Lec01 Trigonometric Integral

难点：

① $\sin x\cos y = \frac{1}{2}(\sin(x+y) + \sin(x-y))$

② $\sin x \sin y = \frac{1}{2}(\cos(x-y) - \cos(x+y))$

③ $\cos x \cos y = \frac{1}{2}(\cos(x+y) + \cos(x-y))$

$\int \sin^n x \cos^n x \, dx$

Case I: at least one odd power

谁奇掏一个谁出来换元

$\sin x dx = -d\cos x$

$\cos x dx = d\sin x$

① if odd power of $\sin x$

$\int \sin^3 x \cos^6 x \, dx$

$= -\int \sin^2 x \cos^6 x \, d\cos x$

$= -\int (1-\cos^2 x)\cos^6 x \, d\cos x$

$u = \cos x$

$= -\int (1-u^2) u^6 \, du$

$\sin x dx = d\cos x$

$\frac{d\cos x}{dx} =$

$\frac{dy}{dx} = \sin x$

$du = -\sin x \, dx$

② if odd power of $\cos x$

$\int_0^{\frac{\pi}{2}} \quad \int \frac{\cos^5 x}{\sqrt{\sin x}} dx$

都偶则降次

Case II : both even

$$\int \sin^4 x \cos^2 x \, dx$$

$$= \int \left(\frac{1-\cos 2x}{2}\right)^2 \frac{1+\cos 2x}{2} \, dx$$

$$= \int \frac{1}{8} (1+\cos 2x - 2\cos 2x - 2\cos^2 2x + \cos^2 2x + \cos^3 2x) \, dx$$

$$= \frac{1}{8} \int (1 - \cos 2x - \cos^2 2x + \cos^3 2x) \, dx$$

$\frac{1}{2}(1+\cos 4x)$

$$= \frac{1}{8} \left(x - \frac{1}{2}\sin 4x - \frac{1}{2}x - \frac{1}{8}\sin 4x - \frac{1}{2}x - \frac{1}{6}x^3 \right)$$

$2\cos 2x dx = d\sin 2x$

$\cos^2 2x \cos 2x \, dx$

$\frac{1}{2} \cos^2 2x \, d\sin 2x$

$\frac{1}{2}(1-\sin^2 2x) \, d\sin 2x$

$\frac{1}{2}(1-u^2) \, du$

$$(\sec x)' = \sec x \tan x$$

$$\int \sec^n x \tan^n x \, dx$$

Case I : even power of sec x

Ex: $\int \sec^4 x \tan^n x \, dx$

$= \int \sec^2 x \tan^n x \, d\tan x$

$= \int (1+\tan^2 x) \tan^n x \, d\tan x$

$\sec^2 x = \frac{1}{c^2} = 1 + \frac{s^2}{c^2}$

$\sec^2 x \, dx = d\tan x$

$\sec^2 x dx = d\tan x$

$\sec x \tan x dx = d\sec x$

$\tan^2 x = (\sec^2 x - 1)$

$-\csc^2 x dx = d\cot x$

$-\csc x \cot x dx = d\csc x$

$\cot^2 x = (\csc^2 x - 1)$

Case II : odd power of tan x and at least one secant

Ex: $\int \sec x \tan^3 x \, dx$

$= \int \tan^2 x \, d\sec x$

$= \int (\sec^2 x - 1) \, d\sec x$

Ex: $\int \tan^2 x \sec x \, dx$

$= \int (\sec^2 x - 1) \sec x \, dx$

$= \int \sec^3 x - \sec x \, dx =$

$\int \tan x \, dx = \ln|\sec x| + C$

$\int \sec x \, dx = \ln|\sec x + \tan x| + C$

$\int \csc x \, dx = \ln|\csc x + \cot x| + C$

Lec 02

Tri ......

