Strain induced SHG

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We present some notes on how one would calculate SHG for a surface with strain field.

I. SHG RADIATION

In this section we derive the formulas required for the calculation of the SHG yield, defined by

$$R(\omega) = I(2\omega)/I^2(\omega), \tag{1}$$

with the intensity

$$I(\omega) = c/8\pi|E(\omega)|^2. \tag{2}$$

There are several ways to calculate R, one of which is the procedure followed by Cini.[1] This approach calculates the non-linear susceptibility and at the same time the radiated fields. However, we present an alternative derivation based in the work of Mizrahi and Sipe,[2] since the derivation of the so called three-layer-model is straightforward. Within our level of approximation this is the best model that we can use. In this scheme, we assume that the SH conversion takes place in a thin layer, just below the surface, that is characterized by a surface dielectric function $\epsilon_\ell(\omega)$. This layer is below vacuum and sits on top of the bulk characterized by $\epsilon_b(\omega)$ (see Fig. 1). The thickness of the layer is much smaller than the wavelength of light, and thus we do not have to take into consideration the multiple reflections inside the layer. The non-linear polarization immersed in the thin layer, will radiate an electric field directly into vacuum and also into the bulk. This bulk directed field, will be reflected back into vacuum. Thus, the total field radiated into vacuum will be the sum of these two contributions (see Fig. 1). We decompose the field into s and p polarizations, then the electric field radiated by a polarization sheet of the form given by

$$\mathcal{P}_i = \chi_{ijk} E_j(\omega) E_k(\omega) \quad \text{(sum over repeated indices)}, \tag{3}$$

is given by,[2]

$$(E_{p\pm}, E_s) = \left(\frac{2\pi i\tilde{\omega}^2}{w}\,\hat{\mathbf{p}}_\pm \cdot \boldsymbol{\mathcal{P}}, \frac{2\pi i\tilde{\omega}^2}{w}\,\hat{\mathbf{s}} \cdot \boldsymbol{\mathcal{P}}\right), \tag{4}$$

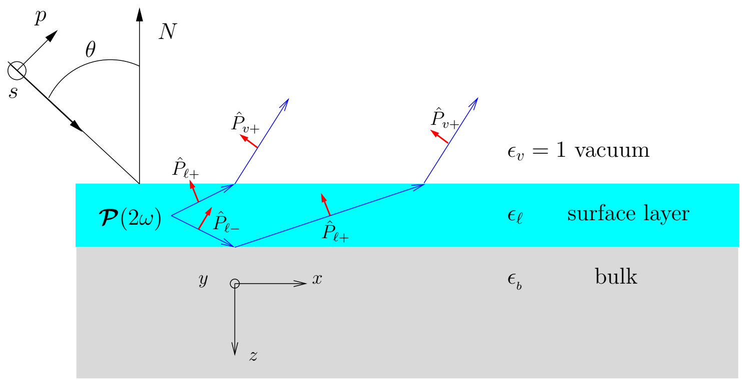

FIG. 1: Sketch of the three layer model for SHG. Vacuum is on top with $\epsilon = 1$, the layer with non-linear polarization $\boldsymbol{\mathcal{P}}(2\omega)$ is characterized with $\epsilon_\ell(\omega)$ and the bulk with $\epsilon_b(\omega)$. In the dipolar approximation the bulk does not radiate SHG. The thin arrows are along the direction of propagation, and the unit vectors for p-polarization are denoted with thick arrows (capital letters denote SH components). The unit vector for s-polarization points along y (out of the page). N is normal to the surface, and θ is the angle of incidence for p or s input polarization.

where $\hat{\mathbf{s}}$ and $\hat{\mathbf{p}}_\pm$ are the unitary vectors for s and p polarization, respectively, and the $\pm$ refers to upward $(+)$ or downward $(-)$ direction of propagation. Also, $\tilde{\omega} = \omega/c$ and $w = \tilde{\omega} k_z$, with

$$k_z(\omega) = \sqrt{\epsilon(\omega) - \sin^2\theta}, \tag{5}$$

and $\hat{\mathbf{p}}_\pm = \mathbf{p}_\pm/\sqrt{\epsilon}$, with

$$\mathbf{p}_\pm = \mp k_z \hat{\mathbf{x}} - \sin\theta \hat{\mathbf{z}}. \tag{6}$$

