Donna Hooshmand **Math282-23580**
Assignment HW2 due 01/20/2021 at 11:59pm PST

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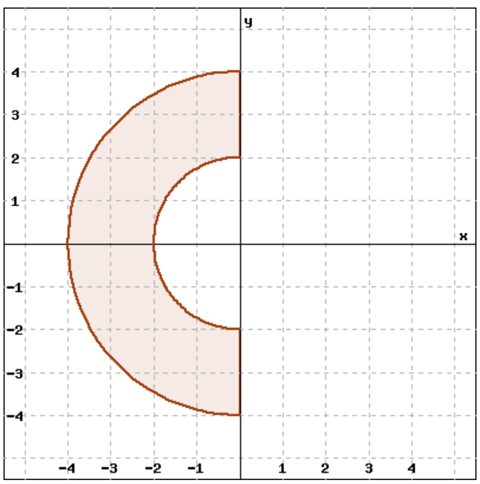
1. (1 point)

Suppose R is the shaded region in the figure. As an iterated integral in polar coordinates,

$$\iint_R f(x,y)\,dA = \int_A^B \int_C^D f(r\cos(\theta), r\sin(\theta))\, r\, dr\, d\theta$$

with limits of integration

A = ________
B = ________
C = ________
D = ________



(Click on graph to enlarge)

Answer(s) submitted:

- pi/2
- 2

(correct)

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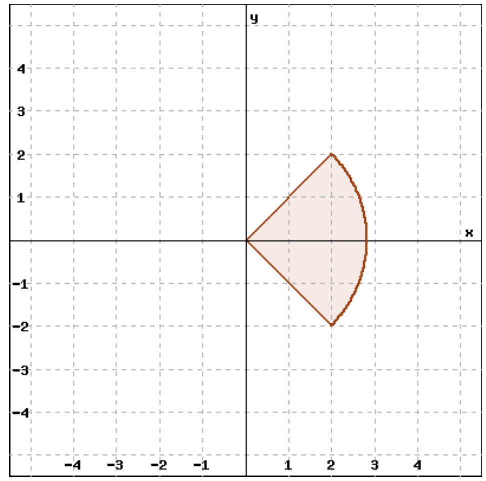
2. (1 point)

Suppose R is the shaded region in the figure. As an iterated integral in polar coordinates,

$$\iint_R f(x,y)\,dA = \int_A^B \int_C^D f(r\cos(\theta), r\sin(\theta))\, r\, dr\, d\theta$$

with limits of integration

A = ________
B = ________
C = ________
D = ________



(Click on graph to enlarge)

Answer(s) submitted:

- -pi/4
- 0

(correct)

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3. (2 points)

Match each double integral in polar with the graph of the region of integration.

? 1. $\int_0^{2\pi} \int_0^1 f(r,\theta)\, r\, dr\, d\theta$

? 2. $\int_0^1 \int_{-\pi/2}^{3\pi/4} f(r,\theta)\, r\, d\theta\, dr$

? 3. $\int_{3\pi/4}^{3\pi/2} \int_0^1 f(r,\theta)\, r\, dr\, d\theta$

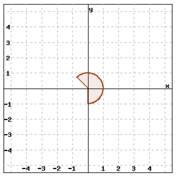
? 4. $\int_{3\pi/2}^{2\pi} \int_0^2 f(r,\theta)\, r\, dr\, d\theta$

? 5. $\int_{-\pi/4}^{3\pi/4} \int_1^2 f(r,\theta)\, r\, dr\, d\theta$

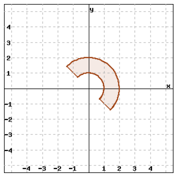
? 6. $\int_1^2 \int_{3\pi/4}^{7\pi/4} f(r,\theta)\, r\, d\theta\, dr$

? 7. $\int_0^1 \int_0^{3\pi/4} f(r,\theta)\, r\, d\theta\, dr$

? 8. $\int_0^{2\pi} \int_1^2 f(r,\theta)\, r\, dr\, d\theta$



A



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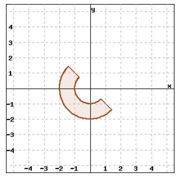

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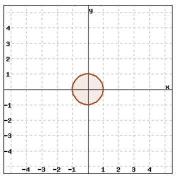

