = w_j^m$ for any two regions $i$ and $j$.

After sensors move toward sinks (according to their weights), some sinks will have sensors while some do not.

In order to capture the presence of a sensor in each sink among the multiple regions (after sensors move), we define the following function:

$$\phi_i^j = \begin{cases} 1, & \mathit{if\ sink\ } s_i^j \mathit{\ has\ a\ sensor}, \\ 0, & \mathit{otherwise}. \end{cases} \quad (4)$$

There is a constraint for the function $\phi$. If $\phi_i^j = 1$, then $\phi_i^m = 1$ for all $m > j$. We are in effect saying here that, if sink $s_i^j$ in region $i$ has a sensor, then each sink $s_i^m$ in region $i$ with larger weights (i.e., $m > j$) should have a sensor. The function $\phi$ captures whether a sink contains a sensor. We define a new metric here called *Score* as follows:

$$Score = \frac{1}{S}\left(\sum_{i=1}^{S}\sum_{j=1}^{\bar{k}} \phi_i^j \times w_i^j\right). \quad (5)$$

The *Score* function is the summation of weights of those sinks (for all regions) that contain a sensor in them. Clearly, the *Score* is larger when there are more sinks containing a sensor. The *Score* function also considers the weight of a sink. As such, in the event that a sensor can move to more than one sink, the *Score* is larger when the sensor moves to the sink with the largest weight. Therefore, during sensor movements, when we attempt to maximize the *Score*, we are in effect ensuring that as many sinks as possible contain a sensor while also ensuring that larger weight sinks always have higher priority compared to smaller weight sinks. We now have the following theorem:

**Theorem 1.** *A sequence of sensor movements that maximizes* Score *will minimize the variance $Var$ and vice versa. (Please see the Appendix for proof.)*

From the above theorem, we can see that our original nonlinear variance objective can be translated to a linear objective. In this paper, we propose three algorithms for our deployment problem following the above methodology. In our algorithms, we create sinks for each region depending on the number of sensors needed. Each sink has a weight associated with it such that, when sensors move, sinks with larger weights have higher priority compared to sinks with smaller weights. The goal of our algorithms is to maximize *Score*, which, according to Theorem 1, minimizes the variance $Var$.

The second objective of our problem is minimizing the total number of sensor movement hops. We achieve this goal by treating sensor movement hops as costs and minimizing such costs in our algorithms. When there are multiple sinks in other regions with same weights, our algorithms will ensure that sensors move to sinks in those regions that are closer in terms of distance to be traversed. Clearly, larger weight sinks are still given priority compared to smaller weight sinks. However, with such movements, the resulting number of overall sensor movement hops is minimized, along with maximizing *Score*. If a sensor in a region does not need to move to another region, we treat the sensor as *virtually* moving to a sink in the same region. Such a movement incurs 0 cost.

# 4 The Optimal Maximum Flow-Based Centralized Algorithm

Our first algorithm is the Optimal Maximum Flow-based (*OMF*) centralized algorithm. In the *OMF* algorithm, the sensor network at initial deployment is translated as a graph structure. The algorithm then determines the minimum cost maximum weighted flow in the graph. The corresponding flow plan in the graph is translated as a movement plan for the sensors in the network.

## 4.1 Description of the Algorithm

In the following, we describe our *OMF* algorithm from the perspective of a base-station executing the algorithm. An alternate approach to executing the *OMF* algorithm is presented in Section 6.1.

**Algorithm 1** Pseudocode of the *OMF* algorithm

1: Collect the information on the number of sensors in each region in the sensor network.
2: Construct a graph $G_V(V_V, E_V)$ using the above region information, desired number of sensors per region $\bar{k}$, and the sensor mobility capacity $H$. $G_V$ models the sensor network at the initial deployment time.
3: Determine the minimum cost maximum weighted flow from source regions to weighted sinks in $G_V$.
4: Determine a movement plan for the sensors in the sensor network based on the above flow plan in $G_V$.
5: Forward the movement plan to sensors in the network.

### 4.1.1 Steps in Algorithm Execution

Algorithm 1 shows the sequence of steps in the *OMF* algorithm. In Step 1, each sensor in the network identifies which region it resides in. Sensors then forward information on the number of sensors in their region toward the base-station. For routing packets toward the base-station, protocols like [17], [18], [9], [19] can be used, where the protocols route packets toward intended destinations in the network (base-station in our case) using shortest paths. The base-station thus obtains information on the number of sensors in all regions in Step 1. As pointed out before, for determining which region a sensor resides in, sensors can be provisioned with GPS devices or methods proposed in [16] can be used where the location of sensors is determined by using sensors themselves as landmarks. Also, we assume that the network is connected without partitions. The issue of partitions is discussed later.

In Step 2, the base-station constructs a *virtual graph* ($G_V$), whose vertices and edges model the regions and sensor movement ability between regions, respectively, at initial deployment. In Step 3, the base-station determines the maximum weighted flow to the sinks in $G_V$ (which maximizes (5)) with minimum cost. In Step 4, the flow plan in the $G_V$ corresponding to the minimum cost maximum weighted flow is translated as a movement plan for the sensors. In Step 5, the base-station forwards the movement plan (which sensors should move and where) to the sensors in the network. We subsequently prove that this movement plan minimizes the variance and overall number of sensor movement hops in the sensor network. Each of Steps 2, 3, 4 and 5 in our *OMF* algorithm is discussed in detail below.

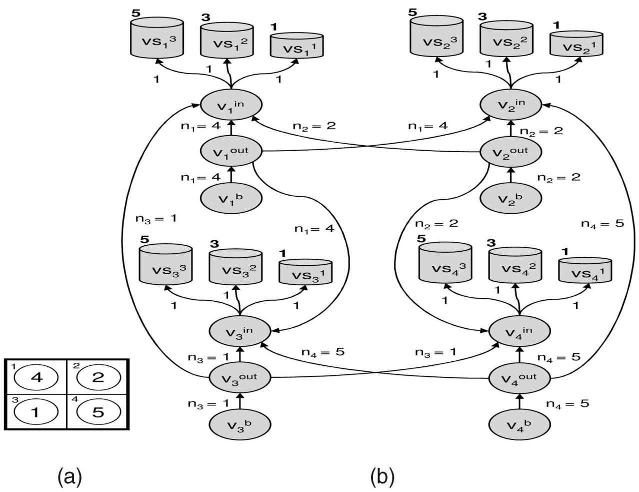


Fig. 3. (a) An instance of the initial network deployment and (b) the corresponding virtual graph $G_V$.

### 4.1.2 Constructing the Virtual Graph $G_V$

We now discuss Step 2, which involves the construction of the virtual graph denoted by $G_V(V_V, E_V)$. Before we discuss $G_V$, we introduce the notation of *reachability* between regions. For any region $i$ in the sensor network, we denote its *reachable* regions as those regions to which a sensor from region $i$ can move. Obviously, the reachable regions depend on the maximum movement hops $H$. We first assume that the regions to which a sensor can move are regions in its adjacent left, right, top, and bottom directions only. Thus, if $H = 1$ in Fig. 1, then the reachable regions for region 1 are regions 2 and 5. If $H = 2$, the reachable regions are regions 2, 3, 5, 6, and 9.

The construction of $G_V$ involves:

1. the establishing of vertices and edges for each region in the sensor network and creation of sinks for each region,
2. the establishing of reachability relationship between the regions,
3. adding weights to sinks following our discussions in Section 3, and
4. adding costs to edges to capture sensor movements across regions.

The objective of this construction is to ensure that $G_V$ models the sensor network and identifies sources, sinks, and reachability relationships among regions. Fig. 3a shows an instance on initial deployment for a $2 \times 2$ network with four regions and 12 sensors and where $\bar{k} = 3$ and $H = 1$. Its corresponding virtual graph $G_V$ is shown in Fig. 3b. The numbers inside the circles in Fig. 3a denote the number of sensors in the corresponding region in the sensor network. In the following, we describe the virtual graph construction process in detail. Let us first describe the establishment of vertices and edge assignment in $G_V$ for one arbitrary region in the sensor network. Without loss of generality, consider region $i$ with $n_i$ sensors initially. For this region, we create a vertex called the *base* vertex of region $i$ (denoted by $v_i^b$) in $G_V$. We create vertex $v_i^{out}$ to keep track of the number of sensors that can move out from region $i$. We then create $\bar{k}$ sink vertices for region $i$ (due to deployment requirement of $\bar{k}$ sensors per region). The sink vertices for region $i$ are denoted by $vs_i^1, vs_i^2, vs_i^3, \ldots, vs_i^{\bar{k}}$. We also create vertex $v_i^{in}$ as a proxy for the $\bar{k}$ sink vertices.

