

CSCI 104 - HWI Part 1

Q3.

(a) void f1(int n)

```
{
    int i = 2;
```

```
    while(i < n) {
```

```
        // do sth A(i) ← A(i)
```

```
        i = i * 2;
```

```
    }
```

3.

By $\Theta = \Theta(\text{while}) \cdot \Theta(1)$

assume $\Theta(\text{while}) = t$

$\therefore i = i \times 2$

$\therefore \text{int } i = 2$

when $i = n$

every turn inside while loop

At initial point $i = 2$

Assume $2^k = n$

$(k \in \mathbb{Z})$ $k = \log n$

$\therefore i = i \times 2$ be executed for t times

$\therefore 2^t = k$ $2^t = \log n$

$t = \log(\log n)$

$\therefore \text{By } \Theta = \Theta(\text{while}) \cdot \Theta(1) = \log(\log n)$

$= \log(\log n)$

b)

```
for (int i = 1; i <= n; i++) {
```

```
    if (i % (int)sqrt(n) == 0) {
```

```
        for (int k = 0; k < pow(i, 3); k++) {
```

```
            A(i)
```

```
        }
```

}

consider. when $i \% (\text{int}) \sqrt{n} == 0$.
when i goes from 1 to n .

$$(\text{int}) \sqrt{n} \approx \sqrt{n} \quad \checkmark, i \% \sqrt{n} == 0 \text{ is equivalent.}$$

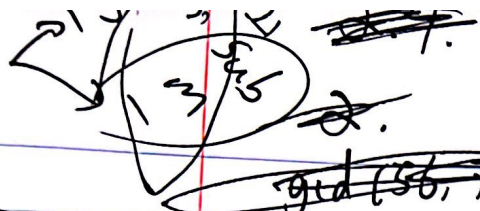
Find ~~the~~ num of $\frac{n}{\sqrt{n}}$ (how many \sqrt{n} inside n).

the num of condition for loop's condition be satisfied

$$\text{is } \frac{n}{\sqrt{n}}$$

- num of Θ (outer for loop) = num (for's condition be satisfied)

$$= \frac{n}{\sqrt{n}} = \sqrt{n} = n^{\frac{1}{2}}$$



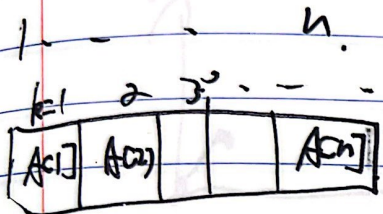
~~gcd(56, 70) = 56 + 70y~~

(b)
P3 Following: i 's value could be the multiples
of \sqrt{n} $i: 1 \times \sqrt{n}, 2\sqrt{n}, 3\sqrt{n}, \dots, \sqrt{n} \cdot \sqrt{n}$.

$$\begin{aligned} \therefore \Theta &= \sum_{i=1}^{\sqrt{n}} (\text{inner loop}) \cdot A(i) = [(1 \cdot \sqrt{n})^3 + (2 \cdot \sqrt{n})^3 + \dots + (\sqrt{n} \cdot \sqrt{n})^3] \cdot A(1) \\ &= [1^3 + 2^3 + \dots + (\sqrt{n})^3] \cdot (\sqrt{n})^3 = \frac{(\sqrt{n})^2 (\sqrt{n}+1)^2}{4} \cdot (n^{\frac{1}{2}})^3 \\ &= \frac{n \cdot (n^{\frac{1}{2}} + 2\sqrt{n} + 1)}{4} \cdot n^{\frac{3}{2}} = \Theta(n^2 \cdot n^{\frac{3}{2}}) = n^{\frac{7}{2}}. \end{aligned}$$

~~$\Theta(n^2) = \Theta(\text{outer for loop}) \cdot \Theta(\text{inner for loop}) \cdot \Theta(1)$~~
 ~~$= n^2 \cdot n^3 \cdot 1 = n^5$~~
 ~~$= \frac{(1^3 + 2^3 + \dots + n^3) \cdot n}{4}$~~

part (c)



inner loop? int m=1; m ≤ n; m = m + 1


$\therefore 1 \cdot 2 \cdot 2 = n$
 inner loop executes t times
 $\therefore 2^t = n$
 $t = \log n$

$\therefore \Theta(\text{inner for loop}) = \log n$

(if $A[k] = z$) $\therefore z$ could be \sqrt{n} (n's different value)

$\therefore A[k]$ also has n values.

each element in A could be equal to z at most $O(\sqrt{n})$.

