9

CHAPTER

VISCOUS FLOW

▶ 9.1 INTRODUCTION

This chapter deals with the flow of fluids which are viscous and flowing at very low velocity. At low velocity the fluid moves in layers. Each layer of fluid slides over the adjacent layer. Due to relative velocity between two layers the velocity gradient $\frac{du}{dy}$ exists and hence a shear stress $\tau = \mu \frac{du}{dy}$ acts on the layers.

The following cases will be considered in this chapter :

1. Flow of viscous fluid through circular pipe.
2. Flow of viscous fluid between two parallel plates.
3. Kinetic energy correction and momentum correction factors.
4. Power absorbed in viscous flow through
   (*a*) Journal bearings, (*b*) Foot-step bearings, and (*c*) Collar bearings.

▶ 9.2 FLOW OF VISCOUS FLUID THROUGH CIRCULAR PIPE

For the flow of viscous fluid through circular pipe, the velocity distribution across a section, the ratio of maximum velocity to average velocity, the shear stress distribution and drop of pressure for a given length is to be determined. The flow through the circular pipe will be viscous or laminar, if the Reynolds number (R_e*) is less than 2000. The expression for Reynold number is given by

$$R_e = \frac{\rho V D}{\mu}$$

where ρ = Density of fluid flowing through pipe

V = Average velocity of fluid

D = Diameter of pipe and

μ = Viscosity of fluid.

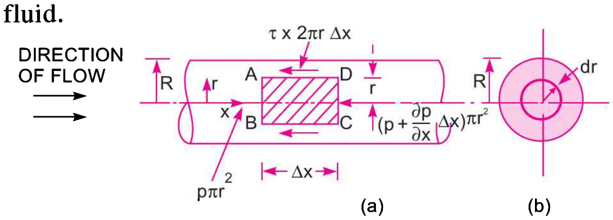

Fig. 9.1 *Viscous flow through a pipe.*

* For derivation, please refer to Art. 12.8.1.

Consider a horizontal pipe of radius R. The viscous fluid is flowing from left to right in the pipe as shown in Fig. 9.1 (a). Consider a fluid element of radius r, sliding in a cylindrical fluid element of radius $(r + dr)$. Let the length of fluid element be Δx. If 'p' is the intensity of pressure on the face AB, then the intensity of pressure on face CD will be $\left(p + \frac{\partial p}{\partial x}\Delta x\right)$. Then the forces acting on the fluid element are :

1. The pressure force, $p \times \pi r^2$ on face AB.
2. The pressure force, $\left(p + \frac{\partial p}{\partial x}\Delta x\right)\pi r^2$ on face CD.
3. The shear force, $\tau \times 2\pi r\Delta x$ on the surface of fluid element. As there is no acceleration, hence the summation of all forces in the direction of flow must be zero *i.e.*,

$$p\pi r^2 - \left(p + \frac{\partial p}{\partial x}\Delta x\right)\pi r^2 - \tau \times 2\pi r \times \Delta x = 0$$

or
$$-\frac{\partial p}{\partial x}\Delta x \pi r^2 - \tau \times 2\pi r \times \Delta x = 0$$

or
$$-\frac{\partial p}{\partial x} . r - 2\tau = 0$$

$\therefore$
$$\tau = -\frac{\partial p}{\partial x}\frac{r}{2} \qquad ...(9.1)$$

The shear stress τ across a section varies with 'r' as $\frac{\partial p}{\partial x}$ across a section is constant. Hence shear stress distribution across a section is linear as shown in Fig. 9.2 (a).

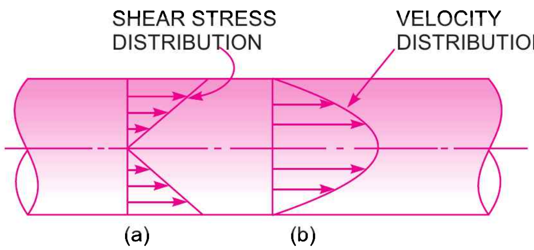

Fig. 9.2 *Shear stress and velocity distribution across a section.*

(i) **Velocity Distribution.** To obtain the velocity distribution across a section, the value of shear stress $\tau = \mu \frac{du}{dy}$ is substituted in equation (9.1).

