

**MATILDA.FT**  
**Mesoscale, Accelerated, Theoretically-Informed, Langevin, Dissipative particle dynamics, and Field**  
**Theory**  
**Documentation**

## 1 Commands

### 1.1 Command line arguments

- `-ft`  
Sets the simulation style to FT
- `-particles`  
Sets the simulation style to TILD
- `-in [file_name]`  
Changes the input script from default `input`. Is used in the command line argument not in the input script.

### 1.2 Input Script Commands

#### 1.2.1 TILD Commands

- `angle [int] [string] arguments`  
Defines angle style to be used. First int: angle type. String: angle style, currently only “wlc” and “harmonic” supported. Float: force constant.  
Worm-like chain (“wlc”) potential:  $u(\theta_{ijk}) = k_{\theta}(1 + \cos(\theta_{ijk}))$ .  
arguments = [float  $k_{\theta}$ ]  
  
Harmonic (“harmonic”) potential:  $u(\theta_{ijk}) = k_{\theta}(\theta_{ijk} - \theta_0)^2$ .  
arguments = [float  $k_{\theta}$ ] [float  $\theta_0$ ]
- `bond [integer bond type] [float prefactor] [float equilibrium bond separation]`  
Parameters for the harmonic bond of the form  $u(r_{ij}) = k(r_{ij} - r_0)^2$ .
- `binary_freq [integer]`  
Frequency for writing to the binary data files
  - `equil_binary_freq [integer] (optional argument)`  
Frequency for writing equilibration data to binary data files. Requires equilibration data collection to be set using `equil_steps` or `prod_steps`. If not used, defaults to `binary_freq`.
- `charges [float] [float]`  
Indicates to read in a charge configuration (follows LAMMPS “charges” format). The first float is the Bjerrum length for electrostatic interactions, the second is the smearing length for the charges (assumed Gaussian distributions).

- compute [style] [arguments]

Enable compute for on-the-fly computations. Currently enabled styles:

- avg\_sk [integer type] [optional arguments]

Optional arguments are: “wait [integer number of time steps]” and “freq [integer number of time steps]”

This compute calculates a running average of the static structure factor on the fly based on the density field of the indicated species type. The “wait” option delays the calculation for some number of time steps (default = 0), and the “freq” option sets the frequency of performing the compute (default = 100 time steps).

- Widom [molec ID] [number configs] This compute uses Widom insertions to grow a molecule to estimate the chemical potential. The topology and particle types are taken from molecule “molec ID”, and “number configs” is the number of trial configurations to generate at each call to this compute.

This compute generates output “Widom.dat” containing the calculated energies of the inserted molecule.

Optional arguments are the same as for avg\_sk.

- chemical\_potential [int min. molec ID] [int max molec ID] [float molec frac] [optional arguments]

This compute calculates the change in potential energy of a system upon removal of one of the molecules in the range given. The minimum and maximum molecule IDs provided are inclusive. The molecule fraction argument is the fraction of molecules in the range to remove (they are chosen at random) at each estimate. This will affect the computational expense of the method.

This compute generates an output “chemical\_potential.dat” containing the change in energy each time the compute was performed.

Optional arguments are the same as for avg\_sk.

- delt [float]

Size of the time step

- diffusivity [int A] [float D]

Sets diffusivity of beads of type A. Only used with GJF integrator

- Dim [integer]

Sets the dimensionality of the system.

- extraforce [string group] [string style] [arguments]

Adds extra forces to the particles in specified group. Supported styles currently include:

- dpd [string neigh\_list] [float sigma] [float  $r_{cutoff}$ ]
- langevin [float gamma]

Adds Langevin noise and friction to the forces, would need to be used with velocity Verlet algorithm to give proper thermostating.

- lewis [string n\_list] [float  $k_{spring}$ ] [float  $e_{bond}$ ] [float  $r_0$ ] [float  $q_{ind}$ ] [int bond\_freq]

Enables dynamic bonding in the system.

- midpush [int axis] [float magnitude]

Adds a force of given magnitude pushing molecules towards the center of the simulation box along the chosen axis. This is useful for creating macrophase separated systems with a particular species in the center of the box. The magnitude of the forces should generally be small.