$\text{num}(\text{Ack} == i) = 2$  $\text{Max}(\text{Ack} == i)$
 contain ~~at least~~ inside it
 assume: $p = \text{num}(\text{Ack} == i)$ be executed.
 $P_{\max} = n$. $P_{\min} = 0$ (no one matches.)

Condition ① $P = P_{\max} = n$.

~~A counter for~~ = p assume: $q = \text{num}(\text{Ack} == i)$ is FALSE.
~~A middle for loop~~ = p $p = \text{num}(\text{Ack} == i)$ is TRUE.

$$p + q = A(\text{counter for}) \cdot A(\text{middle for})$$

$$= n \cdot n = n^2$$

$$\text{If } p = P_{\max} = n \rightarrow q = n^2 - n$$

$$\begin{aligned}
 \theta &= n \cdot \theta(\text{inner for}) \cdot \theta(1) + (n^2 - n) \cdot \theta(1) \\
 &= n \cdot \log(n) \cdot 1 + (n^2 - n) \cdot 1 = n^2 + n \log n - n \\
 &= \theta(n^2)
 \end{aligned}$$

Condition ② $P = P_{\min} = 0$.

Similarly

$$\begin{aligned}
 \theta &= 0 \cdot \theta(\text{inner for loop}) \cdot \theta(1) + (n^2 - 0) \cdot \theta(1) \\
 &= n^2 \cdot 1 = n^2 = \theta(n^2)
 \end{aligned}$$

From Condition ① and ② (Worst and Best case.)
 the Big θ is $\theta(n^2)$.

Part d.)

$\Theta(\text{outer for loop}) = n$ In 2f part. $\text{int newsize} = 3 \cdot \text{size} / 2$.
 $\Theta(\text{inner for loop})$ size increase by multiply $\frac{3}{2}$ every turn.

assume $t = \text{times}(i == \text{size})$ is true.

$\text{int size} = 10$ at starting.

$$10 \left(\frac{3}{2}\right)^t = n \quad t = \log_{\frac{3}{2}} \frac{n}{10} = \log_{\frac{3}{2}} n - \log_{\frac{3}{2}} 10$$

$$= \log n - c \quad (c \text{ is constant})$$

$$\Theta(\text{inner loop}) = \left(\frac{3}{2}\right)^t \cdot 10.$$

Every inner for loop's time will increase by $\frac{3}{2}$.

Σ Time of every inner for loop is the sum of a geometric sequence.

with t items ($t = \log n - c$) (c could be ignored

in following computing), $t = \log n$.

First item in sequence = 10.

$$\begin{aligned} \therefore \text{Sum} &= 10 \cdot \frac{1 - \left(\frac{3}{2}\right)^{\log n}}{1 - \frac{3}{2}} = -20 \left(1 - \left(\frac{3}{2}\right)^{\log n}\right) \\ &= -20 + 20 \left(\frac{3}{2}\right)^{\log n} = 20 \left(\frac{3}{2}\right)^{\log n} - 20. \end{aligned}$$

$\Theta(\text{array's sum})$ content. $\Theta(\text{outside})$ out side if but inside
 $\Theta(\text{outer for loop}) = \Theta(1)$

$$\therefore A = \text{sum} + \Theta(\text{outer for}) \cdot \Theta(1) = c \cdot \left(\frac{3}{2}\right)^{\log n} - c + n.$$

$$\because 2^{\log n} = n \quad \frac{3}{2} < \therefore \left(\frac{3}{2}\right)^{\log n} < n$$

$$\therefore A = \Theta(n).$$