But in the relation $\tau = \mu \frac{du}{dy}$, y is measured from the pipe wall. Hence

$$y = R - r \quad \text{and} \quad dy = -dr$$

$\therefore$
$$\tau = \mu \frac{du}{-dr} = -\mu \frac{du}{dr}$$

Substituting this value in (9.1), we get

$$-\mu \frac{du}{dr} = -\frac{\partial p}{\partial x}\frac{r}{2} \quad \text{or} \quad \frac{du}{dr} = \frac{1}{2\mu}\frac{\partial p}{\partial x} r$$

Integrating this above equation w.r.t. 'r', we get

$$u = \frac{1}{4\mu}\frac{\partial p}{\partial x} r^2 + C \qquad \text{...(9.2)}$$

where C is the constant of integration and its value is obtained from the boundary condition that at $r = R$, $u = 0$.

$$\therefore \qquad 0 = \frac{1}{4\mu}\frac{\partial p}{\partial x} R^2 + C$$

$$\therefore \qquad C = -\frac{1}{4\mu}\frac{\partial p}{\partial x} R^2$$

Substituting this value of C in equation (9.2), we get

$$u = \frac{1}{4\mu}\frac{\partial p}{\partial x} r^2 - \frac{1}{4\mu}\frac{\partial p}{\partial x} R^2$$

$$= -\frac{1}{4\mu}\frac{\partial p}{\partial x}[R^2 - r^2] \qquad \text{...(9.3)}$$

In equation (9.3), values of μ, $\frac{\partial p}{\partial x}$ and R are constant, which means the velocity, u varies with the square of r. Thus equation (9.3) is a equation of parabola. This shows that the velocity distribution across the section of a pipe is parabolic. This velocity distribution is shown in Fig. 9.2 (*b*).

(*ii*) **Ratio of Maximum Velocity to Average Velocity.** The velocity is maximum, when $r = 0$ in equation (9.3). Thus maximum velocity, U_{max} is obtained as

$$U_{max} = -\frac{1}{4\mu}\frac{\partial p}{\partial x} R^2 \qquad \text{...(9.4)}$$

The average velocity, $\bar{u}$, is obtained by dividing the discharge of the fluid across the section by the area of the pipe (πR^2). The discharge (Q) across the section is obtained by considering the flow through a circular ring element of radius r and thickness dr as shown in Fig. 9.1 (*b*). The fluid flowing per second through this elementary ring

$$dQ = \text{velocity at a radius } r \times \text{area of ring element}$$

$$= u \times 2\pi r\, dr$$

$$= -\frac{1}{4\mu}\frac{\partial p}{\partial x}[R^2 - r^2] \times 2\pi r\, dr$$

$$\therefore \qquad Q = \int_0^R dQ = \int_0^R -\frac{1}{4\mu}\frac{\partial p}{\partial x}(R^2 - r^2) \times 2\pi r\, dr$$

$$= \frac{1}{4\mu}\left(\frac{-\partial p}{\partial x}\right) \times 2\pi \int_0^R (R^2 - r^2)\, r dr$$

$$= \frac{1}{4\mu}\left(\frac{-\partial p}{\partial x}\right) \times 2\pi \int_0^R (R^2 r - r^3)\, dr$$

$$= \frac{1}{4\mu}\left(\frac{-\partial p}{\partial x}\right) \times 2\pi \left[\frac{R^2 r^2}{2} - \frac{r^4}{4}\right]_0^R = \frac{1}{4\mu}\left(\frac{-\partial p}{\partial x}\right) \times 2\pi \left[\frac{R^4}{2} - \frac{R^4}{4}\right]$$

$$= \frac{1}{4\mu}\left(\frac{-\partial p}{\partial x}\right) \times 2\pi \times \frac{R^4}{4} = \frac{\pi}{8\mu}\left(\frac{-\partial p}{\partial x}\right) R^4$$