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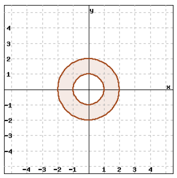

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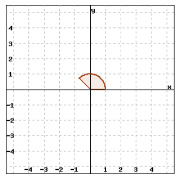

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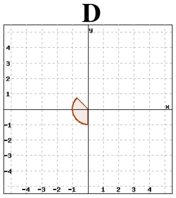

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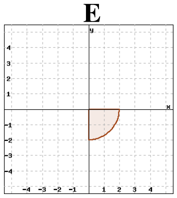

H

Answer(s) submitted:

- D
- A
- G
- H
- B
- C
- F
- E

(correct)

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4. (2 points) Evaluate the double integral $\iint_D \cos\sqrt{x^2+y^2}\,dA$, where D is the disc with center the origin and radius 5, by changing to polar coordinates.

Answer: ________

Answer(s) submitted:

- `2pi(5sin5+cos5-1)`

(correct)

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5. (1 point)
By changing to polar coordinates, evaluate the integral

$$\iint_D (x^2+y^2)^{7/2}\,dxdy$$

where D is the disk $x^2+y^2 \le 25$.

Answer = ____________

Answer(s) submitted:

- `2pi(5^9)/9`

(correct)

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6. (2 points) Evaluate the double integral over the region $D = \{(x,y) | 16 \le x^2+y^2 \le 36, 0 \le y \le x\}$

$$\iint_D \frac{x^2}{x^2+y^2}\,dA = \_\_\_\_\_\_\_\_\_\_\_\_$$

Answer(s) submitted:

- `5pi/4 + 5/2`

(correct)

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7. (2 points) Convert the integral below to polar coordinates and evaluate the integral.

$$\int_0^{3/\sqrt{2}} \int_y^{\sqrt{9-y^2}} xy\,dx\,dy$$

Instructions: Please enter the integrand in the first answer box, typing *theta* for θ. Depending on the order of integration you choose, enter *dr* and *dtheta* in either order into the second and third answer boxes with only one *dr* or *dtheta* in each box. Then, enter the limits of integration and evaluate the integral to find the volume.

$$\int_A^B \int_C^D \_\_\_\_\_\_\_\_\_\_\_\_ \ \_\_\_ \ \_\_\_$$

A = ________
B = ________
C = ________
D = ________

Volume = ____________

Answer(s) submitted:

- `rcos(theta) rsin(theta)r`
- `81/16`

(correct)

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8. (2 points) Evaluate the double integral $\iint_D x^6 y\,dA$, where D is the top half of the disc with center the origin and radius 4, by changing to polar coordinates.

Answer: ________

Answer(s) submitted:

- `2(4^9)/63`

(correct)

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9. (2 points) Find the volume of the region between the graph of $f(x,y) = 9 - x^2 - y^2$ and the xy plane.

volume = ____________

Answer(s) submitted:

- `81pi/2`

(correct)

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10. (2 points) Find the volume of the region below the cone $z = \sqrt{x^2+y^2}$ and above the disk $x^2+y^2 \le 16$ in the xy plane.

volume = ____________

Answer(s) submitted:

- `2pi(64/3)`

(correct)

11. (2 points)

A cylindrical drill with radius 5 is used to bore a hole through the center of a sphere of radius 7. Find the volume of the ring shaped solid that remains.

____

Answer(s) submitted:

- `4pi/3 (49 - 25)^(3/2)`

(correct)

12. (2 points) Find the volume of the ellipsoid $x^2+y^2+5z^2=16$.

____

Answer(s) submitted:

- `256pi/(3sqrt(5))`

(correct)

13. (2 points)

Using polar coordinates, evaluate the integral which gives the area which lies in the first quadrant between the circles $x^2+y^2=256$ and $x^2-16x+y^2=0$.

____

Answer(s) submitted:

- `32pi`

(correct)

14. (2 points)

Using polar coordinates, evaluate the integral which gives the area which lies in the first quadrant below the line $y=1$ and between the circles $x^2+y^2=4$ and $x^2-2x+y^2=0$.

____

Answer(s) submitted:

- `sqrt(3)/2 - pi/6`

(correct)

15. (2 points) A swimming pool is circular with a 40-ft diameter. The depth is constant along the east-west lines and increases linearly from 4-ft at the south end to 9-ft at the north end.