In the above equations z is the direction perpendicular to the surface that points towards the bulk, x is parallel to the surface, and θ is the angle of incidence, where the plane of incidence is chosen as the xz plane (see Fig. 1), thus $\hat{\mathbf{s}} = \hat{\mathbf{y}}$. The function $k_z(\omega)$ is the projection of the wave vector perpendicular to the surface. As we see from Fig. 1, the SH field is refracted at the layer-vacuum interface (ℓv), and reflected from the layer-bulk (ℓb) interface, thus we can define the transmission, $\mathbf{T}$, and reflection, $\mathbf{R}$, tensors as,

$$\mathbf{T}_{\ell v} = \hat{\mathbf{s}} T_s^{\ell v} \hat{\mathbf{s}} + \hat{\mathbf{P}}_{v+} \tilde{T}_p^{\ell v} \hat{\mathbf{P}}_{\ell+}, \tag{7}$$

and

$$\mathbf{R}_{\ell b} = \hat{\mathbf{s}} R_s^{\ell b} \hat{\mathbf{s}} + \hat{\mathbf{P}}_{\ell+} R_p^{\ell b} \hat{\mathbf{P}}_{\ell-}, \tag{8}$$

where variables in capital letters are evaluated at the harmonic frequency 2ω. Notice that since $\hat{\mathbf{s}}$ is independent of ω, then $\hat{\mathbf{s}} = \hat{\mathbf{s}}$. The Fresnel factors, T_i, R_i, and $\tilde{T}_p$, for $i = s, p$

polarization, are evaluated at the appropriate interface ℓv or ℓb, and will be given below. The extra subscript in $\hat{\mathbf{P}}$ denotes the corresponding dielectric function to be used in its evaluation, i.e. $\epsilon_v = 1$ for vacuum (v), ϵ_ℓ for the layer (ℓ), and ϵ_b for the bulk (b). Therefore, the total radiated field at 2ω is

$$\begin{aligned} \mathbf{E}(2\omega) &= E_s(2\omega)\left(\mathbf{T}_{\ell v} + \mathbf{T}_{\ell v}\cdot\mathbf{R}_{\ell b}\right)\cdot\hat{\mathbf{s}} \\ &+ E_{p+}(2\omega)\mathbf{T}_{\ell v}\cdot\hat{\mathbf{P}}_{\ell +} + E_{p-}(2\omega)\mathbf{T}_{\ell v}\cdot\mathbf{R}_{\ell b}\cdot\hat{\mathbf{P}}_{\ell -}. \end{aligned} \tag{9}$$

The first term is the transmitted s-polarized field, the second one is the reflected and then transmitted s-polarized field and the third and fourth terms are the equivalent fields for p-polarization. The transmission is from the layer into vacuum, and the reflection between the layer and the bulk. After some simple algebra, we obtain

$$\mathbf{E}(2\omega) = \frac{4\pi i\tilde{\omega}}{K_{z\ell}}\mathbf{H}\cdot\boldsymbol{\mathcal{P}}, \tag{10}$$

where,

$$\mathbf{H} = \hat{\mathbf{s}}\,T_s^{\ell v}\left(1 + R_s^{\ell b}\right)\hat{\mathbf{s}} + \hat{\mathbf{P}}_{v+}\tilde{T}_p^{\ell v}\left(\hat{\mathbf{P}}_{\ell +} + R_p^{\ell b}\hat{\mathbf{P}}_{\ell -}\right). \tag{11}$$

The magnitude of the radiated field is given by $E(2\omega) = \hat{\mathbf{e}}^{out}\cdot\mathbf{E}(2\omega)$, where $\hat{\mathbf{e}}^{out}$ is the polarization vector of the radiated field, for instance $\hat{\mathbf{s}}$ or $\hat{\mathbf{P}}_{v+}$. Then we write

$$E(2\omega) = \frac{4\pi i\omega}{c}\mathbf{e}^{2\omega}\cdot\boldsymbol{\mathcal{P}}. \tag{12}$$

