Small Variance Nonparametric Clustering on the Hypersphere

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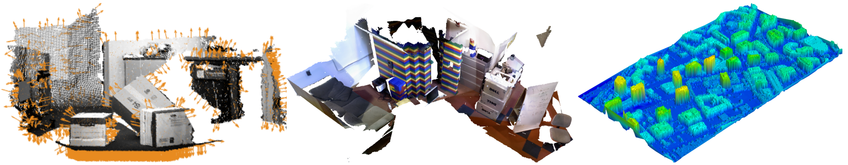
November 21, 2016

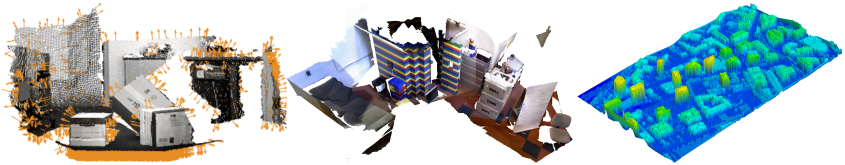
Scene Structure and Distribution of Normals



small scale

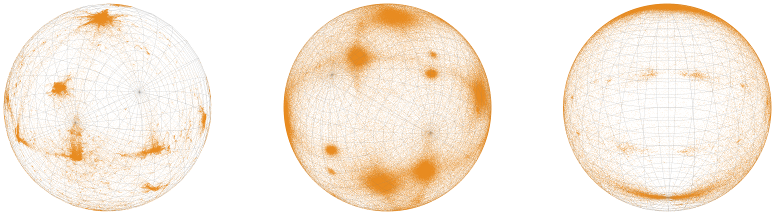
large scale

Scene Structure and Distribution of Normals



small scale

large scale

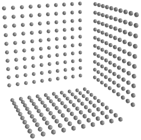


scene structure $\xrightarrow{\text{Gauss Map}}$ structured distribution of surface normals

Different Scene Representations

Real World $\approx$

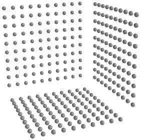
Level-Set
Point-Cloud
Mesh



Different Scene Representations

Real World ≈ Planes | Level-Set, Point-Cloud, Mesh

Planes
[Triebel 2005, Stückler 2008, Holz 2011]



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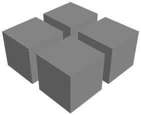
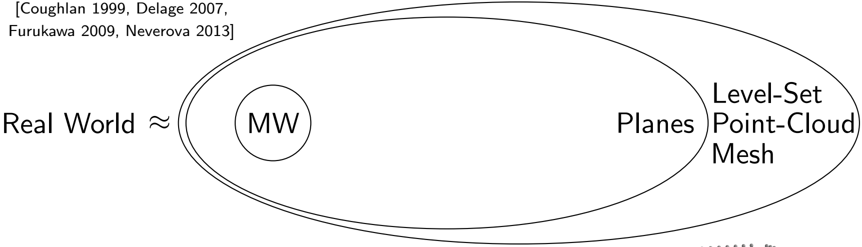


Manhattan World (MW)
[Coughlan 1999, Delage 2007, Furukawa 2009, Neverova 2013]



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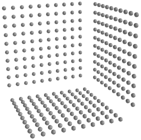

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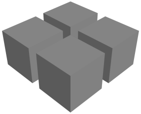

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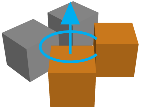
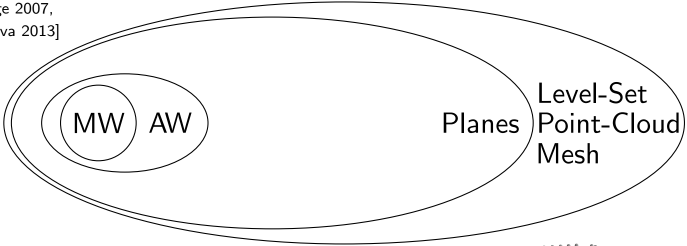


Atlanta World (AW)
[Schindler 2004]

Real World ≈



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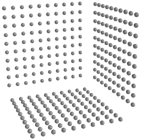

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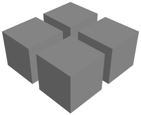

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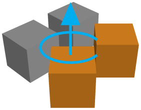

