Chapter 1

Exercise Set 1.1

1. (a) 3 x 3 (b) 3 x 2 (c) 2 x 4 (d) 3 x 1 (e) 3 x 5 (f) 1 x 4

2. 1, 4, 9, -1, 3, 8

3. 4, 5, 6, 7, 2, 3

4. $$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

5. (a) $\begin{bmatrix} 1 & 3 \\ 2 & -5 \end{bmatrix}$ and $\begin{bmatrix} 1 & 3 & 7 \\ 2 & -5 & -3 \end{bmatrix}$

(b) $\begin{bmatrix} 5 & 2 & -4 \\ 1 & 3 & 6 \\ 4 & 6 & -9 \end{bmatrix}$ and $\begin{bmatrix} 5 & 2 & -4 & 8 \\ 1 & 3 & 6 & 4 \\ 4 & 6 & -9 & 7 \end{bmatrix}$

(c) $\begin{bmatrix} -1 & 3 & -5 \\ 2 & -2 & 4 \\ 1 & 3 & 0 \end{bmatrix}$ and $\begin{bmatrix} -1 & 3 & -5 & -3 \\ 2 & -2 & 4 & 8 \\ 1 & 3 & 0 & 6 \end{bmatrix}$

(d) $\begin{bmatrix} 5 & 4 \\ 2 & -8 \\ 1 & 2 \end{bmatrix}$ and $\begin{bmatrix} 5 & 4 & 9 \\ 2 & -8 & -4 \\ 1 & 2 & 3 \end{bmatrix}$

(e) $\begin{bmatrix} 5 & 2 & -4 \\ 0 & 4 & 3 \\ 1 & 0 & -1 \end{bmatrix}$ and $\begin{bmatrix} 5 & 2 & -4 & 8 \\ 0 & 4 & 3 & 0 \\ 1 & 0 & -1 & 7 \end{bmatrix}$

(f) $\begin{bmatrix} -1 & 3 & -9 \\ 1 & 0 & -4 \\ 1 & 8 & 0 \end{bmatrix}$ and $\begin{bmatrix} -1 & 3 & -9 & -4 \\ 1 & 0 & -4 & 11 \\ 1 & 8 & 0 & 1 \end{bmatrix}$

(g) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ and $\begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 12 \\ 0 & 0 & 1 & 8 \end{bmatrix}$

(h) $\begin{bmatrix} -4 & 2 & -9 & 1 \\ 1 & 6 & -8 & -7 \\ 0 & -1 & 3 & -5 \end{bmatrix}$ and $\begin{bmatrix} -4 & 2 & -9 & 1 & -1 \\ 1 & 6 & -8 & -7 & 15 \\ 0 & -1 & 3 & -5 & 0 \end{bmatrix}$

6. (a) $\begin{aligned} x_1 + 2x_2 &= 3 \\ 4x_1 + 5x_2 &= 6 \end{aligned}$ (b) $\begin{aligned} 7x_1 + 9x_2 &= 8 \\ 6x_1 + 4x_2 &= -3 \end{aligned}$ (c) $\begin{aligned} x_1 + 9x_2 &= -3 \\ 5x_1 \quad &= 2 \end{aligned}$

(d) $\begin{aligned} 8x_1 + 7x_2 + 5x_3 &= -1 \\ 4x_1 + 6x_2 + 2x_3 &= 4 \\ 9x_1 + 3x_2 + 7x_3 &= 6 \end{aligned}$

(e) $\begin{aligned} 2x_1 - 3x_2 + 6x_3 &= 4 \\ 7x_1 - 5x_2 - 2x_3 &= 3 \\ 2x_2 + 4x_3 &= 0 \end{aligned}$

(f) $\begin{aligned} -2x_2 &= 4 \\ 5x_1 + 7x_2 &= -3 \\ 6x_1 &= 8 \end{aligned}$

(g) $\begin{aligned} x_1 &= 3 \\ x_2 &= 8 \\ x_3 &= 4 \end{aligned}$

(h) $\begin{aligned} x_1 + 2x_2 - x_3 &= 6 \\ x_2 + 4x_3 &= 5 \\ x_3 &= -2 \end{aligned}$