Using the above equations and the following simple relationships between T and R,

$$T_s^{\ell v} = \frac{K_{z\ell}}{\cos\theta}T_s^{v\ell}, \qquad \tilde{T}_p^{\ell v} = \frac{\sqrt{\epsilon_\ell(2\omega)}K_{z\ell}}{\cos\theta}T_p^{v\ell}, \tag{13}$$

$$1 - R_p^{\ell b} = \frac{\epsilon_\ell(2\omega)K_{zb}}{K_{z\ell}}T_p^{\ell b}, \qquad 1 + R_p^{\ell b} = \epsilon_b(2\omega)T_p^{\ell b}, \tag{14}$$

we obtain

$$\mathbf{e}^{2\omega} = \frac{1}{\cos\theta}\hat{\mathbf{e}}^{out}\cdot\left[\hat{\mathbf{s}}T_s^{v\ell}T_s^{\ell b}\hat{\mathbf{s}} - \hat{\mathbf{P}}_{v+}T_p^{v\ell}T_p^{\ell b}\left(\epsilon_\ell(2\omega)K_{zb}\hat{\mathbf{x}} + \epsilon_b(2\omega)\sin\theta\hat{\mathbf{z}}\right)\right], \tag{15}$$

and then we write from Eq. (12)

$$E_s(2\omega) = \frac{4\pi i\omega}{c\cos\theta}T_s^{v\ell}T_s^{\ell b}\chi_{yij}E_i(\omega)E_j(\omega), \tag{16}$$

and

$$E_p(2\omega) = \frac{-4\pi i\omega}{c\cos\theta}T_p^{v\ell}T_p^{\ell b}\left[\epsilon_\ell(2\omega)K_{zb}\chi_{xij} + \epsilon_b(2\omega)\sin\theta\chi_{zij}\right]E_i(\omega)E_j(\omega). \tag{17}$$

As mentioned before $E_i(\omega)$ is the incident field given by the external field properly screened; then we have

$$\mathbf{E}_s(\omega) = E_o t_s^{v\ell}\left(1 + r_s^{\ell b}\right)\hat{\mathbf{y}}, \tag{18}$$

and

$$\mathbf{E}_p(\omega) = E_o\left[\tilde{t}_p^{v\ell}\left(1 - r_p^{\ell b}\right)\cos\theta_\ell\hat{\mathbf{x}} - \tilde{t}_p^{v\ell}\left(1 + r_p^{\ell b}\right)\sin\theta_\ell\hat{\mathbf{z}}\right], \tag{19}$$

where E_o is the incoming amplitude and θ_ℓ is the angle of refraction in the layer. Notice that the transmitted and reflected fields in the layer are taken into $\mathbf{E}_s$ and $\mathbf{E}_p$. From Eqs. (13-14) we get

$$\mathbf{E}_s(\omega) = E_o t_s^{v\ell} t_s^{\ell b}\hat{\mathbf{y}}, \tag{20}$$

and

$$\mathbf{E}_p(\omega) = E_o t_p^{v\ell} t_p^{\ell b}\left(\epsilon_\ell(\omega)k_{zb}\hat{\mathbf{x}} - \epsilon_b(\omega)\sin\theta\hat{\mathbf{z}}\right). \tag{21}$$

Using Eqs. (16), (17), (20), (21), into R, we finally write

$$R_{iF} = \frac{32\pi^3\omega^2}{(n_o e)^2 c^3\cos^2\theta}\left|T_F^{v\ell}T_F^{\ell b}(t_i^{v\ell}t_i^{\ell b})^2\, r_{iF}\right|^2, \tag{22}$$

where i (lower case) stands for initial polarization and F (upper case) stands for final polarization, with

$$r_{iP} = \left(\epsilon_\ell(2\omega)K_{zb}\chi_{xjk} + \epsilon_b(2\omega)\sin\theta\chi_{zjk}\right)E_j^i E_k^i, \tag{23}$$

and

$$r_{iS} = \chi_{yjk}E_j^i E_k^i, \tag{24}$$

where from Eqs. (20-21),

$$\mathbf{E}^s = \hat{\mathbf{y}} \tag{25a}$$

$$\mathbf{E}^p = \epsilon_\ell(\omega)k_{zb}\hat{\mathbf{x}} - \epsilon_b(\omega)\sin\theta\hat{\mathbf{z}}. \tag{25b}$$

