Technical notes

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1 Ideal-viscous splitting

Here I explain how an updated method of ideal-viscous splitting is implemented in `vHLLE`. To derive it, one starts from Eqs. (11) in [**?**]:

$$
\begin{aligned}
&\tilde{\partial}_\nu(\tau\tilde{T}^{\tau\nu}) + \frac{1}{\tau}(\tau\tilde{T}^{\eta\eta}) = 0, \\
&\tilde{\partial}_\nu(\tau\tilde{T}^{x\nu}) = 0, \\
&\tilde{\partial}_\nu(\tau\tilde{T}^{y\nu}) = 0, \\
&\tilde{\partial}_\nu(\tau\tilde{T}^{\eta\nu}) + \frac{1}{\tau}(\tau\tilde{T}^{\eta\tau}) = 0, \\
&\tilde{\partial}_\nu(\tau\tilde{N}^{\nu}_c) = 0.
\end{aligned} \tag{1}
$$

The numerical finite volume representation of the emergy-momentum conservation part is:

$$
\begin{aligned}
&\frac{1}{\Delta\tau}\Big[(\tau+\Delta\tau)(Q^\mu_{id,n+1}+\delta Q^\mu_{n+1}) - \tau(Q^\mu_{id,n}+\delta Q^\mu_n)\Big] \\
&+\sum_{\alpha=1\dots3}\frac{(\tau+\Delta\tau/2)}{\Delta x_\alpha}\Big[F^{\mu\alpha}_{id,i+1/2}+\delta F^{\mu\alpha}_{i+1/2} - F^{\mu\alpha}_{id,i-1/2} - \delta F^{\mu\alpha}_{i-1/2}\Big] \\
&= (\tau+\Delta\tau/2)(S^\mu_{n+1/2}+\delta S^\mu_{n+1/2}).
\end{aligned} \tag{2}
$$

where second order accurate method is assumed (therefore half-step and cell-edge values), and δQ and δF denote viscous corrections to conserved variables and fluxes, respectively.
The terms in Eqs. **??** can be rearranged as follows:

$$
\begin{aligned}
&\Big[(\tau+\Delta\tau)Q^\mu_{id,n+1} - \tau Q^\mu_{id,n}\Big] + \sum_\alpha \frac{(\tau+\Delta\tau/2)}{\Delta x_\alpha}\Big[F^{\mu\alpha}_{id,i+1/2}+\delta F^{\mu\alpha}_{i+1/2} - F^{\mu\alpha}_{id,i-1/2} - \delta F^{\mu\alpha}_{i-1/2}\Big] \\
&= (\tau+\Delta\tau/2)(S^\mu_{n+1/2}+\delta S^\mu_{n+1/2}) + (\tau\delta Q^\mu_n - (\tau+\Delta\tau)\delta Q^\mu_{n+1})
\end{aligned} \tag{3}
$$

The abouve form of equations mean that that in basic hydrodynamic (i.e. energy-momentum conservation) equations one can follow the evolution of ideal part of the conserved variables Q^μ_{id} only, when extra source terms $(\tau\delta Q^\mu_n - (\tau + \Delta\tau)\delta Q^\mu_{n+1})$ are included in their numerical evolution equations.

References

[1] I. Karpenko, P. Huovinen and M. Bleicher, Comput. Phys. Commun. **185** (2014) 3016