Linear time sort
Count Sort Algorithm

Adapted from lectures of
Prof. Charles Leiserson , MIT &
CLRC textbook 2nd ed. Chapters 8

Lecture Outline

- Linear Time sorting:
  - Non-comparison-based sort: Count Sort
- Analysis of CountSort

Sorting in linear time

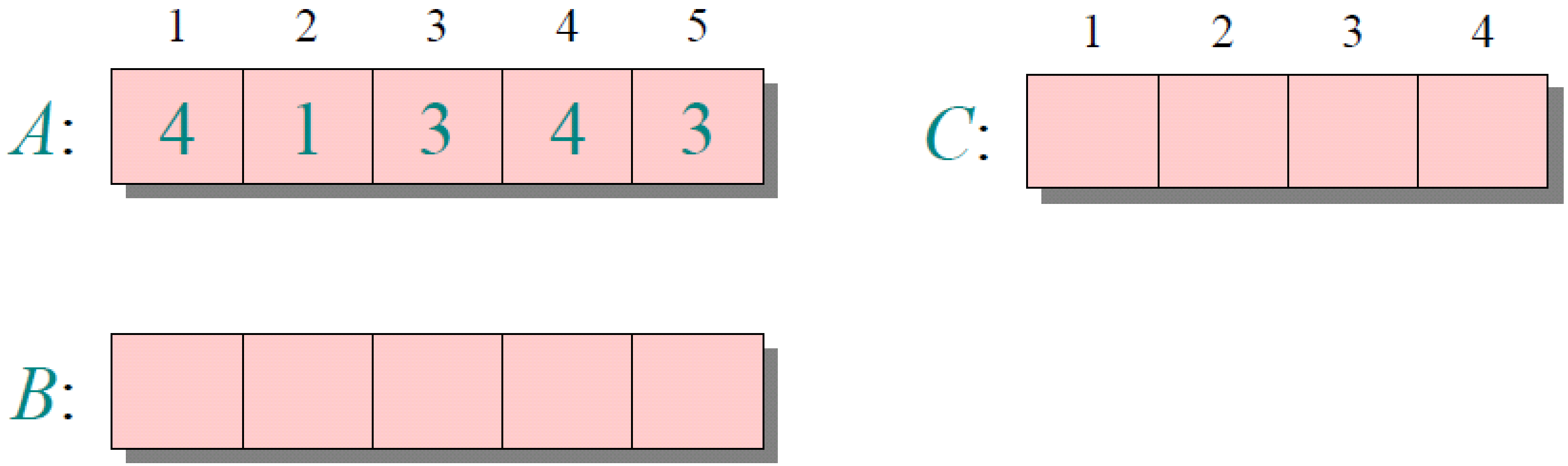
Counting sort: No comparisons between elements.

- ***Input***: $A[1 \,..\, n]$, where $A[j] \in \{1, 2, \ldots, k\}$.
- ***Output***: $B[1 \,..\, n]$, sorted.
- ***Auxiliary storage***: $C[1 \,..\, k]$.

Counting sort

for $i \leftarrow 1$ **to** k
    do $C[i] \leftarrow 0$
for $j \leftarrow 1$ **to** n
    do $C[A[j]] \leftarrow C[A[j]] + 1$ $\triangleright\ C[i] = |\{\text{key} = i\}|$
for $i \leftarrow 2$ **to** k
    do $C[i] \leftarrow C[i] + C[i-1]$ $\triangleright\ C[i] = |\{\text{key} \le i\}|$
for $j \leftarrow n$ **downto** 1
    do $B[C[A[j]]] \leftarrow \mathrm{A}[j]$
        $C[A[j]] \leftarrow C[A[j]] - 1$

Counting sort example



Loop 1

	1	2	3	4	5
A:	4	1	3	4	3

	1	2	3	4
C:	0	0	0	0

B:					

for $i \leftarrow 1$ **to** k
    do $C[i] \leftarrow 0$

Loop 2

A:

1	2	3	4	5
4	1	3	4	3

C:

1	2	3	4
0	0	0	1

B:

for $j \leftarrow 1$ **to** n
    do $C[A[j]] \leftarrow C[A[j]] + 1$ $\quad \triangleright\; C[i] = |\{\text{key} = i\}|$