The $n_o e$ factor in Eq. (22), with n_o the electronic density, renders $\boldsymbol{\chi}$ dimensionless. To complete the required formulas, we write down the Fresnel factors,

$$t_s^{v\ell} = \frac{2\cos\theta}{\cos\theta + k_{z\ell}}, \qquad t_p^{v\ell} = \frac{2\cos\theta}{\epsilon_\ell(\omega)\cos\theta + k_{z\ell}}, \tag{26}$$

$$t_s^{\ell b} = \frac{2k_{z\ell}}{k_{z\ell} + k_{zb}}, \qquad t_p^{\ell b} = \frac{2k_{z\ell}}{\epsilon_b(\omega)k_{z\ell} + \epsilon_s(\omega)k_{zb}}, \tag{27}$$

where the appropriate term $\sqrt{\epsilon(\omega)}$ from the usual definition of t_p,[3] has been taken out to give Eqs. (23) and (24). For a given surface symmetry and its corresponding non-zero tensor elements of χ_{ijk}, Eq. (22) can be calculated explicitly through Eqs. (23) and (24)[4,5]. With the three-layer model we can get two opposite cases, one in which the SH conversion takes places in vacuum for which we simply put $\epsilon_\ell = 1$, and the other case where the layer is identical to the bulk, or $\epsilon_\ell = \epsilon_b$. The former case corresponds to no screening and the latter to the usual Fresnel screening.

II. STRAIN INDUCED SHG

From Ref. 6, it follows that in the presence of elastic strains the χ_{ijk} tensor is written as follows

$$\chi_{ijk} = \chi^{(0)}_{ijk} + p_{ijklm} u_{lm}, \tag{28}$$

where $\chi^{(0)}_{ijk}$ is the non-linear SH surface susceptibility tensor in the absence of the strain field, and p_{ijklm} and u_{lm} are the photoelastic and strain tensors, respectively. Thus, the strain tensor cuples through the photoelsatic tensor, thus giving the coupling of the electric field to the SH polarization, i.e.

$$\mathcal{P}_i(2\omega) = \left(\chi^{(0)}_{ijk} + p_{ijklm} u_{lm} \right) E_j(\omega) E_k(\omega). \tag{29}$$

We mention that $\chi_{ijk} = \chi_{ikj}$, which is the intrinsic symmetry valid for SH. Then, $\chi^{(0)}_{ijk} = \chi^{(0)}_{ikj}$, and $p_{ijklm} = p_{ikjlm}$.

On the other hand, form Ref. 7, the stress field in the matrix induced by a differential thermal load on a through-silicon-vias (TSVs) esctructure can be expressed as

$$\sigma_{xx} = -\sigma_{yy} = -B\Delta\alpha\Delta T R^2 \frac{x^2 - y^2}{2(x^2+y^2)^2} \equiv \xi(x,y) \tag{30a}$$

$$\sigma_{xy} = \sigma_{yx} = -B\Delta\alpha\Delta T R^2 \frac{xy}{2(x^2+y^2)^2} \equiv \zeta(x,y) \tag{30b}$$

$$\sigma_{zz} = \sigma_{zx} = \sigma_{zy} = 0, \tag{30c}$$

where B, $\Delta\alpha$, and ΔT are the biaxial modulus, the mismatch in coefficient of thermal expansion (CTE), and the differential thermal load, respectively. Also, R is the radius of the TSV, and the distance away from the center of TSV is measured by the Cartesian coordinates (x, y). Above formula es valid for $r = \sqrt{x^2 + y^2} \geq R$. The elastic mismatch between TSV and the silicon matrix is neglected for simplicity.

