11

CHAPTER

FLOW THROUGH PIPES

▶ 11.1 INTRODUCTION

In chapters 9 and 10, laminar flow and turbulent flow have been discussed. We have seen that when the Reynolds number is less than 2000 for pipe flow, the flow is known as laminar flow whereas when the Reynolds number is more than 4000, the flow is known as turbulent flow. In this chapter, the turbulent flow of fluids through pipes running full will be considered. If the pipes are partially full as in the case of sewer lines, the pressure inside the pipe is same and equal to atmospheric pressure. Then the flow of fluid in the pipe is not under pressure. This case will be taken in the chapter of flow of water through open channels. Here we will consider flow of fluids through pipes under pressure only.

▶ 11.2 LOSS OF ENERGY IN PIPES

When a fluid is flowing through a pipe, the fluid experiences some resistance due to which some of the energy of fluid is lost. This loss of energy is classified as :

Energy Losses

1. Major Energy Losses

   This is due to friction and it is calculated by the following formulae :
   (*a*) Darcy-Weisbach Formula
   (*b*) Chezy's Formula

2. Minor Energy Losses

   This is due to
   (*a*) Sudden expansion of pipe
   (*b*) Sudden contraction of pipe
   (*c*) Bend in pipe
   (*d*) Pipe fittings etc.
   (*e*) An obstruction in pipe.

▶ 11.3 LOSS OF ENERGY (OR HEAD) DUE TO FRICTION

(*a*) **Darcy-Weisbach Formula.** The loss of head (or energy) in pipes due to friction is calculated from Darcy-Weisbach equation which has been derived in chapter 10 and is given by

$$h_f = \frac{4 . f . L . V^2}{d \times 2g} \qquad \text{...(11.1)}$$

where h_f = loss of head due to friction

f = co-efficient of friction which is a function of Reynolds number

$$= \frac{16}{R_e} \text{ for } R_e < 2000 \text{ (viscous flow)}$$

$$= \frac{0.079}{R_e^{1/4}} \text{ for } R_e \text{ varying from 4000 to } 10^6$$

L = length of pipe,

V = mean velocity of flow,

d = diameter of pipe.

(*b*) **Chezy's Formula for loss of head due to friction in pipes.** Refer to chapter 10 article 10.3.1 in which expression for loss of head due to friction in pipes is derived. Equation (*iii*) of article 10.3.1, is

$$h_f = \frac{f'}{\rho g} \times \frac{P}{A} \times L \times V^2 \qquad \text{...(11.2)}$$

where h_f = loss of head due to friction, P = wetted perimeter of pipe,
A = area of cross-section of pipe, L = length of pipe,
and V = mean velocity of flow.

Now the ratio of $\frac{A}{P} \left(= \frac{\text{Area of flow}}{\text{Perimeter (wetted)}} \right)$ is called hydraulic mean depth or hydraulic radius and is denoted by m.

$\therefore$ Hydraulic mean depth, $m = \frac{A}{P} = \frac{\frac{\pi}{4} d^2}{\pi d} = \frac{d}{4}$

Substituting $\frac{A}{P} = m$ or $\frac{P}{A} = \frac{1}{m}$ in equation (11.2), we get

$$h_f = \frac{f'}{\rho g} \times L \times V^2 \times \frac{1}{m} \text{ or } V^2 = h_f \times \frac{\rho g}{f'} \times m \times \frac{1}{L} = \frac{\rho g}{f'} \times m \times \frac{h_f}{L}$$

$$\therefore \qquad V = \sqrt{\frac{\rho g}{f'} \times m \times \frac{h_f}{L}} = \sqrt{\frac{\rho g}{f'}} \sqrt{m \frac{h_f}{L}} \qquad \text{...(11.3)}$$

Let $\sqrt{\frac{\rho g}{f'}} = C$, where C is a constant known as Chezy's constant and $\frac{h_f}{L} = i$, where i is loss of head per unit length of pipe.

Substituting the values of $\sqrt{\frac{\rho g}{f'}}$ and $\sqrt{\frac{h_f}{L}}$ in equation (11.3), we get

$$V = C\sqrt{mi} \qquad \text{...(11.4)}$$

Equation (11.4) is known as Chezy's formula. Thus the loss of head due to friction in pipe from Chezy's formula can be obtained if the velocity of flow through pipe and also the value of C is known. The value of m for pipe is always equal to $d/4$.

Problem 11.1 *Find the head lost due to friction in a pipe of diameter 300 mm and length 50 m, through which water is flowing at a velocity of 3 m/s using (i) Darcy formula, (ii) Chezy's formula for which C = 60.*

Take ν for water = 0.01 stoke.

Solution. Given :

Dia. of pipe, $d = 300 \text{ mm} = 0.30 \text{ m}$

Length of pipe, $L = 50 \text{ m}$

Velocity of flow, $V = 3 \text{ m/s}$

Chezy's constant, $C = 60$

Kinematic viscosity, $\nu = 0.01 \text{ stoke} = 0.01 \text{ cm}^2/\text{s}$

$= 0.01 \times 10^{-4} \text{ m}^2/\text{s}.$

(*i*) **Darcy Formula** is given by equation (11.1) as

$$h_f = \frac{4 . f . L . V^2}{d \times 2g}$$

where 'f' = co-efficient of friction is a function of Reynolds number, R_e

But R_e is given by $R_e = \dfrac{V \times d}{\nu} = \dfrac{3.0 \times 0.30}{.01 \times 10^{-4}} = 9 \times 10^5$

$\therefore$ Value of $f = \dfrac{0.079}{R_e^{1/4}} = \dfrac{0.079}{\left(9 \times 10^5\right)^{1/4}} = .00256$

$\therefore$ Head lost, $h_f = \dfrac{4 \times .00256 \times 50 \times 3^2}{0.3 \times 2.0 \times 9.81}$ = **.7828 m. Ans.**

(*ii*) **Chezy's Formula.** Using equation (11.4)

$$V = C\sqrt{mi}$$

where $C = 60$, $m = \dfrac{d}{4} = \dfrac{0.30}{4} = 0.075 \text{ m}$

$\therefore$ $3 = 60\sqrt{.075 \times i}$ or $i = \left(\dfrac{3}{60}\right)^2 \times \dfrac{1}{.075} = 0.0333$

But $i = \dfrac{h_f}{L} = \dfrac{h_f}{50}$

Equating the two values of i, we have $\dfrac{h_f}{50} = .0333$

$\therefore$ $h_f = 50 \times .0333$ = **1.665 m. Ans.**

Problem 11.2 *Find the diameter of a pipe of length 2000 m when the rate of flow of water through the pipe is 200 litres/s and the head lost due to friction is 4 m. Take the value of C = 50 in Chezy's formulae.*

Solution. Given :

Length of pipe, $L = 2000$ m

Discharge, $Q = 200$ litre/s $= 0.2$ m^3/s

Head lost due to friction, $h_f = 4$ m

Value of Chezy's constant, $C = 50$

Let the diameter of pipe $= d$

Velocity of flow, $$V = \frac{\text{Discharge}}{\text{Area}} = \frac{Q}{\frac{\pi}{4}d^2} = \frac{0.2}{\frac{\pi}{4}d^2} = \frac{0.2 \times 4}{\pi d^2}$$

Hydraulic mean depth, $$m = \frac{d}{4}$$

Loss of head per unit length, $$i = \frac{h_f}{L} = \frac{4}{2000} = .002$$

Chezy's formula is given by equation (11.4) as $V = C\sqrt{mi}$

Substituting the values of V, m, i and C, we get

$$\frac{0.2 \times 4}{\pi d^2} = 50\sqrt{\frac{d}{4} \times .002} \text{ or } \sqrt{\frac{d}{4} \times .002} = \frac{0.2 \times 4}{\pi d^2 \times 50} = \frac{.00509}{d^2}$$

