Optimization! (4.6 Textbook)

Optimizing a function:

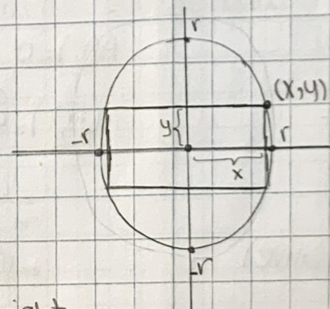
1. understand the Situation.
   - Draw a Picture.
2. Define your variables.
3. what do we want to optimize?
   - write an equation.
4. write down an equation modeling the constraints.
5. Plug the constraints into the O.E.
6. Check the domain.
7. Use calculus on the function to find it's extrema.
   - closed Interval Method
   - 1st Derivative Test
   - 2nd Derivative Test
8. State your conclusion.

"optimizing equation" (step 3)

If there are any! (step 4)

which you always forget!!! (step 8)

Ex: A rectangle is inscribed inside of a circle of radius r. Find the dimensions of the rectangle with greatest area.



r = radius

A = Area of rectangle

(x,y) is a point on the circle.

our point is in QI → Domain: $(0,r]$

A circle whose radius is r and whose center is (0,0).

$$x^2+y^2=r^2$$

$$y^2=r^2-x^2$$

$$y=\pm\sqrt{r^2-x^2}$$

our point is in QI so, we dont want the negative.

$$y=\sqrt{r^2-x^2}$$

The semi-circle of radius r in QI, QII

$$A = \text{width}\cdot\text{height}$$

$$=(2x)(2y)$$

$$=4xy$$

$$A=4x\left(\sqrt{r^2-x^2}\right)$$

$$A'(x)=\frac{d}{dx}\left(4x\left(\sqrt{r^2-x^2}\right)\right)$$

$$A'(x)=4\left[\sqrt{r^2-x^2}+\frac{d}{dx}\left(\sqrt{r^2-x^2}\right)\cdot x\right]$$

$$A'(x)=4\left[(r^2-x^2)^{\frac{1}{2}}+\frac{1}{2}(r^2-x^2)^{-\frac{1}{2}}\cdot(-2x)(x)\right]$$

$$A'(x)=4\left[(r^2-x^2)^{\frac{1}{2}}+\frac{-2x^2}{2(r^2-x^2)^{\frac{1}{2}}}\right]$$

$$A'(x)=\sqrt{r^2-x^2}-\frac{x^2}{\sqrt{r^2-x^2}}$$

$A'(x) = 0$

$\sqrt{r^2 - x^2} - \dfrac{x^2}{\sqrt{r^2 - x^2}} = 0$

$\sqrt{r^2 - x^2} = \dfrac{x^2}{\sqrt{r^2 - x^2}}$

$r^2 - x^2 = x^2$

$r^2 = 2x^2$

$x^2 = \dfrac{r^2}{2}$

$x = \pm\sqrt{\dfrac{r^2}{2}}$) Domain $[0, r]$

critical value! → $x = \sqrt{\dfrac{r^2}{2}}$

Plug the edges + critical value(s)...

$A(0) = 0$

$A(r) = 0$

$A\left(\sqrt{\dfrac{r^2}{2}}\right) = 4\left(\sqrt{\dfrac{r^2}{2}}\right)\left(\sqrt{r^2 - \left(\sqrt{\dfrac{r^2}{2}}\right)^2}\right) =$

$= 4\left(\sqrt{\dfrac{r^2}{2}}\right)\left(\sqrt{\dfrac{2r^2 - r^2}{2}}\right)$

(Positive) → $= 2r^2$

Therefore the maximum area occurs when $x = \sqrt{\dfrac{r^2}{2}}$

$A = 4xy$

$x^2 + y^2 = r^2$

$\left(\sqrt{\dfrac{r^2}{2}}\right)^2 + y^2 = r^2$

$y^2 = r^2 - \dfrac{r^2}{2}$

$y^2 = \dfrac{r^2}{2}$

$y = \pm\sqrt{\dfrac{r^2}{2}}$

$y = +\sqrt{\dfrac{r^2}{2}}$

The dimensions of the maximal rectangle are $2\sqrt{\dfrac{r^2}{2}} \times 2\sqrt{\dfrac{r^2}{2}}$