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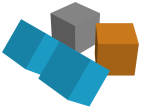
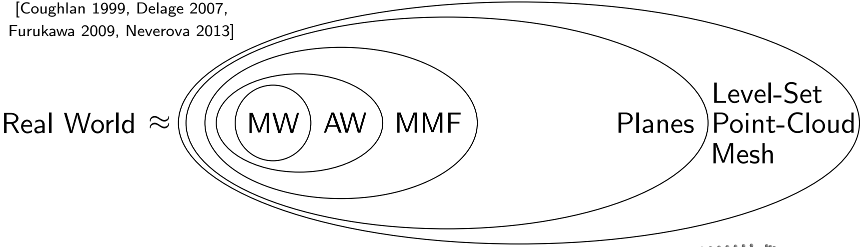


Mixture of Manhattan Frames (MMF)
[Straub 2014]



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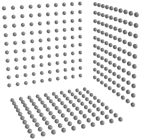

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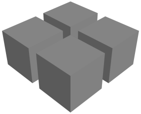

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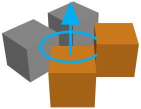

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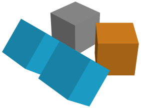

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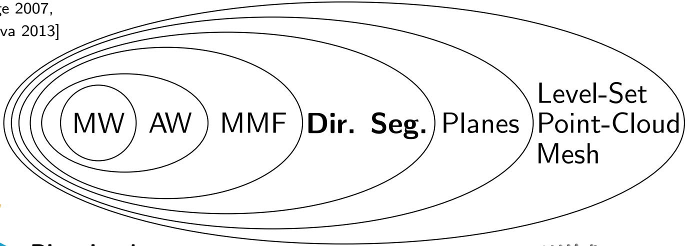
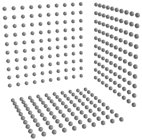


Directional Segmentation (this work)

Planes
[Triebel 2005, Stückler 2008, Holz 2011]



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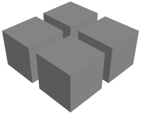

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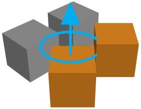

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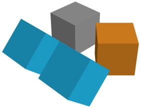

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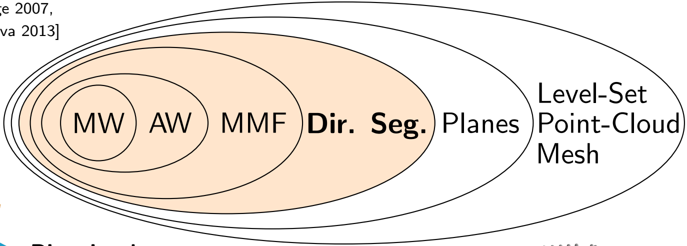

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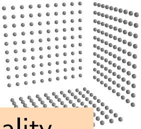
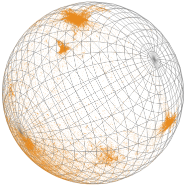


Directional Segmentation relaxes orthogonality constraints to describe complex man-made scenes.

Directional Segmentation

directional scene segmentation = clustering of scene's surface normals

image of scene



surface normals

Directional Segmentation

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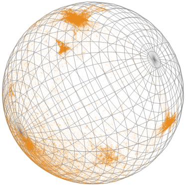

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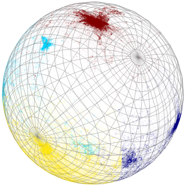
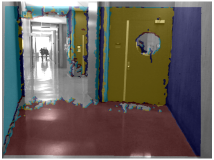


surface normal clustering



directional segmentation

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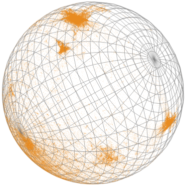

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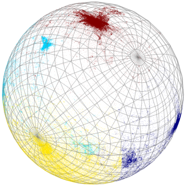

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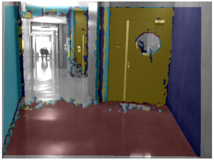


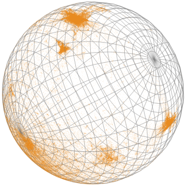
directional segmentation

- **Goals**:
  - manifold-aware (wrap-around)
  - fast and adaptive inference
  - temporal consistency for streaming data

Directional Segmentation

directional scene segmentation = clustering of scene's surface normals

image of scene



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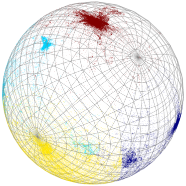

