CSCI104 Counting Problems

1: unusual; unique(subsets of 5 letters.)

① no u inside; only 4 letters; not satisfied.

② 1. u inside: $\cancel{C_3^1 \times 1 = 3}$ $C_4^4 = 1$

③. 2. u inside. $\cancel{C_3^2} \times C_4^{5-2} = \cancel{C_3^2} \times C_4^3 = \cancel{\frac{3\times2\times1}{2\times1}} \times \cancel{\frac{4\times3\times2}{3\times2\times1}}$

$= \frac{4\times3\times2}{3!} = 4.$

$\cancel{= 3 \times 4 = 12}$

④ 3 u inside: $\cancel{C_3^3} \times C_4^2 = \cancel{1} \times \frac{4\times3}{2} = 6.$

$\therefore$ Total: $0 + 1 + 4 + 6 = 11$.

Counting Problems

1. (2): Different strings

For: ~~unsal~~ unsal: $5\times4\times3\times2\times1 = 120$

For uu___ $4\times\frac{5\times4\times3\times2\times1}{2\times1} = 240$

For: uuu__ $6\times\frac{5\times4\times3\times2\times1}{3\times2\times1} = 120$

Total strings $= 120+240+120$

$= 480$

2. 5 card with 2 pairs.

First pair: 13 choices;

Second pair: 12 choice

single card: 11 choice.

$\therefore 13 \times 12 \times 11 = 1716$.

3.

~~17 positions~~

~~|0|0|1|0|0|0|0|0 0 0 1 0 1 0 1 0 0 1 0 1 0 0 1~~

~~put 6 blocks into 17 positions.~~

Condition ① Don't play songs for the fighting couple ⇒, the songs for 6 couples.

16

$\overbrace{0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0}^{17 \text{ positions}}$

put $(6-1)=5$ blocks into 17 positions.

$$C_{17}^{5} = \frac{17\times 16\times 15\times 14\times 13}{5\times 4\times 3\times 2\times 1} = 6188.$$

② Give one song to the fighting couple, dividing 15 songs to other 6 couples

$\underbrace{0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0\,0}_{16 \text{ positions}}$

put $6-1=5$ blocks into 16 positions

$$C_{16}^{5} = \frac{16\times 15\times 14\times 13\times 12}{5\times 4\times 3\times 2\times 1} = 4368$$

Total: $6188 + 4368 = 10556$.

4. BST with 12 Nodes 1-12.

root: 3

1, 2. 2 : nodes

right child

9

4 5 6 7 8 — 5 nodes

10 11 12 — 3 nodes.

for n numbers, there are 2×2 positions, then chose n (nums).

To eliminating repetition conditions: divided C_{2n}^{n} by $n+1$

for 1, 2: $n=2$. $C_{2}^{2\times 2} \div (2+1)$

$$= \frac{C_2^4}{3} = \frac{\frac{4\times 3}{2}}{3} = 2.$$

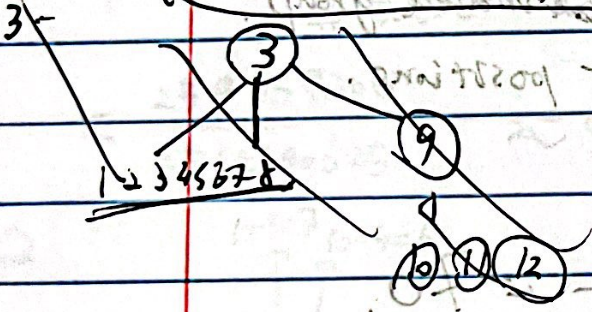
16 (17)

45,678: n=5 $C^{2\times5}_{5} \div (5+1)$

$= C^{10}_{5}/6 = \frac{10\cdot9\cdot8\cdot7\cdot6}{5\cdot4\cdot3\cdot2\cdot1\cdot6} = 42.$

10,11,12: n=3 $C^{2\times3}_{3} \div (3+1) = \frac{6\times5\times4}{3!} \div 4 = 5.$

$\boxed{\text{Binary: } 2\times42\times5 = 420.}$



for 10, 11, 12:

$C^{3\times3}_{3} \div (3+1)$

$= \frac{9\times8\times7}{6} \div 4 = 21.$

$C^{4}_{2} \div 3$

$C^{6}_{3} \div 4$

$C^{8}_{4} \div 5$

$\frac{8\times7\times6\times5}{4\times3\times2\times1} \div 5$

$C^{10}_{5} \div 6$

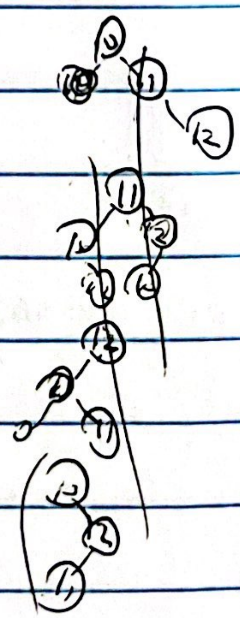
Ways:

2 node trees: 2.

3 node trees: 5.

4 node trees: 14.

5 Node trees: 42.



84

28

112

122

128

5.

From the Given Information, at least one friend will be vacanated by 3 doctors who do not have break.

Therefore, firstly assigned these 3 friends to the 3 doctor's queue.

Then, consider the other 7 friends, Assign them in to 5 groups: First doctor, second doctor, third doctor and the fourth doctor, and unvaccinated group.

put 5-1=4 blocks into (7+1)=8 positions.

$_1 O _2 O _3 O _4 O _5 O _6 O _7 O _8$

$$\therefore C_8^4 = \frac{8\times 7\times 6\times 5}{4\times 3\times 2\times 1} = 70$$

So There 70 combinations.