The next step is adding edges between vertices for this region. An edge of capacity $n_i$ is added from $v_i^b$ to $v_i^{out}$. This means that up to $n_i$ sensors can move from region $i$. Since $v_i^{in}$ is a proxy for the sink vertices, the capacity from $v_i^{out}$ to $v_i^{in}$ is also $n_i$. From $v_i^{in}$, an edge is added to each of the vertices $vs_i^1, vs_i^2, vs_i^3, \ldots, vs_i^{\bar{k}}$ with capacity 1. Since the deployment requirement is $\bar{k}$ sensors per region, we allow up to *one* sensor to move to each sink (for $\bar{k}$ such sinks). All other regions are treated similarly in $G_V$. For example, for region 1 in $G_V$ in Fig. 3b, we create six vertices corresponding to the *base* vertex ($v_1^b$), *in* vertex ($v_1^{in}$), *out* vertex ($v_1^{out}$), and $\bar{k} = 3$ sink vertices ($vs_1^1$, $vs_1^2$, and $vs_1^3$). Edges between the vertices and their capacities for region 1 are also shown. All other regions are treated similarly.

The second step is establishing a reachability relationship among the regions into $G_V$. Let us consider two arbitrary regions $i$ and $j$ that are reachable from each other. In $G_V$, edges are added from $v_i^{out}$ to $v_j^{in}$, with edge capacity $n_i$, which is the number of sensors in region $i$. This is to allow up to $n_i$ sensors to move from region $i$ to region $j$. Correspondingly, edges are added from $v_j^{out}$ to $v_i^{in}$ with capacity $n_j$. For example, in Fig. 3b, there is an edge from $v_1^{out}$ to $v_2^{in}$ with capacity $n_1 = 4$ and an edge from $v_2^{out}$ to $v_1^{in}$ with capacity $n_2 = 2$ since regions 1 and 2 are reachable from each other.

The next steps are weight assignment to sinks and cost assignment to edges. Consider region $i$ again. For sinks $vs_i^1$, $vs_i^2$, $vs_i^3, \ldots, vs_i^{\bar{k}}$ in region $i$, we denote their weights as $w_i^1, w_i^2, w_i^3, \ldots, w_i^{\bar{k}}$. Following from the discussions in Section 3, the values for the weights are $1, 3, 5, \ldots, 2\bar{k} - 1$, respectively. Since $\bar{k} = 3$ in the example in Fig. 3, we have weights 1, 3, and 5 for the sinks (shown alongside the sink vertices). Note that $w_i^m$ is larger than $w_i^j$ if $m > j$. We now discuss costs for edges between regions in $G_V$ in order to capture number of sensor movements. If a sensor moves from its region to its adjacent region, then it denotes one hop made by the sensor. Let us consider two regions $i$ and $j$ in the sensor network that are reachable from each other. Let the distance between them in terms of number of hops be $d_{i,j}$. That is, $d_{i,j}$ denotes the minimum number of hops required for a sensor in region $i$ to move to region $j$ (or vice versa). For instance, in Fig. 1a, $d_{1,3} = d_{3,1} = 1$. Obviously, $d_{i,j} \leq H$ if regions $i$ and $j$ are reachable from each other. To incorporate this in $G_V$, between any two reachable regions $i$ and $j$, the costs of edges from $v_i^{out}$ to $v_j^{in}$ and the costs of edges from $v_j^{out}$ to $v_i^{in}$ are assigned as $d_{i,j}$. Apart from the above, the only remaining edges in $G_V$ are the ones from $v_i^b$ to $v_i^{out}$, from $v_i^{out}$ to $v_i^{in}$ , and from $v_i^{in}$ to $vs_i^1, vs_i^2, vs_i^3, \ldots, vs_i^{\bar{k}}$ (for all regions $i$). These edges denote internal movements within a region and the cost for these edges is set as 0. The costs of edges in $G_V$ are not shown in Fig. 3.

At this point, Step 2 of our *OMF* algorithm is completed. The base-station has constructed $G_V$, which models the sensor network at initial deployment. Before proceeding to Step 3, we define a flow plan $Z$ in $G_V$ and a metric $W$. $Z$ is the sequence of flows (in $G_V$) that meets the following condition: $W = \sum_{i=1}^{S} \sum_{j=1}^{\bar{k}} (f_i^j * w_i^j)$ is maximized, where $f_i^j$ is the subflow to sink $vs_i^j$ in flow plan $Z$. We call $Z$ a maximum weighted flow plan in $G_V$. If the cost of $Z$ is minimized, $Z$ is called a *minimum cost maximum weighted flow* plan. With sinks in $G_V$ having weights associated with them, a maximum weighted flow plan must maximize the number of sink vertices that receive a flow and prioritize flows to larger weight sinks first compared to smaller weight sinks in $G_V$. Since the capacity of the edge from $v_i^{in}$ to $vs_i^j$ in $G_V$ is 1, $f_i^j$ meets the constraint of function $\phi$ defined in (4). Since $G_V$ is a translation of the sensor network, the flow plan $Z$ in $G_V$ can be translated as a corresponding movement plan for sensors in the sensor network (exactly how this is done is discussed in Section 4.1.4). From the definition of *Score* in (5) and $W$ above, the corresponding sensor movement plan maximizes the *Score* with minimum cost which, in turn, minimizes $Var$ (from Theorem 1) with minimum cost. To summarize, with the construction of $G_V$ in place, the variance/movement minimization problem now becomes one, where the weighted flow to sinks in $G_V$ is to be maximized with minimum cost.

#### 4.1.3 Computing the Minimum Cost Maximum Weighted Flow in the Virtual Graph $G_V$

We now proceed to Step 3 in the *OMF* algorithm, where we determine the minimum cost maximum weighted flow in $G_V$. In the following, we present our algorithm to determine the minimum cost maximum weighted flow in $G_V$. Our approach to maximize the weighted flow is to translate larger weight sink vertices as lower cost sink edges. Thus, prioritizing flows to large weight sinks now becomes prioritizing flows through lower cost edges. This is the crux of our algorithm described below.

**Algorithm 2** Pseudocode for computing the minimum cost maximum weighted flow in $G_V$

1: Input: $G_V(V_V, E_V)$, $H$, $S$, and $\bar{k}$
2: Output: Graph $G_V^m(\bar{V}, \bar{E})$ and Minimum Cost Maximum Weighted Flow Plan $Z$ in $G_V$
3: $|E_V| =$ No. of Edges in $G_V$, $|V_V| =$ No. of Vertices in $G_V$
4: $|\bar{E}| = |E_V| + S \times (\bar{k} + 1)$
5: $|\bar{V}| = |V_V| + 2$
6: Add vertices $S_{source}^v$ and $S_{sink}^v$ to $G_V$ to create graph $G_V^m$
7: **for** each region $i$ **do**
8:   **for** $j$ from 1 to $\bar{k}$ **do**
9:     Add edge from sink $vs_i^j$ to $S_{sink}^v$
10:     Assign corresponding edge capacity as 1
11:     Assign corresponding edge cost as $-(2j-1) \times H \times |\bar{E}|$
12:   **end for**
13:   Add edge from $S_{source}^v$ to $v_i^b$
14:   Assign corresponding edge capacity as $\infty$
15:   Assign corresponding edge cost as 0
16: **end for**
17: Determine the maximum flow value $|\bar{Z}|$ from $S_{source}^v$ to $S_{sink}^v$ in $G_V^m$
18: Determine the minimum cost flow plan $Z$ (for flow value $|\bar{Z}|$) from $S_{source}^v$ to $S_{sink}^v$ in $G_V^m$

Algorithm 2 is the pseudocode to determine the minimum cost maximum weighted flow in the virtual graph $G_V$. The input is $G_V(V_V, E_V)$, $H$, number of regions $S$, and $\bar{k}$. We first create a new graph from $G_V$ called $G_V^m(\bar{V}, \bar{E})$ as follows: We first create two new vertices called *Super Source* and *Super Sink*, denoted by $S_{source}^v$ and $S_{sink}^v$, respectively. Edges are added from each sink vertex to $S_{sink}^v$ with capacity 1 to allow only one sensor to move from each sink toward $S_{sink}^v$. The cost of the edges from sinks $vs_i^1, vs_i^2, vs_i^3, \ldots, vs_i^{\bar{k}}$ to $S_{sink}^v$ (for all regions $i$) are set as

$$-H \times |\bar{E}|, -3 \times H \times |\bar{E}|, -5 \times H \times |\bar{E}|, \ldots,$$
$$-(2\bar{k}-1) \times H \times |\bar{E}|,$$

respectively, where $|\bar{E}|$ is defined in Algorithm 2.[2] Finally, edges are added from $S_{source}^v$ to all base vertices (i.e., $v_i^b$ for all $i$) with capacity $\infty$ to allow any amount of flow from $S_{source}^v$. The costs for these edges are set as 0 since the flows through such edges are not actual sensor movements.