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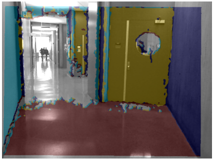

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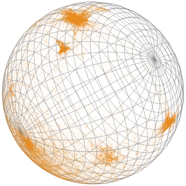
➤ DP-vMF-means

➤ DDP-vMF-means

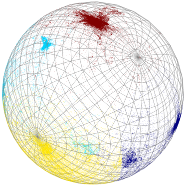
Directional Segmentation

directional scene segmentation = clustering of scene's surface normals

image of scene



surface normals



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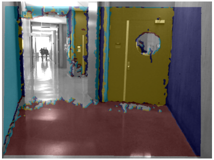

directional segmentation

- **Goals**:
  - manifold-aware (wrap-around)
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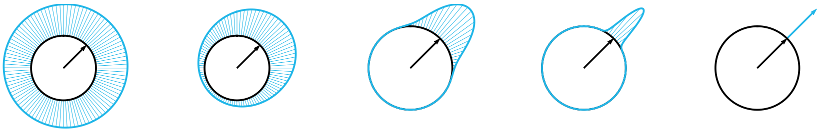
➤ **DP-vMF-means**
➤ **DDP-vMF-means**

- **Approach**:
  - Dirichlet Process (DP) mixture model
  - von-Mises-Fisher (vMF) distribution to model clusters on the sphere
  - k-means-like algorithms via small-variance asymptotics

von-Mises Fisher (vMF) Distribution



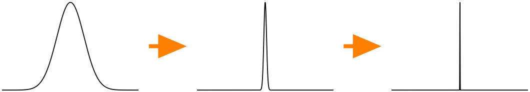
(a) $\tau = 0.1$ (b) $\tau = 1$ (c) $\tau = 10$ (d) $\tau = 100$ (e) $\tau \to \infty$

$$\text{vMF}(x; \mu, \tau) \propto \exp(\tau \mu^T x)$$

mean μ on the sphere

concentration $\tau > 0$

Small-Variance Asymptotics

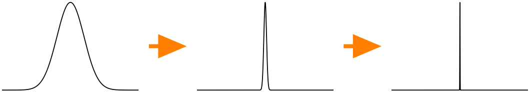


Gaussian mixture model (GMM) $\rightarrow$ k-means [Bishop 2006]

Dirichlet process GMM (DP-GMM) $\rightarrow$ DP-means [Kulis, Jordan 2012]

Dependent DP-GMM (DDP-GMM) $\rightarrow$ Dynamic Means [Campbell 2013]

Small-Variance Asymptotics



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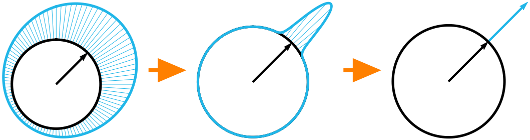
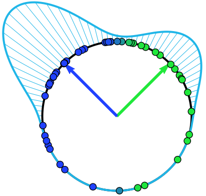


von-Mises-Fisher MM (vMF-MM) → spherical k-means [Banerjee 2005]
DP-vMF-MM → **DP-vMF-means (this work)**
DDP-vMF-MM → **DDP-vMF-means (this work)**

Dirichlet Process vMF Mixture Model Intuition



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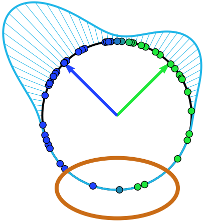

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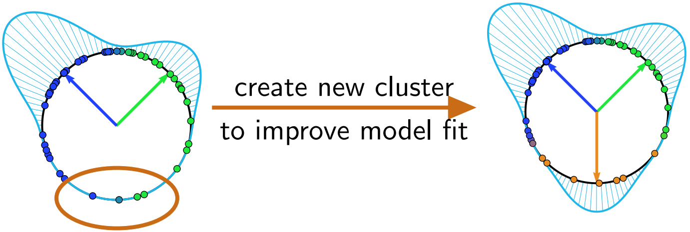