The stress, σ, is proportional to the forze applied on a body and the strain, u,measures the deformation of a body. Within the elastic regime, a body will recover its original shape once the strain is zero, i.e. no deformations will remain. We can assume that the Si matrix will be elastically strained by the TVS, and since Si is isotropic, we can use Hooke's law, to calculate the relationship between σ_{ij} and u_{ij}. Indeed, one can show that[9,10]

$$u_{xx} = \frac{1}{E}\left(\sigma_{xx} - \nu\left(\sigma_{yy} + \sigma_{zz}\right)\right) \tag{31a}$$

$$u_{yy} = \frac{1}{E}\left(\sigma_{yy} - \nu\left(\sigma_{xx} + \sigma_{zz}\right)\right) \tag{31b}$$

$$u_{zz} = \frac{1}{E}\left(\sigma_{zz} - \nu\left(\sigma_{xx} + \sigma_{yy}\right)\right) \tag{31c}$$

$$u_{ij} = \frac{1+\nu}{E}\sigma_{ij} \quad i \neq j, \tag{31d}$$

where E is Young's modulus and ν is Poisson's coefficient. From Eq. (30) we obtain

$$u_{xx} = -u_{yy} = \frac{1+\nu}{E}\xi(x, y) \tag{32a}$$

$$u_{xy} = \frac{1+\nu}{E}\zeta(x, y) \tag{32b}$$

$$u_{zz} = u_{zx,xz} = u_{zy,yz} = 0, \tag{32c}$$

The TSV are built on a Si(001) surface, with a C_{4v} symmetric class, which implies that only the following components of $\chi^{(0)}_{ijk}$ are different from zero,

$$\chi^{(0)}_{zzz}, \quad \chi^{(0)}_{xxz} = \chi^{(0)}_{yyz}, \quad \chi^{(0)}_{zxx} = \chi^{(0)}_{zyy}. \tag{33}$$

The components of p_{ijklm} different from zero for the same C_{4v} symmetry class, are given in Ref. 8, page 140-141 under the Class 4mm (equivalent to C_{4v}). Using Eq. (23)-(25), we arrive at the following expressions for r_{iF},

$$\begin{aligned} r_{pP} = {}& \epsilon_b(2\omega)\sin\theta\left(\epsilon_\ell^2(\omega)k_{zb}^2\,\chi_{zxx} + \epsilon_b^2(\omega)\sin^2\theta\,\chi_{zzz} - 2\epsilon_\ell(\omega)\epsilon_b(\omega)k_{zb}\sin\theta\,\chi_{zxz}\right) \\ & + \epsilon_\ell(\omega)K_{zb}\left(\epsilon_\ell^2(\omega)k_{zb}^2\,\chi_{xxx} + \epsilon_b^2(\omega)\sin^2\theta\,\chi_{xzz} - 2\epsilon_\ell(\omega)\epsilon_b(\omega)k_{zb}\,\chi_{xxz}\right) \end{aligned} \tag{34a}$$

$$r_{pS} = \epsilon_\ell^2(\omega)k_{zb}^2\chi_{xyy} + \epsilon_b^2(\omega)\sin^2\theta\,\chi_{yzz} - 2\epsilon_\ell(\omega)\epsilon_b(\omega)k_{zb}\sin\theta\,\chi_{yxz} \tag{34b}$$

$$r_{sP} = \epsilon_\ell(2\omega)K_{zb}\,\chi_{xyy} + \epsilon_b(2\omega)\sin\theta\,\chi_{zyy} \tag{34c}$$

$$r_{sS} = \chi_{yyy} \tag{34d}$$

The different components of χ_{ijk} that follow from Eq. (28)-(33) and Ref. [8] are

$$\chi_{zxx} = \chi^{(0)}_{zxx} + p_{zxxxx}u_{xx} + p_{zxxyy}u_{yy} \quad (p_{zxxzz}) \tag{35a}$$

$$\chi_{zzz} = \chi^{(0)}_{zzz} + p_{zzzxx}\left(u_{xx} + u_{yy}\right) \quad (p_{zzzzz}) \tag{35b}$$

$$\chi_{zxz} = 0 \quad (p_{zzxxz,zxzxz,zxzzx}) \tag{35c}$$

$$\chi_{xxx} = 0 \quad (p_{xxxxz,xxxzx}) \tag{35d}$$

$$\chi_{xzz} = 0 \quad (p_{xzzxz,xzzzx}) \tag{35e}$$

$$\chi_{xxz} = \chi^{(0)}_{xxz} + 2\left(p_{xxzxx}u_{xx} + p_{xxzyy}u_{yy}\right) \quad (p_{xxzzz=xzxzz}) \tag{35f}$$