$Ex: \int \sqrt{1-x^2}\, dx$

$x = \sin\theta \qquad dx = \cos\theta\, d\theta$

$(-\frac{\pi}{2}, \frac{\pi}{2})$

$x = \sin\theta \in [-1, 1]$

$\cos\theta \in [0, 1]$

$= \int \sqrt{1-\sin^2\theta}\, \cos\theta\, d\theta$

$= \int \cos^2\theta\, d\theta$

$= \frac{1}{2}\theta + \frac{1}{4}\sin 2\theta + C$

$= \frac{1}{2}\arcsin\theta + x\sqrt{1-x^2} + C$

$dx = a\cos\theta\, d\theta$

$\int \sqrt{a^2-x^2}\, dx$

$\Rightarrow x = a\sin\theta$

$\Rightarrow \int a\cos\theta \cdot a\cos\theta\, d\theta$

$\sqrt{a^2-x^2}$

$x = a\sin\theta$

$= a\cos\theta$

$Ex\, 2: \int \frac{x^2}{(9-x^2)^{\frac{3}{2}}}\, dx$

$x = 3\sin\theta \qquad dx = 3\cos\theta\, d\theta$

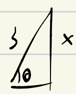
$= \int \frac{9\sin^2\theta}{(9-9\sin^2\theta)^{\frac{3}{2}}} 3\cos\theta\, d\theta$