Loop 2

	1	2	3	4	5
A:	4	1	3	4	3

	1	2	3	4
C:	1	0	0	1

B:					

for $j \leftarrow 1$ **to** n

  do $C[A[j]] \leftarrow C[A[j]] + 1 \quad \triangleright\ C[i] = |\{\text{key} = i\}|$

A:

1	2	3	4	5
4	1	3	4	3

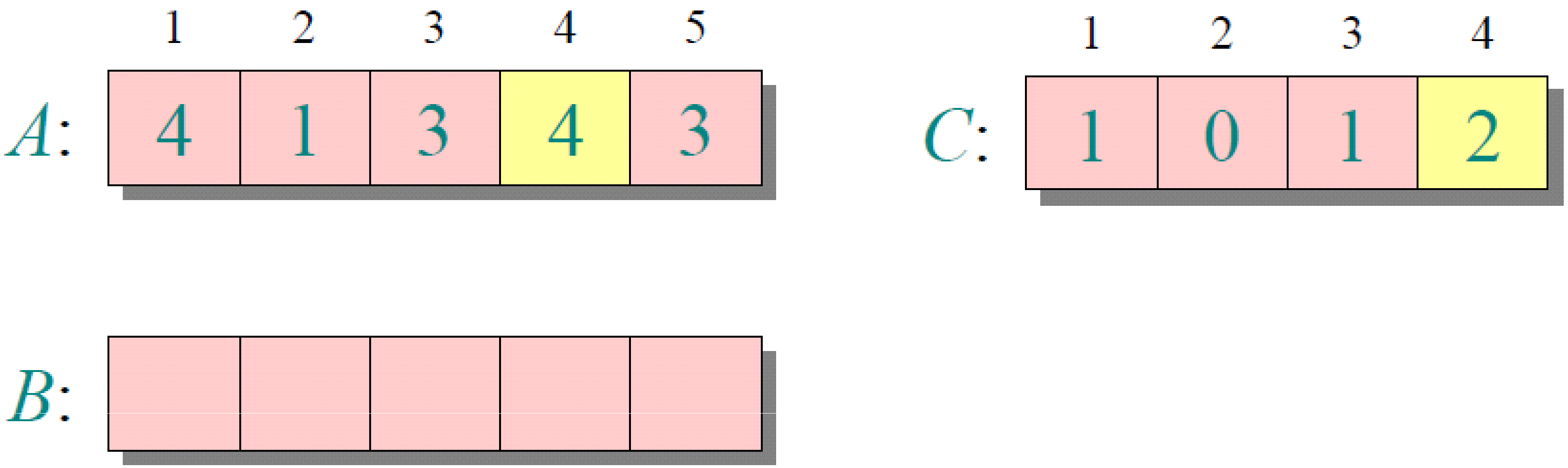
C:

1	2	3	4
1	0	1	1

B:

for $j \leftarrow 1$ **to** n

  do $C[A[j]] \leftarrow C[A[j]] + 1$ $\quad \triangleright\ C[i] = |\{\text{key} = i\}|$



for $j \leftarrow 1$ **to** n

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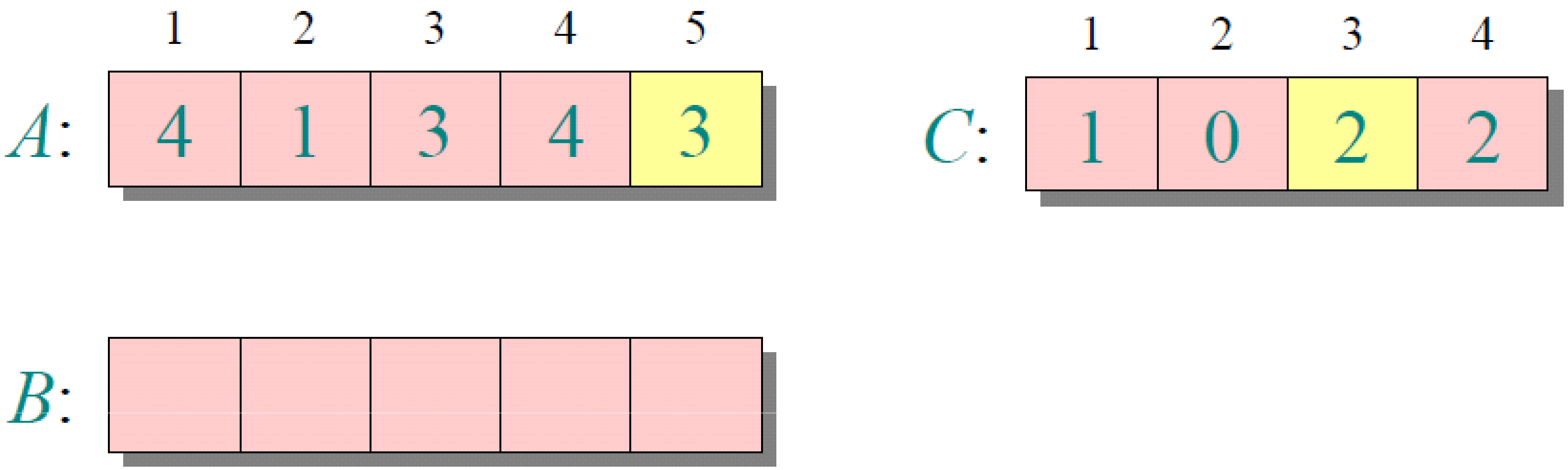


