Coherent Detection Example: Binary Phase Shift Keying (BPSK)

Transmitted Signal
The transmitted signal for Binary Phase Shift Keying (BPSK) is given by:
$s(t) = A \cos(2\pi f_c t + \theta_i)$
where:
- A is the amplitude
- f_c is the carrier frequency
- θ_i is the phase shift, which can take values 0 or π depending on whether the transmitted symbol is 0 or 1.

Thus, the two possible transmitted signals are:
- For symbol 0 ($\theta_i = 0$): $s_0(t) = A \cos(2\pi f_c t)$
- For symbol 1 ($\theta_i = \pi$): $s_1(t) = -A \cos(2\pi f_c t)$

Received Signal
At the receiver, the received signal $r(t)$ is the transmitted signal plus any noise ($n(t)$):
$r(t) = s(t) + n(t)$
For simplicity, we assume no noise, so $r(t) = s(t)$.

Coherent Detection Process
The receiver multiplies the received signal by a synchronized carrier $\cos(2\pi f_c t)$ to recover the original signal.
- **Multiplication**: The receiver multiplies $r(t)$ by $\cos(2\pi f_c t)$:
$r(t) * \cos(2\pi f_c t)$

Case 1: Symbol 0 Transmitted ($s(t) = A \cos(2\pi f_c t)$)
Multiplying $r(t) = A \cos(2\pi f_c t)$ by $\cos(2\pi f_c t)$:
$r(t) * \cos(2\pi f_c t) = A \cos(2\pi f_c t) * \cos(2\pi f_c t)$
Using the identity: $\cos(\theta) \cos(\theta) = (1/2)(1 + \cos(2\theta))$
We get:
$A \cos(2\pi f_c t) * \cos(2\pi f_c t) = (A/2) (1 + \cos(4\pi f_c t))$

Case 2: Symbol 1 Transmitted ($s(t) = -A \cos(2\pi f_c t)$)
Multiplying $r(t) = -A \cos(2\pi f_c t)$ by $\cos(2\pi f_c t)$:
$r(t) * \cos(2\pi f_c t) = -A \cos(2\pi f_c t) * \cos(2\pi f_c t)$
Using the same identity:
$-A \cos(2\pi f_c t) * \cos(2\pi f_c t) = -(A/2)(1 + \cos(4\pi f_c t))$

Low-Pass Filtering
The result of the multiplication is passed through a **low-pass filter (LPF)** that removes the high-frequency component ($\cos(4\pi f_c t)$), leaving only the low-frequency term.

After filtering, we are left with:
- For symbol 0: (A/2)
- For symbol 1: -(A/2)

Decision Rule
The demodulator applies a **threshold** decision rule:
- If the filtered output is greater than 0, decide symbol 0 ($\theta = 0$).
- If the filtered output is less than 0, decide symbol 1 ($\theta = \pi$).
Thus, based on the filtered signal:
- For symbol 0, the output is (A/2), which is positive.
- For symbol 1, the output is -(A/2), which is negative.

Final Output
- **Symbol 0**: The demodulator detects (A/2) and decides 0.
- **Symbol 1**: The demodulator detects -(A/2) and decides 1.