7. (a) $\begin{bmatrix} 1 & 3 & -2 & 0 \\ 1 & 2 & -3 & 6 \\ 8 & 3 & 2 & 5 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & 7 & 5 & 1 \\ 0 & -8 & 4 & 3 \\ 3 & -5 & 8 & 9 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 2 & 3 & -1 \\ 0 & 3 & 10 & 0 \\ 0 & -8 & -1 & -1 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 0 & -1 & -6 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 11 & -1 \end{bmatrix}$ (e) $\begin{bmatrix} 1 & 0 & 0 & -23 \\ 0 & 1 & 0 & 17 \\ 0 & 0 & 1 & 5 \end{bmatrix}$ (f) $\begin{bmatrix} 1 & 0 & 2 & 7 \\ 0 & 1 & 5 & -3 \\ 0 & 0 & 1 & -4 \end{bmatrix}$

8. (a) Create zeros below the leading 1 in the first column.
x_1 is eliminated from all equations except the first.

(b) Normalize the (2,2) element, i.e., make the (2,2) element 1. This becomes a leading 1.
It is now possible to have x_2 in the second equation with coefficient 1.

(c) Need to have the leading 1 in row 2 to the left of leading 1 in row 3.
The second equation now contains an x_2 term.

(d) Create zeros above and below the leading 1 in row 2.
x_2 is eliminated from all equations except the second.

9. (a) Create zeros above the leading 1 in column 3.
x_3 is eliminated from all equations except the third.

(b) Need to have the leading 1 in row 1 to the left of leading 1s in other rows.
It is now possible to have x_1 in Equation 1 with leading coefficient 1.

(c) Normalize the (3,3) element, i.e., make the (3,3) element 1. This becomes a leading 1.
The coefficient of x_3 in the third equation becomes 1.

(d) Create zeros above the leading 1 in column 3.
x_3 is eliminated from all equations except the third.

10. (a) $\begin{bmatrix} 1 & -2 & -8 \\ 2 & -3 & -11 \end{bmatrix} \underset{R2+(-2)R1}{\approx} \begin{bmatrix} 1 & -2 & -8 \\ 0 & 1 & 5 \end{bmatrix} \underset{R1+(2)R2}{\approx} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 5 \end{bmatrix}$,

so the solution is $x_1 = 2$ and $x_2 = 5$.

(b) $\begin{bmatrix} 2 & 2 & 4 \\ 3 & 2 & 3 \end{bmatrix} \underset{(1/2)R1}{\approx} \begin{bmatrix} 1 & 1 & 2 \\ 3 & 2 & 3 \end{bmatrix} \underset{R2+(-3)R1}{\approx} \begin{bmatrix} 1 & 1 & 2 \\ 0 & -1 & -3 \end{bmatrix} \underset{(-1)R2}{\approx} \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 3 \end{bmatrix}$

$\underset{R1+(-1)R2}{\approx} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 3 \end{bmatrix}$, so the solution is $x_1 = -1$, $x_2 = 3$.

(c) $\begin{bmatrix} 1 & 0 & 1 & 3 \\ 0 & 2 & -2 & -4 \\ 0 & 1 & -2 & 5 \end{bmatrix} \underset{(1/2)R2}{\approx} \begin{bmatrix} 1 & 0 & 1 & 3 \\ 0 & 1 & -1 & -2 \\ 0 & 1 & -2 & 5 \end{bmatrix} \underset{R3+(-1)R2}{\approx} \begin{bmatrix} 1 & 0 & 1 & 3 \\ 0 & 1 & -1 & -2 \\ 0 & 0 & -1 & 7 \end{bmatrix}$

$\underset{(-1)R3}{\approx} \begin{bmatrix} 1 & 0 & 1 & 3 \\ 0 & 1 & -1 & -2 \\ 0 & 0 & 1 & -7 \end{bmatrix} \underset{\substack{R1+(-1)R3 \\ R2+R3}}{\approx} \begin{bmatrix} 1 & 0 & 0 & 10 \\ 0 & 1 & 0 & -9 \\ 0 & 0 & 1 & -7 \end{bmatrix}$,

so the solution is $x_1 = 10$, $x_2 = -9$, $x_3 = -7$.

(d) $\begin{bmatrix} 1 & 1 & 3 & 6 \\ 1 & 2 & 4 & 9 \\ 2 & 1 & 6 & 11 \end{bmatrix} \underset{\substack{R2+(-1)R1 \\ R3+(-2)R1}}{\approx} \begin{bmatrix} 1 & 1 & 3 & 6 \\ 0 & 1 & 1 & 3 \\ 0 & -1 & 0 & -1 \end{bmatrix} \underset{\substack{R1+(-1)R2 \\ R3+R2}}{\approx} \begin{bmatrix} 1 & 0 & 2 & 3 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 1 & 2 \end{bmatrix}$

$\underset{\substack{R1+(-2)R3 \\ R2+(-1)R3}}{\approx} \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$, so the solution is $x_1 = -1$, $x_2 = 1$, $x_3 = 2$.