$$\chi_{xyy} = 0 \quad (p_{xyyxz,xyyzx}) \tag{35g}$$

$$\chi_{zyy} = \chi^{(0)}_{zyy} + p_{zyyxx}u_{xx} + p_{zyyyy}u_{yy} \quad (p_{zyyzz}) \tag{35h}$$

$$\chi_{yxx} = 0 \quad (p_{yxxyz,yxxzy}) \tag{35i}$$

$$\chi_{yzz} = 0 \quad (p_{yzzzy,yzzyz}) \tag{35j}$$

$$\chi_{yxz} = 2\left(p_{yxzxy} + p_{yxzyx}\right)u_{xy} \tag{35k}$$

$$\chi_{yyy} = 0 \quad (p_{yyyzy,yyyyz}) \tag{35l}$$

where in parenthesis are the components of p_{ijklm} different from zero that do not contribute since $u_{zm,mz} = 0$. From Eq. (32) we finally get

$$\chi_{zxx} = \chi^{(0)}_{zxx} + \frac{1+\nu}{E}\left(p_{zxxxx} - p_{zxxyy}\right)\xi(x,y) = \chi^{(0)}_{zxx} + A\xi(x,y) \tag{36a}$$

$$\chi_{zzz} = \chi^{(0)}_{zzz} \tag{36b}$$

$$\chi_{xxz} = \chi^{(0)}_{xxz} + 2\frac{1+\nu}{E}\left(p_{xxzxx} - p_{xxzyy}\right)\xi(x,y) = \chi^{(0)}_{xxz} + B\xi(x,y) \tag{36c}$$

$$\chi_{zyy} = \chi^{(0)}_{zyy} + \frac{1+\nu}{E}\left(p_{zyyxx} - p_{zyyyy}\right)\xi(x,y) = \chi^{(0)}_{zyy} + C\xi(x,y) \tag{36d}$$

$$\chi_{yxz} = 2\frac{1+\nu}{E}\left(p_{yxzxy} + p_{yxzyx}\right)\zeta(x,y) = D\zeta(x,y) \tag{36e}$$

There is no a priori reason why the $p's$ in above expressions should cancel. We treat A, B, C, D as numerical constants. Eq. (34) becomes,

$$\begin{aligned} r_{pP} &= \epsilon_b(2\omega)\sin\theta\left(\epsilon^2_\ell(\omega)k^2_{zb}\left(\chi^{(0)}_{zxx} + A\xi(x,y)\right) + \epsilon^2_b(\omega)\sin^2\theta\,\chi^{(0)}_{zzz}\right) \\ &\quad - 2\epsilon^2_\ell(\omega)\epsilon_b(\omega)K_{zb}k_{zb}\left(\chi^{(0)}_{xxz} + B\xi(x,y)\right) \end{aligned} \tag{37a}$$

$$r_{pS} = -2\epsilon_\ell(\omega)\epsilon_b(\omega)k_{zb}\sin\theta\, D\zeta(x,y) \tag{37b}$$

$$r_{sP} = \epsilon_b(2\omega)\sin\theta\left(\chi^{(0)}_{zyy} + C\xi(x,y)\right) \tag{37c}$$

$$r_{sS} = 0 \tag{37d}$$

Therefore we see that $p \to P$ and $s \to P$, that is P-output polarization is a mixture of unstrained and strained contributions to SHG, whereas $p \to S$ is only generated by the strain induced contribution. Finally, $s \to S$ is dipolar forbidden, thus it will only originate from the bulk quadrupolar contribution.

We simplify things by considering $\epsilon_b = \epsilon_\ell = \epsilon$, i.e. just a vacuum-bulk interface, then

$$\begin{aligned} r_{pP} &= \epsilon(2\omega)\epsilon^2(\omega)\sin\theta\left[(\epsilon(2\omega)-\sin^2\theta)\,\left(\chi^{(0)}_{zxx}+A\xi(x,y)\right)+\sin^2\theta\,\chi^{(0)}_{zzz}\right] \\ &\quad - 2\epsilon^3(\omega)\sqrt{\epsilon(2\omega)-\sin^2\theta}\sqrt{\epsilon(\omega)-\sin^2\theta}\left(\chi^{(0)}_{xxz}+B\xi(x,y)\right). \end{aligned} \tag{38}$$