$= \int \frac{9\sin^2\theta}{9\cos^2\theta}\, d\theta$

$= \int (\sec^2\theta - 1)\, d\theta$

$= \tan\theta - \theta + C$

$= \frac{x}{\sqrt{9-x^2}} - \arcsin\frac{1}{3}x + C$



$\sin\theta =$

$dx = a\sec^2\theta\, d\theta$

$\sqrt{a^2+x^2}$

$\Rightarrow x = a\tan\theta$

$\Rightarrow a\sec\theta$

$\sqrt{a^2+x^2}$

$x = a\tan\theta$

$= a|\sec\theta|$

$Ex: \int \frac{1}{\sqrt{4+x^2}}\, dx$

$x = 2\tan\theta \qquad dx = 2\sec^2\theta\, d\theta$

$= \int \frac{1}{2\sec\theta} 2\sec^2\theta\, d\theta$

$= \int \sec\theta\, d\theta$

$= \ln|\sec\theta + \tan\theta| + C$

$= \ln\left|\frac{1}{2}x + \frac{2}{\sqrt{x^2+4}}\right| + C$

Case IV: $\sqrt{x^2-a^2}$

$x = a\sec\theta \quad = a|\tan\theta|$

$\sec\theta \geq 1$

$\theta \in (-\frac{\pi}{2}, \frac{\pi}{2})$

$\theta \in [0, \frac{\pi}{2}) \cup [\pi, \frac{3}{2}\pi)$

$d\theta = a\tan\theta\sec\theta\, d\theta$

$\int\sqrt{x^2-a^2}\,dx$

$\Rightarrow x = a\sec\theta$

$\Rightarrow a\tan\theta$

$\Rightarrow \int a^2\tan^2\theta\sec\theta\, d\theta$

Ex: $\int \frac{\sqrt{x^2-25}}{x}dx$

$x = 5\sec\theta$

$dx = 5\sec\theta\tan\theta\, d\theta$

$= \int 5\tan^2\theta\, d\theta$

$= 5\int \sec^2\theta - 1\, d\theta$

$\sqrt{x^2+4x-5}$

$\Rightarrow \sqrt{(x+2)^2-9}$

$\Rightarrow x+2 = 3\sec\theta$

Ex: $\int \frac{x}{\sqrt{x^2+4x-5}}dx$

$\Rightarrow \int \frac{x}{\sqrt{(x+2)^2-9}}$

$x+2 = 3\sec\theta \qquad dx = 3\sec\theta\tan\theta\, d\theta$

$= \int \frac{3\sec\theta-2}{3\tan\theta} 3\sec\theta\tan\theta\, d\theta$

$= \int 3\sec^2\theta - 2\sec\theta\, d\theta$

$= 3\tan\theta - 2\ln|\sec\theta + \tan\theta| + C$

$\sqrt[n]{ax+b}$
$u = \sqrt[n]{ax+b}$

$\sqrt[n]{ax+b}$

Ex: $\int \frac{x^2+3x}{\sqrt{x+4}} dx$

$u = \sqrt{x+4}$ $\quad$ $du = \frac{1}{2\sqrt{x+4}} dx$

$u^2 = x+4$ $\quad$ $2u\,du = dx$

$\int \frac{x^2+3x}{u} 2u\,du$

$= 2\int (u^2-4)^2 + 3(u^2-4)\, du$

Lec 03

能拆则拆
不能拆则三角

$$\int \frac{1}{a^2+b^2x^2}\,dx = \frac{1}{ab}\arctan\left(\frac{bx}{a}\right)$$

$$\int \frac{3x+2}{x^2+x}\,dx = \int \frac{2x+2+x}{x(x+1)}\,dx = \int \frac{1}{x+1}+\frac{2}{x}\,dx$$

Ex:
$$\frac{x-7}{x^2+x-12} = \frac{x-7}{(x-4)(x+3)}$$
$$= \frac{-\frac{3}{7}}{x-4} + \frac{\frac{12}{7}}{x+3}$$

method: 1
$A+B=1$
$-4A+3B=-7$
$A=\frac{12}{7}$
$B=-\frac{3}{7}$

Ex 2:
$$\int \frac{5}{x^2+2x+4}\,dx$$
$$= \int \frac{5}{(x+1)^2+3}\,dx = \int \frac{5}{3\left(\frac{x+1}{\sqrt{3}}\right)^2+3}\,dx$$
$x+1=\sqrt{3}\tan\theta$
$$\Rightarrow \int \frac{5}{3\sec^2\theta}\sqrt{3}\sec^2\theta\,d\theta$$
$$= \frac{5}{\sqrt{3}}\arctan\frac{x+1}{\sqrt{3}}$$

Case I:
$$\frac{x^2+1}{x(x-1)(x+1)} = \frac{A}{x}+\frac{B}{x-1}+\frac{C}{x+1}$$

Case II:
$$\frac{x^3+1}{x(x-1)^2(x+1)^3} = \frac{A}{x}+\frac{B}{x-1}+\frac{C}{(x-1)^2}+\frac{E}{x+1}+\frac{F}{(x+1)^2}+\frac{G}{(x+1)^3}$$

Case III:
$$\frac{A}{x}+\frac{Bx+C}{x^2+2x+2} \Rightarrow \int \frac{2x+3}{x^2+2x+2}\,dx$$
$$= \int \frac{2x+2}{x^2+2x+2}\,dx + \int \frac{1}{x^2+2x+2}\,dx$$
$$= \ln(x^2+2x+2) + \int \frac{1}{(x+1)^2+1}\,dx \Rightarrow \arctan(x+1)$$

Case IV:
$$\frac{7}{(x+1)(x^2+1)^2} = \frac{A}{x+1}+\frac{Bx+C}{x^2+1}+\frac{Dx+E}{(x^2+1)^2}$$

Ex 3: $\dfrac{x^4+7x^2}{x^2(x-1)(x^2+5)^2}$

Ex 4: $\displaystyle\int \frac{\cos x}{\sin^4 x-16}\,dx$

$u = \sin x$

$= \displaystyle\int \frac{1}{u^4-16}\,du$

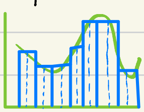
$= \displaystyle\int \frac{1}{(u^2+4)(u-2)(u+2)}\,du$

Lec 04 Approximate integral

1. $\sin^n x \cos^n x$, $\sec^n x \tan^n x$
2. $\sqrt{1-x^2}$ $\sin x$, $\sqrt{1+x^2}$ $\tan x$, $\sqrt{x^2-1}$ $\sec x$
3. rational function

Ex: $\int \sqrt{\frac{1-x}{1+x}}\,dx$
$= \int \frac{\sqrt{1-x^2}}{1-x}\,dx$

1. Midpoint rule



$E = \frac{K(b-a)^3}{24n^2}$

$K = |f''(x)|_{max}$

2. Trapezoidal rule

$E \leq \frac{K(b-a)^3}{12n^2}$

$$\int_a^b f(x)\,dx = \frac{f(x_0)+f(x_1)}{2}\Delta x + \frac{f(x_1)+f(x_2)}{2}\Delta x \dots$$
$$= \frac{\Delta x}{2}\left(f(x_0) + 2f(x_1) + \dots 2f(x_{n-1}) + f(x_n)\right)$$

周老师说不用记
但我觉得还是记
一下好

3. Simpson's rule

1. n = even
2.
3.