(e) $\begin{bmatrix} 1 & -1 & 3 & 3 \\ 2 & -1 & 2 & 2 \\ 3 & 1 & -2 & 3 \end{bmatrix} \underset{\substack{R2+(-2)R1 \\ R3+(-3)R1}}{\approx} \begin{bmatrix} 1 & -1 & 3 & 3 \\ 0 & 1 & -4 & -4 \\ 0 & 4 & -11 & -6 \end{bmatrix} \underset{\substack{R1+R2 \\ R3+(-4)R2}}{\approx} \begin{bmatrix} 1 & 0 & -1 & -1 \\ 0 & 1 & -4 & -4 \\ 0 & 0 & 5 & 10 \end{bmatrix}$

$$\underset{(1/5)R3}{\approx}\begin{bmatrix}1&0&-1&-1\\0&1&-4&-4\\0&0&1&2\end{bmatrix}\underset{\substack{R1+R3\\R2+(4)R3}}{\approx}\begin{bmatrix}1&0&0&1\\0&1&0&4\\0&0&1&2\end{bmatrix},$$

so the solution is $x_1 = 1, x_2 = 4, x_3 = 2$.

(f) $$\begin{bmatrix}-1&1&-1&-2\\3&1&1&10\\4&2&3&14\end{bmatrix}\underset{(-1)R1}{\approx}\begin{bmatrix}1&-1&1&2\\3&1&1&10\\4&2&3&14\end{bmatrix}\underset{\substack{R2+(-3)R1\\R3+(-4)R1}}{\approx}\begin{bmatrix}1&-1&1&2\\0&4&-2&4\\0&6&-1&6\end{bmatrix}$$

$$\underset{(1/4)R2}{\approx}\begin{bmatrix}1&-1&1&2\\0&1&-1/2&1\\0&6&-1&6\end{bmatrix}\underset{\substack{R1+R2\\R3+(-6)R2}}{\approx}\begin{bmatrix}1&0&1/2&3\\0&1&-1/2&1\\0&0&2&0\end{bmatrix}\underset{(1/2)R3}{\approx}\begin{bmatrix}1&0&1/2&3\\0&1&-1/2&1\\0&0&1&0\end{bmatrix}$$

$$\underset{\substack{R1+(-1/2)R3\\R2+(1/2)R3}}{\approx}\begin{bmatrix}1&0&0&3\\0&1&0&1\\0&0&1&0\end{bmatrix},$$ so the solution is $x_1 = 3, x_2 = 1, x_3 = 0$.

11. (a) $$\begin{bmatrix}1&2&3&14\\2&5&8&36\\1&-1&0&-4\end{bmatrix}\underset{\substack{R2+(-2)R1\\R3+(-1)R1}}{\approx}\begin{bmatrix}1&2&3&14\\0&1&2&8\\0&-3&-3&-18\end{bmatrix}\underset{\substack{R1+(-2)R2\\R3+(3)R2}}{\approx}\begin{bmatrix}1&0&-1&-2\\0&1&2&8\\0&0&3&6\end{bmatrix}$$

$$\underset{(1/3)R3}{\approx}\begin{bmatrix}1&0&-1&-2\\0&1&2&8\\0&0&1&2\end{bmatrix}\underset{\substack{R1+R3\\R2+(-2)R3}}{\approx}\begin{bmatrix}1&0&0&0\\0&1&0&4\\0&0&1&2\end{bmatrix},$$

so the solution is $x_1 = 0$, $x_2 = 4$, $x_3 = 2$.