Squaring both sides, $$\frac{d}{4} \times .002 = \frac{.00509^2}{d^4} = \frac{.0000259}{d^4} \text{ or } d^5 = \frac{4 \times .0000259}{.002} = 0.0518$$

$\therefore$ $$d = \sqrt[5]{0.0518} = (.0518)^{1/5} = 0.553 \text{ m} = \mathbf{553\ mm.\ Ans.}$$

Problem 11.3 *A crude oil of kinematic viscosity 0.4 stoke is flowing through a pipe of diameter 300 mm at the rate of 300 litres per sec. Find the head lost due to friction for a length of 50 m of the pipe.*

Solution. Given :

Kinematic viscosity, $\nu = 0.4$ stoke $= 0.4$ cm^2/s $= .4 \times 10^{-4}$ m^2/s

Dia. of pipe, $d = 300$ mm $= 0.30$ m

Discharge, $Q = 300$ litres/s $= 0.3$ m^3/s

Length of pipe, $L = 50$ m

Velocity of flow, $$V = \frac{Q}{\text{Area}} = \frac{0.3}{\frac{\pi}{4}(0.3)^2} = 4.24 \text{ m/s}$$

$\therefore$ Reynolds number, $$R_e = \frac{V \times d}{\nu} = \frac{4.24 \times 0.30}{0.4 \times 10^{-4}} = 3.18 \times 10^4$$

As R_e lies between 4000 and 100000, the value of f is given by

$$f = \frac{.079}{(R_e)^{1/4}} = \frac{.079}{(3.18 \times 10^4)^{1/4}} = .00591$$

$\therefore$ Head lost due to friction, $h_f = \dfrac{4 \,.\, f \,.\, L \,.\, V^2}{d \times 2g} = \dfrac{4 \times .00591 \times 50 \times 4.24^2}{0.3 \times 2 \times 9.81}$ = **3.61 m. Ans.**

Problem 11.4 *An oil of sp. gr. 0.7 is flowing through a pipe of diameter 300 mm at the rate of 500 litres/s. Find the head lost due to friction and power required to maintain the flow for a length of 1000 m. Take ν = .29 stokes.*

Solution. Given :

Sp. gr. of oil, $S = 0.7$

Dia. of pipe, d = 300 mm = 0.3 m

Discharge, Q = 500 litres/s = 0.5 m^3/s

Length of pipe, L = 1000 m

Velocity, $$V = \frac{Q}{\text{Area}} = \frac{0.5}{\frac{\pi}{4} d^2} = \frac{0.5 \times 4}{\pi \times 0.3^2} = 7.073 \text{ m/s}$$

$\therefore$ Reynolds number, $$R_e = \frac{V \times d}{\nu} = \frac{7.073 \times 0.3}{0.29 \times 10^{-4}} = 7.316 \times (10)^4$$

$\therefore$ Co-efficient of friction, $$f = \frac{.079}{R_e^{1/4}} = \frac{0.79}{\left(7.316 \times 10^4\right)^{1/4}} = .0048$$

$\therefore$ Head lost due to friction, $$h_f = \frac{4 \times f \times L \times V^2}{d \times 2g} = \frac{4 \times .0048 \times 1000 \times 7.073^2}{0.3 \times 2 \times 9.81} = 163.18 \text{ m}$$

Power required $$= \frac{\rho g \,.\, Q \,.\, h_f}{1000} \text{ kW}$$

where ρ = density of oil = 0.7 × 1000 = 700 kg/m^3

$\therefore$ Power required $= \dfrac{700 \times 9.81 \times 0.5 \times 163.18}{1000}$ = **560.28 kW. Ans.**

Problem 11.5 *Calculate the discharge through a pipe of diameter 200 mm when the difference of pressure head between the two ends of a pipe 500 m apart is 4 m of water. Take the value of 'f' = 0.009 in the formula* $h_f = \dfrac{4 \,.\, f \,.\, L \,.\, V^2}{d \times 2g}$.

Solution. Given :

Dia. of pipe, d = 200 mm = 0.20 m

Length of pipe, L = 500 m

Difference of pressure head, h_f = 4 m of water

f = .009

Using equation (11.1), we have $$h_f = \frac{4 \times f \times L \times V^2}{d \times 2g}$$

or $$4.0 = \frac{4 \times .009 \times 500 \times V^2}{0.2 \times 2 \times 9.81} \text{ or } V^2 = \frac{4.0 \times 0.2 \times 2 \times 9.81}{4.0 \times .009 \times 500} = 0.872$$

$\therefore$ $V = \sqrt{0.872} = 0.9338 \simeq 0.934$ m/s

$\therefore$ Discharge, Q = velocity × area

$$= 0.934 \times \frac{\pi}{4} d^2 = 0.934 \times \frac{\pi}{4} (0.2)^2$$

$= 0.0293$ m^3/s = **29.3 litres/s. Ans.**

Problem 11.6 *Water is flowing through a pipe of diameter 200 mm with a velocity of 3 m/s. Find the head lost due to friction for a length of 5 m if the co-efficient of friction is given by $f = 0.02 + \frac{.09}{R_e^{0.3}}$, where R_e is Reynolds number. The kinematic viscosity of water = .01 stoke.*

Solution. Given :

Dia. of pipe, $d = 200$ mm $= 0.20$ m

Velocity, $V = 3$ m/s

Length, $L = 5$ m

Kinematic viscosity, $\nu = 0.01$ stoke $= .01 \times 10^{-4}$ m^2/s

$\therefore$ Reynolds number, $R_e = \frac{V \times d}{\nu} = \frac{3 \times 0.20}{.01 \times 10^{-4}} = 6 \times 10^5$

Value of $f = .02 + \frac{0.9}{R_e^{0.3}} = .02 + \frac{.09}{(6 \times 10^5)^{.3}} = .02 + \frac{0.09}{54.13}$

$= .02 + .00166 = 0.02166$

$\therefore$ Head lost due to friction, $h_f = \frac{4 \times f \times L \times V^2}{d \times 2g} = \frac{4.0 \times .02166 \times 5.0 \times 3^2}{0.20 \times 2.0 \times 9.81}$

= **0.993 m of water. Ans.**

Problem 11.7 *An oil of sp. gr. 0.9 and viscosity 0.06 poise is flowing through a pipe of diameter 200 mm at the rate of 60 litres/s. Find the head lost due to friction for a 500 m length of pipe. Find the power required to maintain this flow.*

Solution. Given :

Sp. gr. of oil $= 0.9$

Viscosity, $\mu = 0.06$ poise $= \frac{0.06}{10}$ Ns/m^2

Dia. of pipe, $d = 200$ mm $= 0.2$ m

Discharge, $Q = 60$ litres/s $= 0.06$ m^3/s

Length, $L = 500$ m

Density $\rho = 0.9 \times 1000 = 900$ kg/m^3

$\therefore$ Reynolds number, $R_e = \frac{\rho V d}{\mu} = 900 \times \frac{V \times 0.2}{\frac{0.06}{10}}$

where $V = \frac{Q}{\text{Area}} = \frac{0.06}{\frac{\pi}{4} d^2} = \frac{0.06}{\frac{\pi}{4}(.2)^2} = 1.909$ m/s $\simeq 1.91$ m/s

$$\therefore \qquad R_e = 900 \times \frac{1.91 \times 0.2 \times 10}{0.06} = 57300$$

As R_e lies between 4000 and 10^5, the value of co-efficient of friction, f is given by

$$f = \frac{0.079}{R_e^{0.25}} = \frac{0.079}{(57300)^{0.25}} = .0051$$

$$\therefore \text{ Head lost due to friction, } h_f = \frac{4 \times f \times L \times V^2}{d \times 2g} = \frac{4 \times .0051 \times 500 \times 1.91^2}{0.2 \times 2 \times 9.81}$$

$$= \mathbf{9.48 \text{ m of water. Ans.}}$$

$$\therefore \text{ Power required } = \frac{\rho g \cdot Q \cdot h_f}{1000} = \frac{900 \times 9.81 \times 0.06 \times 9.48}{1000} = \mathbf{5.02 \text{ kW. Ans.}}$$

▶ 11.4 MINOR ENERGY (HEAD) LOSSES

The loss of head or energy due to friction in a pipe is known as major loss while the loss of energy due to change of velocity of the following fluid in magnitude or direction is called minor loss of energy. The minor loss of energy (or head) includes the following cases :

1. Loss of head due to sudden enlargement,
2. Loss of head due to sudden contraction,
3. Loss of head at the entrance of a pipe,
4. Loss of head at the exit of a pipe,
5. Loss of head due to an obstruction in a pipe,
6. Loss of head due to bend in the pipe,
7. Loss of head in various pipe fittings.

