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Work

Newton's 2nd Law of motion →

If an object moves along a straight path with position function $s(t)$, then the force F on the object in the same direction is

$F = ma$ (mass)(acceleration)

$F = m\frac{d^2s}{dt^2}$

Units:

	metric system	US
mass	kg	
time	seconds	seconds
Length	$kg.\frac{m}{s^2} = N$	Pounds (lb)
work	Joules (J)	ft-lbs

Defn → If the accelerations is constant, then force is constant. we define the work W, done by the force as $W = F.d$

where d is the distance the object moves.

work is measured in Newton-meters. (Joules)

Ex: How much work is required to be done in lifting a 1.2 kg book off the floor to put it on a desk 0.7 m high? ($g = 9.8\ m/s^2$)

$W = F.d$

$= m.a.d$

$= (1.2)(9.8)(0.7)$

kg m/s^2 m

$= 8.232$ J

when acceleration is not constant → Force is not constant

Force, Position of object

$W = \sum f(x)\ \Delta x$

$W = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\ \Delta x$

$= \int_a^b f(x)dx$

Ex: A particle is x feet from the origin. A force of $x^2 + 2x$ pounds acts on the particle. How much work is done by the force in moving the object from $x=1$ to $x=3$?

$W = \int_a^b f(x)dx \longrightarrow = \int_1^3 (x^2 + 2x)dx$

$= \left[\frac{1}{3}x^3 + x^2\right]_1^3$

$= \left[\frac{1}{3}(3)^3 + (3)^2\right] - \left[\frac{1}{3} + 1\right]$

$= [9+9] - 4/3$

$= 50/3$ ft-lbs

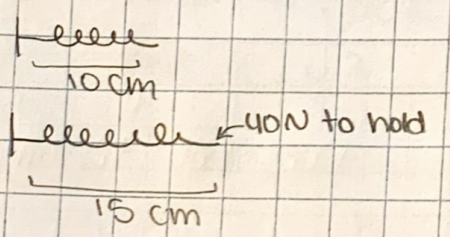
Hook's Law:

The force required to maintain a spring stretched x units beyond it's natural length is proportional to x. That is,

$$f(x) = kx$$

where $k > 0$ is called the **spring constant**.
Hook's Law applies as long as x is not too large.

Ex: A force of 40 N is required to hold a spring that has been stretched from its natural length of 10 cm to a length of 15 cm. How much work is done in stretching the spring from 15 cm to 18 cm?



$f(x) = kx$

$40 = k(15-10)$

$40 = k \cdot .5\text{ cm}$ (kg·m/s²)

$40 = 0.5\text{m}\,k$ (N)

$k = 800$

Force is not constant so we need an integral.

we went to write it in relevance to it's natural length

$$W = \int_a^b f(x)\,dx \quad \begin{matrix} 18\text{cm} \\ 15\text{cm} \end{matrix}$$

$$W = \int_{0.05}^{0.08} 800x\,dx$$

$$= \left[400x^2\right]_{0.05}^{0.08}$$

$$= 400(0.08)^2 - 400(0.05)^2$$

$$= 1.56\text{ J}$$

(5.5) Substitution