Robust Gaussian Filter

July 23, 2021

1 1D Example

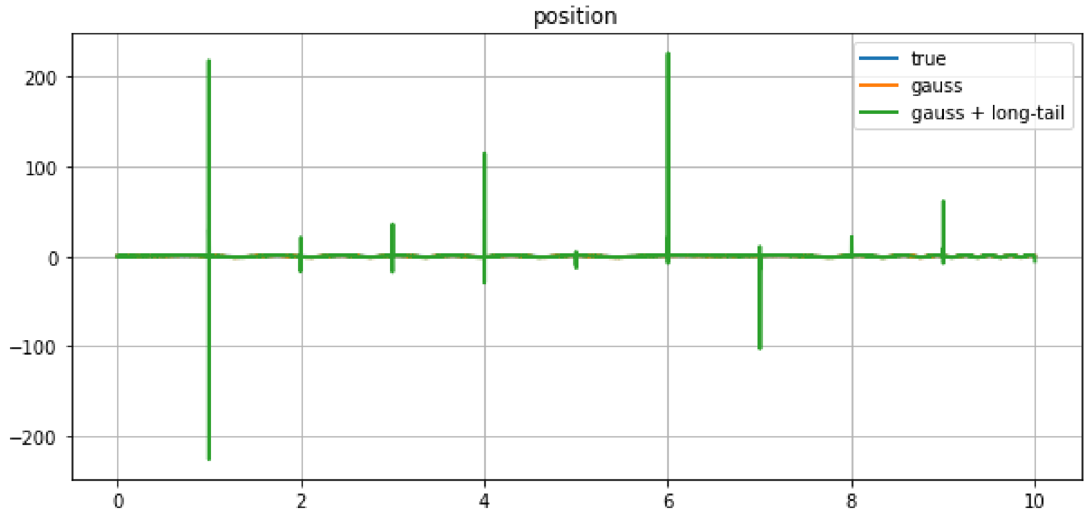
- Transition(Dynamics) Model

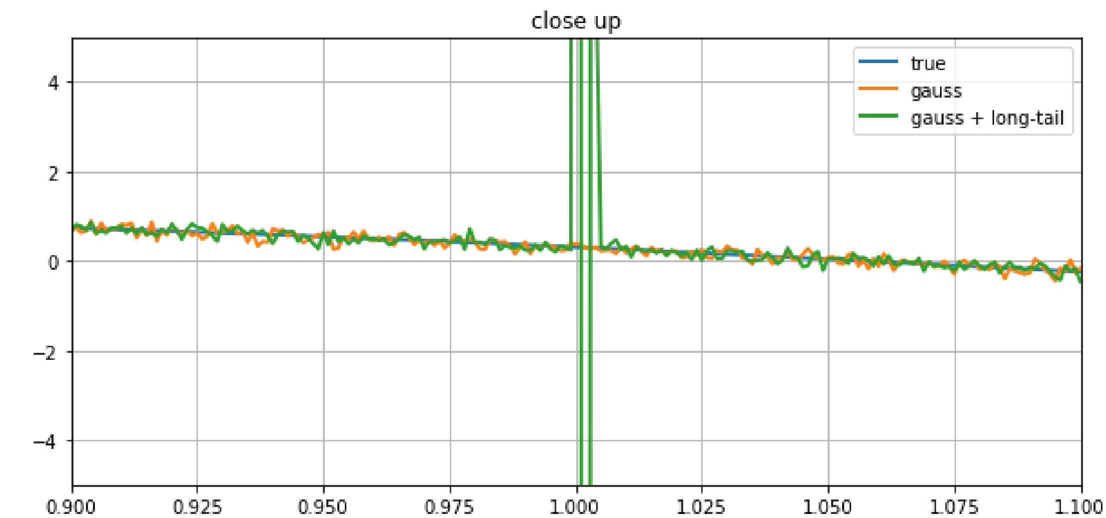
$$x_t = x_{t-1} + v_t * dt \tag{1}$$
$$v_t = v_{t-1} + w_t \tag{2}$$
$$w_t \sim \mathcal{N}(0, Q) \tag{3}$$