(b) $$\begin{bmatrix}1&-1&-1&-1\\-2&6&10&14\\2&1&6&9\end{bmatrix}\underset{\substack{R2+(2)R1\\R3+(-2)R1}}{\approx}\begin{bmatrix}1&-1&-1&-1\\0&4&8&12\\0&3&8&11\end{bmatrix}\underset{(1/4)R2}{\approx}\begin{bmatrix}1&-1&-1&-1\\0&1&2&3\\0&3&8&11\end{bmatrix}$$

$$\underset{\substack{R1+R2\\R3+(-3)R2}}{\approx}\begin{bmatrix}1&0&1&2\\0&1&2&3\\0&0&2&2\end{bmatrix}\underset{(1/2)R3}{\approx}\begin{bmatrix}1&0&1&2\\0&1&2&3\\0&0&1&1\end{bmatrix}\underset{\substack{R1+(-1)R3\\R2+(-2)R3}}{\approx}\begin{bmatrix}1&0&0&1\\0&1&0&1\\0&0&1&1\end{bmatrix},$$

so the solution is $x_1 = 1$, $x_2 = 1$, $x_3 = 1$.

(c) $\begin{bmatrix} 2 & 2 & -4 & 14 \\ 3 & 1 & 1 & 8 \\ 2 & -1 & 2 & -1 \end{bmatrix} \underset{(1/2)R1}{\approx} \begin{bmatrix} 1 & 1 & -2 & 7 \\ 3 & 1 & 1 & 8 \\ 2 & -1 & 2 & -1 \end{bmatrix} \underset{\substack{R2+(-3)R1 \\ R3+(-2)R1}}{\approx} \begin{bmatrix} 1 & 1 & -2 & 7 \\ 0 & -2 & 7 & -13 \\ 0 & -3 & 6 & -15 \end{bmatrix}$

Let us swap rows 2 and rows 2 and 3 to avoid awkward fractions at the next step.

$\underset{R2 \Leftrightarrow R3}{\approx} \begin{bmatrix} 1 & 1 & -2 & 7 \\ 0 & -3 & 6 & -15 \\ 0 & -2 & 7 & -13 \end{bmatrix} \underset{(-1/3)R2}{\approx} \begin{bmatrix} 1 & 1 & -2 & 7 \\ 0 & 1 & -2 & 5 \\ 0 & -2 & 7 & -13 \end{bmatrix}$

$\underset{\substack{R1+(-1)R2 \\ R3+(2)R2}}{\approx} \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & -2 & 5 \\ 0 & 0 & 3 & -3 \end{bmatrix} \underset{(1/3)R3}{\approx} \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & -2 & 5 \\ 0 & 0 & 1 & -1 \end{bmatrix} \underset{R2+(2)R3}{\approx} \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -1 \end{bmatrix}$,

so the solution is $x_1 = 2$, $x_2 = 3$, $x_3 = -1$.

(d) $\begin{bmatrix} 0 & 2 & 4 & 8 \\ 2 & 2 & 0 & 6 \\ 1 & 1 & 1 & 5 \end{bmatrix} \underset{R1 \Leftrightarrow R2}{\approx} \begin{bmatrix} 2 & 2 & 0 & 6 \\ 0 & 2 & 4 & 8 \\ 1 & 1 & 1 & 5 \end{bmatrix} \underset{(1/2)R1}{\approx} \begin{bmatrix} 1 & 1 & 0 & 3 \\ 0 & 2 & 4 & 8 \\ 1 & 1 & 1 & 5 \end{bmatrix}$

$\underset{R3+(-1)R1}{\approx} \begin{bmatrix} 1 & 1 & 0 & 3 \\ 0 & 2 & 4 & 8 \\ 0 & 0 & 1 & 2 \end{bmatrix} \underset{(1/2)R2}{\approx} \begin{bmatrix} 1 & 1 & 0 & 3 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & 1 & 2 \end{bmatrix} \underset{R1+(-1)R2}{\approx} \begin{bmatrix} 1 & 0 & -2 & -1 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & 1 & 2 \end{bmatrix}$

$\underset{\substack{R1+(2)R3 \\ R2+(-2)R3}}{\approx} \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \end{bmatrix}$, so the solution is $x_1 = 3$, $x_2 = 0$, $x_3 = 2$.

(e) $\begin{bmatrix} 1 & 0 & -1 & 3 \\ -1 & 0 & 2 & -8 \\ 3 & 1 & -1 & 0 \end{bmatrix} \underset{\substack{R2+R1 \\ R2+(-3)R1}}{\approx} \begin{bmatrix} 1 & 0 & -1 & 3 \\ 0 & 0 & 1 & -5 \\ 0 & 1 & 2 & -9 \end{bmatrix} \underset{R2 \Leftrightarrow R3}{\approx} \begin{bmatrix} 1 & 0 & -1 & 3 \\ 0 & 1 & 2 & -9 \\ 0 & 0 & 1 & -5 \end{bmatrix}$

$\underset{\substack{R1+R3 \\ R2+(-2)R3}}{\approx} \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -5 \end{bmatrix}$, so the solution is $x_1 = -2$, $x_2 = 1$, $x_3 = -5$.