At this point (Step 16 in Algorithm 2), $G_V^m$ has been constructed. Determining the flow plan to maximize weighted flow to sinks with minimum cost in $G_V^m$ is a two-step process (Steps 17 and 18 in Algorithm 2). The base-station will first determine the maximum flow value ($|\bar{Z}|$) from $S_{source}^v$ to $S_{sink}^v$ in $G_V^m$. The maximum flow value $|\bar{Z}|$ indicates the maximum number of sinks that can get a sensor in $G_V^m$. However, this only indicates the maximum *number* of sinks. The determination of the maximum flow value does not consider the fact that sinks have different weights and larger weight sinks need to be accorded higher priority. Our objective, however, is to determine the *flow plan Z* (the actual flow among the edges) in $G_V^m$ such that a *weighted* flow to sinks is maximized with minimum cost. We do this in Step 18 by determining the minimum cost flow plan $Z$ (for maximum flow value $|\bar{Z}|$) in $G_V^m$, discussed further below.

We know that, when executing the minimum cost flow algorithm on any graph, flow is prioritized through edges with lower cost. By setting the edge costs from sinks to $S_{sink}^v$ as the negative of weights of the corresponding sink, we will achieve our objective of prioritizing flow to sinks with larger weights in determining the minimum cost flow to $S_{sink}^v$. There is one issue we have to resolve during cost assignment. Recall that sensor movements between reachable regions are considered as costs in $G_V^m$. Clearly, these costs will affect the minimum cost flow plan when determining flows to sinks with minimum cost in $G_V^m$. To prevent this from happening, the costs from sinks to $S_{sink}^v$ is assigned as the negative of the sink weights multiplied by a large constant (namely, $H \times |\bar{E}|$). This constant is large enough to ensure that the flow plan ($Z$) to maximize weighted flow in $G_V^m$ is not affected by the costs between reachable regions while still minimizing costs between reachable regions (that denote sensor movements). Before discussing how to translate this flow plan $Z$ into a sensor movement plan, we state the following theorem showing the relationship between $G_V^m$ and $G_V$:

**Theorem 2.** *The flow plan corresponding to the minimum cost maximum flow in $G_V^m$ is the flow plan corresponding to the*

2. The interpretation of $|\bar{E}|$ is discussed subsequently.

*minimum cost maximum weighted flow in $G_V$ (Please see the Appendix for proof).*

#### 4.1.4 Determining the Optimal Movement Plan from the Virtual Graph $G_V$

Once the minimum cost maximum weighted flow to each sink in $G_V$ (and the corresponding flow plan in all edges in $G_V$) is obtained, we proceed to Step 4 in Algorithm 1. In Step 4, we translate the flow plan from Step 3 into actual sensor movements as follows: Let $Z^V$ denote the flow plan (a set of flows) corresponding to the minimum cost maximum weighted flow algorithm in $G_V$, where the capacity of each flow is 1. Each flow $z^V(v_i^b, vs_j^x) \in Z^V$ is a flow from $v_i^b$ to $vs_j^x$ in $G_V$. The flow $z^V(v_i^b, vs_j^x)$ is of the form $\langle v_i^b, v_i^{out}, v_j^{in}, vs_j^x \rangle$. Thus, for the flow plan $Z^V$, we can map it to a corresponding movement plan $Z^S$ (set of movement sequences for sensors) in the sensor network. That is, for each $z^V(v_i^b, vs_j^x)$ $(\in Z^V)$ of the form $\langle v_i^b, v_i^{out}, v_j^{in}, vs_j^x \rangle$, the corresponding $z^S(i,j)$ $(\in Z^S)$ is of the form $\langle i, j \rangle$. Physically, this means that one sensor should move from region $i$ to region $j$. The sensor movement plan $Z^S$ (consisting of the set of all such $z^S$, obtained from $z^V$) is our output. This movement plan, which indicates which sensors should move and where to, is forwarded by the base-station to the sensors in the network.

### 4.2 Optimality of Our *OMF* Algorithm

Before discussing optimality, we first introduce the concept of feasible flows and movement sequences. We call a flow $z^V(v_i^b, vs_j^x)$ of the form $\langle v_i^b, v_i^{out}, v_j^{in}, vs_j^x, \rangle$ feasible in $G_V$ if there exists positive edge capacities from vertices $v_i^b$ to $v_i^{out}$, $v_i^{out}$ to $v_j^{in}$, and $v_j^{in}$ to $vs_j^x$. We call a movement sequence $z^S(i,j)$ of the form $\langle i, j \rangle$ feasible in the sensor network if there is at least one mobile sensor in region $i$ that can move to region $j$. We have the following lemma for a flow in $G_V$ and a sensor movement sequence:

**Lemma 1.** *A flow $z^V(v_i^b, vs_j^x)$ in $G_V$ is feasible if and only if the corresponding movement sequence $z^S(i,j)$ is feasible in the sensor network (Please see the Appendix for proof).*

We obtain the following corollary from Lemma 1:

**Corollary 1.** *For a feasible flow plan $\bar{Z}^V$ (set of all $z^V$) in $G_V$, a corresponding feasible sensor movement sequence plan $\bar{Z}^S$ (set of all $z^S$) can be found in the sensor network and vice versa (for proof, please refer to [20]).*

The following theorem shows that the movement plan obtained by our *OMF* algorithm optimizes both variance and the number of sensor movement hops:

**Theorem 3.** *Let $Z_{opt}^V$ be the minimum cost maximum weighted flow plan in $G_V$. Its corresponding movement plan $Z_{opt}^S$ will minimize variance and the number of sensor movement hops in the sensor network (please see the Appendix for proof).*

We now discuss the time complexity of the *OMF* algorithm. There are three phases in our algorithm in determining the optimal movement plan. The first is construction of $G_V(V_V, E_V)$ and $G_V^m(\bar{V}, \bar{E})$, the second is determining the maximum flow in $G_V^m$, and the third is determining the minimum cost flow in $G_V^m$. The time complexity is dominated by determining the maximum flow and minimum cost flow in $G_V^m$. Our implementation of the maximum flow algorithm is the Edmonds-Karp algorithm [21] and the minimum cost flow algorithm is the one in [22]. The resulting time complexity is $O(max(|\bar{V}||\bar{E}|^2, |\bar{V}|^2|\bar{E}|log|\bar{V}|))$. Here, $|\bar{V}|$ and $|\bar{E}|$ denote the number of vertices and edges in $G_V^m$ and are given by $|\bar{V}| = O(\bar{k}(\lceil \frac{Q}{R} \rceil^2))$ and $|\bar{E}| = O(\bar{k}H^2(\lceil \frac{Q}{R} \rceil^2))$, in which $Q$ is the sensor network size and $R$ is the region size.

## 5 The Simple Peak-Pit-Based Distributed Algorithm

In the above, we presented a centralized and optimal *OMF* algorithm to our deployment using our weight-based methodology. We now present the Simple Peak-Pit-based (*SPP*) algorithm to our deployment problem, which is local, light-weight, and purely distributed. In the *SPP* algorithm, regions request sensors from adjacent regions with weights attached to each request. As before, requests with larger weights are given higher priority when compared to requests with smaller weights, while simultaneously preferring shorter movement hops to satisfy requests. We first discuss some important notations used in the algorithm description. Regions in the network are classified into three types: *pits*, *peaks*, and *forwarders*. A *pit* is a region whose number of sensors is less than $\bar{k}$ and not more than any of its neighboring regions. A *peak* is a region whose number of sensors is larger than any of its neighboring regions. All other regions are *forwarders*. We define an *over-$\bar{k}$ forwarder* as a *forwarder* with more than $\bar{k}$ sensors and denote the *richest* neighbor of a region as a neighbor with the largest number of sensors.