- Observaton Model

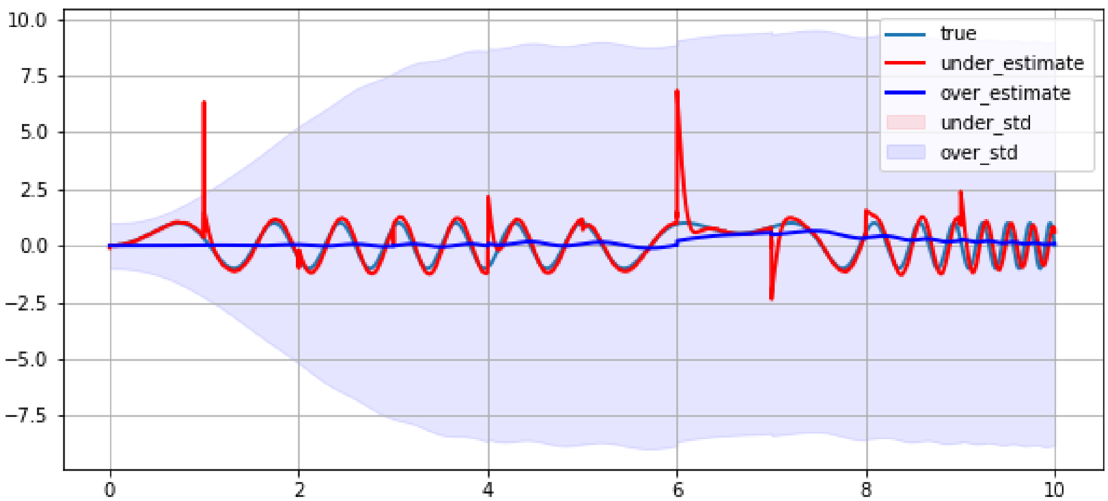
$$y_t = x_t + n_t \tag{4}$$
$$n_t \sim (1-\alpha)\mathcal{N}(0, R) + \alpha\mathcal{C}(0, R') \;\; \alpha = [0, 1] \tag{5}$$

