Integrating by Parts

(5.5)
Substitution
↕ undoes
(3.4)
Chain rule

IBM (5.6)
↕
(3.2)
Product Rule

$$\frac{d}{dx}[f(x)\,g(x)] = f'(x)\,g(x) + f(x)\,g'(x)$$

integrate

$$f(x)\,g(x) = \int [f'(x)g(x) + f(x)g'(x)]\,dx$$

$$f(x)\,g(x) = \int f'(x)g(x)\,dx + \int f(x)g'(x)\,dx$$

$$f(x)\,g(x) - \int g(x)f'(x)\,dx = \int f(x)\,g'(x)\,dx$$

$$\int \underbrace{f(x)}_{u}\underbrace{g'(x)\,dx}_{dv} = \underbrace{f(x)}_{u}\,\underbrace{g(x)}_{v} - \int \underbrace{g(x)}_{v}\underbrace{f'(x)\,dx}_{du}$$

If $u = f(x)$ and $dv = g'(x)\,dx$

$du = f'(x)\,dx$ $\qquad$ $v = g(x)$

$$\int u\,dv = uv - \int v\,du$$

Ex: $\int x^4 \cos x \, dx$

a) $u = x^4$ $\quad dv = \cos x \, dx$

$du = 4x^3 dx \quad v = \sin x$

$\int u\,dv = uv - \int v\,du$

$\int x^4 . \cos x dx = x^4 \sin x - \int \sin x . 4x^3 dx$

degree 3 Polynomial

Integrate by Parts

Terminates

b) $u = \cos x$ $\quad dv = x^4 dx$

$du = -\sin x . dx \quad v = \frac{1}{5}x^5$

$\int u.dv = uv - \int v du$

$\int \cos x . x^4 dx = \frac{1}{5}x^5 \cos x - \int \frac{1}{5}x^5 . -\sin x dx$

degree 5 Polynomial

Does not terminate

05/21/2018

Thought process:
1) Do you know this integral?
2) Try a Substitution
3) Integration by Parts.

Ex: Find $\int t e^{-3t} dt$.

$u = t \quad dv = e^{-3t} dt$

$du = dt \quad v = -\frac{1}{3}e^{-3t}$

$\int u.dv = u.v - \int v du$

$\int t e^{-3t} dt = t . -\frac{1}{3}e^{-3t} - \int -\frac{1}{3}e^{-3t} dt$

$= -\frac{1}{3}te^{-3t} + \frac{1}{3}\int e^{-3t} dt$

$= -\frac{1}{3}te^{-3t} - \frac{1}{9}e^{-3t} + C$

$= -\frac{1}{3}e^{-3t}\left(t + \frac{1}{3}\right) + C$

- What if it was a definite integral? Just evaluate it on the given integral.

Ex: Find $\int e^{2x} \cos x \, dx$.

$u = e^{2x} \quad dv = \cos x dx$

$du = 2e^{2x} dx \quad v = \sin x$

$\int u.dv = u.v - \int v du$

$\int e^{2x} \cos x dx = e^{2x} . \sin x - \int 2e^{2x} \sin x dx$

$= e^{2x} \sin x - 2\int e^{2x} \sin x dx$

$u = e^{2x} \quad dv = \sin x dx$

$du = 2e^{2x} dx \quad v = -\cos x$

$\int u dv = u.v - \int v du$

$\int e^{2x} \sin x dx = -e^{2x} \cos x - \int -\cos x . 2e^{2x} dx$

$= -e^{2x} \cos x + 2\int e^{2x} \cos x dx$

$\int e^{2x} \cos x dx = e^{2x} \sin x + 2e^{2x} \cos x - 4\int e^{2x} \cos x dx$

$5\int e^{2x} \cos x dx = e^{2x} \sin x + 2e^{2x} \cos x$

$\int e^{2x} \cos x dx = \frac{1}{5}e^{2x} \sin x + \frac{2}{5}e^{2x} \cos x + C$