In case of long pipe the above losses are small as compared with the loss of head due to friction and hence they are called minor losses and even may be neglected without serious error. But in case of a short pipe, these losses are comparable with the loss of head due to friction.

11.4.1 Loss of Head Due to Sudden Enlargement. Consider a liquid flowing through a pipe which has sudden enlargement as shown in Fig. 11.1. Consider two sections (1)-(1) and (2)-(2) before and after the enlargement.

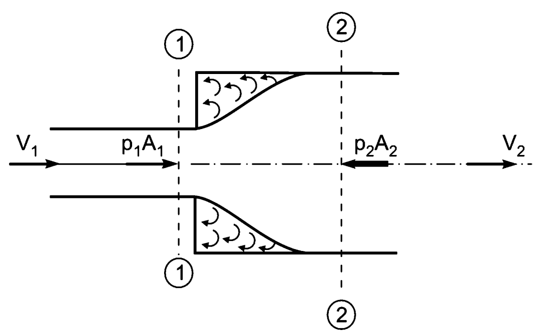

Fig. 11.1 *Sudden enlargement.*

Let p_1 = pressure intensity at section 1-1,

V_1 = velocity of flow at section 1-1,

A_1 = area of pipe at section 1-1,

p_2, V_2 and A_2 = corresponding values at section 2-2.

Due to sudden change of diameter of the pipe from D_1 to D_2, the liquid flowing from the smaller pipe is not able to follow the abrupt change of the boundary. Thus the flow separates from the boundary and turbulent eddies are formed as shown in Fig. 11.1. The loss of head (or energy) takes place due to the formation of these eddies.

Let p' = pressure intensity of the liquid eddies on the area $(A_2 - A_1)$

h_e = loss of head due to sudden enlargement

Applying Bernoulli's equation at sections 1-1 and 2-2,

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + \text{loss of head due to sudden enlargement}$$

But $z_1 = z_2$ as pipe is horizontal

$$\therefore \quad \frac{p_1}{\rho g} + \frac{V_1^2}{2g} = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + h_e$$

or
$$h_e = \left(\frac{p_1}{\rho g} - \frac{p_2}{\rho g}\right) + \left(\frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right) \quad \ldots(i)$$

Consider the control volume of liquid between sections 1-1 and 2-2. Then the force acting on the liquid in the control volume in the direction of flow is given by

$$F_x = p_1 A_1 + p'(A_2 - A_1) - p_2 A_2$$

But experimentally it is found that $p' = p_1$

$$\therefore \quad F_x = p_1 A_1 + p_1(A_2 - A_1) - p_2 A_2 = p_1 A_2 - p_2 A_2$$

$$= (p_1 - p_2) A_2 \quad \ldots(ii)$$

Momentum of liquid/sec at section 1-1 = mass × velocity

$$= \rho A_1 V_1 \times V_1 = \rho A_1 V_1^2$$

Momentum of liquid/sec at section 2-2 = $\rho A_2 V_2 \times V_2 = \rho A_2 V_2^2$

$\therefore$ Change of momentum/sec = $\rho A_2 V_2^2 - \rho A_1 V_1^2$

But from continuity equation, we have

$$A_1 V_1 = A_2 V_2 \text{ or } A_1 = \frac{A_2 V_2}{V_1}$$

$$\therefore \quad \text{Change of momentum/sec} = \rho A_2 V_2^2 - \rho \times \frac{A_2 V_2}{V_1} \times V_1^2 = \rho A_2 V_2^2 - \rho A_2 V_1 V_2$$

$$= \rho A_2 [V_2^2 - V_1 V_2] \quad \ldots(iii)$$

Now net force acting on the control volume in the direction of flow must be equal to the rate of change of momentum or change of momentum per second. Hence equating (*ii*) and (*iii*)

$$(p_1 - p_2) A_2 = \rho A_2 [V_2^2 - V_1 V_2]$$

or
$$\frac{p_1 - p_2}{\rho} = V_2^2 - V_1 V_2$$

Dividing by g on both sides, we have $\dfrac{p_1 - p_2}{\rho g} = \dfrac{V_2^2 - V_1 V_2}{g}$ or $\dfrac{p_1}{\rho g} - \dfrac{p_2}{\rho g} = \dfrac{V_2^2 - V_1 V_2}{g}$

Substituting the value of $\left(\frac{p_1}{\rho g} - \frac{p_2}{\rho g}\right)$ in equation (i), we get

$$h_e = \frac{V_2^2 - V_1V_2}{g} + \frac{V_1^2}{2g} - \frac{V_2^2}{2g} = \frac{2V_2^2 - 2V_1V_2 + V_1^2 - V_2^2}{2g}$$

$$= \frac{V_2^2 + V_1^2 - 2V_1V_2}{2g} = \left(\frac{V_1 - V_2}{2g}\right)^2$$

$$\therefore \qquad h_e = \frac{(V_1 - V_2)^2}{2g}. \qquad ...(11.5)$$

11.4.2 Loss of Head due to Sudden Contraction. Consider a liquid flowing in a pipe which has a sudden contraction in area as shown in Fig. 11.2. Consider two sections 1-1 and 2-2 before and after contraction. As the liquid flows from large pipe to smaller pipe, the area of flow goes on decreasing and becomes minimum at a section C-C as shown in Fig. 11.2. This section C-C is called Vena-contracta. After section C-C, a sudden enlargement of the area takes place. The loss of head due to sudden contraction is actually due to sudden enlargement from Vena-contracta to smaller pipe.

Let A_c = Area of flow at section C-C

V_c = Velocity of flow at section C-C

A_2 = Area of flow at section 2-2

V_2 = Velocity of flow at section 2-2

h_c = Loss of head due to sudden contraction.