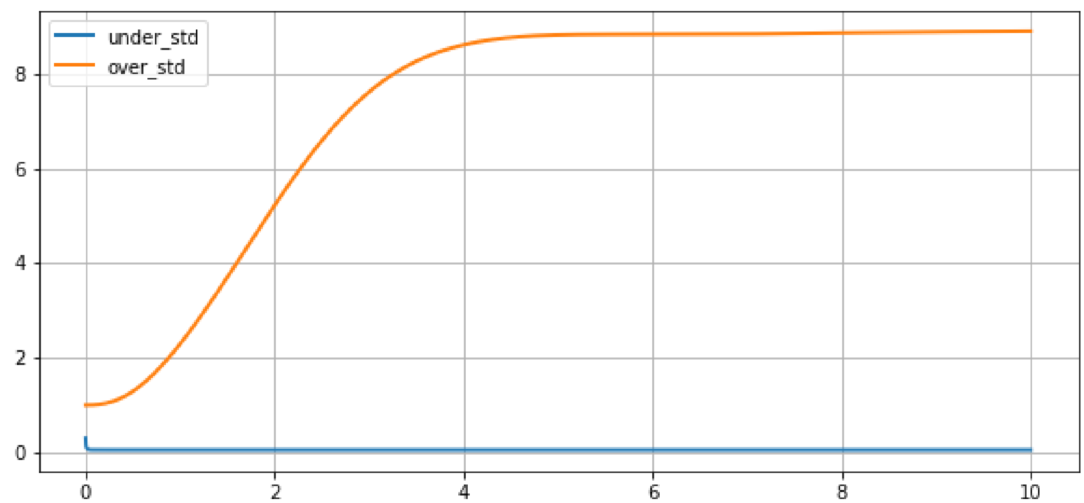
1.0.1 Overall Data





2 Kalman Filter on Long-tail Data

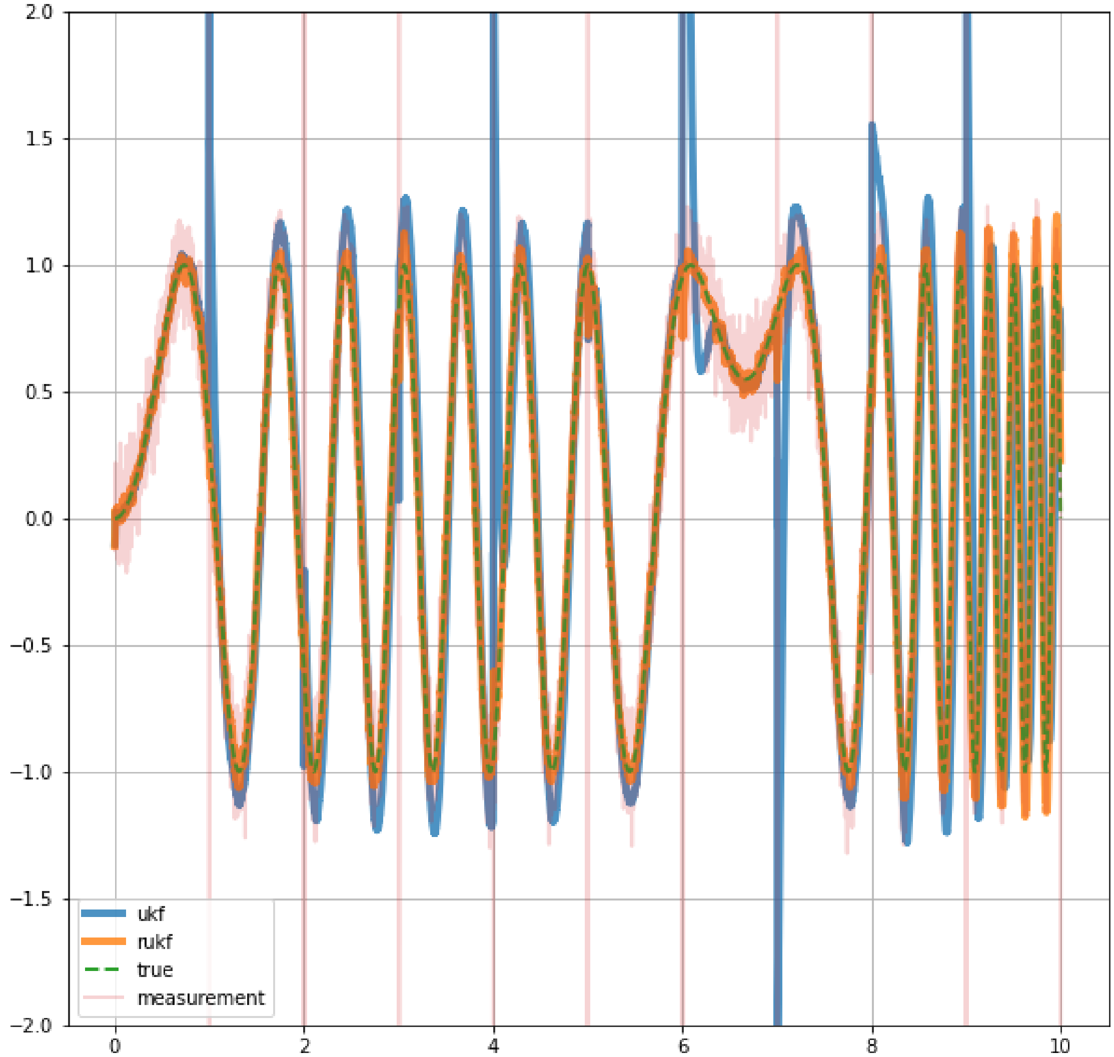




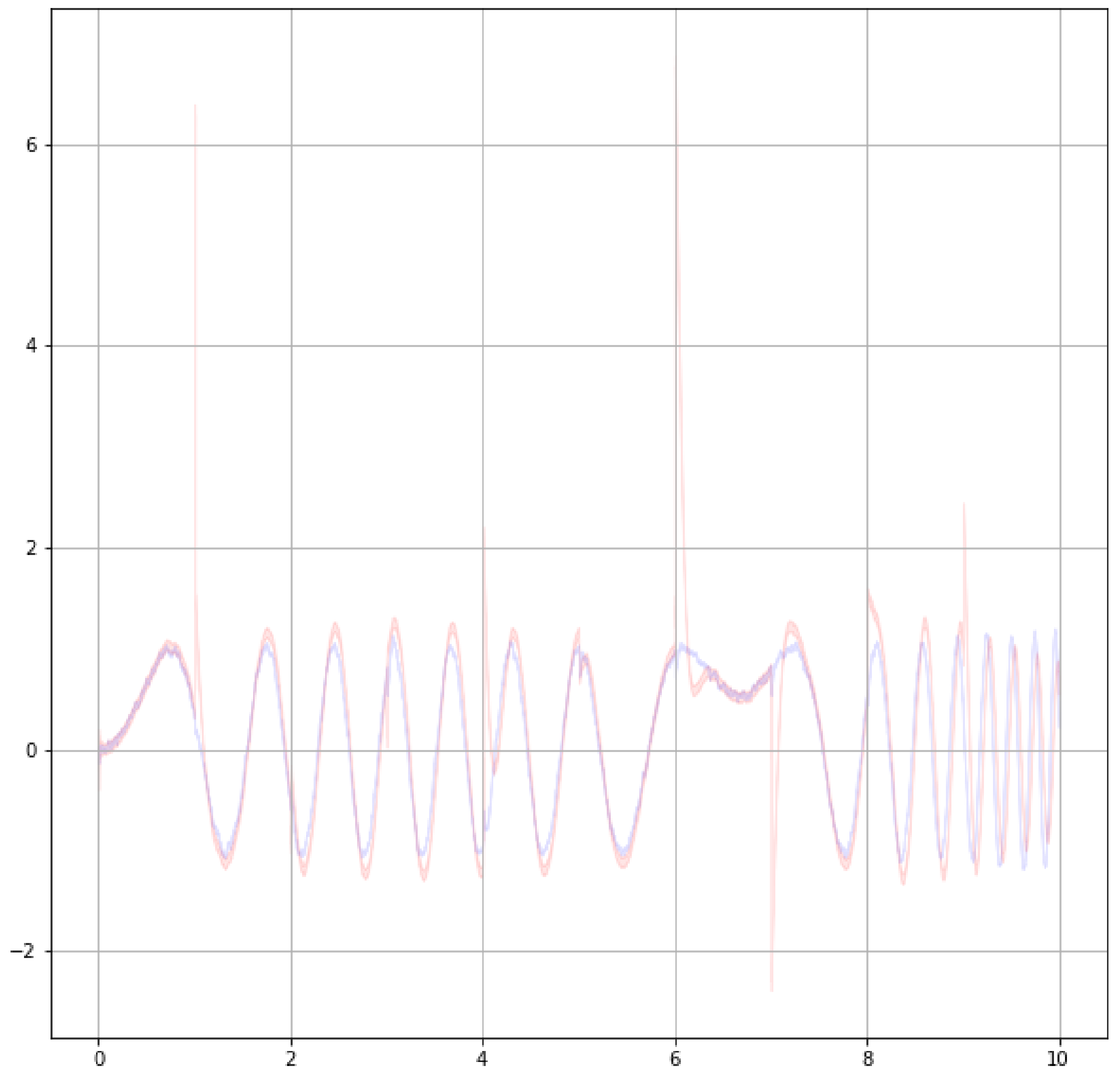
2.1 Robust Kalman Filter

modeling fat(long)-tail distribution in observation model

M. Wuthrich, S. Trimpe, D. Kappler, S. Schaal. A New Perspective and Extension of the Gaussian Filter. In Robotics: Science and Systems, 2015. http://arxiv.org/abs/1504.07941