In the *SPP* algorithm, a *pit* $i$ will request $\bar{k} - n_i$ sensors in its request (*REQ*). The *pit* $i$ will assign different weights to each of the $\bar{k} - n_i$ requested sensors as $w_i^j = 1, 3, 5, \ldots, 2j-1$, where $1 \leq j \leq (\bar{k} - n_i)$ (as before). Here, we let only *pits* send *REQs* so that *nonpit* regions will not compete with *pits* during requests to ensure that more deficient regions will be given priority. An *REQ* generated will be forwarded toward progressively richest neighbors to increase likelihood of *REQs* arriving at *over-$\bar{k}$ forwarders* or *peaks* on shorter paths. Recipients receiving *REQs* will sort all the requested sensors in the *REQs* by *weights* and serve those with larger *weights* first. Ties are broken by fulfilling requests with shorter paths first. We call the neighbors chosen for the next hop *tried* neighbors.

Algorithm 3 shows the pseudocode of our *SPP* algorithm. It is executed by each region $i$ independently and is event-driven. Using interregion communications, a region leader will be elected for coordination. Each leader obtains the number of sensors in its region and its four adjacent neighboring regions. The region leader of each *pit* will send an *REQ* to its *richest* neighbor, requesting the number of sensors needed. If multiple *richest* neighbors exist, ties are broken randomly. If some regions have no sensors, they can be assisted by neighboring leaders in sending our requests. We discuss this issue in further detail later.

**Algorithm 3** Pseudocode of the *SPP* algorithm run by region $i$

```
Region leader selection
while TRUE do
  switch type of event
  case region i becomes a pit:
    send REQ to richest untried neighbor;
  case receive REQ:
    put REQ into Queue(i);
    if region i is an over-k̄ forwarder then
      select REQs in Queue(i) to serve by weights and path
      lengths;
      send ACKs, move sensors and/or forward REQs
      accordingly;
    else if region i is a peak then
      select REQs in Queue(i) to serve by weights and path
      lengths;
      send ACKs, move sensors, and/or send FAILs
      accordingly;
    else
      forward REQ to richest neighbor;
  case receive ACK for pit j:
    forward ACK to j if i ≠ j;
  case receive FAIL for pit j:
    resend REQ to richest untried neighbor;
  case detect hole neighboring region j:
    if region i can provide sensor then
      move a sensor to j after random delay;
  end switch
end while
```

Due to limited mobility, when requests are sent out, it is important that path feasibility should be maintained during the selection of next hop *forwarder*. This means there should exist at least one mobile sensor on any continuous $H$ hop segment of the path a *REQ* traverses. Otherwise, mobile sensors on the other side of the segment will not be able to move back to the requesting *pit* due to limited mobility. In case there are not enough mobile sensors in a certain segment with $H$ hops on the path, the requested number of sensors in the *REQ* message should be adjusted since we can never move enough sensors back on the path. All the intermediate *forwarders* will reserve enough mobile sensors to guarantee the feasibility of the path.

When *REQs* are forwarded to *over-$\bar{k}$ forwarders* or *peaks*, some of them may or may not get served. Considering that the *REQ* with largest *weight* requested sensor may not always come first, the *over-$\bar{k}$ forwarder* or *peak* will put the *REQs* into its queue and serve them in periodic intervals of time. When serving multiple requested sensors with the same *weights*, those with shorter paths will be served first. An *over-$\bar{k}$ forwarder* will send *ACKs* back to the *pits* whose *REQs* contains sensors that will be served and forward the *REQs* if not all sensors can be served. Those forwarded *REQs* will be updated if part of the requested sensors are served eventually. A *peak* will send *ACKs* back to the *pits* whose *REQs* contains sensors that will be served and send *FAILs* back to *pits* if not all requests can be served.[3] Sensors will start moving after *ACKs* are sent following the reserved paths of the corresponding *REQs*.

After receiving the $ACK(s)$ and mobile sensor(s), each *pit* will inform its neighbors its new sensor number and *REQs* are generated if need be. After a *pit* or *forwarder* receives a $FAIL$, it will release the reserved path and resend *REQs* to its *richest untried* neighbor and so on. The algorithm terminates when each *pit* has either obtained at least $\bar{k}$ sensors or expiration of movement capability of the sensors, or if a certain number of requests have been tried without success by requesting sensors.[4]

It may happen that some regions have no sensors in them (i.e., *holes*) after initial deployment. A hole can be filled by one of its neighbors with extra mobile sensors after coordination by other nonempty neighbors. In case a hole cannot be filled directly due to none of its neighbors being able to provide an extra sensor, one of its neighbors can become its proxy region via the same mechanism discussed above. In the extreme case when all of a hole's neighbors are empty, the hole may be filled by sensors or have a proxy region leader later when some of its neighbors get sensors during the *SPP* algorithm execution

## 6 Discussions

In the above, we presented an optimal centralized *OMF* and a distributed *SPP* algorithm for our deployment problem. We now discuss execution of our algorithms, extensions to nonuniform scenarios, the issues of arbitrary sensor movement directions, and network partitions.

### 6.1 Executing Our Algorithms

The algorithms we proposed above can be executed in more than one way. We first discuss a semidistributed version of the the *OMF* algorithm, called the *Domain-based OMF* (*D-OMF*) algorithm. Here, the sensor network is divided into multiple domains and each domain contains multiple regions. We let each domain obtain region information (number of sensors) *only* in their domain. The movement plan for variance minimization in each domain is independently determined with this information (without exchanging information with other domains) using the *OMF* algorithm. The base-station can do this for each domain or a special sensor in each domain can do so. Note that the *D-OMF* algorithm being semidistributed has lower messaging and computational complexity than the *OMF* algorithm. But, optimality is compromised since the *D-OMF* algorithm achieves local optima in each domain and cannot guarantee global optima. Note that this trade-off depends on the domain size, uniformity of initial deployment, and sensor mobility capacity. Conducting an analytical comparison of performance of *D-OMF* and *OMF* algorithms is too difficult, if not impossible. We study this using extensive simulations in Section 7. Furthermore, we point out that our proposed algorithms can also be combinedly executed. A simple instance is one where the *OMF* algorithm is executed first to optimize deployment and, at later stages, the lightweight distributed algorithms can be executed to repair deployment under faults, failures, etc.

3. We do not let recipients of requests choose shortest return paths as such paths may be *blocked* due to mobility limitations.

4. The number of unsuccessful requests per sensor is application decided.

## 6.2 Extensions to Nonuniform Scenarios

### 6.2.1 Nonuniformity at the Sensor Side

We have so far assumed that all sensors are homogeneous in their mobility capacity. In many scenarios, due to deployment costs, faults in sensors, etc., it may happen that only a subset of the deployed sensors is mobile. Our solutions can be extended in such scenarios. The weight assignment rule is still the same. In the *OMF* algorithm, we have to modify reachability information in $G_V$. For example, say only *one* sensor in region 2 in Fig. 3 is mobile. Then, the edges from region 2 to its reachable regions 1 and 4 (i.e., from $v_2^{out}$ to $v_1^{in}$ and from $v_2^{out}$ to $v_4^{in}$) each have capacity *one*. This allows up to only *one* sensor to move out from region 2. Other construction rules remain the same. The resulting solution is still optimal. The *SPP* algorithm needs no changes. Only, fewer paths will be feasible now due to not all sensors being mobile.

### 6.2.2 Nonuniformity at the Deployment Area Side

In our discussions above, we focused on uniform deployment areas, where $\bar{k}$ is the same for all regions. However, in many situations, the deployment area can be nonuniform. Examples are certain sensitive zones that need to be sensed to a higher degree, which means $\bar{k}$ is more in such zones than others; certain hostile zones like lakes, fires, etc., can destroy sensors, which means $\bar{k} = 0$ for such zones. For addressing such requirements, our weight assignment rule is still the same as in (3). However, the number of sinks created per region and their corresponding weights will be different depending on the desired $\bar{k}$ per region in the *OMF* algorithm. In the *SPP* algorithm, the number of requests generated and their weights are modified accordingly. The rest of our solutions are still the same.