- wall [string wallstyle] [args]

Adds impermeable walls to the simulation box. Walls repel particles towards the center of the enclosed space. Arguments are passed as [int dim] [args], optional [int dim] [args] ..., where [dim] is the axis on which the walls will be placed x = 0, y = 1, z = 2 Different wall styles have different repulsive potentials. Available styles include:

- \* hard [int dim] [float low] [float high]

Creates hard walls with hard repulsive potential

Example: extraforce all wall hard 1 3.0 15.5 2 2.5 4.5 will place confining walls along the y-axis, one at y = 3.0 and another at y = 15.5.

A second set of walls will be added along the z-axis at z = 2.5 and 4.5

- \* rinv [int dim] [float A] [float low] [float high]

Repels particles with potential  $V = \frac{A}{r}$ , where  $r$  is the distance of particle's COM from the corresponding wall

- \* exp [int dim] [float A] [float low] [float high]

Repels particles with potential  $V = Ae^r$ , where  $r$  is the distance of particle's COM from the corresponding wall

- grid\_freq [integer]

Frequency of writing the grid data to a human-readable text file. Use sparingly.

- equil\_grid\_freq [integer] (optional argument)

Frequency for writing equilibration grid data to a human-readable text file. Requires equilibration data collection to be set using equil\_steps or prod\_steps. If not used, defaults to grid\_freq.

- gsd\_freq [integer]

Frequency of writing the trajectory to the GSD file format, written by the Glotzer lab.

- gsd\_name [string]

Changes the name of writing GSD file. Default is "traj.gsd".

- group [string name] [string style] [style options...]

– type [int type1 type2 ...]

Makes a group containing all particles of the listed types.

– id [string file]

Initializes the group based on global particle IDs (matching the data file) from a plain text file.

- integrator [group\_name] [style] [options]

Integration scheme to employ; current options are VV (Velocity Verlet) or GJF (Grønbech-Jensen and Farago)

– GJF

– VV [float v\_max] [int distribution]

c\_max - max magnitude of the initial velocity; distribution - distribution of velocities (0 - normal, 1 - uniform)

- log\_freq [integer]

Frequency of writing to data.dat file. Requires host-device communication.

– equil\_log\_freq [integer] (optional argument)

Frequency for writing equilibration data to equil\_data.dat file. Requires equilibration data collection to be set using equil\_steps or prod\_steps. If not used, defaults to log\_freq.

- max\_steps [integer]

Total number of time steps

– prod\_steps [integer] (optional argument)

Sets number of production steps. If called with max\_steps, sets equil\_steps to the difference and enables equilibration data collection. If called with equil\_steps, sets max\_steps to the sum. Note- you cannot call max\_steps, prod\_steps, and equil\_steps in the same input script, or prod\_steps greater than max\_steps.

– prod\_steps [integer] (optional argument)

Sets number of equilibration steps. If called with max\_steps, sets prod\_steps to the difference and enables equilibration data collection. If called with prod\_steps, sets max\_steps to the sum and enables equilibration data collection. Note- you cannot call max\_steps, prod\_steps, and equil\_steps in the same input script, or equil\_steps greater than max\_steps.

- n\_gaussians [integer]

Number of Gaussian non-bonded interaction pairs,  $n_G$ . This line **must** be immediately followed by  $n_G$  lines of the form:

gaussian [integer type 1] [integer type 2] [float prefactor] [float std deviation]

This defines a non-bonded interaction between particles of type 1 and 2 of the form

$$u_G(r_{ij}) = \frac{A}{(2\pi\sigma^2)^{\mathbb{D}/2}} e^{-|r_{ij}|^2/2\sigma^2}$$

Employs the `ramp` keyword.