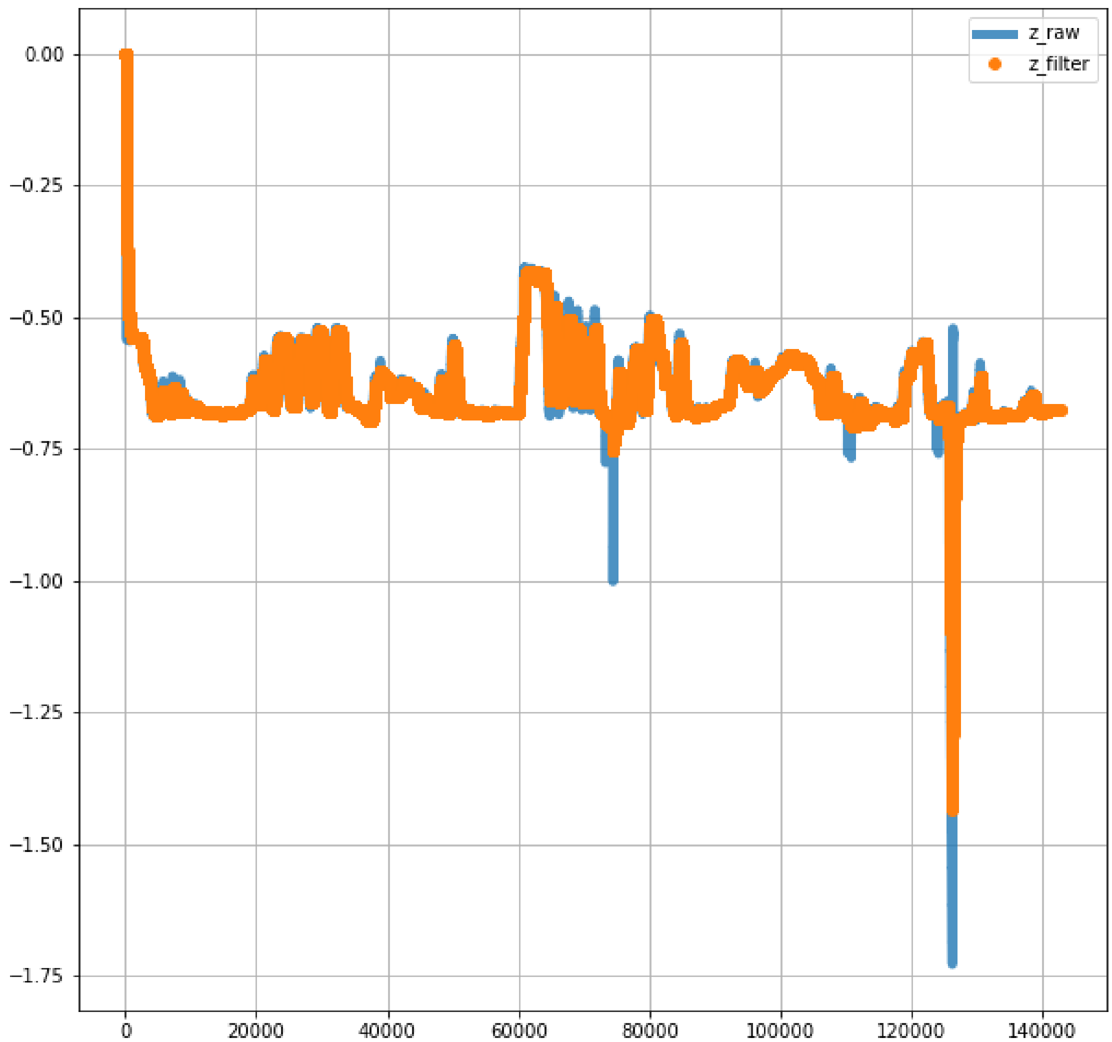
[35]: <matplotlib.collections.PolyCollection at 0x7f564109d7c0>