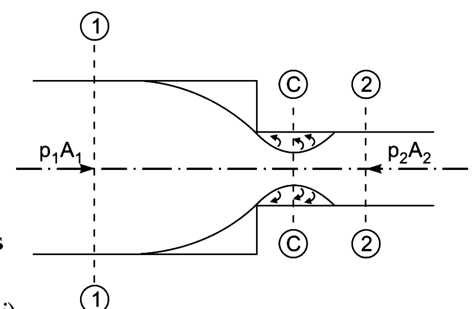

Fig. 11.2 *Sudden contraction.*

Now h_c = actual loss of head due to enlargement from section C-C to section 2-2 and is given by equation (11.5) as

$$= \frac{(V_c - V_2)^2}{2g} = \frac{V_2^2}{2g}\left[\frac{V_c}{V_2} - 1\right]^2 \qquad ...(i)$$

From continuity equation, we have

$$A_cV_c = A_2V_2 \quad \text{or} \quad \frac{V_c}{V_2} = \frac{A_2}{A_c} = \frac{1}{(A_c / A_2)} = \frac{1}{C_c} \qquad \left[\because C_c = \frac{A_c}{A_2}\right]$$

Substituting the value of $\frac{V_c}{V_2}$ in (i), we get

$$h_c = \frac{V_2^2}{2g}\left[\frac{1}{C_c} - 1\right]^2 \qquad ...(11.6)$$

$$= \frac{kV_2^2}{2g}, \text{ where } k = \left[\frac{1}{C_c} - 1\right]^2$$

If the value of C_c is assumed to be equal to 0.62, then

$$k = \left[\frac{1}{0.62} - 1\right]^2 = 0.375$$

Then h_c becomes as $$h_c = \frac{kV_2^2}{2g} = 0.375\,\frac{V_2^2}{2g}$$

If the value of C_c is not given then the head loss due to contraction is taken as

$$= 0.5\,\frac{V_2^2}{2g} \text{ or } h_c = 0.5\,\frac{V_2^2}{2g}. \qquad ...(11.7)$$

Problem 11.8 *Find the loss of head when a pipe of diameter 200 mm is suddenly enlarged to a diameter of 400 mm. The rate of flow of water through the pipe is 250 litres/s.*

Solution. Given :

Dia. of smaller pipe, $D_1 = 200$ mm $= 0.20$ m

$\therefore$ Area, $$A_1 = \frac{\pi}{4} D_1^{\,2} = \frac{\pi}{4}(.2)^2 = 0.03141 \text{ m}^2$$

Dia. of large pipe, $D_2 = 400$ mm $= 0.4$ m

$\therefore$ Area, $$A_2 = \frac{\pi}{4} \times (0.4)^2 = 0.12564 \text{ m}^2$$

Discharge, $Q = 250$ litres/s $= 0.25$ m^3/s

Velocity, $$V_1 = \frac{Q}{A_1} = \frac{0.25}{.03141} = 7.96 \text{ m/s}$$

Velocity, $$V_2 = \frac{Q}{A_2} = \frac{0.25}{.12564} = 1.99 \text{ m/s}$$

Loss of head due to enlargement is given by equation (11.5) as

$$h_e = \frac{(V_1 - V_2)^2}{2g} = \frac{(7.96 - 1.99)^2}{2g} = \textbf{1.816 m of water. Ans.}$$

Problem 11.9 *At a sudden enlargement of a water main from 240 mm to 480 mm diameter, the hydraulic gradient rises by 10 mm. Estimate the rate of flow.* (J.N.T.U., S 2002)

Solution. Given :

Dia. of smaller pipe, $D_1 = 240$ mm $= 0.24$ m

$\therefore$ Area, $$A_1 = \frac{\pi}{4} D_1^{\,2} = \frac{\pi}{4}(.24)^2$$

Dia. of large pipe, $D_2 = 480$ mm $= 0.48$ m

$\therefore$ Area, $$A_2 = \frac{\pi}{4}(0.48)^2$$

Rise of hydraulic gradient*, *i.e.*, $$\left(z_2 + \frac{p_2}{\rho g}\right) - \left(\frac{p_1}{\rho g} + z_1\right) = 10 \text{ mm} = \frac{10}{1000} = \frac{1}{100} \text{ m}$$

Let the rate of flow $= Q$

Applying Bernoulli's equation to both sections, *i.e.*, smaller pipe section, and large pipe section.

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + \text{Head loss due to enlargement} \qquad ...(i)$$

* Please refer Art. 11.5.1.

But head loss due to enlargement,

$$h_e = \frac{(V_1 - V_2)^2}{2g} \quad \text{...}(ii)$$

From continuity equation, we have $A_1V_1 = A_2V_2$

$$\therefore \quad V_1 = \frac{A_2V_2}{A_1} = \frac{\frac{\pi}{4}D_2^2 \times V_2}{\frac{\pi}{4}D_1^2} = \left(\frac{D_2}{D_1}\right)^2 \times V_2 = \left(\frac{.48}{.24}\right)^2 \times V_2 = 2^2 \times V_2 = 4V_2$$

Substituting this value in (*ii*), we get

$$h_e = \frac{(4V_2 - V_2)^2}{2g} = \frac{(3V_2)^2}{2g} = \frac{9V_2^2}{2g}$$

Now substituting the value of h_e and V_1 in equation (*i*),

$$\frac{p_1}{\rho g} + \frac{(4V_2)^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + \frac{9V_2^2}{2g}$$

or
$$\frac{16V_2^2}{2g} - \frac{V_2^2}{2g} - \frac{9V_2^2}{2g} = \left(\frac{p_2}{\rho g} + z_2\right) - \left(\frac{p_1}{\rho g} + z_1\right)$$

But hydraulic gradient rise $= \left(\frac{p_2}{\rho g} + z_2\right) - \left(\frac{p_1}{\rho g} + z_1\right) = \frac{1}{100}$

$$\therefore \quad \frac{16V_2^2}{2g} - \frac{V_2^2}{2g} - \frac{9V_2^2}{2g} = \frac{1}{100} \text{ or } \frac{6V_2^2}{2g} = \frac{1}{100}$$

$$\therefore \quad V_2 = \sqrt{\frac{2 \times 9.81}{6 \times 100}} = 0.1808 \simeq 0.181 \text{ m/s}$$

$\therefore$ Discharge, $Q = A_2 \times V_2$

$$= \frac{\pi}{4} D_2^2 \times V_2 = \frac{\pi}{4} (.48)^2 \times .181 = 0.03275 \text{ m}^3\text{/s}$$

$= \mathbf{32.75}$ **litres/s. Ans.**

Problem 11.10 *The rate of flow of water through a horizontal pipe is 0.25 m³/s. The diameter of the pipe which is 200 mm is suddenly enlarged to 400 mm. The pressure intensity in the smaller pipe is 11.772 N/cm². Determine :*

(*i*) *loss of head due to sudden enlargement,* (*ii*) *pressure intensity in the large pipe,*
(*iii*) *power lost due to enlargement.*

Solution. Given :

Discharge, $Q = 0.25 \text{ m}^3\text{/s}$

Dia. of smaller pipe, $D_1 = 200 \text{ mm} = 0.20 \text{ m}$

∴ Area, $A_1 = \frac{\pi}{4}(.2)^2 = 0.03141\ \text{m}^2$

Dia. of large pipe, $D_2 = 400$ mm $= 0.40$ m

∴ Area, $A_2 = \frac{\pi}{4}(0.4)^2 = 0.12566\ \text{m}^2$

Pressure in smaller pipe, $p_1 = 11.772\ \text{N/cm}^2 = 11.772 \times 10^4\ \text{N/m}^2$

Now velocity, $V_1 = \frac{Q}{A_1} = \frac{0.25}{.03141} = 7.96$ m/s

Velocity, $V_2 = \frac{Q}{A_2} = \frac{0.25}{.12566} = 1.99$ m/s

(*i*) Loss of head due to sudden enlargement,

$$h_e = \frac{(V_1 - V_2)^2}{2g} = \frac{(7.96 - 1.99)^2}{2 \times 9.81} = \mathbf{1.816\ m.\ Ans.}$$

(*ii*) Let the pressure intensity in large pipe $= p_2$.