Find a function defining the depth of water in the pool.

$f(x,y) =$ ______________

Find the volume of water in the pool.

$V =$ ______________

Answer(s) submitted:

- `1/8 y + 13/2`
- `2600pi`

(correct)

16. (2 points)

A sprinkler distributes water in a circular pattern, supplying water to a depth of e^{-r} feet per hour at a distance of r feet from the sprinkler.

A. What is the total amount of water supplied per hour inside of a circle of radius 2?

______________ ft^3 per hour

B. What is the total amount of water that goes through the sprinkler per hour?

______________ ft^3 per hour

Answer(s) submitted:

- `2pi (-3e^(-2) + 1)`
- `2pi`

(correct)

17. (2 points) Convert the integral

$$I = \int_0^{6/\sqrt{2}} \int_y^{\sqrt{36-y^2}} e^{7x^2+7y^2}\, dx\, dy$$

to polar coordinates, getting

$$\int_C^D \int_A^B h(r,\theta)\, dr\, d\theta,$$

where

$h(r,\theta) =$ ______________

$A =$ ______

$B =$ ______

$C =$ ______

$D =$ ______

and then evaluate the resulting integral to get

$I =$ ______________.

Answer(s) submitted:

- `re^(7r^2)`
- `0`
- `6`
- `0`
- `pi/4`
- `pi/4 (-1/14 + (e^252/14))`

(correct)

18. (1 point) Sketch the region of integration for the following integral.

$\int_0^{\pi/4} \int_0^{4/\cos(\theta)} f(r,\theta)\, r\, dr\, d\theta$

The region of integration is bounded by

- A. $y=0, y=\sqrt{16-x^2},$ and $x=4$
- B. $y=0, y=x,$ and $y=4$
- C. $y=0, x=\sqrt{16-y^2},$ and $y=4$
- D. $y=0, y=x,$ and $x=4$
- E. None of the above

Answer(s) submitted:

- `D`

(correct)

19. (2 points) Convert the integral

$$\int_0^{\sqrt{4}} \int_{-x}^{x} dy\,dx$$

to polar coordinates and evaluate it (use t for θ):
With $a =$ ____, $b =$ ____, $c =$ ____ and $d =$ ____,
$\int_0^{\sqrt{4}} \int_{-x}^{x} dy\,dx = \int_a^b \int_c^d$ ___ $dr\,dt$

$$= \int_a^b \_\_\_\_\_\_\_\_\_\_ \; dt$$
$$= \_\_\_\_\_\_\_\_\_\_ \Big|_a^b$$
$$= \_\_\_\_\_\_\_\_\_\_ .$$

Answer(s) submitted:

- `-pi/4`
- `pi/4`
- `0`
- `sqrt(4) sect`
- `r`
- `2sec^2(t)`
- `2tan(t)`
- `4`

(correct)

20. (2 points) Find the mass of the triangular region with vertices (0, 0), (1, 0), and (0, 3), with density function $\rho(x,y) = x^2 + y^2$.

____

Answer(s) submitted:

- `1/3 (-9+30-81/2+27)`

(correct)

21. (2 points)

A lamina occupies the region bounded by $y = x^2$ and $y = x+2$ and the density at each point is given by the function $\rho(x,y) = y$.

A. What is the total mass? ________

B. What is the moment about the x-axis? ________

C. What is the moment about the y-axis? ________

D. Where is the center of mass? (________, ________)

Answer(s) submitted:

- `1/2 (3+18-33/5)`
- `1/3 (64 -11/28 - (2^7)/7)`
- `1/2(10+15/12)`
- `(1/2(10+15/12))/(1/2 (3+18-33/5))`
- `(1/3 (64 -11/28 - (2^7)/7))/(1/2 (3+18-33/5))`

(correct)

22. (2 points)

A lamina occupies the part of the disk $x^2 + y^2 \leq 16$ in the first quadrant and the density at each point is given by the function $\rho(x,y) = 3(x^2 + y^2)$.

A. What is the total mass? ________

D. Where is the center of mass? (________, ________)

Answer(s) submitted:

- `96pi`
- `1(3072)/(96*5*pi)`
- `1(3072)/(96*5*pi)`

(correct)