∴ Average velocity,
$$\bar{u} = \frac{Q}{\text{Area}} = \frac{\frac{\pi}{8\mu}\left(\frac{-\partial p}{\partial x}\right) R^4}{\pi R^2}$$

or
$$\bar{u} = \frac{1}{8\mu}\left(\frac{-\partial p}{\partial x}\right) R^2 \qquad \text{...(9.5)}$$

Dividing equation (9.4) by equation (9.5),

$$\frac{U_{max}}{\bar{u}} = \frac{-\frac{1}{4\mu}\frac{\partial p}{\partial x} R^2}{\frac{1}{8\mu}\left(-\frac{\partial p}{\partial x}\right) R^2} = 2.0$$

∴ Ratio of maximum velocity to average velocity = 2.0.

(*iii*) **Drop of Pressure for a given Length (*L*) of a pipe**

From equation (9.5), we have

$$\bar{u} = \frac{1}{8\mu}\left(\frac{-\partial p}{\partial x}\right) R^2 \quad \text{or} \quad \left(\frac{-\partial p}{\partial x}\right) = \frac{8\mu\bar{u}}{R^2}$$

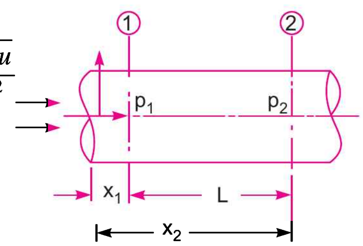

Fig. 9.3

Integrating the above equation w.r.t. *x*, we get

$$-\int_2^1 dp = \int_2^1 \frac{8\mu\bar{u}}{R^2}\, dx$$

∴
$$-[p_1 - p_2] = \frac{8\mu\bar{u}}{R^2}[x_1 - x_2] \text{ or } (p_1 - p_2) = \frac{8\mu\bar{u}}{R^2}[x_2 - x_1]$$

$$= \frac{8\mu\bar{u}}{R^2} L \qquad \{\because \ x_2 - x_1 = L \text{ from Fig. 9.3}\}$$

$$= \frac{8\mu\bar{u}L}{(D/2)^2} \qquad \left\{\because \ R = \frac{D}{2}\right\}$$

or
$$(p_1 - p_2) = \frac{32\mu\bar{u}L}{D^2},$$ where $p_1 - p_2$ is the drop of pressure.

∴ Loss of pressure head $= \dfrac{p_1 - p_2}{\rho g}$

∴
$$\frac{p_1 - p_2}{\rho g} = h_f = \frac{32\mu\bar{u}L}{\rho g D^2} \qquad \text{...(9.6)}$$

Equation (9.6) is called **Hagen Poiseuille Formula.**

Problem 9.1 *A crude oil of viscosity 0.97 poise and relative density 0.9 is flowing through a horizontal circular pipe of diameter 100 mm and of length 10 m. Calculate the difference of pressure at the two ends of the pipe, if 100 kg of the oil is collected in a tank in 30 seconds.*

Solution. Given : $\mu = 0.97 \text{ poise} = \frac{0.97}{10} = 0.097 \text{ Ns/m}^2$

Relative density $= 0.9$

$\therefore$ ρ_0, or density, $= 0.9 \times 1000 = 900 \text{ kg/m}^3$

Dia. of pipe, $D = 100 \text{ mm} = 0.1 \text{ m}$

$L = 10 \text{ m}$

Mass of oil collected, $M = 100 \text{ kg}$

Time, $t = 30 \text{ seconds}$

Calculate difference of pressure or $(p_1 - p_2)$.

