# Logic

**Symbols:** $\geq \leq \neq \neg \wedge \vee \oplus \equiv \rightarrow \leftrightarrow$ □$\exists$ $\forall$

**Identities:**

| | | |
|---|---|---|
| $\neg(\neg p) \equiv p$ | | Double Negation |
| $p \wedge \mathbf{T} \equiv p$ | $p \vee \mathbf{F} \equiv p$ | Identity |
| $p \vee \mathbf{T} \equiv \mathbf{T}$ | $p \wedge \mathbf{F} \equiv \mathbf{F}$ | Domination |
| $p \wedge p \equiv p$ | $p \vee p \equiv p$ | Idempotent |
| $p \vee q \equiv q \vee p$ | $p \wedge q \equiv q \wedge p$ | Commutative |
| $(p \vee q) \vee r \equiv p \vee (q \vee r)$ | | Associative |
| $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$ | | “ |
| $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ | | Distributive |
| $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$ | | “ |
| $\neg(p \wedge q) \equiv \neg p \vee \neg q$ | | DeMorgan's |
| $\neg(p \vee q) \equiv \neg p \wedge \neg q$ | | “ |
| $p \vee (p \wedge q) \equiv p$ | | Absorption |
| $p \wedge (p \vee q) \equiv p$ | | “ |
| $p \rightarrow q \equiv \neg q \rightarrow \neg p$ | | Contrapositive |
| $p \oplus q \equiv q \oplus p$ | | “ |
| $p \rightarrow q \equiv \neg p \vee q$ | | Implication |
| $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$ | | Biconditional Equivalence |
| $(p \wedge q) \rightarrow r \equiv p \rightarrow (q \rightarrow r)$ | | Exportation |
| $(p \rightarrow q) \wedge (p \rightarrow \neg q) \equiv \neg p$ | | Absurdity |
| $p \vee q \equiv \neg p \rightarrow q$ | | Alternate Implication |
| $p \wedge q \equiv \neg(p \rightarrow \neg q)$ | | “ |
| $\neg(p \rightarrow q) \equiv p \wedge \neg q$ | | “ |
| $\neg\forall x\, P(x) \equiv \exists x\, \neg P(x)$ | | DeMorgan's for Quantifiers |
| $\neg\exists x\, Q(x) \equiv \forall x\, \neg Q(x)$ | | “ |

**Proofs**

Direct Assume P and prove Q

Contrapositive: Assume Not Q and prove Not P

Contradiction: Assume P and Not Q and prove a contradiction

Induction: Prove base(s), assume P(m), prove P(k+1)

# Sets

**Symbols:** $\in \notin \subseteq \subset \supseteq \supset \varnothing \cup \cap \times$

**Common Sets:**

$\mathbf{N}$ = {0, 1, 2, …} = natural numbers

$\mathbf{Z}$ = {…, -1, 0, 1, 2, …} = integers

$\mathbf{Z}^+$ = {1, 2, 3, …} = positive integers

$\mathbf{Q} = \{p/q \mid p \in \mathbf{Z}, q \in \mathbf{Z}, q \neq 0\}$ rational numbers

$\mathbf{U}$ = {*} universal

**Identities:**

| | | |
|---|---|---|
| $A \cup \varnothing = A$ | $A \cap \mathbf{U} = A$ | Identity |
| $A \cup \mathbf{U} = \mathbf{U}$ | $A \cap \varnothing = \varnothing$ | Domination |
| $A \cup A = A$ | $A \cap A = A$ | Idempotent |
| $A \cup A^C = \varnothing$ | $A \cap A^C = \mathbf{U}$ | Complement |
| $A \cup B = B \cup A$ | $A \cap B = B \cap A$ | Commutative |
| $(A \cup B) \cup C = A \cup (B \cup C)$ | | Associative |
| $(A \cap B) \cap C = A \cap (B \cap C)$ | | " |
| $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ | | Distributive |
| $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ | | " |
| $(A \cup B)^C = A^C \cap B^C$ | | DeMorgan's |
| $(A \cap B)^C = A^C \cup B^C$ | | " |
| $A \cup (A \cap B) = A$ | | Absorption |
| $A \cap (A \cup B) = A$ | | " |

**Proofs**

Structural Induction: Prove P(e) for base(s), assume P(e), prove P(f) for any new element added

# Series & Sums

**Symbols:** ∑ ·

**Sum equations:**

Geometric Progression $a_n = a \cdot r^n$ $$\sum_{k=0}^{n} r^k = \frac{\left(r^{(n+1)} - 1\right)}{(r-1)}$$

Arithmetic Progression $a_n = a + d \cdot n$ $$\sum_{k=1}^{n} k = \frac{1}{2}\left(n^2 + n\right)$$

# Counting

**Symbols:** λ

**Equations:**

Sum Rule: for a union of disjoint sets the total number of elements is:

$S(m) = |S_1| \cdot |S_2| \cdot \ldots \cdot |S_m|$

Product Rule: for a sequence of k choices the total number of elements is:

$P(m) = |P_1| \cdot |P_2| \cdot \ldots \cdot |P_m|$

Subset Exclusion: when a set to be counted is the difference of two sets A – B, where A ⊆B

$C(m) = |A| - |B|$

Inclusion/Exclusion: when a set to be counted is the union of non-disjoint sets A and B

$D(m) = |A| + |B| - |A \cap B|$