## 6.3 Arbitrary Sensor Movement Directions and Network Partitions

In Section 2, we assumed that sensors can move only to regions in their adjacent left, right, top, and bottom directions. We now discuss the case of arbitrary sensor movement directions. For *OMF* and *D-OMF* algorithms, only virtual graph $(G_V)$ construction changes. In $G_V$, we now have to add new edges (with corresponding costs and capacities) from a region to all newly *reachable* regions corresponding to arbitrary movement directions. In the *SPP* algorithm, there are now more neighbor choices to forward a request and extra feasible paths can be reserved while sensors move to satisfy requests.

In Section 2, we assumed that the sensor network is not partitioned. In some situations, it may happen that sensors in one part of the network may not be able to communicate with sensors in another part. In such cases, we have to repair such partitions while still being constrained by mobility distance. In the approach proposed by Wu and Wang [5], empty holes are filled by placing a *seed* from a nonempty region to a hole. We can apply the algorithms in [5] to repair partitions in our case. However, we are still constrained by the mobility in sensors. Addressing the issue of repairing network partitions optimally using limited mobile sensors is a part of our ongoing work.

# 7 Performance Evaluations

In this section, we report our experimental data to study the performance of our *OMF*, *D-OMF*, and *SPP* algorithms under various sensor and network parameters. We also simulate the well-known VORonoi-based Virtual Force (*VOR*) algorithm proposed in [4] and compare its performance with our algorithms.

## 7.1 Performance Metrics and Evaluation Environment

### 7.1.1 Performance Metrics

We have three major performance metrics in this paper. The first is the *Variance Improvement* (denoted by $VI$) at final deployment after sensors have finished movements. It is defined as $VI = (\frac{Var_{in} - Var_{out}}{Var_{in}}) \times 100$, where $Var_{in}$ is the variance at initial deployment and $Var_{out}$ is the variance at final deployment. Our second metric is the number of sensor *Movement Hops* per percent variance improvement (denoted by $MH$). It is defined as $MH = \frac{M}{VI}$, where $M$ denotes the total number of sensor movement hops. The reason we define $MH$ as a ratio is because it is more fair to compare number of hops per improvement in variance than just the number of hops.

Our third metric is the messaging overhead incurred by our algorithms, which is defined as the *Packet Number* per region (denoted by $PN$). Denoting $P$ as the total number of packets (or messages) sent and denoting $S$ as the number of regions, we have $PN = \frac{P}{S}$. Physically speaking, $VI$ captures the improvement in deployment as a result of our algorithms, while $MH$ and $PN$ reflect the overhead in terms of sensor movement hops and messaging overhead. The packet number for our *OMF* algorithm is calculated based on a simple protocol. After initial deployment, an elected region-head in each region sends a packet to a base-station (located in the center of the network) with information on the number of sensors in its region. The packets are forwarded along shortest paths through other regions toward the base-station. After the base-station receives all packets and determines a movement plan, it sends one packet to each region in the reverse path, informing regions of its movement plan. A similar protocol is assumed for the *D-OMF* algorithm, where the regions in each domain will forward packets to a special sensor in the domain which executes the algorithm and forwards a movement plan to each region in the domain. Note that there can be other versions of the above protocols, like direct relaying of messages, row-wise (or column-wise) message delivery, etc.

### 7.1.2 Evaluation Environment

We denote the number of regions in the network as $n \times n$ (represented in the figures as simply $n$). Our default value is $8 \times 8$. The default desired number of sensors per region is $\bar{k} = 3$ and the maximum number of hops a sensor can move is $H = 3$. By default, the number of sensors initially deployed is $n \times n \times \bar{k}$ and all sensors in the network are mobile by default. For the *D-OMF* algorithm, we choose the domain size $D$ as $D = \frac{n}{2}$. Our implementation of the maximum flow algorithm is the Edmonds-Karp algorithm [21] and the minimum cost flow algorithm is the one in [22]. In the *SPP* algorithm, a *peak* and $over\text{-}\bar{k}$ forwarder will batch up the coming *REQs* in a time period to serve. In our simulation, the time period is given by $t_u \times n$, in which $t_u$ is the message transmission delay between two neighboring regions. For comparisons, we also simulate the

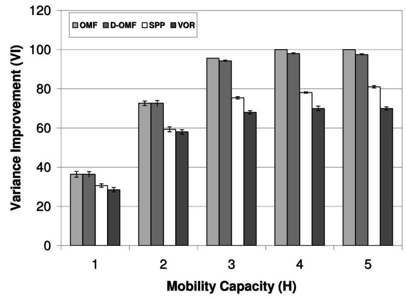


Fig. 4. Sensitivity of $VI$ to $H$ for all four algorithms.

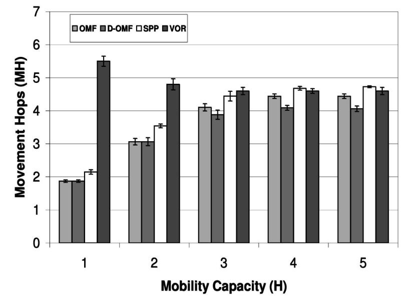


Fig. 5. Sensitivity of $MH$ to $H$ for all four algorithms.

VORonoi-based virtual force (*VOR*) algorithm [4], the basic idea of which was discussed in Section 1. The termination condition for the *SPP* algorithm is when each *pit* has either obtained at least $\bar{k}$ sensors or expiration of sensor movement capability, or if a certain number of requests have been tried without success by requesting sensors. By default, the number of requests per sensor without success is set as 3. For the *VOR* algorithm, the termination condition was each region obtaining at least $\bar{k}$ sensors or expiration of sensor movement capability.

We conduct our simulations on a custom simulator.[5] For initial deployment, our simulator uses a topology generator for 2D-Normal distribution [23]. A 2D-Normal distribution involves two random variables, $x$ and $y$, with mean values $\mu_x$ and $\mu_y$. The mean values corresponding to each variable can be written as a vector $\mathbf{u} = (\mu_x, \mu_y)^T$. Each variable will have a variance $\sigma_x$ and $\sigma_y$. However, it may happen that the variables are related to each other, in which case, there will be covariances $\sigma_{xy}$ and $\sigma_{yx}$ with $\sigma_{xy} = \sigma_{yx}$, all of which can be incorporated into a variance-covariance matrix:

$$\mathbf{v} = \begin{pmatrix} \sigma_x^2 & \sigma_{xy}^2 \\ \sigma_{yx}^2 & \sigma_y^2 \end{pmatrix}.$$

The 2D-Normal distribution is then given by

$$P(\mathbf{z}) = \frac{1}{2\pi\sqrt{|\mathbf{v}|}} e^{[-\frac{1}{2}(\mathbf{z}-\mathbf{u})^T \mathbf{v}^{-1}(\mathbf{z}-\mathbf{u})]},$$

where $|\mathbf{v}|$ is the determinant of $\mathbf{v}$. In our simulations, we set $\sigma_{xy} = \sigma_{yx} = 0$, which means the location at the $x$ and $y$ axis are independent when sensors are deployed. We let $\sigma^2 = 1/\sigma_x^2 = 1/\sigma_y^2$. Hence, when $\sigma$ increases, sensors will be more concentrated at the center and when $\sigma$ tends to 0, sensors are more uniformly distributed. For our simulations, by default, $\sigma = 4$. All data reported here were collected across 10 iterations and averaged.[6]

## 7.2 Performance Results

### 7.2.1 Performance Comparison of All Algorithms

We first study the sensitivity of $VI$, $MH$, and $PN$ to mobility capacity $H$ for the *OMF*, *D-OMF*, *SPP*, and *VOR* algorithms. We assume that the sensors are initially deployed as a one time step targeted toward the center of the network. All other settings are defaults. From Fig. 4, we observe that the *OMF* algorithm (being optimal) performs best in terms of $VI$. We observe that the *D-OMF* algorithm performs quite close to the *OMF* algorithm in all cases, while the performance of the *SPP* and *VOR* algorithms are quite good for smaller values of $H$. Among all algorithms, the virtual force (*VOR*) algorithm has the poorest performance. Since sensors in the *VOR* algorithm attempt to achieve local force balance between themselves, they incur several back and forth movements that rapidly deplete overall sensor movement capability resulting in overall poor variance improvement. Fig. 4 also shows the effects of limited mobility on $VI$. When $H$ increases, $VI$ increases in all algorithms as increased movement ability in general helps to move sensors to needy regions farther away. The improvement stays constant when $H \geq 4$ for the *OMF* algorithm and for $H \geq 3$ in the other algorithms. This demonstrates that mobility beyond a certain point does not bring in further benefits. Note that, when $H \geq 4$, the *OMF* algorithm achieves the upper bound of 100 percent $VI$ because movement choices can be *optimally* exploited by the *OMF* algorithm with larger $H$, unlike other algorithms.