The difference of pressure $(p_1 - p_2)$ for viscous or laminar flow is given by

$$p_1 - p_2 = \frac{32\mu \bar{u} L}{D^2}, \text{ where } \bar{u} = \text{average velocity} = \frac{Q}{\text{Area}}$$

Now, mass of oil/sec $= \frac{100}{30}$ kg/s

$= \rho_0 \times Q = 900 \times Q \qquad (\because \rho_0 = 900)$

$$\therefore \qquad \frac{100}{30} = 900 \times Q$$

$$\therefore \qquad Q = \frac{100}{30} \times \frac{1}{900} = 0.0037 \text{ m}^3/\text{s}$$

$$\therefore \qquad \bar{u} = \frac{Q}{\text{Area}} = \frac{.0037}{\frac{\pi}{4} D^2} = \frac{.0037}{\frac{\pi}{4} (.1)^2} = 0.471 \text{ m/s.}$$

For laminar or viscous flow, the Reynolds number (R_e) is less than 2000. Let us calculate the Reynolds number for this problem.

Reynolds number, $R_e{}^* = \frac{\rho V D}{\mu}$

where $\rho = \rho_0 = 900$, $V = \bar{u} = 0.471$, $D = 0.1$ m, $\mu = 0.097$

$$\therefore \qquad R_e = 900 \times \frac{.471 \times 0.1}{0.097} = 436.91$$

As Reynolds number is less than 2000, the flow is laminar.

$$\therefore \qquad p_1 - p_2 = \frac{32\mu \bar{u} L}{D^2} = \frac{32 \times 0.097 \times .471 \times 10}{(.1)^2} \text{ N/m}^2$$

$= 1462.28 \text{ N/m}^2 = 1462.28 \times 10^{-4} \text{ N/cm}^2 =$ $\mathbf{0.1462\ N/cm^2}$. **Ans.**

* For derivation, please refer to Art. 12.8.1

Problem 9.2 *An oil of viscosity 0.1 Ns/m² and relative density 0.9 is flowing through a circular pipe of diameter 50 mm and of length 300 m. The rate of flow of fluid through the pipe is 3.5 litres/s. Find the pressure drop in a length of 300 m and also the shear stress at the pipe wall.*

Solution. Given : Viscosity, $\mu = 0.1$ Ns/m²

Relative density $= 0.9$

$\therefore$ ρ_0 or density of oil $= 0.9 \times 1000 = 900$ kg/m³ ($\because$ Density of water = 1000 kg/m³)

$D = 50$ mm $= .05$ m

$L = 300$ m

$$Q = 3.5 \text{ litres/s} = \frac{3.5}{1000} = .0035 \text{ m}^3\text{/s}$$

Find (*i*) Pressure drop, $p_1 - p_2$

(*ii*) Shear stress at pipe wall, τ_0

(*i*) **Pressure drop ($\mathbf{p_1 - p_2}$)** $= \dfrac{32\mu\bar{u}L}{D^2}$, where $\bar{u} = \dfrac{Q}{\text{Area}} = \dfrac{.0035}{\frac{\pi}{4}D^2} = \dfrac{.0035}{\frac{\pi}{4}(.05)^2} = 1.782$ m/s

The Reynolds number (R_e) is given by, $R_e = \dfrac{\rho VD}{\mu}$

where $\rho = 900$ kg/m³, V = average velocity = $\bar{u}$ = 1.782 m/s

$$\therefore \quad R_e = 900 \times \frac{1.782 \times .05}{0.1} = 801.9$$

As Reynolds number is less than 2000, the flow is viscous or laminar

$$\therefore \quad p_1 - p_2 = \frac{32 \times 0.1 \times 1.782 \times 3000}{(.05)^2}$$

$= 684288$ N/m² $= 68428 \times 10^{-4}$ N/cm² $=$ **68.43 N/cm². Ans.**

(*ii*) **Shear Stress at the pipe wall (τ_0)**

The shear stress at any radius r is given by the equation (9.1)

i.e.,
$$\tau = -\frac{\partial p}{\partial x}\frac{r}{2}$$

$\therefore$ Shear stress at pipe wall, where $r = R$ is given by

$$\tau_0 = \frac{-\partial p}{\partial x}\frac{R}{2}$$

Now
$$\frac{-\partial p}{\partial x} = \frac{-(p_2 - p_1)}{x_2 - x_1} = \frac{p_1 - p_2}{x_2 - x_1} = \frac{p_1 - p_2}{L}$$

$$= \frac{684288}{300}\frac{\text{N/m}^2}{\text{m}} = 2280.96 \text{ N/m}^3$$

and
$$R = \frac{D}{2} = \frac{.05}{2} = .025 \text{ m}$$

$$\tau_0 = 2280.96 \times \frac{.025}{2}\frac{\text{N}}{\text{m}^2} = \mathbf{28.512\ N/m^2.\ Ans.}$$