12 (a) $\begin{bmatrix} 3/2 & 0 & 3 & 15 \\ -1 & 7 & -9 & -45 \\ 2 & 0 & 5 & 22 \end{bmatrix} \underset{(2/3)R1}{\approx} \begin{bmatrix} 1 & 0 & 2 & 10 \\ -1 & 7 & -9 & -45 \\ 2 & 0 & 5 & 22 \end{bmatrix} \underset{\substack{R2+R1 \\ R3+(-2)R1}}{\approx} \begin{bmatrix} 1 & 0 & 2 & 10 \\ 0 & 7 & -7 & -35 \\ 0 & 0 & 1 & 2 \end{bmatrix}$

$$\underset{(1/7)R2}{\approx} \begin{bmatrix} 1 & 0 & 2 & 10 \\ 0 & 1 & -1 & -5 \\ 0 & 0 & 1 & 2 \end{bmatrix} \underset{\substack{R1+(-2)R3 \\ R2+R3}}{\approx} \begin{bmatrix} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 2 \end{bmatrix},$$

so the solution is $x_1 = 6$, $x_2 = -3$, $x_3 = 2$.

(b) $$\begin{bmatrix} -3 & -6 & -15 & -3 \\ 2 & 3 & 9 & 1 \\ -4 & -7 & -17 & -4 \end{bmatrix} \underset{(-1/3)R1}{\approx} \begin{bmatrix} 1 & 2 & 5 & 1 \\ 2 & 3 & 9 & 1 \\ -4 & -7 & -17 & -4 \end{bmatrix} \underset{\substack{R2+(-2)R1 \\ R3+(4)R1}}{\approx} \begin{bmatrix} 1 & 2 & 5 & 1 \\ 0 & -1 & -1 & -1 \\ 0 & 1 & 3 & 0 \end{bmatrix}$$

$$\underset{(-1)R2}{\approx} \begin{bmatrix} 1 & 2 & 5 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 3 & 0 \end{bmatrix} \underset{\substack{R1+(-2)R2 \\ R3+(-1)R2}}{\approx} \begin{bmatrix} 1 & 0 & 3 & -1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 2 & -1 \end{bmatrix} \underset{(1/2)R3}{\approx} \begin{bmatrix} 1 & 0 & 3 & -1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & -1/2 \end{bmatrix}$$

$$\underset{\substack{R1+(-3)R3 \\ R2+(-1)R3}}{\approx} \begin{bmatrix} 1 & 0 & 0 & 1/2 \\ 0 & 1 & 0 & 3/2 \\ 0 & 0 & 1 & -1/2 \end{bmatrix},$$ so the solution is $x_1 = 1/2$, $x_2 = 3/2$, $x_3 = -1/2$.

(c) $$\begin{bmatrix} 3 & 6 & 0 & -3 & 3 \\ 1 & 3 & -1 & -4 & -12 \\ 1 & -1 & 1 & 2 & 8 \\ 2 & 3 & 0 & 0 & 8 \end{bmatrix} \underset{(1/3)R1}{\approx} \begin{bmatrix} 1 & 2 & 0 & -1 & 1 \\ 1 & 3 & -1 & -4 & -12 \\ 1 & -1 & 1 & 2 & 8 \\ 2 & 3 & 0 & 0 & 8 \end{bmatrix}$$

$$\underset{\substack{R2+(-1)R1 \\ R3+(-1)R1 \\ R4+(-2)R1}}{\approx} \begin{bmatrix} 1 & 2 & 0 & -1 & 1 \\ 0 & 1 & -1 & -3 & -13 \\ 0 & -3 & 1 & 3 & 7 \\ 0 & -1 & 0 & 2 & 6 \end{bmatrix} \underset{\substack{R1+(-2)R2 \\ R3+(3)R2 \\ R4+R2}}{\approx} \begin{bmatrix} 1 & 0 & 2 & 5 & 27 \\ 0 & 1 & -1 & -3 & -13 \\ 0 & 0 & -2 & -6 & -32 \\ 0 & 0 & -1 & -1 & -7 \end{bmatrix}$$