for $j \leftarrow 1$ **to** n

  do $C[A[j]] \leftarrow C[A[j]] + 1 \quad \triangleright\ C[i] = |\{\text{key} = i\}|$

Loop 3

A:

1	2	3	4	5
4	1	3	4	3

C:

1	2	3	4
1	0	2	2

B:

C':

1	1	2	2

for $i \leftarrow 2$ **to** k

  do $C[i] \leftarrow C[i] + C[i-1]$ $\triangleright$ $C[i] = |\{\text{key} \leq i\}|$

A:

1	2	3	4	5
4	1	3	4	3

C:

1	2	3	4
1	0	2	2

B:

C':

1	1	3	2

for $i \leftarrow 2$ **to** k

do $C[i] \leftarrow C[i] + C[i-1]$ $\quad \triangleright\ C[i] = |\{\text{key} \leq i\}|$

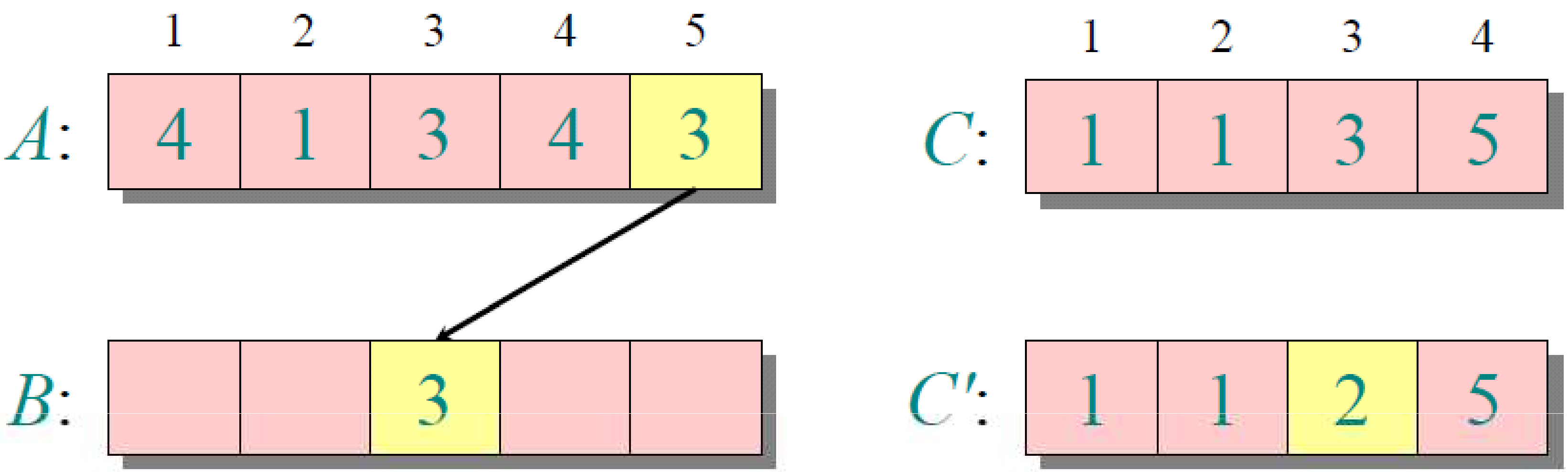
	1	2	3	4	5
A:	4	1	3	4	3
B:					