Problem 9.3 *A laminar flow is taking place in a pipe of diameter 200 mm. The maximum velocity is 1.5 m/s. Find the mean velocity and the radius at which this occurs. Also calculate the velocity at 4 cm from the wall of the pipe.*

Solution. Given : Dia. of pipe, D = 200 mm = 0.20 m

$$U_{max} = 1.5 \text{ m/s}$$

Find (*i*) Mean velocity, $\bar{u}$

(*ii*) Radius at which $\bar{u}$ occurs

(*iii*) Velocity at 4 cm from the wall.

(*i*) **Mean velocity,** $\bar{u}$

Ratio of $$\frac{U_{\max}}{\bar{u}} = 2.0 \quad \text{or} \quad \frac{1.5}{\bar{u}} = 2.0 \quad \therefore \quad \bar{u} = \frac{1.5}{2.0} = \mathbf{0.75\ m/s.\ Ans.}$$

(*ii*) **Radius at which** $\bar{u}$ occurs

The velocity, u, at any radius 'r' is given by (9.3)

$$u = -\frac{1}{4\mu}\frac{\partial p}{\partial x}[R^2 - r^2] = -\frac{1}{4\mu}\frac{\partial p}{\partial x} R^2\left[1 - \frac{r^2}{R^2}\right]$$

But from equation (9.4) $U_{\max}$ is given by

$$U_{\max} = -\frac{1}{4\mu}\frac{\partial p}{\partial x} R^2 \quad \therefore \quad u = U_{\max}\left[1 - \left(\frac{r}{R}\right)^2\right] \qquad \text{...(1)}$$

Now, the radius r at which $u = \bar{u}$ = 0.75 m/s

$$\therefore \quad 0.75 = 1.5\left[1 - \left(\frac{r}{D/2}\right)^2\right]$$

$$= 1.5\left[1 - \left(\frac{r}{0.2/2}\right)^2\right] = 1.5\left[1 - \left(\frac{r}{0.1}\right)^2\right]$$

$$\therefore \quad \frac{0.75}{0.50} = 1 - \left(\frac{r}{0.1}\right)^2$$

$$\therefore \quad \left(\frac{r}{0.1}\right)^2 = 1 - \frac{.75}{1.50} = 1 - \frac{1}{2} = \frac{1}{2}$$

$$\therefore \quad \frac{r}{0.1} = \sqrt{\frac{1}{2}} = \sqrt{0.5}$$

$$\therefore \quad r = 0.1 \times \sqrt{.5} = 0.1 \times .707 = .0707 \text{ m}$$

$$= \mathbf{70.7\ mm.\ Ans.}$$

(*iii*) **Velocity at 4 cm from the wall**

$$r = R - 4.0 = 10 - 4.0 = 6.0 \text{ cm} = 0.06 \text{ m}$$

∴ The velocity at a radius = 0.06 m

or 4 cm from pipe wall is given by equation (1)

$$= U_{max}\left[1-\left(\frac{r}{R}\right)^2\right] = 1.5\left[1-\left(\frac{.06}{.1}\right)^2\right]$$

$$= 1.5[1.0 - .36] = 1.5 \times .64 = \mathbf{0.96\ m/s.\ Ans.}$$

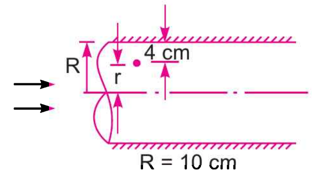

Fig. 9.4

Problem 9.4 *Crude oil of μ = 1.5 poise and relative density 0.9 flows through a 20 mm diameter vertical pipe. The pressure gauges fixed 20 m apart read 58.86 N/cm² and 19.62 N/cm² as shown in Fig. 9.5. Find the direction and rate of flow through the pipe.*