3 Real Data

- real data contains outlier also bias in measurement

[37]: <matplotlib.legend.Legend at 0x7f5641653f10>



[41]: <matplotlib.legend.Legend at 0x7f5679f075e0>

[42]: <matplotlib.legend.Legend at 0x7f56416b5940>

4 Bias Augmented Model

$$\begin{bmatrix} x_k \\ b_k \end{bmatrix} = \begin{bmatrix} F_k & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} x_{k-1} \\ b_{k-1} \end{bmatrix} + \begin{bmatrix} \sqsupseteq_k \\ \zeta_k \end{bmatrix} \quad \sqsupseteq_k \sim \mathcal{N}(0, Q_k^x) \;\; \zeta_k \sim \mathcal{N}(0, Q_k^b) \tag{6}$$

$$y_k = \begin{bmatrix} H_k & C_k \end{bmatrix} + v_k \quad \sqsubseteq_k \sim \mathcal{N}(0, R_k) \tag{7}$$

[52]: [<matplotlib.lines.Line2D at 0x7f564dc81ca0>]

5 Uncertainty Modeling in Lie Group(Manifold; non-Euclidean space)

5.0.1 Notation

$$T = \begin{bmatrix} R & p \\ \mathbf{0} & 1 \end{bmatrix} \tau = [v, w] \tag{8}$$

$$\mathcal{T} \in SE(3) \xrightarrow{Log(\bullet)} \tau \in R^6 \tag{9}$$

$$\tau \in R^6 \xrightarrow{Exp(\bullet)} \mathcal{T} \in SE(3) \tag{10}$$

5.0.2 Uncertainty on Lie Group

Setup

$$\zeta \sim \mathcal{N}(0, P), \zeta \in R^6 \tag{11}$$

$$T = \bar{T} Exp(\zeta) \tag{12}$$

$$T' = \bar{T} Exp(\zeta + \delta\zeta) \tag{13}$$

$$T' \approx \bar{T} Exp(\zeta) Exp(\mathbf{J_r}(\zeta)\delta\zeta) \tag{14}$$

$$T' \approx T Exp(\mathbf{J_r}(\zeta)\delta\zeta) \tag{15}$$

$$\delta T = \mathbf{J_r}(\zeta)\delta\zeta \tag{16}$$

Sum up to 1

$$1 = \int p(\zeta)\, d\zeta \tag{17}$$

$$= \int \eta \exp\Big(-\frac{1}{2}\zeta^T P^{-1}\zeta\Big)\, d\zeta \tag{18}$$

$$= \int \beta \exp\Big(-\frac{1}{2}\mathbf{Log}(\bar{T}^{-1}T) P^{-1} \mathbf{Log}(\bar{T}^{-1}T)\Big) dT \tag{19}$$

Mean & Covariance

$$0 = \int \zeta p(\zeta)\, d\zeta \tag{20}$$

$$= \int \mathbf{Log}(\bar{T}^{-1}T) p(T) dT \tag{21}$$

$$\tag{22}$$

$$P = \int \zeta\zeta^T p(\zeta)\, d\zeta \tag{23}$$

$$= \int \mathbf{Log}(\bar{T}^{-1}T)\mathbf{Log}(\bar{T}^{-1}T)^T p(T) dT \tag{24}$$

$$\tag{25}$$