$E \leq \frac{K(b-a)^5}{180n^4}$ $K = |f^{(4)}(x)|_{max}$

$\Delta x_i = \frac{\Delta x}{3}[f(x_{i-1}) + 4f(x_i) + f(x_{i+1})]$

$$\int_a^b f(x)\,dx \approx \frac{\Delta x}{3}\left(f(x_0) + f(x_n) + 4(f(x_1) + f(x_3) \dots) + 2(f(x_2) + f(x_4) + \dots)\right]$$

Lec 05

Ex1: $\int_0^{\infty} e^{-x} dx = \lim_{t\to\infty} \int_0^t e^{-x} dx$

$= -e^{-x} \Big|_0^t$

$= 1$

Ex2: $\int_{-\infty}^{b} f(x)\,dx$

$\lim_{t\to-\infty} \int_t^b f(x)\,dx$

limit exists — convergent

limit DNE — divergent

Case IV: $\int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{a} f(x)\,dx + \int_a^{\infty} f(x)\,dx$

$= \int_{-t}^{a} f(x)\,dx + \int_a^{t} f(x)\,dx$

split at any point a

$$\boxed{\int_{-\infty}^{\infty} f(x)\,dx = \lim_{t\to\infty} \int_{-t}^{t} f(x)\,dx \quad \text{False}}$$

这个等式是错的，因为对于广义积分 $\int_{-\infty}^{\infty} f(x)dx$ 的定义，应该是分别取上下限的极限，而不是对称地取极限。正确的定义应该是：

$$\int_{-\infty}^{\infty} f(x)dx = \lim_{a\to-\infty} \lim_{b\to\infty} \int_a^b f(x)dx$$

而你写的等式是：

$$\int_{-\infty}^{\infty} f(x)dx = \lim_{t\to\infty} \int_{-t}^{t} f(x)dx$$

这种对称地取极限的方式并不总是正确的，因为它假设了积分在对称区间上的极限存在，而实际上，广义积分的收敛性需要分别考虑上下限的极限。例如，如果函数 $f(x)$ 在 $+\infty$ 和 $-\infty$ 处的行为不同，这种对称极限可能不存在或不等于分别取极限的结果。

举个例子，考虑函数 $f(x)=1$，显然：

$$\lim_{t\to\infty} \int_{-t}^{t} 1dx = \lim_{t\to\infty}(t-(-t)) = \lim_{t\to\infty} 2t = \infty$$

而按照正确的定义：

$$\int_{-\infty}^{\infty} 1dx = \lim_{a\to-\infty} \lim_{b\to\infty} \int_a^b 1dx = \lim_{a\to-\infty} \lim_{b\to\infty} (b-a) = \infty$$

虽然在这个例子中两种方法都得到发散的结果，但在某些情况下，对称极限可能存在而分别取极限的结果不存在，或者两者不相等。因此，正确的定义是分别取上下限的极限。

$\frac{1}{x^p}$

$p=1$: $\lim_{t\to\infty} \int_a^t \frac{1}{x}dx = \ln x \Big|_a^t = \ln t - \ln a$

So $\int_a^{\infty} \frac{1}{x}dx$ is divergent

$p>1$: $\int_a^{\infty} \frac{1}{x^p} dx = \frac{1}{1-p} x^{1-p} \Big|_a^t = \frac{1}{1-a} x^{1-a}$

So $\int_a^{\infty} \frac{1}{x^p} dx$ is convergent

$\int_{-\infty}^{\infty} \cos 2x\,dx = 2\int_0^{\infty} \cos 2x\,dx = \sin 2x \Big|_0^t = \sin 2t - 0$