Figs. 4 and 5 show that, when $VI$ increases, $MH$ also increases for the *OMF*, *D-OMF*, and *SPP* algorithms. In our algorithms, each sensor movement typically results in a more balanced deployment for the network. Also, it could happen that, for improvement in variance, many potentially longer paths are found in our algorithms. Consequently, when $VI$ increases, $MH$ also increases in our algorithms while staying a constant when $VI$ stops increasing. On the other hand, we observe that the $MH$ for the *VOR* algorithm is initially a higher value, then decreases and stays constant. When $H$ is small, $VI$ is quite low for the *VOR* algorithm due to more stringent mobility limitations, which makes $MH$ higher for smaller $H$. As $VI$ improves further, $MH$ decreases for the *VOR* algorithm in Fig. 5. Since $VI$ stays constant beyond $H > 2$, $MH$ also stays constant. Note that, when $H$ increases, the $MH$ in the *D-OMF*, *SPP*, and VOR algorithms are quite close to the optimal *OMF* algorithm (while even being smaller in some cases). As pointed out before, this is because of the improved $VI$ that can be achieved by the *OMF* algorithm by exploiting several additional movement choices compared to the other algorithms, which increases the number of movements and, hence, $MH$ in the *OMF* algorithm.

In Fig. 6, we can see that $PN$ in the *OMF* and *D-OMF* algorithms are constant since the packet number does not depend on $H$ for these algorithms. The messaging overhead in the VOR algorithm is the maximum because of many local message exchanges caused by several back and forth

5. Code can be obtained by contacting the primary author of the paper.
6. The standard deviations for all data sets are shown in the corresponding figures.

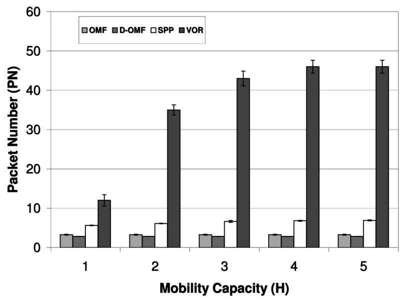


Fig. 6. Sensitivity of $PN$ to $H$ for all four algorithms.

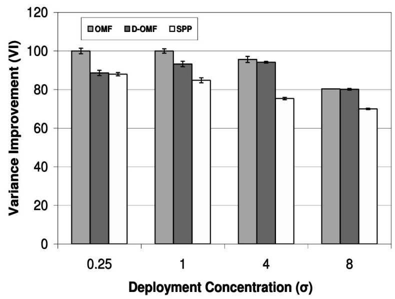


Fig. 7. Sensitivity of $VI$ to $\sigma$.

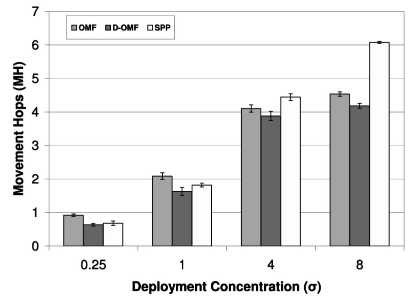


Fig. 8. Sensitivity of $MH$ to $\sigma$.

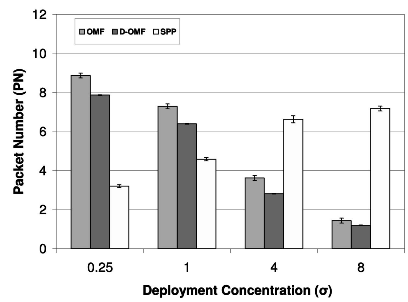


Fig. 9. Sensitivity of $PN$ to $\sigma$.

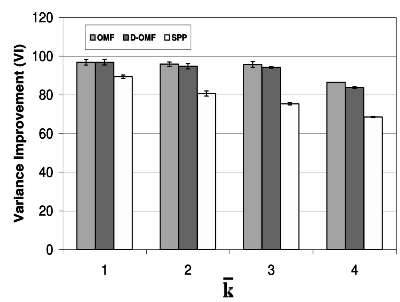


Fig. 10. Sensitivity of $VI$ to $\bar{k}$.

sensor movements. The $PN$ in the *SPP* algorithm takes a middle ground, since only deficient regions send out requests and those toward richer regions only, while responses always aim to take shorter movement paths. In both the *SPP* and VOR algorithms, $PN$ increases with $H$ due to more movement choices when $H$ increases.

Figs. 7, 8, and 9 show the sensitivity of our performance metrics to $\sigma$ for our algorithms. In Fig. 7, $VI$ decreases when $\sigma$ increases for the *OMF* and *SPP* algorithms. Since larger $\sigma$ implies more concentrated initial deployment, it is harder for regions near the boundary to find sensors under mobility constraints, which decreases $VI$ for the *OMF* and *SPP* algorithms. We also see that the $VI$ of the *D-OMF* and *SPP* algorithms become closer to that of the *OMF* algorithm as $\sigma$ increases. This is because the number of mobility choices that the *OMF* algorithm can exploit is not significantly more than that of the other algorithms when deployment is highly concentrated.

In Fig. 8, we see that $MH$ increases as $\sigma$ increases for our algorithms. This is mainly because of the reduction in $VI$ with increasing $\sigma$. Note here that $MH$ is lower for the *SPP* algorithm when $\sigma$ is small. This is because, when deployment is more uniform (smaller $\sigma$), more *pits* can find enough *over-$\bar{k}$ forwarders* or *peaks* nearby, which causes a reduction in overall sensor movements. In Fig. 9, we see that the $PN$ for the *OMF* and *D-OMF* algorithms decreases with $\sigma$ since the number of sensors farther away from the center of the network decreases with increased $\sigma$. On the other hand, $PN$ increases with $\sigma$ in the case of the *SPP* algorithm since the increase in $\sigma$ means that the bias increases, resulting in more requests and responses. We also observe that, when $\sigma$ is less, the $PN$ in the *SPP* algorithm is lower than that of the *OMF* and *D-OMF* algorithms. This is because more *pits* can find enough *over-$\bar{k}$ forwarders* or *peaks* in the *SPP* algorithm when the deployment is more uniform, further highlighting the fact that the distributed *SPP* algorithm achieves less overhead under favorable deployment conditions.

We now study the sensitivity of our performance metrics to $\bar{k}$ for our algorithms. In order to compare the sensitivity to $\bar{k}$ fairly, the number of sensors initially deployed is fixed as $8 \times 8 \times 3 = 192$ for all cases (all other settings are default). From Figs. 10 and 11, we can see that an increase in $\bar{k}$ causes a decrease in $VI$ and an increase in $MH$ in our algorithms. When $\bar{k}$ increases, the objective becomes harder, which causes this trend. We can also see that the *D-OMF* algorithm performs quite close to the *OMF* algorithm in all cases. An interesting observation here is that, when $\bar{k}$ is small, the performance of the *SPP* algorithm in all metrics compares quite favorably with the other algorithms. This is because, when the deployment objective is relatively mild (less $\bar{k}$), local requests and responses suffice for good performance. Once again, the $PN$ for the *OMF* and *D-OMF* algorithms in Fig. 12 is independent of $\bar{k}$ and, hence, is constant. The $PN$ of the *SPP* algorithm is similar to the other algorithms for less $\bar{k}$ and increases with increasing $\bar{k}$ since more requests and responses are generated when $\bar{k}$ increases.

In Figs. 13, 14, and 15, we can see that, as $n$ increases, $VI$ decreases and both $MH$ and $PN$ increase for our algorithms. A larger $n$ implies a larger network, which makes more regions near the boundary of the network unable to

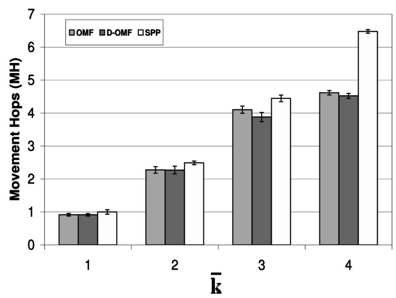


Fig. 11. Sensitivity of $MH$ to $\bar{k}$.

