$$\therefore \quad \frac{p_A}{\rho g} + 0 = \frac{19.62 \times 10^4}{1000 \times 9.81} + 30 + \frac{4 \times .008 \times 400 \times .7074^2}{0.3 \times 2 \times 9.81}$$

$$= 20 + 30 + 1.088 = 51.088$$

$$\therefore \quad p_A = 51.088 \times 1000 \times 9.81 \text{ N/m}^2$$

$$= \frac{51.088 \times 1000 \times 9.81}{10^4}$$

$$= \mathbf{50.12 \ N/cm^2. \ Ans.}$$

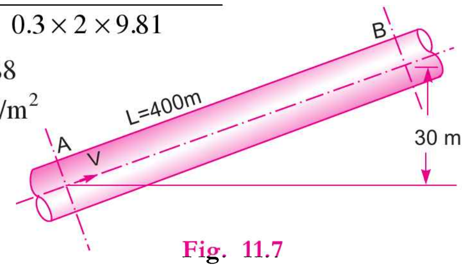

Fig. 11.7

▶ 11.5 HYDRAULIC GRADIENT AND TOTAL ENERGY LINE

The concept of hydraulic gradient line and total energy line is very useful in the study of flow of fluids through pipes. They are defined as :

11.5.1 Hydraulic Gradient Line. It is defined as the line which gives the sum of pressure head $\left(\frac{p}{w}\right)$ and datum head (z) of a flowing fluid in a pipe with respect to some reference line or it is the line which is obtained by joining the top of all vertical ordinates, showing the pressure head (p/w) of a flowing fluid in a pipe from the centre of the pipe. It is briefly written as H.G.L. (Hydraulic Gradient Line).

11.5.2 Total Energy Line. It is defined as the line which gives the sum of pressure head, datum head and kinetic head of a flowing fluid in a pipe with respect to some reference line. It is also defined as the line which is obtained by joining the tops of all vertical ordinates showing the sum of pressure head and kinetic head from the centre of the pipe. It is briefly written as T.E.L. (Total Energy Line).

Problem 11.22 *For the problem 11.16, draw the Hydraulic Gradient Line (H.G.L.) and Total Energy Line (T.E.L.).*

Solution. Given :

$$L = 50 \text{ m}, \; d = 200 \text{ mm} = 0.2 \text{ m}$$

$$H = 4 \text{ m}, \; f = .009$$

Velocity, V through pipe is calculated in problem 11.16 and its value is $V = 2.734$ m/s

Now, h_i = Head lost at entrance of pipe

$$= 0.5 \frac{V^2}{2g} + \frac{0.5 \times 2.734^2}{2 \times 9.81} = 0.19 \text{ m}$$

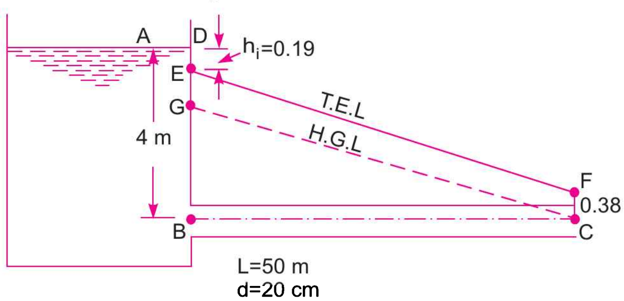

Fig. 11.8

and h_f = Head loss due to friction

$$= \frac{4 \times f \times L \times V^2}{d \times 2g} = \frac{4 \times 0.009 \times 50 \times (2.734)^2}{0.2 \times 2 \times 9.81} = 3.428 \text{ m.}$$

(*a*) **Total Energy Line (T.E.L.).** Consider three points, A, B and C on the free surface of water in the tank, at the inlet of the pipe and at the outlet of the pipe respectively as shown in Fig. 11.8. Let us find total energy at these points, taking the centre of pipe as reference line.

1. Total energy at $A = \frac{p}{\rho g} + \frac{V^2}{2g} + z = 0 + 0 + 4.0 = 4$ m
2. Total energy at B = Total energy at $A - h_i = 4.0 - 0.19 = 3.81$ m
3. Total energy at $C = \frac{p_c}{\rho g} + \frac{V_c^2}{2g} + z_c = 0 + \frac{V^2}{2g} + 0 = \frac{2.734^2}{2 \times 9.81} = 0.38$ m.

Hence total energy line will coincide with free surface of water in the tank. At the inlet of the pipe, it will decrease by h_i (= 0.19 m) from free surface and at outlet of pipe total energy is 0.38 m. Hence in Fig. 11.8,

(*i*) Point D represents total energy at A

(*ii*) Point E, where $DE = h_i$, represents total energy at inlet of the pipe

(*iii*) Point F, where $CF = 0.38$ represents total energy at outlet of pipe.