Here we would have to use the values of $\epsilon(\omega)$ and θ to see how the prefactors of the different susceptibilities commensurate. In Fig. 2 we show the prefactors of χ_{ijk}, from where we see that at the experiment's ω = 780 nm=1.6 eV, the prefactor of χ_{zxx} is twice of χ_{xxz} while that of χ_{zzz} is negligible. The prefactors are in general complex, so the absolute value is plotted. However for $\hbar\omega < 1.6$ both $\epsilon(\omega)$ and $\epsilon(2\omega)$ are real. The angle of incidence used so far is $\theta = 45°$, but by using an smaller angles, a much higher contrast can be obtained between the prefactors of χ_{zxx} and χ_{xxz}. From Fig. 3, we see that going to smaller θ and smaller $\hbar\omega$ this ratio is amplified, thus making the assumption of only one dominating term in r_{pP} more robust, so giving room for better experimental conditions. We simplify the above expression and only keep χ_{xx},

$$\begin{aligned} r_{pP} &\sim \chi^{(0)}_{zxx} + A\xi(x,y) \\ R_{pP} &\sim |\chi^{(0)}_{zxx} + A\xi(x,y)|^2 \\ &= |\chi^{(0)}_{zxx}|^2 + |A\xi(x,y)|^2 + 2\mathrm{Re}[A^*\chi^{(0)}_{zxx}]\xi(x,y) \\ &\sim |\chi^{(0)}_{zxx}|^2 + 2\mathrm{Re}[A^*\chi^{(0)}_{zxx}]\xi(x,y) \\ R_{pP} - R^{(0)}_{pP} &\sim 2\mathrm{Re}[A^*\chi^{(0)}_{zxx}]\xi(x,y), \end{aligned} \tag{39}$$

since we expect the effect of the strain field to be small, and $R^{(0)}_{pP} \sim |\chi^{(0)}_{xxz}|^2$. A more careful calculation would also involve the contribution from χ_{xxz} (I'll do it later, now I have to drive back home). From Eq. (37c) we see that there is only one component of $\chi^{(0)}_{ijk}$, thus the following equation gives all that there is for $s \to P$,

$$R_{sP} - R^{(0)}_{sP} \sim 2\mathrm{Re}[C^*\chi^{(0)}_{zyy}]\xi(x,y), \tag{40}$$

and of course

$$R_{pS} \sim |D|^2\zeta^2(x,y). \tag{41}$$

In Figs. 4 and 5 we show $\xi(x,y)$ and $\zeta^2(x,y)$, respectively.

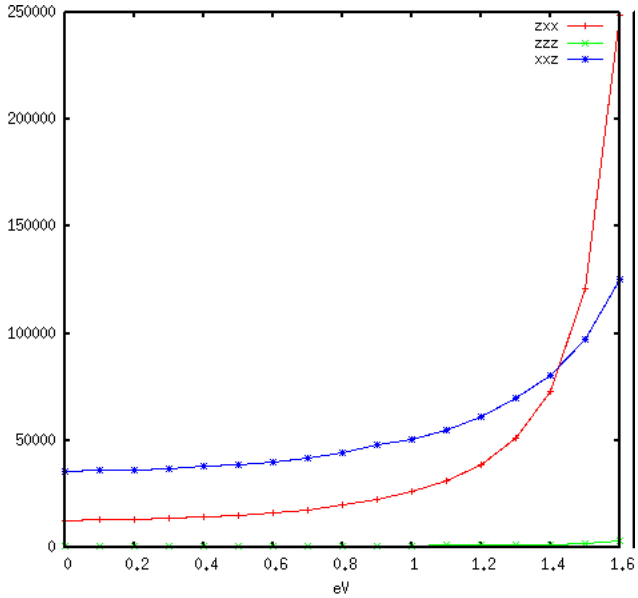

FIG. 2: Prefactors of χ_{ijk} in Eq. (38), where that of χ_{zzz} is negligible, as compared with those of χ_{zxx} and χ_{xxz}. The expriments are done at 780 nm=1.6 eV, for which the prefactor of χ_{zxx} is twice of that of χ_{xxz}. $\theta = 45°$.