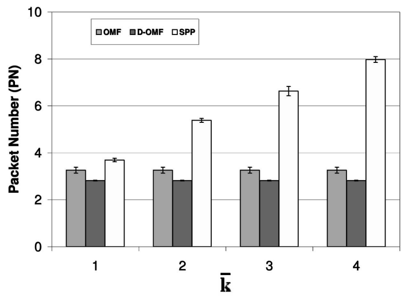


Fig. 12. Sensitivity of $PN$ to $\bar{k}$.

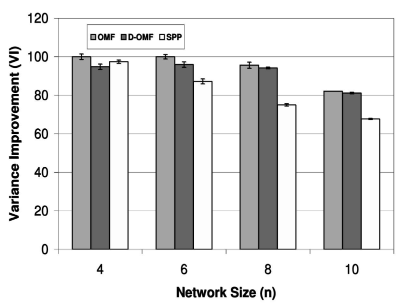


Fig. 13. Sensitivity of $VI$ to $n$.

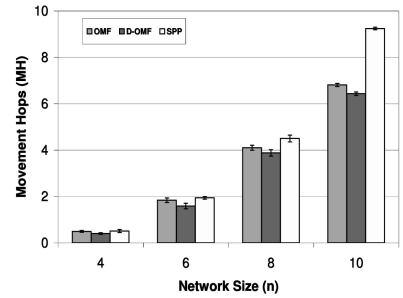


Fig. 14. Sensitivity of $MH$ to $n$.

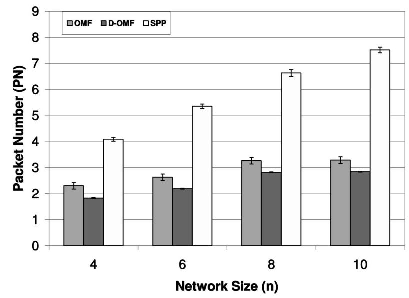


Fig. 15. Sensitivity of $PN$ to $n$.

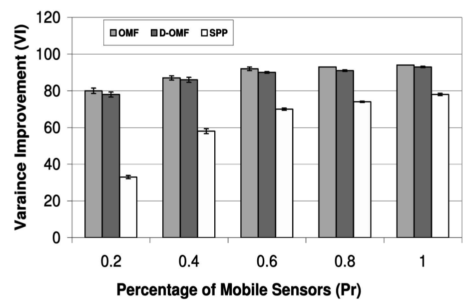


Fig. 16. Sensitivity of $VI$ to $Pr$.

get sensors and, thus, $VI$ decreases. Also, sensors need to travel longer distances, which increases $MH$ and $PN$.

### 7.2.2 Performance When Only a Subset of Sensors Are Mobile

Our default case above consisted of all sensors in the network being capable of being limitedly mobile. We now demonstrate the sensitivity of performance ($VI$) of our algorithms under different sensor mobility capacity ($H$) when only a subset of sensors is mobile (as discussed in Section 6.2.1). The value of $H$ is set as 3. All other settings are defaults. In Fig. 16, the term $P_r$ on the X-axis denotes the percentage of sensors that are mobile. For instance, if $P_r = 0.2$, then only 20 percent of the sensors are mobile. We observe that, while $VI$ improves with increasing $P_r$, there is a threshold beyond which an increase in $VI$ is negligible in all agorithms. For the case when $H = 3$ in Fig. 16, the threshold is around 60 percent. The threshold in fact depends on $H$ and decreases as $H$ increases and vice versa.[7] This demonstrates that not all sensors in the network need to be mobile. Depending on the mobility capacity $H$, there is a threshold beyond which deployment quality cannot be enhnaced significantly with more mobile sensors.

7. We do not report data for other values of $H$ due to space limitations.

### 7.2.3 Convergence Time of the SPP and VOR Algorithms

In Fig. 17, we study the sensitivity of convergence time of the *SPP* and *VOR* algorithms to $H$. All other settings are defaults. For the *SPP* algorithm, the convergence time (in terms of rounds) is obtained as follows: We denote one unit time as the time taken by a sensor to perform local computations and send a packet to a sensor in a neighboring region. The number of rounds is the total number of unit times spent to complete execution of the *SPP* algorithm in the network. For the *VOR* algorithm, it is simply the number of rounds it takes for the algorithm to terminate similar to the definition in [4]. From Fig. 17, we see the convergence time increases with $H$ for both algorithms, since more movement choices are available with increasing $H$. We also observe that the convergence time begins to saturate with increasing $H$, demonstrating that, beyond a certain point, an increase in mobility does not help deployment much. Note that the number of rounds in the *VOR* algorithm is much lower than the *SPP* algorithm. However, this should not be construed as better

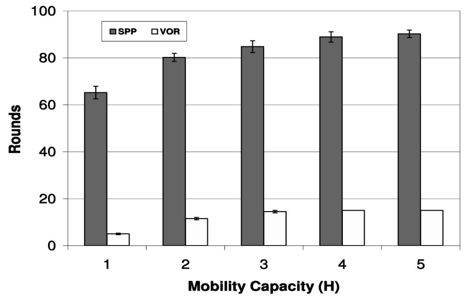


Fig. 17. Sensitivity of convergence time to $H$ in the *SPP* and *VOR* algorithm.

performance by the *VOR* algorithm. Rather, it is due to the faster depletion of sensor mobility capacity and the lower $VI$ in the *VOR* algorithm compared to the *SPP* algorithm. The sensitivity of the number of rounds to $\bar{k}$, $\sigma$, and $n$ follow expected trends and are not reported here (due to space limitations).

## 8 Related Work

In this paper, we addressed a sensor networks deployment problem using limited mobility sensors. While a host of works have appeared on deployment [4], [6], [15], [5], [24], [8], [25], [26], [27], [28], [29], [30], [31], [32], [33], in this section, we particularly discuss related work in the areas of mobility-assisted sensor networks deployment.

In [4], [6], and [15], the goal is uniform coverage of the network. This means that every point in the network is covered by at least *one* sensor. The approach in [4], [6], and [15] is balancing sensor virtual forces. Two sensors may repel or attract each other based on the distance between them. At each iteration, sensors move to achieve a better force balance and sensors stop moving when a force equilibrium is reached. However, under hard mobility constraints, two sensors may not be able to achieve force balance if the distance required to be traversed is too large. Second, the virtual force approach will result in several back and forth sensor movements during force balancing, which across many iterations will rapidly deplete the mobility capacity of sensors. Another difference is that all of the above works focus on *one*-coverage of the sensor network, while we are addressing a general variance minimization problem. Another mobility-assisted deployment work is [5], where the objective is load balancing sensor deployment. In [5], the sensor network is initially divided into 2D clusters. The problem is to ensure that, starting from an initial deployment, the number of sensors in all clusters in the sensor network be the same. Based on efficiently scanning the clusters in two stages (row-wise and column-wise), sensors determine to which cluster they have to move. One drawback of [5] is that the ratio of number of hops in their algorithm and the optimal case is bounded by a factor of 2. Limited mobility sensors cannot tolerate so many unwanted moves. Also, the problem in [5] is a special case of our general deployment problem in this paper.

Other works in this area include [24], where algorithms are proposed to let sensors relocate to new positions in a 2D grid-based network. The relocation process is event-driven (new events, sensor failures, faults, etc.). However, such relocation is done without compromising existing functionality of the network. In [8], algorithms are designed to enable a sensor to move to events in the field. The algorithms are designed to be less energy consuming and computationally mild for the sensors. Both the above works are event-driven and also do not consider hard mobility limitations on sensors.

We have done some prior work in limited mobility deployment in [25]. There, our problem was maximizing the number of regions in the network with at least *one* sensor, where the sensors can *hop* only *once* to a *fixed* distance. The problem we address in this paper is minimizing variance, which is a nonlinear objective. The corresponding methodology and algorithms in this paper are different from [25]. Also, we have a more general limited mobility model in this paper, where only the maximum sensor movement distance is limited. This paper also proposes a distributed algorithm unlike our work in [25].