- n\_erfs [integer  $n_{\text{erf}}$ ]

Number of Erf non-bonded interaction pairs,  $n_{\text{erf}}$ . This line **must** be immediately followed by  $n_{\text{erf}}$  lines of the form:

gaussian [integer type 1] [integer type 2] [float prefactor] [float std deviation]

This defines a non-bonded interaction between particles of type 1 and 2 of the form

$$u(r) = A \operatorname{erf}\left(\frac{|\mathbf{r}| - R_p}{\sigma}\right) * \operatorname{erf}\left(\frac{|\mathbf{r}| - R_p}{\sigma}\right)$$

Employs the `ramp` keyword

- n\_gaussian\_erfs [integer  $n_{\text{G-erf}}$ ]

Number of Gaussian-erf cross potential interaction pairs,  $n_{\text{G-erf}}$ . This line **must** be immediately followed by  $n_{\text{G-erf}}$  lines of the form:

gaussian [integer type 1] [integer type 2] [float prefactor] [float std deviation]

This defines a non-bonded interaction between particles of type 1 and 2 of the form

$$u(r) = A \frac{1}{(2\pi\sigma^2)^{D/2}} e^{-|\mathbf{r}|^2/2\sigma^2} * \operatorname{erf}\left(\frac{|\mathbf{r}| - R_p}{\sigma}\right)$$

Employs the `ramp` keyword

- n\_fieldphases [integer  $n_f$ ]

Number of external applied fields with which the particles should interact, see section 3.4 above. This line **must** be immediately followed by  $n_f$  lines of the form:

fieldphase [int type 1] [int type 2] [float prefactor] [int phase] [int dir] [int n periods]

For the phase: 0 = Lamellar, 1 = BCC (NOT YET IMPLEMENTED), 2 = CYL, 3 = GYR. Direction is the direction normal to lamellae, along the cylinders, and is ignore for GYR and BCC.

Employs the `ramp` keyword

- nlist [string group] [string style] [string name] [float  $r_n$ ] [float  $r_{\text{skin}}$ ] [float density] [float frequency] [options]

Constructs a neighbour list for a given group of particles.  $r_n$  is the cut-off radius of the associated force.  $r_{\text{skin}}$  is the skin radius. Density specifies the capacity of each cell on the grid. Frequency is the update frequency of the neighbour list. Positive integer specifies the number of time-steps after which the update occurs. -1 triggers automatic, displacement based update.

- bonding [string file\_name]

Creates a neighbour list to be associated with lewis force (dynamic bonding). Additional file maps particles to donor and acceptor types. The donors [type 1] and acceptors [type 0] are specified in the additional file that matches type to the id within the group.

- half\_distance

Creates a neighbour list in which particles only store the information about their neighbours with a lower index than their own.

- Nx [integer]

Ny [integer]

Nz [integer]

Sets the grid points in the x-, y-, and z-directions

- pmeorder [integer]

Should be an integer 1, 2, 3, or 4. Determines the order of the spline used to map particle densities to the grid.

- rand\_seed [integer]

seed for the random number generator

- read\_data [string]

Name of the input configuration file, should be in roughly LAMMPS data file format.

- read\_resume [string]

Must be supplied along with the read\_data command to read atom data and bonds.

Name of the resume file to set particle positions, assumed to be a lammprstj frame format.

- read\_gsd [string]

Allows you to read in a GSD file, which is a binary format for storing molecular dynamics trajectories. This is useful for reading in a trajectory from another simulation, or for reading in a configuration file. The GSD file format is written by the Glotzer lab.

Optional keywords: `frame` [integer] to read in a specific frame. `resume` to read in a GSD and only use the particle positions and timesteps. `read_config` to read in a GSD, using all information natively (including box size, particle types, bonds, angles, etc.) and use this as the initial configuration. `read_config` and `resume` are mutually exclusive.

Defaults: `frame` is -1, which will resume from the final frame of the GSD file. The code is set up to read resume by default. The code searches the inputted frame for the following information: particle positions, particle types, box size, timesteps, and bonds. If any of these are not found, the code will instead search the first frame for all that information. This behavior can be changed upon request.

- set\_timestep [intn]

Sets the global timestep of the system; useful for keeping track of simulations that are resumed from a previous run.

- tensor\_pair\_style [style] [additional arguments] Currently supported style is “MaierSaupe”

Additional arguments are [lc.input filename] [prefactor] [length scale]

lc.input is an input file that defines the pairs to create orientation vectors.

prefactor is the ratio of  $\mu/\rho_0$  used in the Maier-Saupe potential and length scale is the smearing length of the Gaussian. See Section 4 below for more details and the example “MaierSaupe” in examples folder.

- threads [integer]

Number of threads per GPU block to employ. Default is 512.

