* Def : Survival Function

Reliability an survival of an item at the time point "t" is defined as the probability that item survives at least at the time "t". Let T be the lifetime random variable of the item with distribution function F(.) which is absolutely.

$$S(t) = \mathbb{P}(T \geq t) = 1 - F_T(t) = \bar{F}(t).$$

* Def : Lehmann Alternatives

$G(x) = [F(x)]^{\alpha}$, $\alpha > 0$ is a distribution function.

* Distribution Functions :

1) Exponential : $F_E(x) = 1 - e^{-\lambda x}$

$f_E(x) = \lambda e^{-\lambda x}$

2) Weibull : $F_W(x) = 1 - e^{-(\lambda x)^{\alpha}}$

$f_W(x) = \alpha \lambda (\lambda \alpha)^{\alpha - 1} e^{-(\lambda x)^{\alpha}}$

3) Gamma : $f_G(x) = \dfrac{\beta^{\alpha}}{\Gamma(\alpha)} x^{\alpha-1} . e^{-\lambda x}$

4) Exponentiated Exponential : $F_X(x) = (1 - e^{-\lambda x})^{\alpha}$,

$f_X(x) = \alpha \lambda (1 - e^{-\lambda x})^{\alpha - 1} e^{-\lambda x}$

5) Marshall Olkin : Conforming continuous distribution with

a discrete distribution.

Suppose that $X_1, \dots, X_N \overset{iid}{\sim} F(.)$ and assume that $N \sim Geo(\theta)$

$$\mathbb{P}(N = n) = \theta(1-\theta)^{n-1}, \quad 0 < \theta < 1.$$

Consider $Y = \min\{X_1, \dots, X_n\}$

$$\begin{aligned} S_Y(y) &= \mathbb{P}(Y \geq y) \\ &= \sum_{n \geq 1} \mathbb{P}(Y \geq y \mid N = n)\, \mathbb{P}(N = n) \quad \text{Total probability} \\ &= \sum_{n \geq 1} (1 - F(y))^n \cdot \theta(1-\theta)^{n-1} \\ &= \theta(1 - F(y)) \sum_{n \geq 1} ((1 - F(y))(1-\theta))^n \end{aligned}$$

$$G(x) = \frac{F(x)}{F(x) + \theta S(x)} \qquad S(x) = \frac{\theta(1 - F(x))}{1 - (1-\theta)(1 - F(x))}$$

$$f_{MOE}(x, \lambda, \alpha, \theta) = \frac{\alpha \lambda \theta e^{-\lambda x} (1 - e^{-\lambda x})^{\alpha - 1}}{(1 - (1-\theta)(1 - (1 - e^{-\lambda x})^{\alpha}))^2} \mathbb{1}(x \geq 0)$$

6) Weighted Exponential distribution : $f_{WE}(x, \alpha, \lambda) = \frac{\alpha}{\alpha + 1} \lambda e^{-\lambda x} (1 - e^{-\alpha \lambda x}) \mathbb{1}(x \geq 0)$

7) α - Power Transformation Method (APT Method) :

Given data : plot density

Fit CDF / SF

MLE estimates

Goodness of fit test result (loglikelihood / KS / p-value)

Simulation from the distribution

Package name : mapt

$$F(x) = \Big\{ \frac{\alpha^{F(x)} - 1}{\quad}, \quad \alpha > 0, \alpha \neq 1$$

$$
\begin{cases} \quad \alpha - 1 & \\ F(n) \quad , & \alpha = 1 \end{cases}
$$

APT

$$
f_{APT}(n) = \begin{cases} \dfrac{f(n) \log(\alpha) \, \alpha^{F(n)}}{\alpha - 1} \quad , & \alpha > 0 \; , \; \alpha \neq 1 \\ f(n) \quad , & \alpha = 1 \end{cases}
$$

$$
\ell(\alpha, \lambda) = n \log \alpha + n \log\left(\frac{\log(\alpha)}{\alpha - 1}\right) + n \log \lambda - \lambda \sum_{i=1}^{n} y_i - \log(\alpha) \sum_{i=1}^{n} e^{-\lambda y_i}
$$