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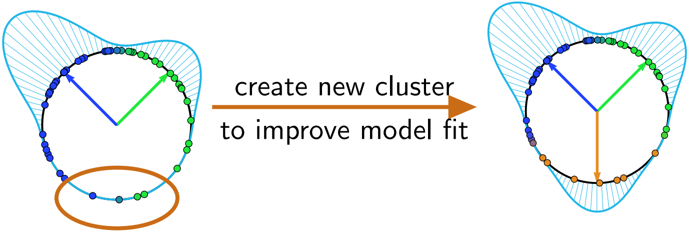
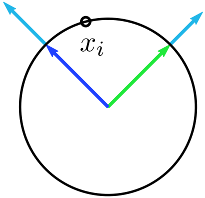


Gibbs sampling inference in DP-vMF-MM $\overset{\tau\to\infty}{\longrightarrow}$ DP-vMF-means

DP-vMF-means Label Update

$$z_i = \underset{k \in \{1,\ldots,K+1\}}{\arg\min} \begin{cases} \phi_k = \arccos(x_i^T \mu_k) & k \leq K \\ \phi_\lambda = \arccos(\lambda + 1) & k = K + 1 \end{cases}$$



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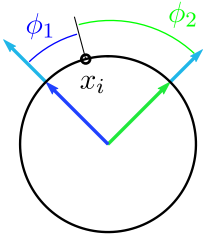

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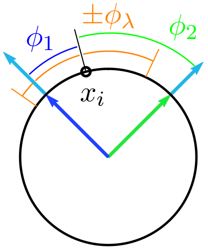

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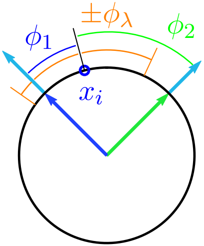

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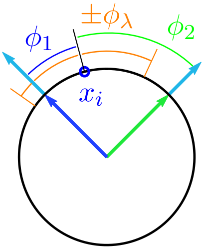

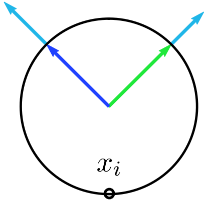

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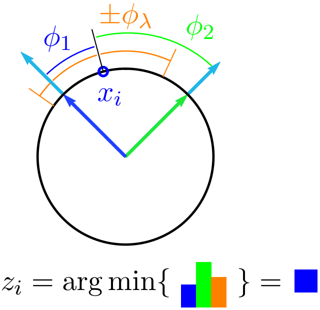

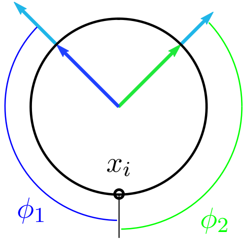

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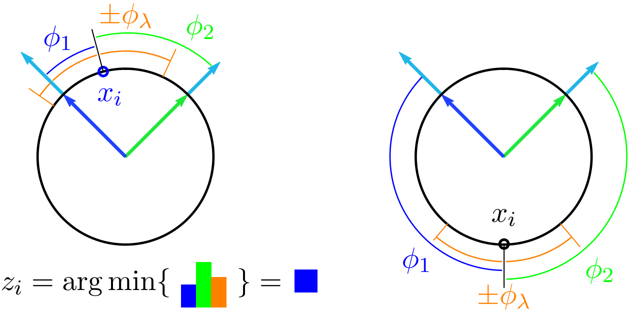
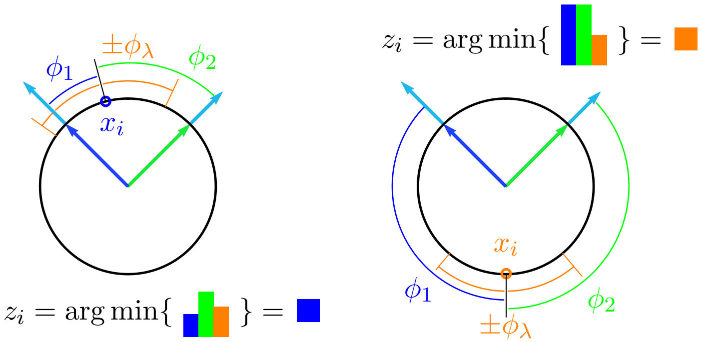


DP-vMF-means Label Update

$$z_i = \underset{k \in \{1,\dots,K+1\}}{\arg\min} \begin{cases} \phi_k = \arccos(x_i^T \mu_k) & k \le K \\ \phi_\lambda = \arccos(\lambda + 1) & k = K+1 \end{cases}$$