Solution. Given : $\mu = 1.5$ poise $= \frac{1.5}{10} = 0.15$ Ns/m²

Relative density = 0.9

∴ Density of oil = 0.9 × 1000 = 900 kg/m³

Dia. of pipe, D = 20 mm = 0.02 m

L = 20 m

$p_A = 58.86$ N/cm² $= 58.86 \times 10^4$ N/m²

$p_B = 19.62$ N/cm² $= 19.62 \times 10^4$ N/m².

Find (*i*) Direction of flow

(*ii*) Rate of flow.

(*i*) **Direction of flow.** To find the direction of flow, the total energy $\left(\frac{p}{\rho g}+\frac{v^2}{2g}+Z\right)$ at the lower end A and at the upper end B is to be calculated. The direction of flow will be given from the higher energy to the lower energy. As here the diameter of the pipe is same and hence kinetic energy at A and B will be same. Hence to find the direction of flow, calculate $\left(\frac{p}{\rho g}+Z\right)$ at A and B.

Taking the level at A as datum. The value of $\left(\frac{p_A}{\rho g}+Z\right)$ at

$$A = \frac{p_A}{\rho g} + Z_A$$

$$= \frac{6\times10^4\times9.81}{900\times9.81} + 0 \quad \{\because\ r = 900\ \text{kg/cm}^2\}$$

$$= 66.67\ \text{m}$$

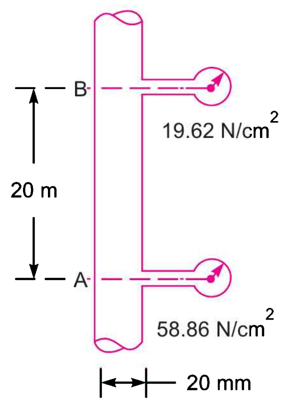

Fig. 9.5

The value of $\left(\frac{p}{\rho g}+Z\right)$ at $B = \frac{p_B}{\rho g} + Z_B$

$$= \frac{2\times10^4\times9.81}{900\times9.81} + 20 = 22.22 + 20 = 42.22\ \text{m}$$

As the value of $\left(\frac{p}{\rho g}+Z\right)$ is higher at A and hence flow takes place from A to B. **Ans.**

(*ii*) **Rate of flow.** The loss of pressure head for viscous flow through circular pipe is given by

$$h_f = \frac{32\mu\bar{u}L}{\rho g D^2}$$

For a vertical pipe h_f = Loss of peizometric head

$$= \left(\frac{p_A}{\rho g} + Z_A\right) - \left(\frac{p_B}{\rho g} + Z_B\right) = 66.67 - 42.22 = 24.45 \text{ m}$$

$$\therefore \quad 24.45 = \frac{32 \times 0.15 \times \bar{u} \times 20.0}{900 \times 9.81 \times (.02)^2}$$

or

$$\bar{u} = \frac{24.45 \times 900 \times 9.81 \times .0004}{32 \times 0.15 \times 20.0} = 0.889 \simeq 0.9 \text{ m/s.}$$

The Reynolds number should be calculated. If Reynolds number is less than 2000, the flow will be laminar and the above expression for loss of pressure head for laminar flow can be used.

Now Reynolds number $= \dfrac{\rho V D}{\mu}$

where $\rho = 900$ kg/m^3 and $V = \bar{u}$

$\therefore$ Reynolds number $= 900 \times \dfrac{0.9 \times .02}{0.15} = 108$

As Reynolds number is less than 2000, the flow is laminar.