- traj\_freq [integer]

Frequency for writing to lammpstrj text file. Use sparingly.

- equil\_traj\_freq [integer] (optional argument)

Frequency for writing equilibration data to lammpstrj text file. Requires equilibration data collection to be set using equil\_steps or prod\_steps. If not used, defaults to traj\_freq.

### **1.2.2 FT Commands**

(coming soon...)

## 2 Utilities

In the directory “utils”, there are several files that can be used to post-process or generate initial configurations.

### 2.1 utils/post-proc

dump\_grid\_densities.cpp converts the given frames into .tec files for view in ParaView.

avg-grid-densities.cpp does not appear to actually average as promised...

calc\_sk/ subdirectory. Uses FFTW to calculate  $S(k)$  from the binary files.

dump\_particle\_posits.cpp dumps the binary particle positions into a lammpstrj file.



### 3 Interaction Potentials

#### 3.1 Gaussians

A standard Gaussian governs the non-bonded interactions,

$$u(r) = \frac{A}{(2\pi\sigma^2)^{D/2}} \exp(-r^2/2\sigma^2) \quad (1)$$

$$= A u_G(r) \quad (2)$$

where  $A$  and  $\sigma$  are the amplitude and standard deviation of the Gaussian potential and  $u_G$  is a normalized Gaussian distribution in  $D$  dimensions. Entering the potential parameters in the input file is of the form

```
pair_style gaussian Itype Jtype A sigma
```

to calculate a potential energy of the form

$$U = A \int d\mathbf{r} \int d\mathbf{r}' \rho_I(\mathbf{r}) u_G(|\mathbf{r} - \mathbf{r}'|) \rho_J(\mathbf{r}'). \quad (3)$$

##### 3.1.1 Relation to other field theory forms

Koski et al. [JCP 2013] and Villet and Fredrickson [JCP 2014] have shown the relationship between the “smearing functions” commonly used to regularize field-theoretic simulations (FTS) and non-bonded potentials,  $u(r) = A(h_I * h_J)(r)$ , where the  $*$  indicates a convolution, and  $h_I(r)$  and  $h_J(r)$  are the normalized smearing functions for species  $I$  and  $J$ . Smearing with a unit Gaussian is commonly used in FTS, which leads to an effective potential in Fourier space

$$\begin{aligned} \tilde{u}(k) &= A \tilde{h}_I(k) \tilde{h}_J(k) \\ &= A e^{-k^2 a_I^2/2} e^{-k^2 a_J^2/2} \\ &= A e^{-k^2 (a_I^2 + a_J^2)/2}. \end{aligned} \quad (4)$$

Or, in real space, the potential can be related to the smearing functions

$$u(r) = \frac{A}{(2\pi(a_I^2 + a_J^2))^{D/2}} e^{-r^2/2(a_I^2 + a_J^2)} \quad (5)$$

##### 3.1.2 Gaussian-Regularized Model A implementation

To implement a Gaussian regularized Edwards model (Model A from ETIP), the effective potential should simply be

$$u(r) = \frac{u_0}{2} u_G(r) \quad (6)$$

and so treating  $A = u_0/2$  in Eq. 2 gives the standard Model A.

##### 3.1.3 Compressible Blend (Model D)

The blend requires two components  $A$  and  $B$ , and for this section I’ll assume the range of the Gaussian potential ( $\sigma$ ) on all components is identical. Model D has the total interaction energy

$$\begin{aligned} U &= \frac{\kappa}{2\rho_0} \int d\mathbf{r} \int d\mathbf{r}' [\rho_+(\mathbf{r}) - \rho_0] u_G(|\mathbf{r} - \mathbf{r}'|) [\rho_+(\mathbf{r}') - \rho_0] \\ &\quad + \frac{\chi}{\rho_0} \int d\mathbf{r} \int d\mathbf{r}' \rho_A(\mathbf{r}) u_G(|\mathbf{r} - \mathbf{r}'|) \rho_B(\mathbf{r}'), \end{aligned} \quad (7)$$

where  $\rho_+ = \rho_A + \rho_B$ . Writing out  $\rho_+$  and collecting terms that are quadratic in the spatially-varying density, we arrive at the form typically used in TILD,