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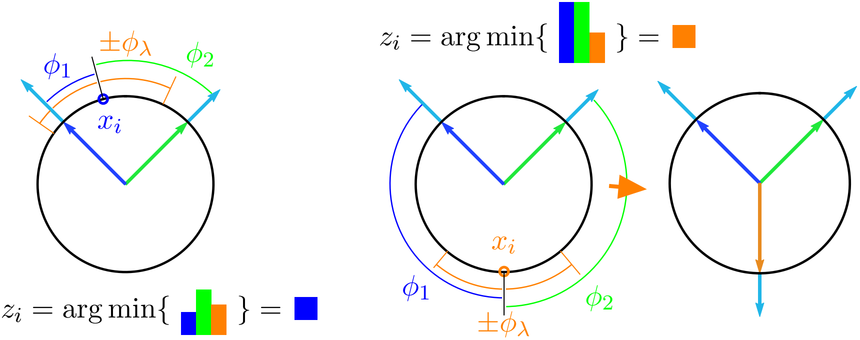
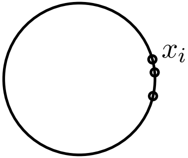


optimistic iterated restarts (OIR) to parallelize the label assignment

DP-vMF-means Parameter Update and Cost Fct.

parameter update: $\mu_k = \frac{\sum_{i \in \mathcal{I}_k} x_i}{\| \sum_{i \in \mathcal{I}_k} x_i \|_2} \quad \forall k \in \{1, \ldots, K\}$



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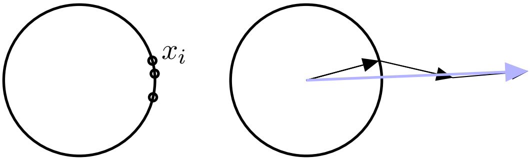

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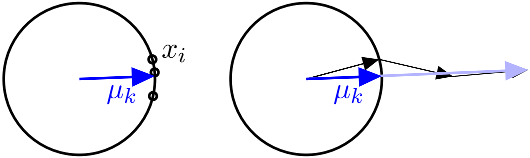

DP-vMF-means Parameter Update and Cost Fct.

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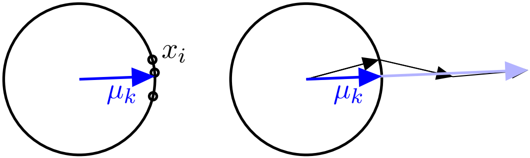

cost function: $J_{\text{DP-vMF}} = \underbrace{\sum_{k=1}^{K} \sum_{i \in \mathcal{I}_k} x_i^T \mu_k}_{\text{spherical } k\text{-means cost}}$

DP-vMF-means Parameter Update and Cost Fct.

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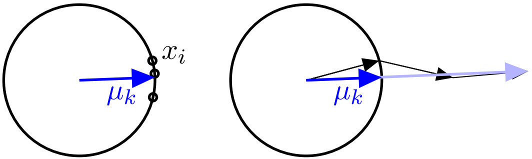
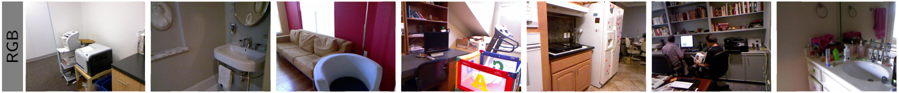
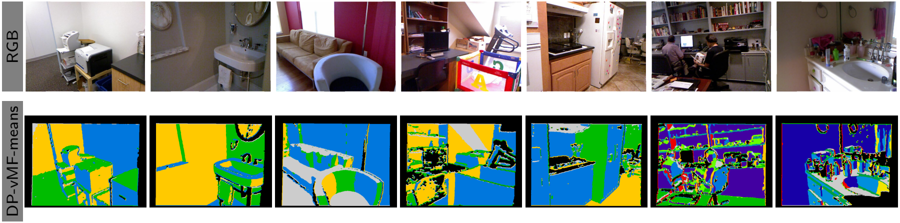


cost function: $J_{\text{DP-vMF}} = \underbrace{\sum_{k=1}^{K} \sum_{i \in \mathcal{I}_k} x_i^T \mu_k}_{\text{spherical } k\text{-means cost}} + \underbrace{\lambda K}_{\text{penalty for adding } K \text{ clusters}}$