$$\underset{(-1/2)R3}{\approx} \begin{bmatrix} 1 & 0 & 2 & 5 & 27 \\ 0 & 1 & -1 & -3 & -13 \\ 0 & 0 & 1 & 3 & 16 \\ 0 & 0 & -1 & -1 & -7 \end{bmatrix} \underset{\substack{R1+(-2)R3 \\ R2+R3 \\ R4+R3}}{\approx} \begin{bmatrix} 1 & 0 & 0 & -1 & -5 \\ 0 & 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 3 & 16 \\ 0 & 0 & 0 & 2 & 9 \end{bmatrix}$$

$$\underset{(1/2)R4}{\approx} \begin{bmatrix} 1 & 0 & 0 & -1 & -5 \\ 0 & 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 3 & 16 \\ 0 & 0 & 0 & 1 & 9/2 \end{bmatrix} \underset{\substack{R1+R4 \\ R3+(-3)R4}}{\approx} \begin{bmatrix} 1 & 0 & 0 & 0 & -1/2 \\ 0 & 1 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 & 5/2 \\ 0 & 0 & 0 & 1 & 9/2 \end{bmatrix},$$

so the solution is $x_1 = -1/2$, $x_2 = 3$, $x_3 = 5/2$, $x_4 = 9/2$.

(d) $$\begin{bmatrix} 1 & 2 & 2 & 5 & 11 \\ 2 & 4 & 2 & 8 & 14 \\ 1 & 3 & 4 & 8 & 19 \\ 1 & -1 & 1 & 0 & 2 \end{bmatrix} \underset{\substack{R2+(-2)R1 \\ R3+(-1)R1 \\ R4+(-1)R1}}{\approx} \begin{bmatrix} 1 & 2 & 2 & 5 & 11 \\ 0 & 0 & -2 & -2 & -8 \\ 0 & 1 & 2 & 3 & 8 \\ 0 & -3 & -1 & -5 & -9 \end{bmatrix}$$

$$\underset{R2 \Leftrightarrow R3}{\approx} \begin{bmatrix} 1 & 2 & 2 & 5 & 11 \\ 0 & 1 & 2 & 3 & 8 \\ 0 & 0 & -2 & -2 & -8 \\ 0 & -3 & -1 & -5 & -9 \end{bmatrix} \underset{\substack{R1+(-2)R2 \\ R4+(3)R2}}{\approx} \begin{bmatrix} 1 & 0 & -2 & -1 & -5 \\ 0 & 1 & 2 & 3 & 8 \\ 0 & 0 & -2 & -2 & -8 \\ 0 & 0 & 5 & 4 & 15 \end{bmatrix}$$

$$\underset{(-1/2)R3}{\approx} \begin{bmatrix} 1 & 0 & -2 & -1 & -5 \\ 0 & 1 & 2 & 3 & 8 \\ 0 & 0 & 1 & 1 & 4 \\ 0 & 0 & 5 & 4 & 15 \end{bmatrix} \underset{\substack{R1+(2)R3 \\ R2+(-2)R3 \\ R4+(-5)R3}}{\approx} \begin{bmatrix} 1 & 0 & 0 & 1 & 3 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 4 \\ 0 & 0 & 0 & -1 & -5 \end{bmatrix}$$

$$\underset{(-1)R4}{\approx} \begin{bmatrix} 1 & 0 & 0 & 1 & 3 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 4 \\ 0 & 0 & 0 & 1 & 5 \end{bmatrix} \underset{\substack{R1+(-1)R4 \\ R2+(-1)R4 \\ R3+(-1)R4}}{\approx} \begin{bmatrix} 1 & 0 & 0 & 0 & -2 \\ 0 & 1 & 0 & 0 & -5 \\ 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 5 \end{bmatrix},$$

so the solution is $x_1 = -2$, $x_2 = -5$, $x_3 = -1$, $x_4 = 5$.