Then applying Bernoulli's equation before and after the sudden enlargement,

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_e$$

But $z_1 = z_2$ (Given horizontal pipe)

$$\therefore \quad \frac{p_1}{\rho g} + \frac{V_1^2}{2g} = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + h_e \text{ or } \frac{p_2}{\rho g} = \frac{p_1}{\rho g} + \frac{V_1^2}{2g} - \frac{V_2^2}{2g} - h_e$$

$$= \frac{11.772 \times 10^4}{1000 \times 9.81} + \frac{7.96^2}{2 \times 9.81} - \frac{1.99^2}{2 \times 9.81} - 1.816$$

$$= 12.0 + 3.229 - 0.2018 - 1.8160$$

$$= 15.229 - 2.0178 = 13.21 \text{ m of water}$$

$$\therefore \quad p_2 = 13.21 \times \rho g = 13.21 \times 1000 \times 9.81\ \text{N/m}^2$$

$$= 13.21 \times 1000 \times 9.81 \times 10^{-4}\ \text{N/cm}^2 = \mathbf{12.96\ N/cm^2.\ Ans.}$$

(*iii*) Power lost due to sudden enlargement,

$$P = \frac{\rho g \cdot Q \cdot h_e}{1000} = \frac{1000 \times 9.81 \times 0.25 \times 1.816}{1000} = \mathbf{4.453\ kW.\ Ans.}$$

Problem 11.11 *A horizontal pipe of diameter 500 mm is suddenly contracted to a diameter of 250 mm. The pressure intensities in the large and smaller pipe is given as 13.734 N/cm² and 11.772 N/cm² respectively. Find the loss of head due to contraction if $C_c = 0.62$. Also determine the rate of flow of water.*

Solution. Given :

Dia. of large pipe, $D_1 = 500$ mm $= 0.5$ m

Area, $A_1 = \frac{\pi}{4}(0.5)^2 = 0.1963\ \text{m}^2$

Dia. of smaller pipe, $D_2 = 250 \text{ mm} = 0.25 \text{ m}$

$\therefore$ Area, $A_2 = \frac{\pi}{4} (.25)^2 = 0.04908 \text{ m}^2$

Pressure in large pipe, $p_1 = 13.734 \text{ N/cm}^2 = 13.734 \times 10^4 \text{ N/m}^2$

Pressure in smaller pipe, $p_2 = 11.772 \text{ N/cm}^2 = 11.772 \times 10^4 \text{ N/m}^2$

$C_c = 0.62$

Head lost due to contraction $= \frac{V_2^2}{2g}\left[\frac{1}{C_c} - 1.0\right]^2 = \frac{V_2^2}{2g}\left[\frac{1}{0.62} - 1.0\right]^2 = 0.375 \frac{V_2^2}{2g}$

From continuity equation, we have $A_1V_1 = A_2V_2$

or
$$V_1 = \frac{A_2V_2}{A_1} = \frac{\frac{\pi}{4}D_2^2 \times V_2}{\frac{\pi}{4}D_1^2} = \left(\frac{D_2}{D_1}\right)^2 \times V_2 = \left(\frac{0.25}{0.50}\right)^2 V_2 = \frac{V_2}{4}$$

Applying Bernoulli's equation before and after contraction,

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + h_c$$

But $z_1 = z_2$ (pipe is horizontal)

$$\therefore \quad \frac{p_1}{\rho g} + \frac{V_1^2}{2g} = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + h_c$$

But $h_c = 0.375 \frac{V_2^2}{2g}$ and $V_1 = \frac{V_2}{4}$

Substituting these values in the above equation, we get

$$\frac{13.734 \times 10^4}{9.81 \times 1000} + \frac{(V_2/4)^2}{2g} = \frac{11.772 \times 10^4}{1000 \times 9.81} + \frac{V_2^2}{2g} + 0.375 \frac{V_2^2}{2g}$$

or
$$14.0 + \frac{V_2^2}{16 \times 2g} = 12.0 + 1.375 \frac{V_2^2}{2g}$$

or
$$14 - 12 = 1.375 \frac{V_2^2}{2g} - \frac{1}{16}\frac{V_2^2}{2g} = 1.3125 \frac{V_2^2}{2g}$$

or
$$2.0 = 1.3125 \times \frac{V_2^2}{2g} \text{ or } V_2 = \sqrt{\frac{2.0 \times 2 \times 9.81}{1.3125}} = 5.467 \text{ m/s.}$$

(*i*) Loss of head due to contraction, $h_c = 0.375 \frac{V_2^2}{2g} = \frac{0.375 \times (5.467)^2}{2 \times 9.81}$ = **0.571 m. Ans.**

(*ii*) Rate of flow of water, $Q = A_2V_2 = 0.04908 \times 5.467 = 0.2683 \text{ m}^3\text{/s}$ = **268.3 lit/s. Ans.**

Problem 11.12 *If in the problem 11.11, the rate of flow of water is 300 litres/s, other data remaining the same, find the value of co-efficient of contraction, C_c.*

Solution. Given :

$$D_1 = 0.5 \text{ m}, D_2 = 0.25 \text{ m}, p_1 = 13.734 \times 10^4 \text{ N/m}^2,$$

$$p_2 = 11.772 \times 10^4 \text{ N/m}^2, Q = 300 \text{ lit/s} = 0.3 \text{ m}^3\text{/s}$$

Also from Problem 11.11, $V_1 = \dfrac{V_2}{4}$, where $V_1 = \dfrac{Q}{A_1} = \dfrac{0.30}{\dfrac{\pi}{4}(0.5)^2} = 1.528$ m/s

$\therefore$ $$V_2 = 4 \times V_1 = 4 \times 1.528 = 6.112 \text{ m/s}$$

From Bernoulli's equation, we have

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + h_c. \qquad [\because z_1 = z_2]$$

or $$\frac{13.734 \times 10^4}{9.81 \times 1000} + \frac{(1.528)^2}{2 \times 9.81} = \frac{11.772 \times (10)^4}{9.81 \times 1000} + \frac{(6.112)^2}{2 \times 9.81} + h_c$$

or $$14.0 + 0.119 = 12.0 + 1.904 + h_c$$

$$14.119 = 13.904 + h_c$$

$\therefore$ $$h_c = 14.119 - 13.904 = 0.215$$

But from equation (11.6), $h_c = \dfrac{V_2^2}{2g}\left[\dfrac{1}{C_c} - 1\right]^2$

Hence equating the two values of h_c, we get

$$\frac{V_2^2}{2g}\left[\frac{1}{C_c} - 1\right]^2 = 0.215$$

$$V_2 = 6.112, \therefore \frac{6.112^2}{2 \times 9.81}\left[\frac{1}{C_c} - 1\right]^2 = 0.215$$

or $$\left[\frac{1}{C_c} - 1\right]^2 = \frac{0.215 \times 2.0 \times 9.81}{6.112 \times 6.112} = 0.1129$$

or $$\frac{1.0}{C_c} - 1.0 = \sqrt{0.1129} = 0.336 \text{ or } \frac{1.0}{C_c} = 1.0 + 0.336 = 1.336$$

$\therefore$ $$C_c = \frac{1.0}{1.336} = \mathbf{0.748.\ Ans.}$$

Problem 11.13 *A 150 mm diameter pipe reduces in diameter abruptly to 100 mm diameter. If the pipe carries water at 30 litres per second, calculate the pressure loss across the contraction. Take the co-efficient of contraction as 0.6.*

Solution. Given :

Dia. of large pipe, $D_1 = 150 \text{ mm} = 0.15 \text{ m}$

Area of large pipe, $A_1 = \frac{\pi}{4}(.15)^2 = 0.01767 \text{ m}^2$

Dia. of smaller pipe, $D_2 = 100 \text{ mm} = 1.10 \text{ m}$

Area of smaller pipe, $A_2 = \frac{\pi}{4}(.10)^2 = 0.007854 \text{ m}^2$

Discharge, $Q = 30 \text{ litres/s} = .03 \text{ m}^3\text{/s}$

Co-efficient of contraction, $C_c = 0.6$

From continuity equation, we have $A_1V_1 = A_2V_2 = Q$

$$\therefore \quad V_1 = \frac{Q}{A_1} = \frac{0.03}{.01767} = 1.697 \text{ m/s}$$

and
$$V_2 = \frac{Q}{A_2} = \frac{.03}{.007854} = 3.82 \text{ m/s}$$

Applying Bernoulli's equation before and after contraction,

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + h_c \qquad ...(i)$$

But $Z_1 = Z_2$

and h_c, the head loss due to contraction is given by equation (11.6) as

$$h_c = \frac{V_2^2}{2g}\left[\frac{1}{C_c} - 1\right]^2 = \frac{3.82^2}{2 \times 9.81}\left[\frac{1}{0.6} - 1\right]^2 = 0.33$$