$\therefore$ Rate of flow = average velocity × area

$$= \bar{u} \times \frac{\pi}{4} D^2 = 0.9 \times \frac{\pi}{4} \times (.02)^2 \text{ m}^3\text{/s} = 2.827 \times 10^{-4} \text{ m}^3\text{/s}$$

= 0.2827 litres/s. Ans. $(\because \quad 10^{-3} \text{ m}^3 = 1 \text{ litre})$

Problem 9.5 *A fluid of viscosity 0.7 Ns/m^2 and specific gravity 1.3 is flowing through a circular pipe of diameter 100 mm. The maximum shear stress at the pipe wall is given as 196.2 N/m^2, find (i) the pressure gradient, (ii) the average velocity, and (iii) Reynolds number of the flow.*

Solution. Given : $\mu = 0.7 \dfrac{\text{Ns}}{\text{m}^2}$

Sp. gr. = 1.3

$\therefore$ Density $= 1.3 \times 1000 = 1300$ kg/m^3

Dia. of pipe, $D = 100$ mm $= 0.1$ m

Shear stress, $\tau_0 = 196.2$ N/m^2

Find (*i*) Pressure gradient, $\dfrac{dp}{dx}$

(*ii*) Average velocity, $\bar{u}$

(*iii*) Reynolds number, R_e

(*i*) **Pressure gradient,** $\dfrac{dp}{dx}$

The maximum shear stress (τ_0) is given by

$$\tau_0 = -\frac{\partial p}{\partial x}\frac{R}{2} \text{ or } 196.2 = -\frac{\partial p}{\partial x} \times \frac{D}{4} = -\frac{\partial p}{\partial x} \times \frac{0.1}{4}$$

$$\therefore \qquad \frac{\partial p}{\partial x} = -\frac{196.2 \times 4}{0.1} = -7848 \text{ N/m}^2 \text{ per m}$$

$\therefore$ Pressure Gradient = **– 7848 N/m² per m. Ans.**

(*ii*) **Average velocity,** $\bar{u}$

$$\bar{u} = \frac{1}{2} U_{max} = \frac{1}{2}\left[-\frac{1}{4\mu}\frac{\partial p}{\partial x} R^2\right] \qquad \left\{\because \; U_{max} = -\frac{1}{8\mu}\frac{\partial p}{\partial x} R^2\right\}$$

$$= \frac{1}{8\mu} \times \left(-\frac{\partial p}{\partial x}\right) R^2$$

$$= \frac{1}{8 \times 0.7} \times (7848) \times (.05)^2 \qquad \left\{\because \; R = \frac{D}{2} = \frac{1}{2} = .05\right\}$$

$$= 3.50 \text{ m/s}$$

(*iii*) **Reynolds number,** R_e

$$R_e = \frac{\bar{u} \times D}{\nu} = \frac{\bar{u} \times D}{\mu / \rho} = \frac{\rho \times \bar{u} \times D}{\mu}$$

$$= 1300 \times \frac{3.50 \times 0.1}{0.7} = \textbf{650.00. Ans.}$$

Problem 9.6 *What power is required per kilometre of a line to overcome the viscous resistance to the flow of glycerine through a horizontal pipe of diameter 100 mm at the rate of 10 litres/s ? Take μ = 8 poise and kinematic viscosity (ν) = 6.0 stokes.*

Solution. Given :

Length of pipe, $L = 1 \text{ km} = 1000 \text{ m}$

Dia. of pipe, $D = 100 \text{ mm} = 0.1 \text{ m}$

Discharge, $Q = 10 \text{ litres/s} = \frac{10}{1000} \text{ m}^3\text{/s} = .01 \text{ m}^3\text{/s}$

Viscosity, $\mu = 8 \text{ poise} = \frac{8}{10}\frac{\text{Ns}}{\text{m}^2} = 0.8 \text{ N s/m}^2$

