Probubility

1.

$\# total = 15^8$ (every Q has 15 choice).

\#. no one answers more than one q.

$= 15 \times 14 \times 13 \times 12 \times 11 \times 10 \times 9 \times 8$.

$$\therefore P = \frac{\# \text{no one answer more than one q}}{\# \text{total}} = \frac{15\times14\times13\times12\times11\times10\times9\times8}{15^8}$$

$$= \frac{259459200}{2562890625} \approx 0.1012.$$

2. \# Interested Number

$10-3=7$, $10-4=6$

5 choices (1, 3, 5, 7, 9), 4 choices, 0 2 4 6 8 5 choices.

$= 5 \times 4 \times 7 \times 6 \times 5$

$= 4200$.

$\# total = 99999 - 0 + 1 = 100000 = 10^5$.

~~\# total combinations~~ $\cancel{= C^{8}_{10^5}}$

~~\# 5 inpot~~ $\cancel{= C^{5}_{4200} \times C^{8-5}_{10^5-4200}}$

$\cancel{= C^{5}_{4200} \times C^{3}_{95800}}$

P(select a Interested num)

$$= \frac{4200}{10^5} = 0.042$$

$$P = \binom{8}{5} (0.042)^5 \cdot (1-0.042)^{8-5}$$

$$= \frac{\cancel{8\times7\times6\times5\times4}}{\cancel{5\times4\times3\times2\times1}} \times (0.042)^5 \cdot (0.958)^3 = 0.0000064347$$

$\approx 6.4347 \times 10^{-6}$.

$3\times3\times3 = 27$

$3\times3\times3\times3 = 81$

3.

$P(A)=$

$\#\text{total} = 6^3 = 216.$

$\#\,A\text{ event} = \cancel{3\times3\times6\times3 = 162}$

$\#$ 2 dice show ≥ 4

$+ \#$ 3 dice show $\geq 4 = C_3^2 \times \underset{\text{choose from } 4,5,6}{3\times3} \times \underset{\text{chose from } 1,2,3}{3} + 3\times3\times3$

(3 choice of unrestricted dice)

$= \frac{3\times2}{2}\times27+27 = 3\times27+27 = 108.$

$\#B\text{ event} = 6.$ $\quad \therefore P(A)=\frac{\#A}{\#\text{total}} = \frac{108}{216} = \frac{1}{2}.$

$P(B)=\frac{\#B}{\#\text{total}} = \frac{6}{216} = \frac{1}{36}$ $\quad P(A)\cdot P(B) = \frac{1}{2}\cdot\frac{1}{36} = \frac{1}{72}.$

$P(A\cap B) \Rightarrow \#A\cap B = 3$ $\quad (\{4,4,4\}, \{5,5,5\}, \{6,6,6\})$

$P(A\cap B) = \frac{\#A\cap B}{\#\text{total}} = \frac{3}{216} = \frac{1}{72}.$

$\because P(A)\cdot P(B) = P(A\cap B)$ $\quad \therefore$ A and B are independent.

4.

For each deck, 4 suits, each 13 cards.

$\#$ combination of 5 cards $= C_{52}^5 = \frac{52\times51\times50\times49\times48}{5!}$

$= 2598960$

$\#\text{ flush} = 4\cdot C_{13}^5 = 4\times\frac{13\times12\times11\times10\times9}{5!}$

$= 4\times1287 = 5148$

$\therefore P(\text{get a flush in one deck}) = \frac{5148}{2598960} \approx 0.0020$

$E(\text{hands}) = \frac{1}{P_{\text{flush}}} = \frac{1}{0.0020} = 500$

$3\times3\times3$

$\to 81$

$\to 81.$

27

5.

$P(win \mid super) = 0.7. \quad P(win \mid non\text{-}super) = 0.5.$

$P(super) = 0.75$

$P(win\ 4\ in\ 5 \mid super)$

$= C_5^4 \cdot 0.7^4 \cdot 0.3^1 \approx 0.3602.$

$P(win\ 4\ in\ 5 \mid non\text{-}super) = C_5^4 \cdot 0.5^4 \cdot 0.5^1$

≈ 0.1563

$P(win\ 4/5) = P(win\ 4\ in\ 5 \mid super) \cdot P(super)$

$+ P(win\ 4\ in\ 5 \mid non\text{-}super) \cdot P(non\text{-}super)$

$= 0.3602 \times 0.75 + 0.1563(1-0.75) \approx 0.3092$

$$P(super \mid win\ 4\ in\ 5) = \frac{P(win\ 4\ in\ 5 \mid super) \cdot P(super)}{P(win\ 4\ in\ 5)}$$

$$= \frac{0.3602 \times 0.75}{0.3092} \approx 0.8737.$$