Substituting these values in equation (*i*), we get

$$\frac{p_1}{\rho g} + \frac{1.697^2}{2 \times 9.81} = \frac{p_2}{\rho g} + \frac{3.82^2}{2 \times 9.81} + 0.33$$

or
$$\frac{p_1}{\rho g} + 0.1467 = \frac{p_2}{\rho g} + .7438 + .33$$

$$\therefore \quad \frac{p_1}{\rho g} - \frac{p_2}{\rho g} = .7438 + .33 - .1467 = 0.9271 \text{ m of water}$$

$$\therefore \quad (p_1 - p_2) = \rho g \times 0.9271 = 1000 \times 9.81 \times 0.9271 \text{ N/m}^2$$
$$= 0.909 \times 10^4 \text{ N/m}^2 = 0.909 \text{ N/cm}^2$$

$\therefore$ Pressure loss across contraction
$= p_1 - p_2 =$ **0.909 N/cm². Ans.**

Problem 11.14 *In Fig. 11.3 below, when a sudden contraction is introduced in a horizontal pipe line from 50 cm to 25 cm, the pressure changes from 10,500 kg/m^2 (103005 N/m^2) to 6900 kg/m^2 (67689 N/m^2). Calculate the rate of flow. Assume co-efficient of contraction of jet to be 0.65.*

Following this if there is a sudden enlargement from 25 cm to 50 cm and if the pressure at the 25 cm section is 6900 kg/m^2 (67689 N/m^2) what is the pressure at the 50 cm enlarged section ?

Solution. Given :

Dia. of large pipe, $D_1 = 50 \text{ cm} = 0.5 \text{ m}$

Area, $A_1 = \dfrac{\pi}{4}(.5)^2 = 0.1963 \text{ m}^2$

Dia. of smaller pipe, $D_2 = 25 \text{ cm} = 0.25 \text{ m}$

$\therefore$ Area, $A_2 = \dfrac{\pi}{4}(.25)^2 = 0.04908 \text{ m}^2$

Pressure in large pipe, $p_1 = 10500 \text{ kg/m}^2$ or 103005 N/m^2

Pressure in smaller pipe, $p_2 = 6900 \text{ kg/m}^2$ or 67689 N/m^2

Co-efficient of contraction, $C_c = 0.65$

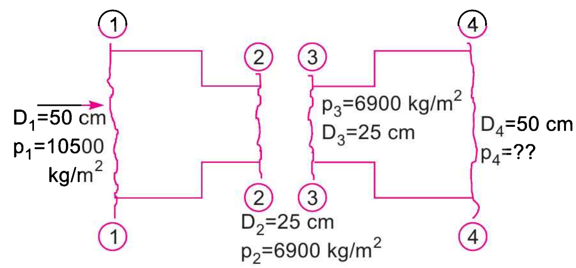

Fig. 11.3

Head lost due to contraction is given by equation (11.6),

$$h_c = \frac{V_2^2}{2g}\left(\frac{1}{C_c} - 1.0\right)^2 = \frac{V_2^2}{2g}\left(\frac{1}{0.65} - 1\right)^2 = 0.2899\,\frac{V_2^2}{2g} \qquad \ldots(i)$$

From continuity equation, we have

$$A_1V_1 = A_2V_2 \text{ or } V_1 = \frac{A_2V_2}{A_1} = \frac{\frac{\pi}{4}D_2^2 \times V_2}{\frac{\pi}{4}D_1^2}$$

$$= \left(\frac{D_2}{D_1}\right)^2 \times V_2 = \left(\frac{0.50}{0.25}\right)^2 \times V_2 = \frac{V_2}{4} \qquad \ldots(ii)$$

Applying Bernoulli's equation at sections 1-1 and 2-2,

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + Z_2 + h_c$$

But $Z_1 = Z_2$ (as pipe is horizontal)

$$\therefore \qquad \frac{p_1}{\rho g} + \frac{V_1^2}{2g} = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + h_c$$

Substituting the values of p_1, p_2, h_c and V_1, we get

$$\frac{103005}{1000 \times 9.81} + \frac{(V_2/4)^2}{2g} = \frac{67689}{1000 \times 9.81} + \frac{V_2^2}{2g} + .2899\,\frac{V_2^2}{2g}$$

or
$$10.5 + \frac{V_2^2}{16 \times 2g} = 6.9 + 1.2899\,\frac{V_2^2}{2g}$$

or
$$10.5 - 6.9 = 1.2899\,\frac{V_2^2}{2g} - \frac{1}{16} \times \frac{V_2^2}{2g} = 1.2274\,\frac{V_2^2}{2g}$$

or
$$3.6 = 1.2274 \times \frac{V_2^2}{2g}$$

$$\therefore \qquad V_2 = \sqrt{\frac{3.6 \times 2 \times 9.81}{1.2274}} = 7.586 \text{ m/s}$$

(*i*) Rate of flow of water, $Q = A_2V_2 = 0.04908 \times 7.586$

$= \mathbf{0.3723\ m^3/s\ or\ 372.3\ lit/s.}$ **Ans.**

(*ii*) Applying Bernoulli's equation to sections 3-3 and 4-4,

$$\frac{p_3}{\rho g} + \frac{V_3^2}{2g} + Z_3 = \frac{p_4}{\rho g} + \frac{V_4^2}{2g} + Z_4 + \text{head loss due to sudden enlargement } (h_e)$$

But
$$p_3 = 6900 \text{ kg/m}^2, \text{ or } 67689 \text{ N/m}^2$$
$$V_3 = V_2 = 7.586 \text{ m/s}$$
$$V_4 = V_1 = \frac{V_2}{4} = \frac{7.586}{4} = 1.8965$$
$$Z_3 = Z_4$$

And head loss due to sudden enlargement is given by equation (11.5) as

$$h_e = \frac{(V_3 - V_4)^2}{2g} = \frac{(7.586 - 1.8965)^2}{2 \times 9.81} = 1.65 \text{ m}$$

Substituting these values in Bernoulli's equation, we get

$$\frac{67689}{1000 \times 9.81} + \frac{7.586^2}{2 \times 9.81} = \frac{p_4}{1000 \times 9.81} + \frac{1.8965^2}{2 \times 9.81} + 1.65$$

or
$$6.9 + 2.933 = \frac{p_4}{1000 \times 9.81} + 0.183 + 1.65$$

$$\therefore \qquad \frac{p_4}{1000 \times 9.81} = 6.9 + 2.933 - 0.183 - 1.65 = 9.833 - 1.833 = 8.00$$

$$\therefore \qquad p_4 = 8 \times 1000 \times 9.81 = \mathbf{78480\ N/m^2.\ Ans.}$$

11.4.3 Loss of Head at the Entrance of a Pipe. This is the loss of energy which occurs when a liquid enters a pipe which is connected to a large tank or reservoir. This loss is similar to the loss of head due to sudden contraction. This loss depends on the form of entrance. For a sharp edge entrance, this loss is slightly more than a rounded or bell mouthed entrance. In practice the value of loss of head at the entrance (or inlet) of a pipe with sharp cornered entrance is taken = $0.5 \frac{V^2}{2g}$, where V = velocity of liquid in pipe.