## 9 Final Remarks

In this paper, we defined a general sensor network deployment problem under limited mobility sensors and proposed a set of sensor movement algorithms for it. Our ongoing work addresses the issue of repairing network partitions with limited mobility sensors. Also, we plan to study the issues of limited mobility sensors in applications like sensor tracking systems. The challenge is how to design algorithms that can exploit limited mobility in sensors and algorithms for provisioning sensors in the network to improve tracking efficiency throughout the network and some specific hot spots. Finally, we are planning to investigate opportunities and challenges associated with mobility in more complex environments like those where link qualities, sensing ranges, and transmission ranges are nonuniform.

## Appendix

## Proof of Theorem 1 in Section 3

**Proof.** Consider two arbitrary sequences of sensor movements $F$ and $G$ with functions $\{f_i^j\}$ and $\{g_i^j\}$, respectively. Assume there are $m_i$ and $n_i$ sinks in region $i$ that have a sensor at the end of sequences $F$ and $G$, respectively. Recalling the constraint of $\phi_i^j$ in (4), we have

$$f_i^j = \begin{cases} 1, & j > \bar{k} - m_i, \\ 0, & j \leq \bar{k} - m_i, \end{cases} \tag{6}$$

$$g_i^j = \begin{cases} 1, & j > \bar{k} - n_i, \\ 0, & j \leq \bar{k} - n_i. \end{cases} \tag{7}$$

The gain of $Score$ for sequence $F$ compared with $Score$ for sequence $G$ $(Score(F) - Score(G))$ is

$$\begin{aligned} &= \frac{1}{S}\sum_{i=1}^{S}\sum_{j=1}^{\bar{k}}(f_i^j * w_i^j) - \frac{1}{S}\sum_{i=1}^{S}\sum_{j=1}^{\bar{k}}(g_i^j * w_i^j) \\ &= \frac{1}{S}\sum_{i=1}^{S}\big((m_i - n_i) * (2\bar{k} - m_i - n_i)\big). \end{aligned} \tag{8}$$

The loss of variance $Var$ of $F$ compared with that of $G$ $(Var(G) - Var(F))$ is

$$\begin{aligned} &= \frac{1}{S}\sum_{i=1}^{S}(\bar{k}-n_i)^2 - \frac{1}{S}\sum_{i=1}^{S}(\bar{k}-m_i)^2 \\ &= \frac{1}{S}\sum_{i=1}^{S}\big((m_i-n_i)*(2\bar{k}-m_i-n_i)\big). \end{aligned} \tag{9}$$

We can see that the amount of gain in $Score$ for $F$ is the same as the amount of loss in $Var$. Thus, the sequence of sensor movements that maximizes $Score$ simultaneously minimizes $Var$ and vice versa. □

## Proof of Theorem 2 in Section 4.1.3

**Proof.** We first prove that the flow plan corresponding to the minimum cost maximum flow in $G_V^m$ is the flow plan corresponding to the maximum weighted flow in $G_V$. We will prove this by contradiction. Let the minimum cost maximum flow plan in $G_V^m$ be $Z$. Suppose $Z$ does not yield the maximum weighted flow in $G_V$. This means there exists a flow plan $Y$ that has a higher weighted flow than that of $Z$. Let us denote the weighted flow values of $Z$ and $Y$ to sinks in $G_V$ by $W_Z$ and $W_Y$, respectively. We then have $W_Y - W_Z \geq 1$. Denoting $Cost_Z$ and $Cost_Y$ as the cost values of $Z$ and $Y$ in $G_V^m$, respectively, we have

$$Cost_Z = -W_Z * |\bar{E}| * H + Cost'_Z, \tag{10}$$

$$Cost_Y = -W_Y * |\bar{E}| * H + Cost'_Y, \tag{11}$$

in which $Cost'_Z$ and $Cost'_Y$ denote the sum of the edge costs from $v_i^{out}$ to $v_j^{in}$ for all regions $i$ and $j$ in $Z$ and $Y$, respectively. Since $Z$ applies to the minimum cost flow algorithm, we have $Cost_Z < Cost_Y$. However, we can also obtain

$$\begin{aligned} Cost_Z = &-W_Z * |\bar{E}| * H + Cost'_Z \geq -W_Z * |\bar{E}| * H \\ &\geq -W_Y * |\bar{E}| * H + |\bar{E}| * H \\ &> -W_Y * |\bar{E}| * H + Cost'_Y = Cost_Y, \end{aligned}$$

which is a contradiction. Therefore, flow plan $Z$ yields the maximum weighted flow in $G_V$. Since $Z$ is the plan after executing the minimum cost algorithm in $G_V^m$, the costs of flow among edges between reachable regions is minimized in $G_V^m$. $G_V$ is made of exactly the same edges (edges between reachable regions). Therefore, flow plan $Z$ corresponds to the minimum cost maximum weighted flow in $G_V$. □

## Proof of Lemma 1 in Section 4.2

**Proof.** We first prove that, if $z^S(i,j)$ is feasible, then $z^V(v_i^b, vs_j^x)$ is feasible. If $z^S(i,j)$ is feasible, then there is at least one mobile sensor in region $i$, and regions $i$ and $j$ are reachable from each other. That is, the capacities of the edges from $v_i^b$ to $v_i^{out}$ and from $v_i^{out}$ to $v_j^{in}$ are $\geq 1$ and there exists an edge from $v_j^{in}$ to $vs_j^x$, whose capacity is 1 (from Section 4.1.2). Thus, $z^V(v_i^b, vs_j^x)$ is feasible.

We now prove that, if $z^V(v_i^b, vs_j^x)$ is feasible, then $z^S(i,j)$ is feasible. If $z^V(v_i^b, vs_j^x) = \langle v_i^b, v_i^{out}, v_j^{in}, vs_j^x\rangle$ is feasible, then the capacities of the edges from $v_i^b$ to $v_i^{out}$, from $v_i^{out}$ to $v_j^{in}$, and from $v_j^{in}$ to $vs_j^x$ are all $\geq 1$. This implies that there is a sensor in region $i$ and regions $i$ and $j$ are reachable from each other. So, a sensor can move from region $i$ to region $j$. Thus, $z^S(i,j)$ is feasible. □

## Proof of Theorem 3 in Section 4.2

**Proof.** We first prove that our *OMF* algorithm is optimal in terms of minimizing variance. We prove this by contradiction. Consider a sensor movement plan $Z_{opt}^S$ that corresponds to a flow plan $Z_{opt}^V$ determined by executing the minimum cost maximum weighted flow algorithm on $G_V$. Let this movement plan be nonoptimal in terms of variance. This implies that there is a better movement plan, $Z_x^S$, that can further minimize variance in the sensor network. By Corollary 1, a corresponding flow plan $Z_x^V$ can be found in $G_V$. The amount of weighted flow in this plan is larger than the weighted flow achieved using plan $Z_{opt}^V$, which is a contradiction. Hence, $Z_{opt}^S$ is the optimal movement plan for sensors that minimizes variance.

We now prove that our *OMF* algorithm is optimal in terms of minimizing number of sensor movement hops. We prove this by contradiction. Consider a sensor movement plan $Z_{opt}^S$ that corresponds to a flow plan $Z_{opt}^V$ determined by executing the minimum cost maximum weighted flow algorithm on $G_V$. Let this movement plan be nonoptimal in terms of the number of sensor movement hops. This implies that there is a better plan, $Z_x^S$, that can reduce at least one movement in the sensor network. By Corollary 1, a corresponding flow plan $Z_x^V$ can be found in $G_V$. The number of movement hops (or overall cost) in this plan is less than that achieved using $Z_{opt}^V$, which is a contradiction. Hence, $Z_{opt}^S$ is the optimal movement plan that minimizes number of sensor movement hops. □

## Acknowledgments

This work was supported in part by the US National Science Foundation (NSF) under grant no. ACI-0329155 and CAREER Award CCF-0546668 and by the Army Research Office (ARO) under grant no. AMSRD-ACC-R 50521-CI. Any opinions, findings, conclusions, and recommendations in this paper are those of the authors and do not necessarily reflect the views of the funding agencies.

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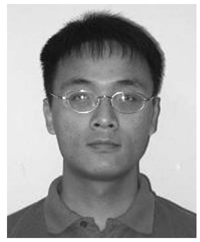

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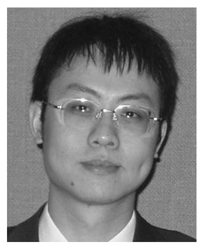

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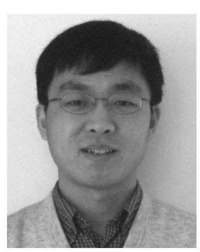

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