

Algorithm Analysis

Mark Sort

```
for(int i=0; i<n-1; i++) {  
    for(int j=i+1; j<n; j++) {  
        If(a[i] > a[j]) {  
            temp = a[i]  
            a[i] = a[j]  
            a[j] = temp;  
        }  
    }  
}
```

O_b O_i O_j PO_s

$$O_b + \sum_{i=0}^{n-2} \left(O_i + \sum_{j=i+1}^{n-1} (O_j + PO_s) \right)$$

O represents operations
which equates $T(O) \rightarrow$ clock cycles

$$\sum_{j=x}^y 1 = (y-x)+1$$

$$\text{Let } O_j + PO_s = O_{js}$$

$$\text{So } \rightarrow \begin{matrix} (n-1) - (i+1) + 1 \\ (n-i-1) \end{matrix} O_{js}$$

$$O_b + \sum_{i=0}^{N-2} (O_i + (N-1-i) O_{i+1})$$

$$\sum_{i=0}^x i = x \cdot (x+1) / 2$$

$$\begin{aligned} \sum_{i=0}^x ((i+1)^2 - i^2) &= \cancel{(1^2 - 0^2)} \\ &+ \cancel{(2^2 - 1^2)} \\ &+ \cancel{(3^2 - 2^2)} \\ &\vdots \\ &+ \cancel{(x^2 - (x-1)^2)} \\ &+ ((x+1)^2 - \cancel{x^2}) \end{aligned}$$

$$\sum_{i=0}^x (\cancel{i^2} + 2i + 1 - \cancel{i^2}) = (x+1)^2$$

$$\sum_{i=0}^x (2i + 1)$$

$$2 \sum_{i=0}^x i + \sum_{i=0}^x 1$$

$$\sum_{i=0}^x i = \frac{(x+1)^2 - (x+1)}{2}$$

$$= \frac{x^2 + 2x + 1 - x - 1}{2}$$

$$= \frac{x^2 + x}{2}$$

$$= \frac{(x+1)(x+1-1)}{2}$$

$$= (x+1)x/2$$

$$O_b + \sum_{i=0}^{N-2} (O_i + (N-1-i) O_{15})$$

$$O_b + \sum_{i=0}^{N-2} (O_i + (N-1) O_{15} - i O_{15})$$

$$O_b + (N-1)(O_i + (N-1) O_{15}) - O_{15} \sum_{i=0}^{N-2} i$$

$$(N-1)^2 O_{15} + (N-1) O_i + O_b - \frac{(N-2)(N+1)}{2} O_{15}$$

$$f(N) = \text{second order polynomial in } N$$

$$\left(O_{15} - \frac{O_{15}}{2}\right) N^2 + \left(-2O_{15} + O_i + \frac{1}{2}O_{15}\right) N + (2O_{15} - O_i + O_b)$$

$$C'' N^2 + C' N + C$$

$$C'' = (O_{15}/2) \quad C' = (O_i - 3/2 O_{15})$$

$$C = (2O_{15} - O_i + O_b)$$