So $\int_{-\infty}^{\infty} \cos 2x\,dx$ is divergent

$\int_0^2 \frac{dx}{(x-1)^2} = \frac{1}{x-1}\Big|_0^2 = 2$ False：x=1处不连续

$\Rightarrow \lim_{a\to1^-}\lim_{b\to1^+} \int_0^a \frac{dx}{(x-1)^2} + \int_b^2 \frac{dx}{(x-1)^2} = -\frac{1}{x-1}\Big|_0^{1^-} + -\frac{1}{x-1}\Big|_{1^+}^2 = \infty - 1 - 1 + \infty$

$= \infty$

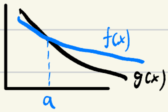
$\int_a^b f(x)\,dx$, $[a,b)$连续，b处不连续

$= \int_a^{b^-} f(x)\,dx$

$(a,b]$ ，a处不连续同理

Comparison Task

f, g continuous with $f(x) \geq g(x) \geq 0$, for $[a, +\infty)$



a. $f(x)$ converge $\Rightarrow$ $g(x)$ converge

b. $g(x)$ diverge $\Rightarrow$ $f(x)$ diverge

Lec 06 arc length (笔者这节课旷了，没上成)

arc length: $\int_a^b \sqrt{1+y'^2}\,dx$

原理:

arc length $= \sum_{i=1}^{\infty} |P_i P_{i+1}|$

$= \int_a^b \sqrt{(dx)^2+(dy)^2}$

$= \int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx \Big/ \int_a^b \sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy$

绕X旋转:

$\int_a^b 2\pi y \sqrt{1+y'^2}\,dx$

绕y旋转

$\int_a^b 2\pi x \sqrt{1+y'^2}\,dx$

Lec07

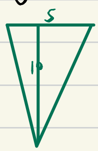
Ex1: aquarium 10m high, 20 m long. force on wall?

$p = hg\rho$

$F = p\cdot \Delta S = hg\rho \cdot 20 \cdot dh$

$\int_0^{10} \rho g h \, 20 \, dh$

Ex2:

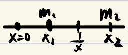


hydrostatic force?

$dF = \rho g h \cdot \frac{10-h}{2} dh$

Ex 3: the turning ability of force $M = Fd$

Generally



$m(\bar{x} - x_1) = m_2(x_2 - \bar{x})$

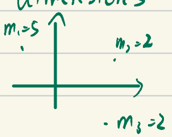
$\bar{x} = \frac{m_2 x_2 + m_1 x_1}{m_1 + m_2}$

M_x means Moment about X

moment $M = \sum m_i x_i$

$\bar{x} = \frac{\text{moment of mass}}{\text{total mass}} = \frac{\sum m_i x_i}{\sum m_i}$

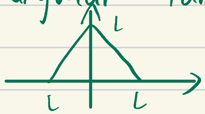
2 dimensions



$M_x = m_i y_i$ $\qquad$ $M_y = \sum m_i x_i$

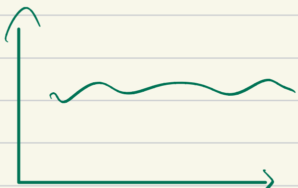
$\bar{y} = \frac{M_x}{m}$ $\qquad$ $\bar{x} = \frac{M_y}{m}$

Triangular lamina of uniform density ρ

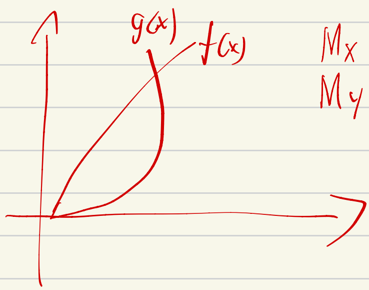


$$M_y = 0$$
$$M_x = \sum m_i y_i$$
$$dM_x = \rho \cdot 2(L-y) \cdot y \, dy$$
$$M_x = \int dM_x = \int_0^L 2\rho y(L-y)\, dy = \frac{\rho L^3}{3}$$

$$m = L^2 \rho \qquad \bar{y} = \frac{M_x}{m} = \frac{L}{3}$$



$$\boxed{\begin{aligned} M_y &= \rho \int_a^b x f(x)\, dx \\ M_x &= \rho \int_a^b \frac{f^2(x)}{2}\, dx \end{aligned}}$$



$$M_x = \int_a^b (f(x)-g(x)) \frac{(f(x)+g(x))}{2}\, dx$$
$$M_y = \int_a^b x\,(f(x)-g(x))\, dx$$
$$m = \int_a^b (f(x)-g(x))\, dx$$