Dir. Seg. of NYU RGB-D Scenes

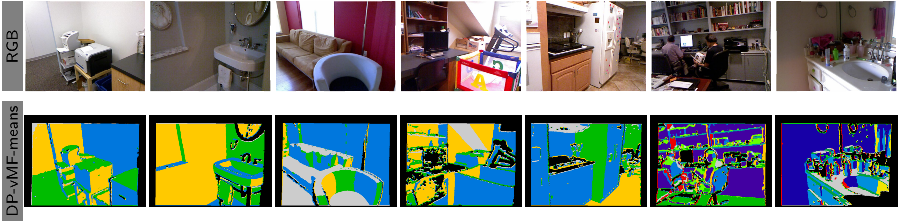


Dir. Seg. of NYU RGB-D Scenes



Black designates missing data due to sensor limitations.

Dir. Seg. of NYU RGB-D Scenes



$$\phi_\lambda^\star = 100^\circ = \underbrace{90^\circ}_{\text{Manhattan World}} + \underbrace{10^\circ}_{\text{noise}}$$

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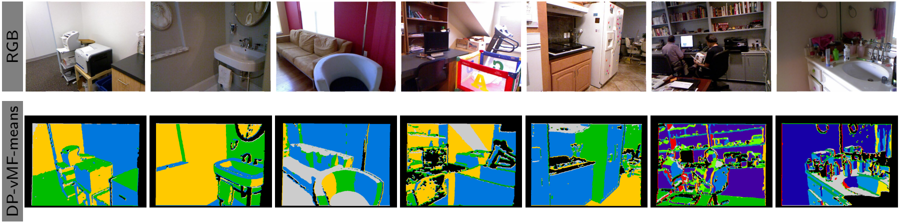

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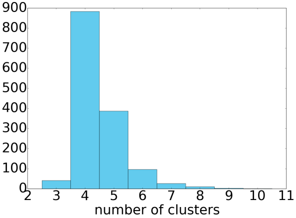

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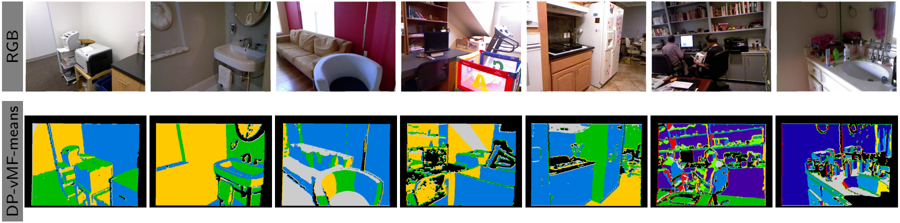
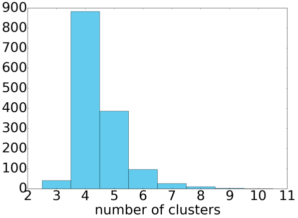
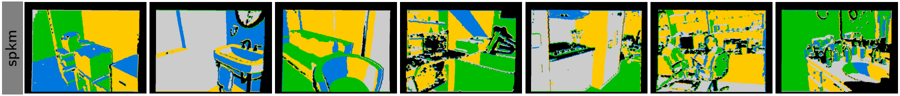
Dir. Seg. of NYU RGB-D Scenes



$$\phi_\lambda^\star = 100^\circ = \underbrace{90^\circ}_{\text{Manhattan World}} + \underbrace{10^\circ}_{\text{noise}}$$



$$K_{\text{spkm}}^\star = 4$$



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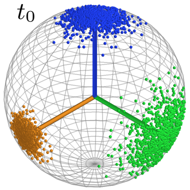
Dependent DP-vMF-means Intuition

Dependent DP-vMF-MM = Markov chain of DP-vMF-MMs

DP-vMF-MM



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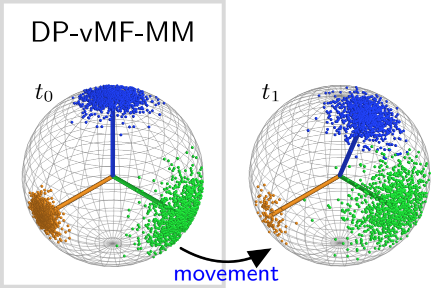