	1	2	3	4
C:	1	0	2	2
C':	1	1	3	5

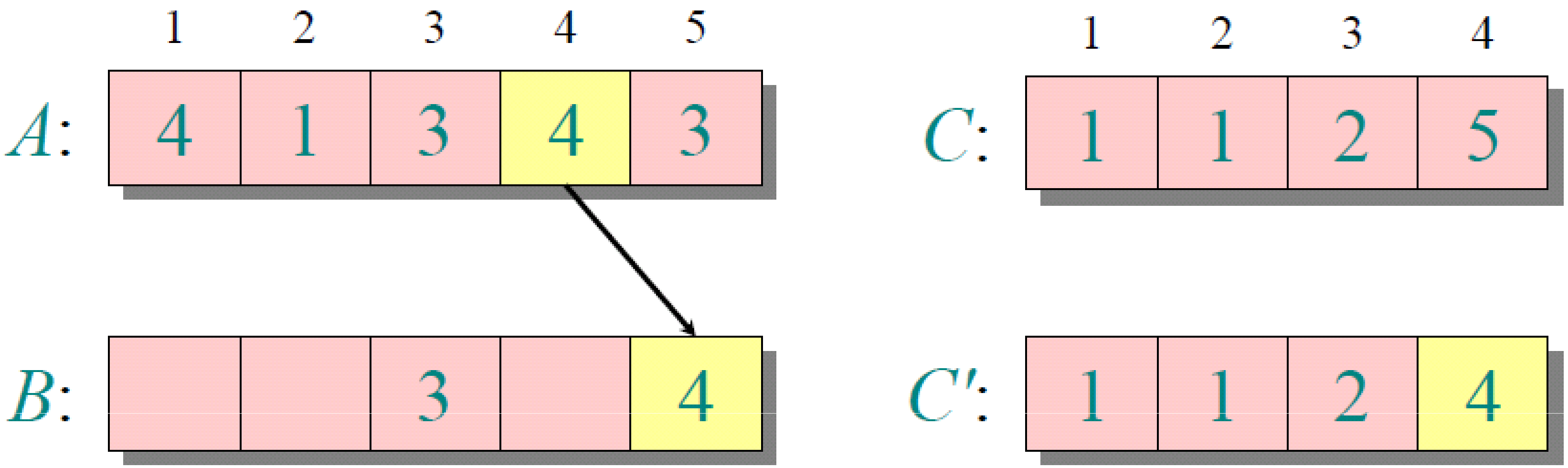
for $i \leftarrow 2$ **to** k

  do $C[i] \leftarrow C[i] + C[i-1]$ $\triangleright$ $C[i] = |\{\text{key} \leq i\}|$

Loop 4



for $j \leftarrow n$ **downto** 1
  do $B[C[A[j]]] \leftarrow \mathrm{A}[j]$
    $C[A[j]] \leftarrow C[A[j]] - 1$