Kinematic Viscosity, $\nu = 6.0$ stokes $\qquad \left(\because \; 1 \text{ poise} = \frac{1}{10} \text{Ns/m}^2\right)$

$$= 6.0 \text{ cm}^2\text{/s} = 6.0 \times 10^{-4} \text{ m}^2\text{/s}$$

Loss of pressure head is given by equation (9.6) as $h_f = \dfrac{32\mu\bar{u}L}{\rho g D^2}$

$$\text{Power required} = W \times h_f \text{ watts} \qquad \ldots(i)$$

where W = weight of oil flowing per sec = $\rho g \times Q$

Substituting the values of W and h_f in equation (*i*),

$$\text{Power required} = (\rho g \times Q) \times \frac{(32\,\mu\bar{u}L)}{\rho g D^2} \text{ watts} = \frac{Q \times 32\,\mu\bar{u}L}{D^2} \qquad \text{(cancelling } \rho g\text{)}$$

$$\text{But} \qquad \bar{u} = \frac{Q}{\text{Area}} = \frac{.01}{\frac{\pi}{4}D^2} = \frac{.01 \times 4}{\pi \times (.1)^2} = 1.273 \text{ m/s}$$

$\therefore$ Power required $= \dfrac{.01 \times 32 \times 0.8 \times 1.273 \times 1000}{(.1)^2}$

$= 32588.8$ W $=$ **32.588 kW. Ans.**

▶ 9.3 FLOW OF VISCOUS FLUID BETWEEN TWO PARALLEL PLATES

In this case also, the shear stress distribution, the velocity distribution across a section ; the ratio of maximum velocity to average velocity and difference of pressure head for a given length of parallel plates, are to be calculated.

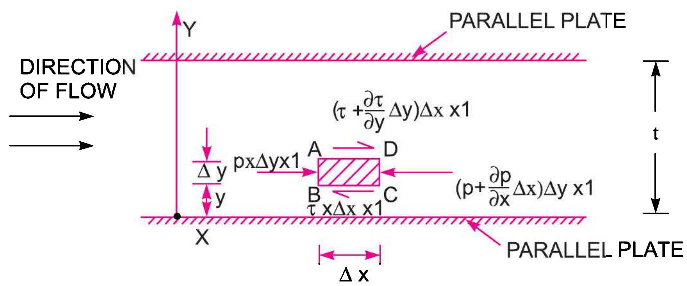

Fig. 9.6 *Viscous flow between two parallel plates.*

Consider two parallel fixed plates kept at a distance 't' apart as shown in Fig. 9.6. A viscous fluid is flowing between these two plates from left to right. Consider a fluid element of length Δx and thickness Δy at a distance y from the lower fixed plate. If p is the intensity of pressure on the face AB of the fluid element then intensity of pressure on the face CD will be $\left(p + \frac{\partial p}{\partial x}\Delta x\right)$. Let τ is the shear stress acting on the face BC then the shear stress on the face AD will be $\left(\tau + \frac{\partial \tau}{\partial y}\Delta y\right)$. If the width of the element in the direction perpendicular to the paper is unity then the forces acting on the fluid element are :

1. The pressure force, $p \times \Delta y \times 1$ on face AB.
2. The pressure force, $\left(p + \frac{\partial p}{\partial x}\Delta x\right)\Delta y \times 1$ on face CD.
3. The shear force, $\tau \times \Delta x \times 1$ on face BC.
4. The shear force, $\left(\tau + \frac{\partial \tau}{\partial y}\Delta y\right)\Delta x \times 1$ on face AD.

For steady and uniform flow, there is no acceleration and hence the resultant force in the direction of flow is zero.

$$\therefore \qquad p\Delta y \times 1 - \left(p + \frac{\partial p}{\partial x}\Delta x\right)\Delta y \times 1 - \tau\Delta x \times 1 + \left(\tau + \frac{\partial \tau}{\partial y}\Delta y\right)\Delta x \times 1 = 0$$

or
$$-\frac{\partial p}{\partial x}\Delta x\Delta y + \frac{\partial \tau}{\partial x}\Delta y\Delta x = 0$$

Dividing by $\Delta x\Delta y$, we get $-\dfrac{\partial p}{\partial x} + \dfrac{\partial \tau}{\partial y} = 0$ or $\dfrac{\partial p}{\partial x} = \dfrac{\partial \tau}{\partial y}$ ...(9.7)