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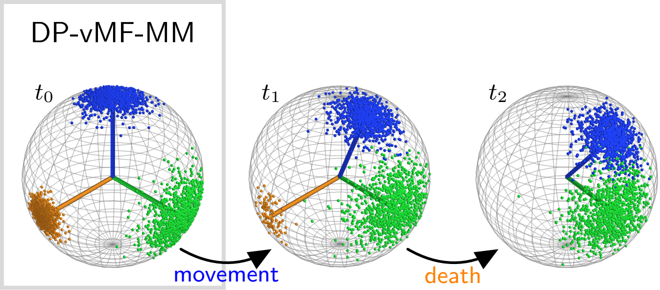

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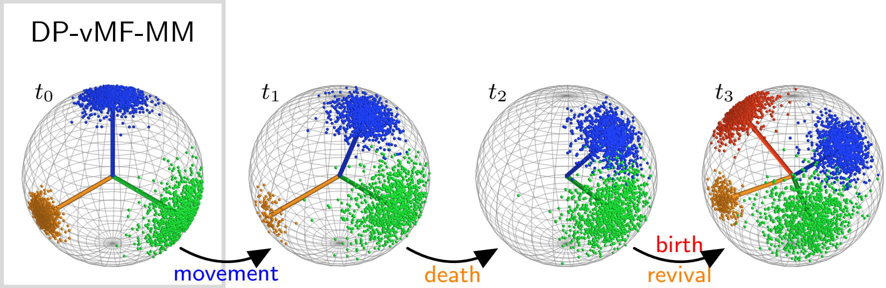

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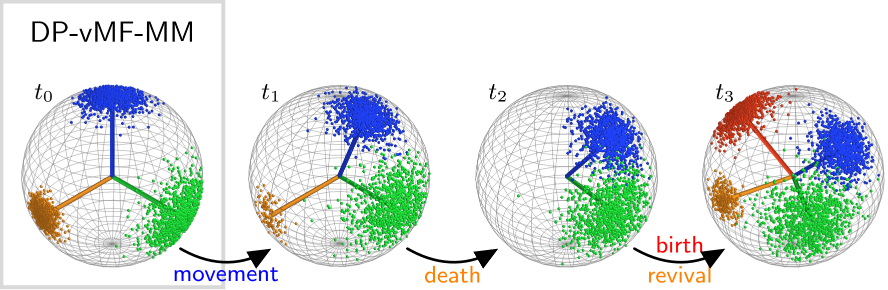
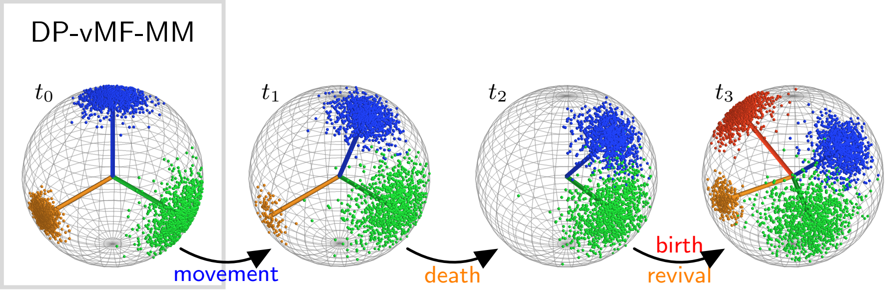


DDP-vMF-MM $\xrightarrow[\text{asymptotic analysis}]{\text{small-variance}}$ DDP-vMF-means

