Optimizing Bidding Curves for Renewable Energy in Two-Settlement Electricity Markets (Supplementary Materials)

Dongwei Zhao*, Stefanos Delikaraoglou*, Vladimir Dvorkin†, Alberto J. Lamadrid L.‡, Audun Botterud*
*Laboratory for Information and Decision Systems, Massachusetts Institute of Technology, Cambridge, MA, USA
†Department of Electrical Engineering and Computer Science, University of Michigan, Ann Arbor, MI, USA
‡Department of Economics and Department of Industrial and Systems Engineering, Lehigh University, Bethlehem, PA, USA

APPENDIX

A. Proof of Theorem 1

Since Problem *BiD-q* is a special case of Problem *BiD*, we always have $S^{\text{BiD}} \leqslant S^{\text{BiD-q}}$. We thus only need to prove $S^{\text{BiD}} \geqslant S^{\text{BiD-q}}$ to arrive at equality. The idea is to show that given the optimal solution to Problem *BiD*, we can always find a feasible solution to Problem *BiD-q*, which achieves the same system cost as Problem *BiD*.

First, we define some notations for the optimal solution to Problem *BiD*. We denote the optimal bidding curve as $(\boldsymbol{C}^{\text{W}\star}, \boldsymbol{W}^{\star})$, the optimal DAM dispatch solution as $\Phi^{\text{DA}\star}$. The latter includes VRE schedule $p^{\text{W}\star}_{k,t,s}$ for any k,t,s. We denote the DAM dispatch solution, except VRE schedule $\boldsymbol{p}^{\text{W}\star} = (p^{\text{W}\star}_{k,t,s}, \forall k,t,s)$, as $\Phi^{\text{DA}\star}_{-W}$, i.e., $\Phi^{\text{DA}\star} = (\boldsymbol{p}^{\text{W}\star}, \Phi^{\text{DA}\star}_{-W})$.

Second, we construct feasible solutions for Problem *BiD-q* based on the optimal solution to Problem *BiD*. Note that the bidding prices satisfy $C^{\star}_{k,t,1} \leqslant C^{\star}_{k,t,2}, ..., \leqslant C^{\star}_{k,t,S}$. Thus, in terms of optimal solutions to Problem *BiD*, there exists an s' such that for any $1 \leqslant s \leqslant s'$, $p^{\text{W}*}_{k,t,s} \geqslant 0$, and for any $s'+1 \leqslant s \leqslant S$, $p^{\text{W}*}_{k,t,s} = 0$. We now construct feasible solutions $\boldsymbol{W}^{\dagger}$ and $\Phi^{\text{DA}\dagger}$ for Problem *BiD-q*. We let

$$W^{\dagger}_{k,t} = p^{\text{W}\dagger}_{k,t} = \sum_{1 \leqslant s \leqslant s'} p^{\text{W}\star}_{k,t,s}, \tag{1}$$

$$\Phi^{\text{DA}\dagger}_{-W} = \Phi^{\text{DA}\star}_{-W}. \tag{2}$$

The constructed solutions $\boldsymbol{W}^{\dagger}$ and $\Phi^{\text{DA}\dagger} = \left(\boldsymbol{p}^{\text{W}\dagger}, \Phi^{\text{DA}\dagger}_{-W}\right)$ satisfy the constraint $\Phi^{\text{DA}} \in \mathcal{X}^{\text{DA}}(\boldsymbol{W}^{\dagger})$ in the lower-level problem of Problem *BiD-q*.

Third, we prove by contradiction that $\Phi^{\text{DA}\dagger} = \left(\boldsymbol{p}^{\text{W}\dagger}, \Phi^{\text{DA}\dagger}_{-W}\right)$ is an optimal solution to the following lower-level problem under the bidding quantity $\boldsymbol{W}^{\dagger}$ in Problem *BiD-q*.

$$\min_{\Phi^{\text{DA}}} \ f_0^{\text{DA}}(\Phi^{\text{DA}}) \tag{3a}$$

$$\text{s.t. } \Phi^{\text{DA}} \in \mathcal{X}^{\text{DA}}(\boldsymbol{W}^{\dagger}). \tag{3b}$$

The information, data, or work presented herein was funded by the Advanced Research Projects Agency-Energy (ARPA-E), U.S. Department of Energy, under Award Number DE-AR0001277. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

To begin with, we assume that the optimal DAM dispatch solution to the above problem (3) is $\Phi^{\text{DA}\ddagger} = \left(\boldsymbol{p}^{\text{W}\ddagger}, \Phi^{\text{DA}\ddagger}_{-W}\right)$. Note that $p^{\text{W}\ddagger}_{k,t} \leqslant W^{\dagger}_{k,t} = p^{\text{W}\dagger}_{k,t}$. If $p^{\text{W}\ddagger}_{k,t} = p^{\text{W}\dagger}_{k,t}$, we easily have $\Phi^{\text{DA}\ddagger}_{-W} = \Phi^{\text{DA}\dagger}_{-W}$.