for $j \leftarrow n$ **downto** 1

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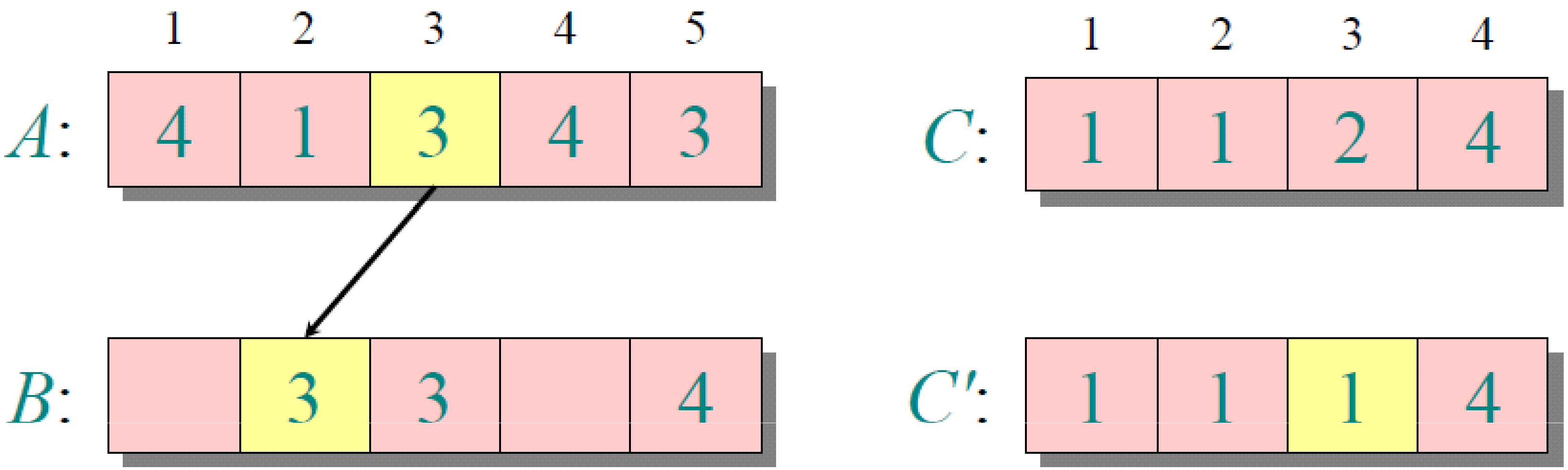

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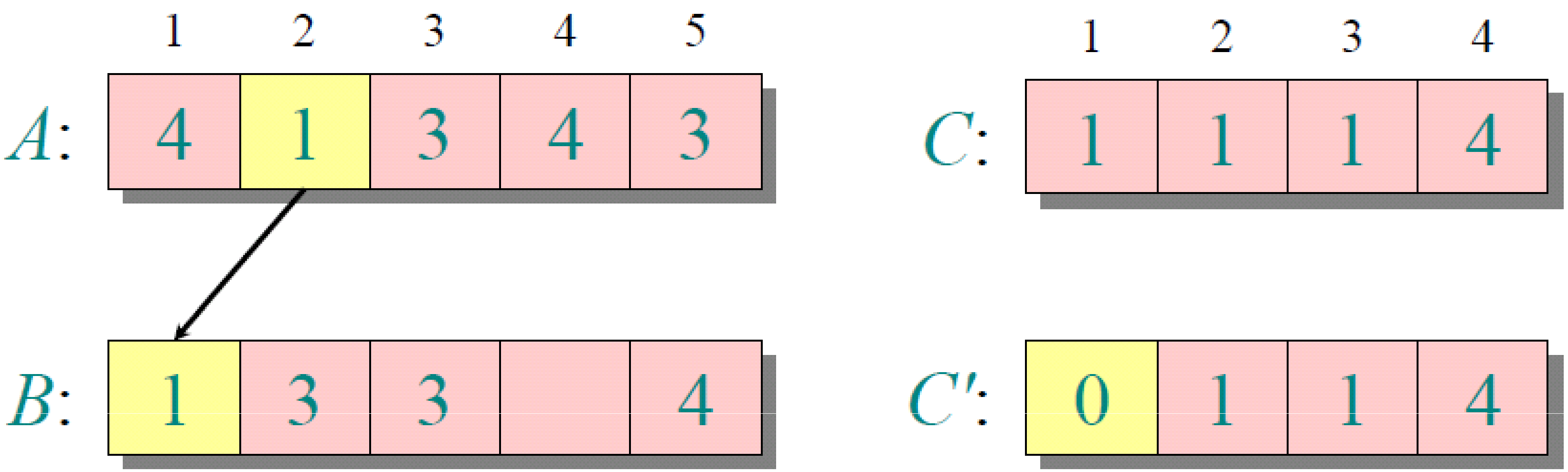