$$\begin{aligned}
U = & \frac{\kappa + \chi}{\rho_0} \int d\mathbf{r} \int d\mathbf{r}' \rho_A(\mathbf{r}) u_G(|\mathbf{r} - \mathbf{r}'|) \rho_B(\mathbf{r}') \\
& + \frac{\kappa}{2\rho_0} \int d\mathbf{r} \int d\mathbf{r}' \rho_A(\mathbf{r}) u_G(|\mathbf{r} - \mathbf{r}'|) \rho_A(\mathbf{r}') \\
& + \frac{\kappa}{2\rho_0} \int d\mathbf{r} \int d\mathbf{r}' \rho_B(\mathbf{r}) u_G(|\mathbf{r} - \mathbf{r}'|) \rho_B(\mathbf{r}') + u_{irr},
\end{aligned} \tag{8}$$

where  $u_{irr}$  are terms that are either constant or linear in the densities and thus have no thermodynamic relevance other than a shift in the energy scale. Thus, using two distinct prefactors in Eq. 2  $A_{II} = \kappa/2\rho_0$  and  $A_{IJ} = (\kappa + \chi)/\rho_0$ , we find

$$\kappa = 2\rho_0 A_{II} \tag{9}$$

$$\chi = \rho_0 (A_{IJ} - 2A_{II}). \tag{10}$$

### Example inputs

Binary mixture with a finite  $\chi$ :

```
gaussian 1 1 2.5 0.5
gaussian 2 2 2.5 0.5
gaussian 1 2 10.0 0.5
```

Two-component system that begins with  $\chi = 0$  in the above equations, then ramps  $\chi$  to larger values linearly over the simulation to a finite value:

```
gaussian 1 1 10.0 0.5
gaussian 1 2 20.0 0.5 ramp 20.4
gaussian 2 2 10.0 0.5
```

## 3.2 Erf Potential

This potential is used to generate smooth spheres within the simulation. The potential is defined as real-space as

$$u(r) = A \text{erfc} \left( \frac{|\mathbf{r}| - R_p}{\sigma} \right) \tag{11}$$

where  $A$  is the amplitude,  $R_p$  is the radius of the nanoparticles,  $\sigma$  controls the width of the nanoparticle. Entering the potential parameters in the input file is of the form

```
pair_style erf IType JType A Rp sigma ramp A_final
```

with `ramp` and `A_final` are optional parameters similar to the Gaussian parameters.

The potential in this case is the convolution between two spheres and thus should not be used between point particles and spheres (which is what “`gaussian_erfbelow`” is for).

The function is defined in kspace since the “erfc” function is the convolution between a Gaussian function and a box function. The kspace potential used is

$$U(k) = A u_G(k) \left( \frac{4\pi(\sin(R_p k) - R_p k \cos(R_p k))}{k^3} \right)^2 \quad (12)$$

To use the potential here

### 3.3 Gaussian-erfc cross potential

To model the interactions between the polymer and the spheres themselves, we use a modified version of the potential called `gaussian_erf`.

$$U(k) = A u_G(k) \frac{4\pi(\sin(R_p k) - R_p k \cos(R_p k))}{k^3} \quad (13)$$

### 3.4 Phase Fields

This potential is a static field that allows biasing into particular phases. The form of the pair style is

$$U = \int d\mathbf{r} [\rho_I(\mathbf{r}) - \rho_J(\mathbf{r})] A_o w(\mathbf{r}), \quad (14)$$

where  $w(\mathbf{r})$  takes values in  $\pm 1$  and has the symmetry of a desired phase. For example, one version might have  $w(\mathbf{r}) = \sin(2\pi x/L_x)$ , a sine wave to induce a single period of a lamellar structure in the  $x$ -direction. The ultimate amplitude is governed by  $A_o$ . One could imagine using such a field to initialize a system, then switching to the Gaussian potentials above to properly equilibrate and test the stability of a phase. While it won't guarantee going into the equilibrium phase, it should enable the formation of defect-free structures.

This potential also includes `ramp` arguments like those of Gaussian and Erf.

NOTE: This should be used in conjunction with some kind of self-repulsion (e.g., a Gaussian potential between  $I - I$  and  $J - J$ ) to prevent large total density fluctuations.