Then, suppose $\left(\boldsymbol{p}^{\text{W}\dagger}, \Phi^{\text{DA}\dagger}_{-W}\right)$ is not an optimal solution to the problem (3), i.e., we can assume that there exist k_α and t_α such that $p^{\text{W}\ddagger}_{k_\alpha,t_\alpha} < p^{\text{W}\dagger}_{k_\alpha,t_\alpha} = W^{\dagger}_{k,t}$. Thus, we have

$$f_0^{\text{DA}}(\Phi^{\text{DA}\ddagger}) < f_0^{\text{DA}}(\Phi^{\text{DA}\dagger}) = f_0^{\text{DA}}(\Phi^{\text{DA}\star}). \tag{4}$$

The latter equality in (4) is because f_0^{DA} is only related to Φ^{DA}_{-W} and we have $\Phi^{\text{DA}\dagger}_{-W} = \Phi^{\text{DA}\star}_{-W}$.

Now, back to Problem *BiD*, there will exist $\mathcal{S}_\alpha$ and $\tilde{p}^{\text{W}\ddagger}_{k,t,s}$ such that (i) $\tilde{p}^{\text{W}\ddagger}_{k_\alpha,t_\alpha,s_\alpha} < p^{\text{W}\star}_{k_\alpha,t_\alpha,s_\alpha}$, $\forall s_\alpha \in \mathcal{S}_\alpha$; (ii) If $k \neq k_\alpha$ or $t \neq t_\alpha$ or $s \notin \mathcal{S}_\alpha$, $\tilde{p}^{\text{W}\ddagger}_{k,t,s} = p^{\text{W}\star}_{k,t,s}$; and (iii) $\sum_{1\leqslant s \leqslant s'} \tilde{p}^{\text{W}\ddagger}_{k,t,s} = p^{\text{W}\ddagger}_{k,t}$. Thus, we have a new feasible solution $\tilde{\Phi}^{\text{DA}\ddagger} = \left(\tilde{\boldsymbol{p}}^{\text{W}\ddagger}, \Phi^{\text{DA}\ddagger}_{-W}\right)$ satisfying the lower problem constraint $\mathcal{X}^{\text{DA}}(\boldsymbol{W}^{\star})$ under Problem *BiD*. Based on (i), (ii), and (4), we have

$$\begin{aligned} &f_0^{\text{DA}}(\tilde{\Phi}^{\text{DA}\ddagger}) + \sum_{t\in\mathcal{T}} \sum_{k\in\mathcal{K}} \sum_{S\in\mathcal{S}} C^{\text{W}\star}_{k,t,s} \cdot p^{\text{W}\ddagger}_{k,s,t} \\ &\quad < f_0^{\text{DA}}(\Phi^{\text{DA}\star}) + \sum_{t\in\mathcal{T}} \sum_{k\in\mathcal{K}} \sum_{S\in\mathcal{S}} C^{\text{W}\star}_{k,t,s} \cdot p^{\text{W}\star}_{k,t,s}. \end{aligned}$$

This contradicts the fact that $\Phi^{\text{DA}\star}$ is the optimal solution to

$$\begin{aligned} &\min_{\Phi^{\text{DA}}} \ f^{\text{DA}}(\Phi^{\text{DA}}) \\ &\text{s.t. } \Phi^{\text{DA}} \in \mathcal{X}^{\text{DA}}(\boldsymbol{W}^{\star}). \end{aligned}$$

Therefore, $\left(\boldsymbol{p}^{\text{W}\dagger}, \Phi^{\text{DA}\dagger}_{-W}\right)$ is an optimal solution to the problem (3), i.e., $\left(\boldsymbol{p}^{\text{W}\dagger}, \Phi^{\text{DA}\dagger}_{-W}\right)$ is feasible to Problem *BiD-q*. Note that we have $f_0^{\text{DA}}(\Phi^{\text{DA}\dagger}) = f_0^{\text{DA}}(\Phi^{\text{DA}\star})$ shown in (4). Based on (1) and (2), the constraint $\Phi^{\text{RT}}_\omega \in \mathcal{X}^{\text{RT}}_\omega(\Phi^{\text{DA}\dagger})$ of Problem *BiD-q* will be the same as $\Phi^{\text{RT}}_\omega \in \mathcal{X}^{\text{RT}}_\omega(\Phi^{\text{DA}\star})$ of Problem *BiD*. Therefore, the objective, i.e., the expected system cost, of Problem *BiD-q* under $\left(\boldsymbol{p}^{\text{W}\dagger}, \Phi^{\text{DA}\dagger}_{-W}\right)$ is equal to that of Problem *BiD* under $\left(\boldsymbol{p}^{\text{W}\star}, \Phi^{\text{DA}\star}_{-W}\right)$, meaning that we always have $S^{\text{BiD}} \geqslant S^{\text{BiD-q}}$. □