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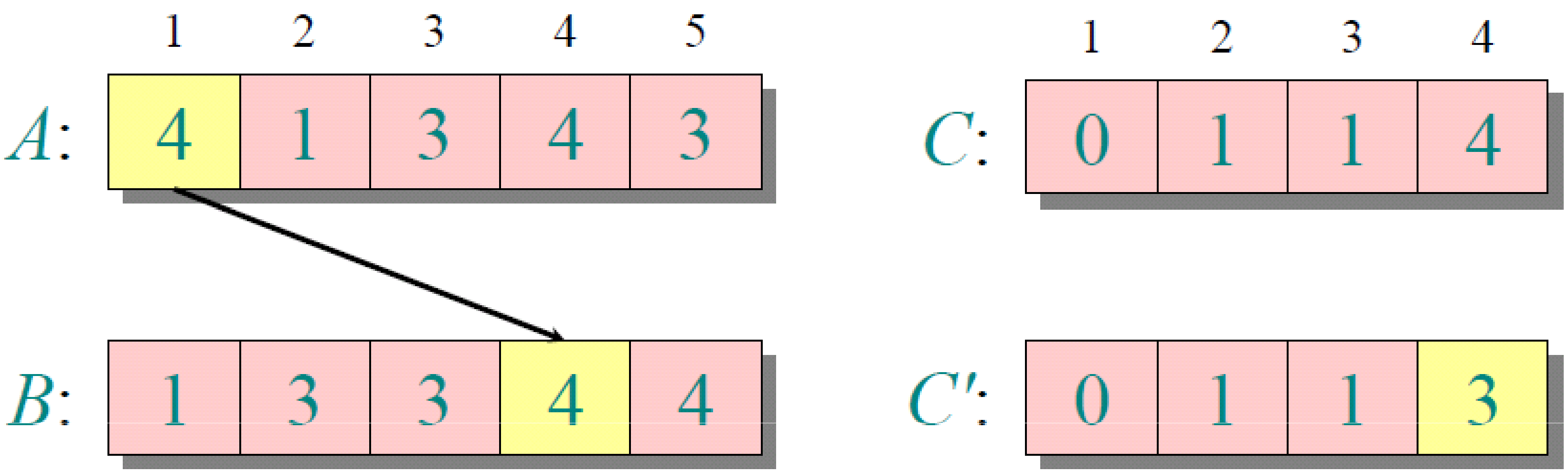
  do $B[C[A[j]]] \leftarrow \mathrm{A}[j]$

    $C[A[j]] \leftarrow C[A[j]] - 1$



for $j \leftarrow n$ **downto** 1
  do $B[C[A[j]]] \leftarrow \mathrm{A}[j]$
    $C[A[j]] \leftarrow C[A[j]] - 1$

Analysis

for $i \leftarrow 1$ **to** k
  do $C[i] \leftarrow 0$

for $j \leftarrow 1$ **to** n
  do $C[A[j]] \leftarrow C[A[j]] + 1$

for $i \leftarrow 2$ **to** k
  do $C[i] \leftarrow C[i] + C[i\text{–}1]$

for $j \leftarrow n$ **downto** 1
  do $B[C[A[j]]] \leftarrow \mathrm{A}[j]$
    $C[A[j]] \leftarrow C[A[j]] - 1$

Analysis

$\Theta(k)$ $\left\{ \begin{array}{l} \textbf{for } i \leftarrow 1 \textbf{ to } k \\ \quad \textbf{do } C[i] \leftarrow 0 \end{array} \right.$

$\Theta(n)$ $\left\{ \begin{array}{l} \textbf{for } j \leftarrow 1 \textbf{ to } n \\ \quad \textbf{do } C[A[j]] \leftarrow C[A[j]] + 1 \end{array} \right.$

$\Theta(k)$ $\left\{ \begin{array}{l} \textbf{for } i \leftarrow 2 \textbf{ to } k \\ \quad \textbf{do } C[i] \leftarrow C[i] + C[i-1] \end{array} \right.$

$\Theta(n)$ $\left\{ \begin{array}{l} \textbf{for } j \leftarrow n \textbf{ downto } 1 \\ \quad \textbf{do } B[C[A[j]]] \leftarrow \mathrm{A}[j] \\ \qquad C[A[j]] \leftarrow C[A[j]] - 1 \end{array} \right.$

$\Theta(n + k)$

Running time

If $k = O(n)$, then counting sort takes $\Theta(n)$ time.

- But, sorting takes $\Omega(n \lg n)$ time!
- Where's the fallacy?

Answer:

- ***Comparison sorting*** takes $\Omega(n \lg n)$ time.
- Counting sort is not a ***comparison sort***.
- In fact, not a single comparison between elements occurs!

Stable Sort

Counting sort is a ***stable*** sort: it preserves the input order among equal elements.