Lec 08 Linear differential equation

Linear differential equation

$$a_0(x)y + a_1(x)y' + a_2(x)y'' + a_3(x)y''' + \cdots a_n(x)y^{(n)} = b(x)$$

$a_0(x), a_1(x), a_2(x), b(x)$ 均不同函数

first order ...

$$a_0(x)y + a_1(x)y' = b(x)$$

Is it linear differential equation

1) $y' + x\sqrt{y} = 3x^2$ X

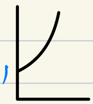
2) $ue^t = 3t + \sqrt{t}\,(u')$

Approximate values

h = step

$x_n = x_{n-1} + h$

$y_n = y_{n-1} + h\,F(x_{n-1}, y_{n-1})$,



$y' = x + y$

$F(x_{n-1}, y_{n-1}) = y'$

n	x_n	y_n	$F(x_{n-1}, y_{n-1})$
0	0	1	$0+1=1$
1	1	2	$1+2=3$
2	2	5	$2+5=7$

Standard form

$$y' + P(x)y = Q(x)$$

$$I(x)y' + P(x)I(x)y = I(x)Q(x)$$

左边变成全导数

$$I(x)P(x) = I'(x)$$

$$(I(x)y)' = I(x)Q(x)$$

$$\int (I(x)y)'\,dx = \int I(x)Q(x)\,dx$$

$$y = \frac{1}{I(x)}\left(\int I(x)Q(x)\,dx + C\right)$$

注意：
y'前系数最好为 1

$$y' + P(x)y = Q(x)$$

$$y = \frac{1}{I(x)}\left(\int I(x)Q(x)\,dx + C\right)$$

$$I(x) = e^{\int P\,dx}$$

$$I(x)P(x) = \frac{dI}{dx}$$

$$P(x)\,dx = \frac{dI}{I(x)}$$

$$\int P(x)\,dx = \int \frac{dI}{I(x)}$$

$$\ln|I| = \int P\,dx + C$$

$$I(x) = e^{\int P\,dx}$$

Ex: $y' + 3x^2 y = 6x^2$

$$I(x) = e^{x^3}$$

$$e^{x^3}y' + (e^{x^3})'y = 6x^2 e^{x^3}$$

$$\int (e^{x^3}y)'\,dx = 2e^{x^3} + C$$

$$y = \frac{1}{e^{x^3}}(2e^{x^3} + C)$$

logistic equation

$$\boxed{\begin{aligned} \frac{dp}{dt} &= rp\left(1-\frac{p}{k}\right) \\ P(t) &= \frac{k}{1+\frac{k-p_0}{p_0}e^{-rt}} \end{aligned}}$$

证明过程:

$$\frac{dp}{p(1-\frac{p}{k})} = r\,dt$$

$$\int \frac{k}{pk-p^2}dp = rt + C$$

$$\int \frac{1}{p} + \frac{1}{k-p}\,dp = rt + C$$

$$\ln|p| - \ln|k-p| = rt + C$$

$$\ln\left|\frac{p}{k-p}\right| = rt + C$$

$$\left|\frac{k-p}{p}\right| = e^{-rt-C}$$

Assume $A = \pm e^{-C}$

$$\frac{k-p}{p} = Ae^{-rt}$$

当 $t=0$, $p=p_0$

$$\frac{k-p_0}{p_0} = A$$

$$\frac{k}{p} = Ae^{-rt}+1$$

$$p = \frac{k}{1+\frac{k-p_0}{p_0}e^{-rt}}$$

下面的没理过

Lec09 midterm review 1

$\frac{1}{x}$

$\int_{-\infty}^{\infty} f(x)\,dx = 1$

p^{-p}

Type1: Infinite integrals

$\int_1^{\infty} \frac{1}{x^p}dx \begin{cases} \frac{1}{p-1} & , p>1 \\ \text{Div} & , p=1 \\ \text{Div} & p<1 \end{cases}$