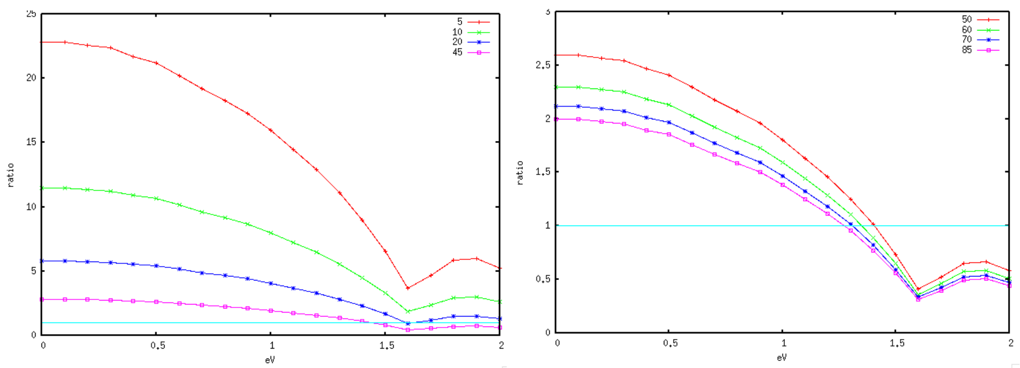

FIG. 3: Ratio of the prefactors of χ_{zxx} and χ_{xxz} of Eq. (38) vs eV and for several values of the angle of incidence θ. Above (bellow) 1 the prefactor of χ_{zxx} is larger (smaller) than that of χ_{xxz}.

That's all for the time being.

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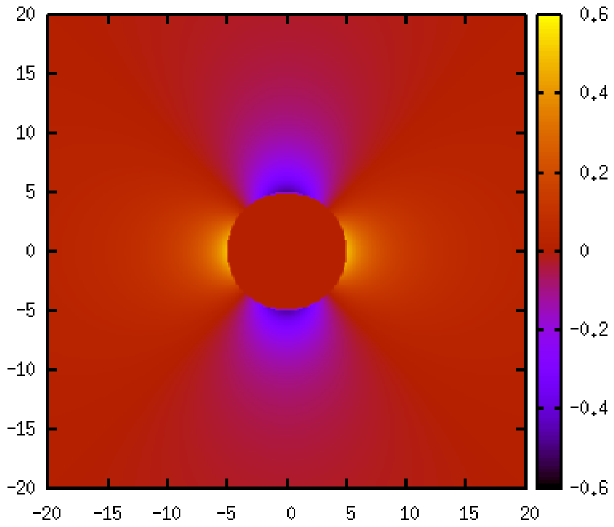

FIG. 4: Polt of $\xi(x,y)$(Eq. (30a)), that according to Eq. (38) or (40), would represent the strain induced SHG radiation pattern for $p \to P$ or $s \to P$ input-Output polarizations.

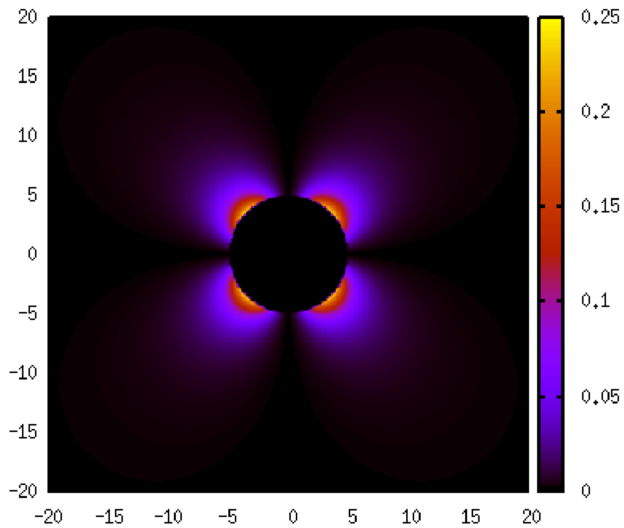

FIG. 5: Polt of $\zeta^2(x,y)$(Eq. (30b)), that according to Eq. (41), would represent the strain induced SHG radiation pattern for $p \to S$ input-Output polarizations.

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