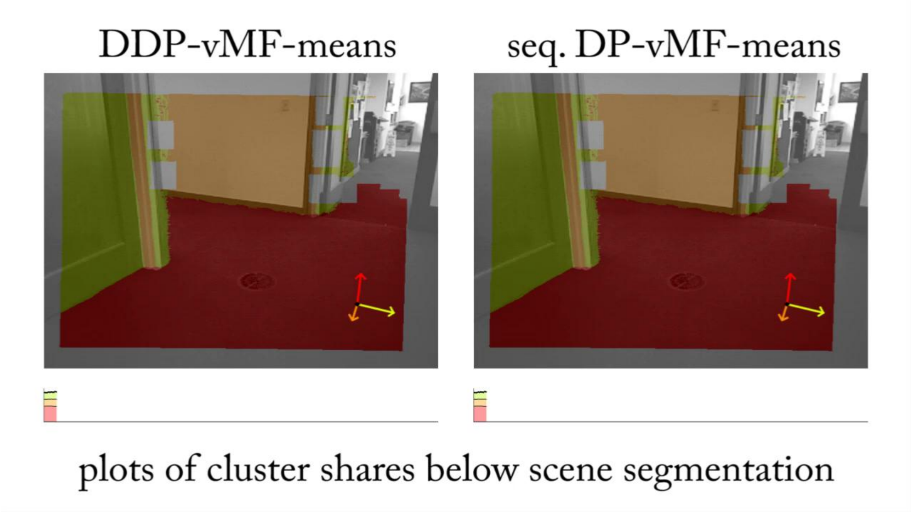
Dependent DP-vMF-means Intuition

Dependent DP-vMF-MM = Markov chain of DP-vMF-MMs

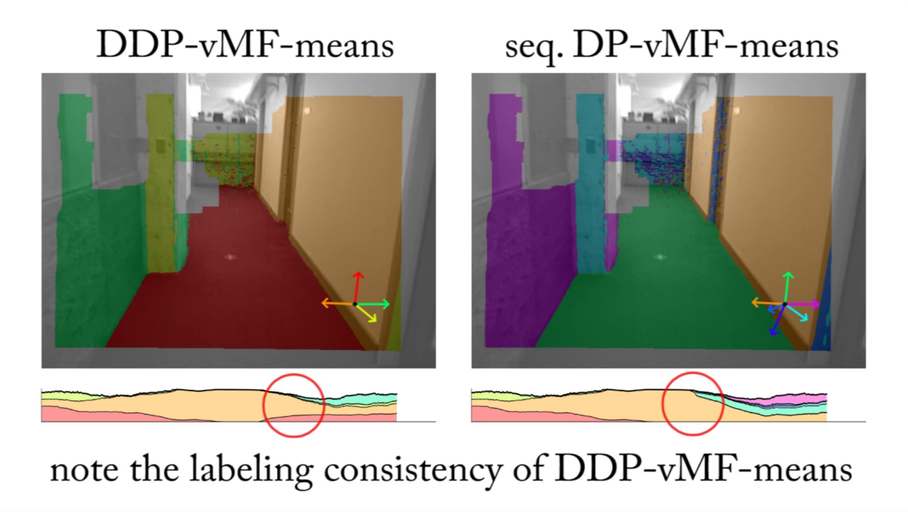


$$\text{DDP-vMF-MM} \xrightarrow[\text{asymptotic analysis}]{\text{small-variance}} \text{DDP-vMF-means}$$

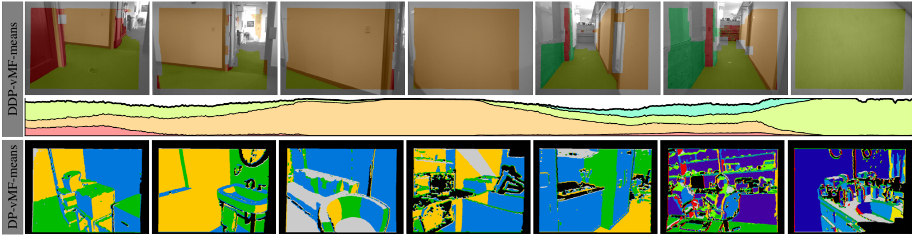
- label updates augmented to allow revival of clusters
- parameter updates account for unobserved motion of old clusters
- temporally consistent clustering


DDP-vMF-means
seq. DP-vMF-means
plots of cluster shares below scene segmentation

Real-time Streaming Directional Segmentation

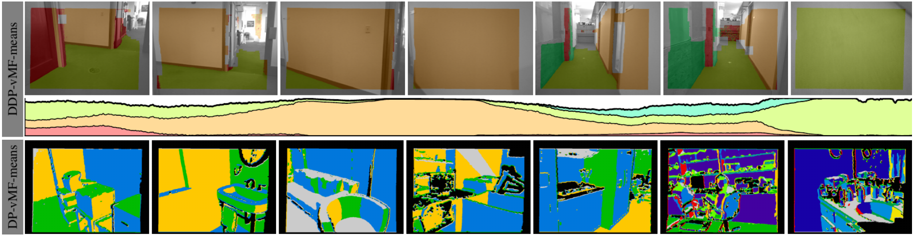


Conclusion



- k-means-like algorithms for nonparametric directional clustering
- principled derivation via small-variance asymptotics
- real-time operation via optimistic iterated restarts
- applicable to other directional data sources

Conclusion



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more details in the paper and at **poster 37**

code is available at `http://people.csail.mit.edu/jstraub/`