This loss is denoted by h_i

$$\therefore \qquad h_i = 0.5 \frac{V^2}{2g} \qquad \text{...(11.8)}$$

11.4.4 Loss of Head at the Exit of Pipe. This is the loss of head (or energy) due to the velocity of liquid at outlet of the pipe which is dissipated either in the form of a free jet (if outlet of the pipe is free) or it is lost in the tank or reservoir (if the outlet of the pipe is connected to the tank or reservoir). This loss is equal to $\frac{V^2}{2g}$, where V is the velocity of liquid at the outlet of pipe. This loss is denoted h_o.

$$\therefore \qquad h_o = \frac{V^2}{2g} \qquad \text{...(11.9)}$$

where V = velocity at outlet of pipe.

11.4.5 Loss of Head Due to an Obstruction in a Pipe. Whenever there is an obstruction in a pipe, the loss of energy takes place due to reduction of the area of the cross-section of the pipe at the place where obstruction is present. There is a sudden enlargement of the area of flow beyond the obstruction due to which loss of head takes place as shown in Fig. 11.3 (*a*)

Consider a pipe of area of cross-section A having an obstruction as shown in Fig. 11.3 (*a*).

Let a = Maximum area of obstruction

A = Area of pipe

V = Velocity of liquid in pipe

Then $(A - a)$ = Area of flow of liquid at section 1-1.

As the liquid flows and passes through section 1-1, a vena-contracta is formed beyond section 1-1, after which the stream of liquid widens again and velocity of flow at section 2-2 becomes uniform and equal to velocity, V in the pipe. This situation is similar to the flow of liquid through sudden enlargement.

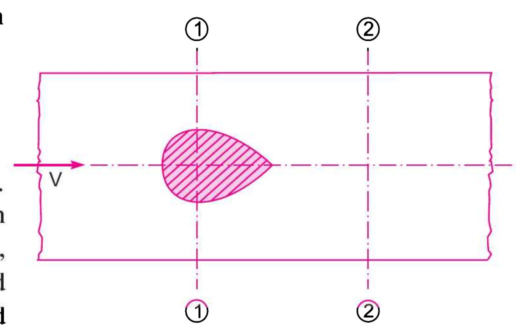

Fig. 11.3 (*a*) *An obstruction in a pipe.*

Let V_c = Velocity of liquid at vena-contracta.

Then loss of head due to obstruction = loss of head due to enlargement from vena-contracta to section 2-2.

$$= \frac{(V_c - V)^2}{2g} \quad ...(i)$$

From continuity, we have $a_c \times V_c = A \times V$ ...(*ii*)

where a_c = area of cross-section at vena-contracta

If C_c = co-efficient of contraction

Then $$C_c = \frac{\text{area at vena-contracta}}{(A - a)} = \frac{a_c}{(A - a)}$$

$\therefore$ $a_c = C_c \times (A - a)$

Substituting this value in (*ii*), we get

$$C_c \times (A - a) \times V_c = A \times V \quad \therefore \quad V_c = \frac{A \times V}{C_c\,(A - a)}$$

Substituting this value of V_c in equation (*i*), we get

$$\text{Head loss due to obstruction} = \frac{(V_c - V)^2}{2g} = \frac{\left(\dfrac{A \times V}{C_c\,(A - a)} - V\right)^2}{2g} = \frac{V^2}{2g}\left(\frac{A}{C_c\,(A - a)} - 1\right)^2 \quad ...(11.10)$$

11.4.6 Loss of Head due to Bend in Pipe. When there is any bend in a pipe, the velocity of flow changes, due to which the separation of the flow from the boundary and also formation of eddies takes place. Thus the energy is lost. Loss of head in pipe due to bend is expressed as

$$h_b = \frac{kV^2}{2g}$$

where h_b = loss of head due to bend, V = velocity of flow, k = co-efficient of bend

The value of k depends on

(*i*) Angle of bend, (*ii*) Radius of curvature of bend, (*iii*) Diameter of pipe.

11.4.7 Loss of Head in Various Pipe Fittings. The loss of head in the various pipe fittings such as valves, couplings etc., is expressed as

$$= \frac{kV^2}{2g} \quad ...(11.11)$$

where V = velocity of flow, k = co-efficient of pipe fitting.

Problem 11.15 *Water is flowing through a horizontal pipe of diameter 200 mm at a velocity of 3 m/s. A circular solid plate of diameter 150 mm is placed in the pipe to obstruct the flow. Find the loss of head due to obstruction in the pipe if C_c = 0.62.*

Solution. Given :

Dia. of pipe, D = 200 mm = 0.20 m

Velocity, V = 3.0 m/s

Area of pipe, $A = \frac{\pi}{4} D^2 = \frac{\pi}{4} (0.2)^2 = 0.03141 \text{ m}^2$

Dia. of obstruction, d = 150 mm = 0.15 m

$\therefore$ Area of obstruction, $a = \frac{\pi}{4} (.15)^2 = 0.01767 \text{ m}^2$

$$C_c = 0.62$$

The head lost due to obstruction is given by equation (11.10) as

$$= \frac{V^2}{2g}\left(\frac{A}{C_c\,(A-a)} - 1.0\right)^2$$

$$= \frac{3\times 3}{2\times 9.81}\left[\frac{.03141}{0.62\,[.03141 - .01767]} - 1.0\right]^2$$

$$= \frac{9}{2\times 9.81}\,[3.687 - 1.0]^2 = \mathbf{3.311\ m.\ Ans.}$$

Problem 11.16 *Determine the rate of flow of water through a pipe of diameter 20 cm and length 50 m when one end of the pipe is connected to a tank and other end of the pipe is open to the atmosphere. The pipe is horizontal and the height of water in the tank is 4 m above the centre of the pipe. Consider all minor losses and take f = .009 in the formula* $h_f = \dfrac{4\,.\,f\,.\,L\,.\,V^2}{d\times 2g}$.

Solution. Dia. of pipe, $d = 20$ cm $= 0.20$ m

Length of pipe, $L = 50$ m

Height of water, $H = 4$ m

Co-efficient of friction, $f = .009$

Let the velocity of water in pipe $= V$ m/s.

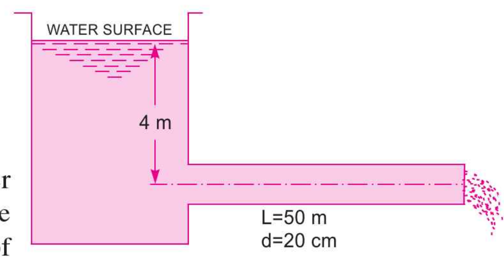

Fig. 11.4

Applying Bernoulli's equation at the top of the water surface in the tank and at the outlet of pipe, we have [Taking point 1 on the top and point 2 at the outlet of pipe].

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 + \text{all losses}$$

Considering datum line passing through the centre of pipe

$$0 + 0 + 4.0 = 0 + \frac{V_2^2}{2g} + 0 + (h_i + h_f)$$

or

$$4.0 = \frac{V_2^2}{2g} + h_i + h_f$$

But the velocity in pipe $= V$, $\quad \therefore V = V_2$

$$\therefore \qquad 4.0 = \frac{V^2}{2g} + h_i + h_f \qquad \ldots(i)$$

From equation (11.8), $h_i = 0.5\,\dfrac{V^2}{2g}$ and h_f from equation (11.1) is given as

$$h_f = \frac{4 \,.\, f \,.\, L \,.\, V^2}{d \times 2g}$$

Substituting these values, we have

$$4.0 = \frac{V^2}{2g} + \frac{0.5\, V^2}{2g} + \frac{4 \,.\, f \,.\, L \,.\, V^2}{d \times 2g}$$

$$= \frac{V^2}{2g}\left[1.0 + 0.5 + \frac{4 \times .009 \times 50}{0.2}\right] = \frac{V^2}{2g}\ [1.0 + 0.5 + 9.0]$$

$$= 10.5 \times \frac{V^2}{2g}$$

$\therefore$ $$V = \sqrt{\frac{4 \times 2 \times 9.81}{10.5}} = 2.734 \text{ m/sec}$$