Join D to E and E to F. Then DEF represents the total energy line.

(*b*) **Hydraulic Gradient Line (H.G.L.).** H.G.L. gives the sum of $(p/w + z)$ with reference to the datum-line. Hence hydraulic gradient line is obtained by subtracting $\frac{V^2}{2g}$ from total energy line. At outlet of the pipe, total energy = $\frac{V^2}{2g}$. By subtracting $\frac{V^2}{2g}$ from total energy at this point, we shall get point C, which lies on the centre line of pipe. From C, draw a line CG parallel to EF. Then CG represents the hydraulic gradient line.

Problem 11.23 *For the problem 11.17, draw the hydraulic gradient and total energy line.*

Solution. Refer to problem 11.17.

Given : $L_1 = 25$ m, $d_1 = 0.15$ m

$L_2 = 15$ m, $d_2 = 0.3$ m, $f = .01$, $H = 8$ m

The velocity V_2 as calculated in problem 11.17 is

$$V_2 = 1.113 \text{ m/s}$$

$$V_1 = 4V_2 = 4 \times 1.113 = 4.452 \text{ m/s}$$

The various head losses are $h_i = 0.5 \times \frac{V_1^2}{2g} = \frac{0.5 \times 4.452^2}{2 \times 9.81} = 0.50$ m

$$h_{f_1} = \frac{4f \times L_1 \times V_1^2}{d_1 \times 2g} = \frac{4 \times .01 \times 25 \times (4.452)^2}{0.15 \times 2 \times 9.81} = 6.73 \text{ m}$$

$$h_e = \frac{(V_1 - V_2)^2}{2g} = \frac{(4.452 - 1.11)^2}{2 \times 9.81} = 0.568 \text{ m}$$

$$h_{f_2} = \frac{4 \times f \times L_2 \times V_2^2}{d_2 \times 2g} = \frac{4 \times .01 \times 15 \times (1.113)^2}{0.3 \times 2 \times 9.81} = 0.126 \text{ m}$$

$$h_o = \frac{V_2^2}{2g} = \frac{1.113^2}{2 \times 9.81} = 0.063 \text{ m}$$

Also $$V_1^2/2g = \frac{4.452^2}{2 \times 9.81} = 1.0 \text{ m.}$$

Total Energy Line

(*i*) Point *A* lies on free surface of water.

(*ii*) Take $AB = h_i = 0.5$ m.

(*iii*) From *B*, draw a horizontal line. Take *BL* equal to the length of pipe, *i.e.*, L_1. From *L* draw a vertical line downward.

(*iv*) Cut the line $LC = h_{f_1} = 6.73$ m.

(*v*) Join the point *B* to *C*. From *C*, take a line *CD* vertically downward equal to $h_e = 0.568$ m.

(*vi*) From *D*, draw *DM* horizontal and from point *F* which is lying on the centre of the pipe, draw a vertical line in the upward direction, meeting at *M*. From *M*, take a distance $ME = h_{f_2} = 0.126$. Join *DE*.

Then line *ABCDE* represents the total energy line.

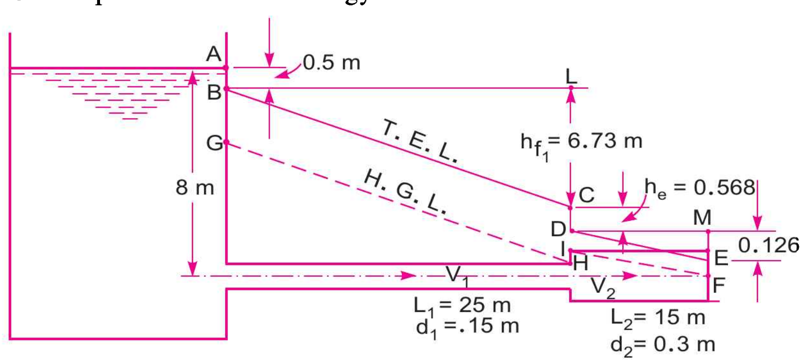

Fig. 11.9

Hydraulic Gradient Line (H.G.L.)

(*i*) From *B*, take $BG = \frac{V_1^2}{2g} = 1.0$ m.

(*ii*) Draw the line *GH* parallel to the line *BC*.

(*iii*) From *F*, draw a line *FI* parallel to the line *ED*.

(*iv*) Join the point *H* and *I*.

Then the line *GHIF* represents the hydraulic gradient line (H.G.L.).

Problem 11.24 *For Problem 11.18, draw the hydraulic gradient and total energy line.*

Solution. Refer to Problem 11.18,

Given : $d = 300$ mm $= 0.3$ m

$L = 400$ m, $Q = 300$ litres/s $= 0.3$ m^3/s

$f = .008$