Type 2:

$\int_0^1 \frac{1}{x^p}dx \begin{cases} \text{Div} & , p>1 \\ \text{Div} & , p=1 \\ \frac{1}{1-p} & , p<1 \end{cases}$

x^{-p}

$\frac{1}{1-p} x^{1-p}$

$\int_0^1 \frac{1}{x^{\frac{1}{2}}}dx$

Ex: $\int_0^{\infty} \frac{40x}{x^3+1}dx = \int_0^1 \frac{40x}{x^3+1}dx + \int_1^{\infty} \frac{40x}{x^3+1}dx$

$= C + \int_1^{\infty} \frac{40x}{x^3+1}dx$

Ex2:

$$\int_1^{\infty} \frac{2+\cos x}{x^2} dx$$

$$\frac{2+\cos x}{x^2} \leq \frac{3}{x^2} \quad \text{converge}$$

so $\frac{2+\cos x}{x^2}$ converge

$$\int_0^1 \frac{2+\cos x}{x^2} dx$$

$$\frac{2+\cos x}{x^2} > \frac{1}{x^2} \quad \} \text{ diverge}$$

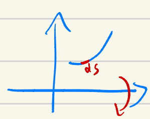
So $\int_0^1 \frac{2+\cos x}{x^2}$ diverge

Ex3: Center of mass for region bounded by $y=x^3$, $y=\sqrt{x}$

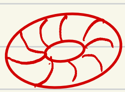
$$M_y = \int x(f(x)-g(x))\,dx$$

$$M_x = \int (f(x)-g(x))\left(\frac{f(x)+g(x)}{2}\right) dx$$

$$\bar{x} = \frac{M_y}{S}, \quad \bar{y} = \frac{M_y}{S}$$



Theorem of pappus:



$$V = \int_a^b 2\pi x(f(x)-g(x))\,dx$$

$$= 2\pi \int_a^b x(f(x)-g(x))dx$$

$$= 2\pi M_y$$

$$= 2\pi S \cdot \bar{x}$$

Center of a curve

$$\bar{x} = \frac{\int x\,ds}{\int ds}, \quad \bar{y} = \frac{\int y\,ds}{\int ds}$$

$$S = \int 2\pi y\,ds$$

$$= 2\pi \int y\,ds = 2\pi \bar{y} L$$

Lec 10 sequence

$$\left\{\frac{1}{2^n}\right\}_{n=0}^{\infty} = \left\{\frac{1}{1}, \frac{1}{2}, \frac{1}{4}, \frac{1}{8} \cdots\cdots\right\}$$

$$\lim_{n\to\infty} |a_n| = 0, \Rightarrow \lim_{n\to\infty} a_n = 0$$

Ex (1) $\left\{\frac{(-1)^n}{n}\right\}_{n=1}^{\infty} = \lim_{n\to\infty} \frac{1}{n} = 0$

(2) $\{$

Simplify

Ex: $\lim_{n\to\infty} \frac{(2n+1)!}{(2n-1)!\,n^2}$

$= \lim_{n\to\infty} \frac{2n(2n+1)}{n^2}$

$= 4 + \lim_{n\to\infty} \frac{2}{n}$

Ex: $\lim_{n\to\infty} \frac{\ln(6n)}{\ln(2n)}$

$= \lim_{n\to\infty} \frac{\ln 2n + \ln 3}{\ln 2n}$

$= \lim_{n\to\infty} 1 + \frac{\ln 3}{\ln 2n}$

$= 1$

Every bounded, monotonic sequence is convergent

Ex1: $\left\{\frac{3^n}{n!}\right\}_{n=3}^{\infty}$