## 4 Maier-Saupe Potential

The Maier-Saupe (MS) potential has been implemented using a modified form of the potential presented in Pryamitsyn and Ganesan, JCP V120 p5824 (2003). The form of the potential is

$$\beta U_{MS} = -\frac{\mu}{2\rho_0} \int d\mathbf{r} \int d\mathbf{r}' \mathbf{S}(\mathbf{r}) : \mathbf{S}(\mathbf{r}') u_G(|\mathbf{r} - \mathbf{r}'|), \quad (15)$$

where  $\mathbf{S}(\mathbf{r})$  is a continuous tensor field that is constructed using information about the particle-level mesogen orientations and  $u_G(r)$  is a unit Gaussian to render the interactions non-local. Each particle  $i$  that represents a mesogenic center (i.e., one that interacts with the MS potential) carries an orientation that is defined using one of the neighboring particles,  $\mathbf{u}_i = \mathbf{r}_{ji}/|\mathbf{r}_{ji}|$  with  $\mathbf{r}_{ji} = \mathbf{r}_j - \mathbf{r}_i$ , and the index  $j$  represents the “partner” particle.

The particle-level tensor  $\mathbf{S}_i$  is defined as

$$\mathbf{S}_i = \mathbf{u}_i \mathbf{u}_i - \frac{\mathbf{I}}{\mathcal{D}}, \quad (16)$$

where  $\mathcal{D}$  is the dimensionality of the system and the product  $\mathbf{u}_i \mathbf{u}_i$  should be interpreted as an outer/tensor/dyadic product. The tensor field is defined on the grid as

$$\mathbf{S}(\mathbf{r}) = \sum_i^{n_{LC}} \delta(\mathbf{r} - \mathbf{r}_i) \mathbf{S}_i, \quad (17)$$

and usual particle-to-mesh schemes are used to discretize the delta function. Explicit differentiation of the force yields two contributions to the total force on particle  $j$  given as

$$\frac{\partial \beta U_{MS}}{\partial \mathbf{r}_j} = \frac{-\mu}{\rho_0} \int d\mathbf{r} \int d\mathbf{r}' \{ \delta(\mathbf{r} - \mathbf{r}_j) \mathbf{S}_j : \mathbf{S}(\mathbf{r}') \nabla_{\mathbf{r}} u(\mathbf{r} - \mathbf{r}') \} \quad (18)$$

$$\begin{aligned} & + \sum_i \delta(\mathbf{r} - \mathbf{r}_i) [\mathbf{I} \mathbf{u}_i + \mathbf{u}_i \mathbf{I}] \cdot \frac{\partial \mathbf{u}_i}{\partial \mathbf{r}_j} : \mathbf{S}(\mathbf{r}') u(\mathbf{r} - \mathbf{r}'), \\ & = \int d\mathbf{r} \int d\mathbf{r}' \{ \delta(\mathbf{r} - \mathbf{r}_j) \mathbf{S}_j : \mathbf{S}(\mathbf{r}') [-\frac{\mu}{\rho_0} \nabla_{\mathbf{r}} u(\mathbf{r} - \mathbf{r}')] \\ & - \sum_i \delta(\mathbf{r} - \mathbf{r}_i) [\mathbf{I} \mathbf{u}_i + \mathbf{u}_i \mathbf{I}] \cdot \frac{\partial \mathbf{u}_i}{\partial \mathbf{r}_j} : \mathbf{S}(\mathbf{r}') [\frac{\mu}{\rho_0} u(\mathbf{r} - \mathbf{r}')] \}. \end{aligned} \quad (19)$$

The first term (first line) represents how the potential changes as the particle carrying the unit vector is moved through space, and an integration by parts was used to get the derivative on the Gaussian potential. The second term represents how the unit vector changes as the particle moves. The sum over  $k$  is a sum over all MS interactions where particle  $j$  is involved in the definition of  $\mathbf{u}$ , including itself ( $k = j$ ). In practice, it is easier to accumulate the second term for  $k \neq j$  when the first term is evaluated for  $k$ . All of the double-dot product terms involve particle-level information on the left of the double dot and field info on the right.

Note: the prefactor read by the input file is the value of  $\mu/2\rho_0$  and a *positive* value should be given to promote nematic order.