(e) $$\begin{bmatrix} 1 & 1 & 2 & 6 & 11 \\ 2 & 3 & 6 & 19 & 36 \\ 0 & 3 & 4 & 15 & 28 \\ 1 & -1 & -1 & -6 & -12 \end{bmatrix} \underset{\substack{R2+(-2)R1 \\ R4+(-1)R1}}{\approx} \begin{bmatrix} 1 & 1 & 2 & 6 & 11 \\ 0 & 1 & 2 & 7 & 14 \\ 0 & 3 & 4 & 15 & 28 \\ 0 & -2 & -3 & -12 & -23 \end{bmatrix}$$

$$\underset{\substack{R1+(-1)R2 \\ R3+(-3)R2 \\ R4+(2)R2}}{\approx} \begin{bmatrix} 1 & 0 & 0 & -1 & -3 \\ 0 & 1 & 2 & 7 & 14 \\ 0 & 0 & -2 & -6 & -14 \\ 0 & 0 & 1 & 2 & 5 \end{bmatrix} \underset{(-1/2)R3}{\approx} \begin{bmatrix} 1 & 0 & 0 & -1 & -3 \\ 0 & 1 & 2 & 7 & 14 \\ 0 & 0 & 1 & 3 & 7 \\ 0 & 0 & 1 & 2 & 5 \end{bmatrix}$$

$$\underset{\substack{R2+(-2)R3\\R4+(-1)R3}}{\approx}\begin{bmatrix}1&0&0&-1&-3\\0&1&0&1&0\\0&0&1&3&7\\0&0&0&-1&-2\end{bmatrix}\underset{(-1)R4}{\approx}\begin{bmatrix}1&0&0&-1&-3\\0&1&0&1&0\\0&0&1&3&7\\0&0&0&1&2\end{bmatrix}$$

$$\underset{\substack{R1+R4\\R2+(-1)R4\\R3+(-3)R4}}{\approx}\begin{bmatrix}1&0&0&0&-1\\0&1&0&0&-2\\0&0&1&0&1\\0&0&0&1&2\end{bmatrix}$$, so the solution is $x_1 = -1$, $x_2 = -2$, $x_3 = 1$, $x_4 = 2$.

13. (a) $\begin{bmatrix}1&2&3&4&3\\3&5&8&9&7\end{bmatrix}\underset{R2+(-3)R1}{\approx}\begin{bmatrix}1&2&3&4&3\\0&-1&-1&-3&-2\end{bmatrix}\underset{(-1)R2}{\approx}\begin{bmatrix}1&2&3&4&3\\0&1&1&3&2\end{bmatrix}\underset{R1-2R2}{\approx}$

$\begin{bmatrix}1&0&1&-2&-1\\0&1&1&3&2\end{bmatrix}$, so $x_1 = 1$, $x_2 = 1$; $x_1 = -2$, $x_2 = 3$; and $x_1 = -1$, $x_2 = 2$.

(b) $\begin{bmatrix}1&1&0&5&1\\2&3&1&13&2\end{bmatrix}\underset{R2+(-2)R1}{\approx}\begin{bmatrix}1&1&0&5&1\\0&1&1&3&0\end{bmatrix}\underset{R1+(-1)R2}{\approx}\begin{bmatrix}1&0&-1&2&1\\0&1&1&3&0\end{bmatrix}$,

so the solutions are in turn $x_1 = -1$, $x_2 = 1$; $x_1 = 2$, $x_2 = 3$; and $x_1 = 1$, $x_2 = 0$.

(c) $\begin{bmatrix}1&-2&3&6&-5&4\\1&-1&2&5&-3&3\\2&-3&6&14&-8&9\end{bmatrix}\underset{\substack{R2+(-1)R1\\R3+(-2)R1}}{\approx}\begin{bmatrix}1&-2&3&6&-5&4\\0&1&-1&-1&2&-1\\0&1&0&2&2&1\end{bmatrix}$

$$\underset{\substack{R1+(2)R2\\R3+(-1)R2}}{\approx}\begin{bmatrix}1&0&1&4&-1&2\\0&1&-1&-1&2&-1\\0&0&1&3&0&2\end{bmatrix}\underset{\substack{R1+(-1)R3\\R2+R3}}{\approx}\begin{bmatrix}1&0&0&1&-1&0\\0&1&0&2&2&1\\0&0&1&3&0&2\end{bmatrix},$$

so the solutions are in turn $x_1 = 1$, $x_2 = 2$, $x_3 = 3$; $x_1 = -1$, $x_2 = 2$, $x_3 = 0$; and $x_1 = 0$, $x_2 = 1$, $x_3 = 2$.

(d) $\begin{bmatrix}1&2&-1&-1&6&0\\-1&-1&1&1&-4&-2\\3&7&-1&-1&18&-4\end{bmatrix}\underset{\substack{R2+R1\\R3+(-3)R1}}{\approx}\begin{bmatrix}1&2&-1&-1&6&0\\0&1&0&0&2&-2\\0&1&2&2&0&-4\end{bmatrix}$