CSCI 104 - HW1 Part 1

Q3.

(a)
```
void f1 (int n)
{
    int i=2;
    while(i<n){
        /* do sth Θ(1) <- Θ(1)
        i=i*i; <- Θ(1)
    }
}
```

$\Theta($while$)$.

Big Θ = Θ(while) $\cdot$ $\Theta(1)$

∴ assume Θ(while) $= t$.

∵ $i = i \times i$. ∵ int $i=2$.

~~when $i^k = n$~~ ∵ $i = i\times i$ will be exerute in every turn inside while loop. ∴

~~Asse~~. ∴ At initial point $i=2$.

assume $2^k = n$ ($k \in \mathbb{Z}$) $k = \log n$.

∵ $i = i \times i$ be executed for t times.

∴ $2^t = k$ $\quad 2^t = \log n$

$t = \log(\log(n))$.

∴ Big Θ = Θ(while) $\cdot$ $\Theta(1)$ = $\log(\log(n)) \cdot 1$

$= \log(\log(n))$.

(b)
```
for (int i=1 i<=n; i++){
    if ( i% (int)sqrt(n)==0) {
        for (int k=0 , k<pow(i,3);k++){
            Θ(1)
        }
    }
}
```

consider. when $i \% (int)\, sqrt(n) == 0$.

when i goes from 1 to n.

$(int)\, sqrt(n) \approx \sqrt{n}$ $\therefore i \% \sqrt{n} == 0$ is equivalent.

~~θ~~ Find ~~num~~ num of $\frac{n}{\sqrt{n}}$. (how many $\sqrt{n}$ inside n).

the num of ~~condition~~ for loop's condition be satisfied

is $\frac{n}{\sqrt{n}}$.

$\cdot$ θ ~~num of~~ θ (outer. for loop) = num (for's condition be satisfied)

$= \frac{n}{\sqrt{n}} = \sqrt{n} = n^{\frac{1}{2}}$.

1, 3, 5

~~gcd(56, 72) = 56x + 72y~~

(b)

P3 Following: i's value could be the multiples of $\sqrt{n}$ i: $1\times\sqrt{n}$, $2\sqrt{n}$, $3\sqrt{n}$ . . . $\sqrt{n}\cdot\sqrt{n}$.

$$\Theta = \sum_{i=1}^{\sqrt{n}} \Theta(\text{inner loop})\,\Theta(1) = \left[(1\cdot\sqrt{n})^3 + (2\cdot\sqrt{n})^3 + \cdots (\sqrt{n}\cdot\sqrt{n})^3\right]\cdot\Theta(1)$$

$$= \left[1^3 + 2^3 + \cdots + (\sqrt{n})^3\right]\cdot(\sqrt{n})^3 = \frac{(\sqrt{n})^2(\sqrt{n}+1)^2}{4}\cdot\left(n^{\frac{1}{2}}\right)^3$$

$$= \frac{n\cdot(n + 2\sqrt{n} + 1)}{4}\cdot n^{\frac{3}{2}} = \Theta\left(n^2\cdot n^{\frac{3}{2}}\right) = n^{\frac{7}{2}}.$$

$\therefore$ Big Θ = Θ(outer for loop) $\cdot$ Θ(inner for loop) $\cdot$ $\Theta(1)$

$= n^{\frac{3}{2}} \cdot n^{\frac{3}{2}} \cdot 1 = n^3$

part (c)

1 . . . n.

k=1 2 3 . . . .

A[1]	A[2]			A[n]

Θ .. Inner loop: int m=1 $m \le n$. $m = 2 \cdot m$.

$\therefore \underbrace{1 \cdot 2 \cdot 2}_{\text{inner executed } t \text{ times}} = n$.

$\therefore 2^t = n$ t $t = \log n$.

$\therefore \Theta$(inner for loop) $= \log n$.

$\therefore$ if (A[k] == i] $\because$ i could be 1 to n. (n is different value)

$\therefore$ A[k] also has. n values.

each element in A could equal to i at most once.

$\because$ num $(A[k]==i)$ ~~Max~~ Max $(A[k]==i)$

content inside if be executed.

assume: p = num ~~A[k]==i be~~ ~~A[k]==i~~

$p_{max} = n$. $p_{min} = 0$ (no one matchs.)

Condition ① $p = p_{max} = n$.

~~θ(outer for) = n~~ assume: q num $(A[k]==i)$ is FALSE.

~~θ(middle for loop) =~~ p: num $(A[k]==i)$ is TRUE.

$p+q = \theta(\text{outer for}) \cdot \theta(\text{middle for})$

$= n \cdot n = n^2$.

If $p = p_{max} = n$ $\Rightarrow q = n^2 - n$.

$\theta = n \cdot \theta(\text{inner for}) \cdot \theta(1) + (n^2-n) \cdot \theta(1)$

$= n \cdot \log(n) \cdot 1 + (n^2-n) \cdot 1 = n^2 + n\log n - n$.

$= \theta(n^2)$.

Condition ② $p = p_{min} = 0$.

$\therefore$ similarly

$\theta = 0 \cdot \theta(\text{inner for loop}) \cdot \theta(1) + (n^2-0) \cdot \theta(1) =$

$= n^2 \cdot 1 = n^2 = \theta(n^2)$

From Condition ① and ② (Worst and Best case.)

the Big θ is $\theta(n^2)$.

Part d.)

θ(outer for loop) = n. In if part.

~~θ(inner for loop) = n~~ int newsize = 3*size/2.

size increase by multiply $\frac{3}{2}$ every turn.

assue t = times (i==size) is true.

$\because$ int size = 10. at starting.

$\therefore 10(\frac{3}{2})^t = n \qquad t = \log_{1.5}\frac{n}{10} = \log_{1.5} n - \log_{1.5} 10$

$= \log n - c$ (c is constant)

θ(inner for loop) $= (\frac{3}{2})^t \cdot 10$.

Every Inner for loop's time will incrise by $\frac{3}{2}$.

$\therefore \Sigma$ Time of every inner for loop is the sum of a geometric sequence.

with t items ($t = \log n - c$) (c could be ignored in following computing) $\therefore t = \log n$.

First ~~elem~~ item in sequence = 10.

$\therefore$ Sum $= 10 \cdot \frac{1-(\frac{3}{2})^{\log n}}{1-\frac{3}{2}} = -20(1-(\frac{3}{2})^{\log n})$

$= -20 + 20(\frac{3}{2})^{\log n} = 20(\frac{3}{2})^{\log n} - 20$.

~~θ(a[i] = i*i)~~ 's sum $\quad \theta$(content ~~after~~ out side if but inside outer for loop) $= \theta(1)$

$\therefore \theta = \text{sum} + \theta(\text{outer for}) \cdot \theta(1) = c \cdot (\frac{3}{2})^{\log n} - c + n$.

$\because 2^{\log n} = n \quad \frac{3}{2} < 2 \quad \therefore (\frac{3}{2})^{\log n} < n$

$\therefore \theta = \theta(n)$.