$\therefore$ Rate of flow, $$Q = A \times V = \frac{\pi}{4} \times (0.2)^2 \times 2.734 = 0.08589 \text{ m}^3\text{/s}$$

$$= \mathbf{85.89 \text{ litres/s. Ans.}}$$

Problem 11.17 *A horizontal pipe line 40 m long is connected to a water tank at one end and discharges freely into the atmosphere at the other end. For the first 25 m of its length from the tank, the pipe is 150 mm diameter and its diameter is suddenly enlarged to 300 mm. The height of water level in the tank is 8 m above the centre of the pipe. Considering all losses of head which occur, determine the rate of flow. Take f = .01 for both sections of the pipe.*

Solution. Given :

Total length of pipe, $L = 40$ m

Length of 1st pipe, $L_1 = 25$ m

Dia. of 1st pipe, $d_1 = 150$ mm $= 0.15$ m

Length of 2nd pipe, $L_2 = 40 - 25 = 15$ m

Dia. of 2nd pipe, $d_2 = 300$ mm $= 0.3$ m

Height of water, $H = 8$ m

Co-efficient of friction, $f = 0.01$

Applying Bernoulli's theorem to the free surface of water in the tank and outlet of pipe as shown in Fig. 11.5 and taking reference line passing through the centre of pipe.

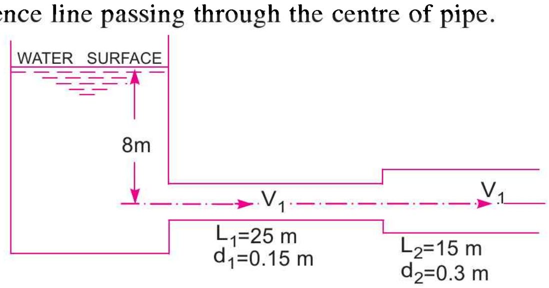

Fig. 11.5

$$0 + 0 + 8 = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + 0 + \text{all losses}$$

or
$$8.0 = 0 + \frac{V_2^2}{2g} + h_i + h_{f_1} + h_e + h_{f_2} \quad \text{...(i)}$$

where h_i = loss of head at entrance = $0.5 \frac{V_1^2}{2g}$

h_{f_1} = head lost due to friction in pipe 1 = $\frac{4 \times f \times L_1 \times V_1^2}{d_1 \times 2g}$

h_e = loss head due to sudden enlargement = $\frac{(V_1 - V_2)^2}{2g}$

h_{f_2} = Head lost due to friction in pipe 2 = $\frac{4 \times f \times L_2 \times V_2^2}{d_2 \times 2g}$

But from continuity equation, we have

$$A_1V_1 = A_2V_2$$

$$\therefore \quad V_1 = \frac{A_2V_2}{A_1} = \frac{\frac{\pi}{4}d_2^2 \times V_2}{\frac{\pi}{4}d_1^2} = \left(\frac{d_2}{d_1}\right)^2 \times V_2 = \left(\frac{0.3}{.15}\right)^2 \times V_2 = 4V_2$$

Substituting the value of V_1 in different head losses, we have

$$h_i = \frac{0.5\,V_1^2}{2g} = \frac{0.5 \times (4V_2)^2}{2g} = \frac{8V_2^2}{2g}$$

$$h_{f_1} = \frac{4 \times 0.01 \times 25 \times (4V_2)^2}{0.15 \times 2 \times g}$$

$$= \frac{4 \times .01 \times 25 \times 16}{0.15} \times \frac{V_2^2}{2g} = 106.67\,\frac{V_2^2}{2g}$$

$$h_e = \frac{(V_1 - V_2)^2}{2g} = \frac{(4V_2 - V_2)^2}{2g} = \frac{9V_2^2}{2g}$$

$$h_{f_2} = \frac{4 \times .01 \times 15 \times V_2^2}{0.3 \times 2g} = \frac{4 \times .01 \times 15}{0.3} \times \frac{V_2^2}{2g} = 2.0 \times \frac{V_2^2}{2g}$$

Substituting the values of these losses in equation (*i*), we get

$$8.0 = \frac{V_2^2}{2g} + \frac{8V_2^2}{2g} + 106.67\,\frac{V_2^2}{2g} + \frac{9V_2^2}{2g} + 2 \times \frac{V_2^2}{2g}$$

$$= \frac{V_2^2}{2g}\,[1 + 8 + 106.67 + 9 + 2] = 126.67\,\frac{V_2^2}{2g}$$

$$\therefore \quad V_2 = \sqrt{\frac{8.0 \times 2 \times g}{126.67}} = \sqrt{\frac{8.0 \times 2 \times 9.81}{126.67}} = \sqrt{1.2391} = 1.113 \text{ m/s}$$

$\therefore$ Rate of flow, $Q = A_2 \times V_2 = \frac{\pi}{4}(0.3)^2 \times 1.113 = 0.07867 \text{ m}^3/\text{s}$ = **78.67 litres/s. Ans.**

Problem 11.18 *Determine the difference in the elevations between the water surfaces in the two tanks which are connected by a horizontal pipe of diameter 300 mm and length 400 m. The rate of flow of water through the pipe is 300 litres/s. Consider all losses and take the value of f = .008.*

Solution. Given :

Dia. of pipe, $d = 300 \text{ mm} = 0.30 \text{ m}$

Length, $L = 400 \text{ m}$

Discharge, $Q = 300 \text{ lit/s} = 0.3 \text{ m}^3/\text{s}$

Co-efficient of friction, $f = 0.008$

Velocity, $$V = \frac{Q}{\text{Area}} = \frac{0.3}{\frac{\pi}{4} \times (.3)^2} = 4.244 \text{ m/s}$$

Let the two tanks are connected by a pipe as shown in Fig. 11.6.

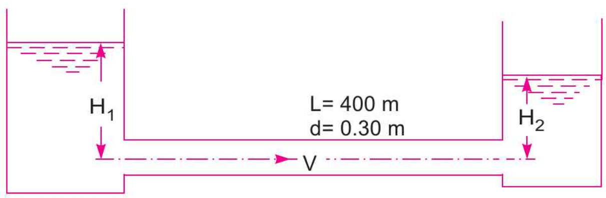

Fig. 11.6

Let H_1 = height of water in 1st tank above the centre of pipe

H_2 = height of water in 2nd tank above the centre of pipe

Then difference in elevations between water surfaces = $H_1 - H_2$

Applying Bernoulli's equation to the free surface of water in the two tanks, we have

$$H_1 = H_2 + \text{losses}$$

$$= H_2 + h_i + H_{f_1} + h_o \quad \text{...}(i)$$

where h_i = Loss of head at entrance $= 0.5 \dfrac{V^2}{2g} = \dfrac{0.5 \times 4.244^2}{2 \times 9.81} = 0.459 \text{ m}$

h_{f_1} = Loss of head due to friction $= \dfrac{4 \times f \times L \times V^2}{d \times 2g} = \dfrac{4 \times .008 \times 400 \times 4.244^2}{0.3 \times 2 \times 9.81} = 39.16 \text{ m}$

h_o = Loss of head at outlet $= \dfrac{V^2}{2g} = \dfrac{4.244^2}{2 \times 9.81} = 0.918 \text{ m}$

Substituting these values in (*i*), we get

$$H_1 = H_2 + 0.459 + 39.16 + 0.918 = H_2 + 40.537$$

$\therefore$ $H_1 - H_2$ = Difference in elevations

= **40.537 m. Ans.**

Problem 11.19 *The friction factor for turbulent flow through rough pipes can be determined by Karman-Prandtl equation,* $\dfrac{1}{\sqrt{f}} = 2 \log_{10} (R_0/k) + 1.74$

where f = friction factor, R_0 = pipe radius, k = average roughness.