

# Root locus preparation of equations for plots generation

## Relevant information:

Using the characteristic equation (CE) for a non-ideal single-ended common-drain Colpitts oscillator, appropriate expression is derived in order to be able to simulate the relevant root locus plots.

Using this expression, the root locus plots are generated in the "RLocus.m" matlab script.

## Relevant variables:

$L$  - LC resonant circuit inductance

$R$  - Total circuit resistance

$g_m$  - Transistor transconductance

$C_1/C_2$  - LC resonant circuit capacitances

$R_p$  - LC inductor equivalent series parasitic resistance

$C_p$  - LC inductor equivalent parallel parasitic capacitance

## 1. Characteristic equation for $R_p$ and $C_p$

$$LR(C_1C_2 + C_1C_p + C_2C_p)s^3 + \left[ L(C_1 + C_p) + RR_p \right] C_p(C_1 + C_2) + C_1C_2 + g_m LR C_p s^2 + \left[ R(C_1 + C_2) + R_p(C_1 + C_p) + g_m RR_p C_p \right] s + g_m R + 1 = 0$$

Extracting  $g_m R$  from relevant terms and refactoring gives:

$$LR(C_1C_2 + C_1C_p + C_2C_p)s^3 + \left[ L(C_1 + C_p) + RR_p \right] C_p(C_1 + C_2) + C_1C_2 s^2 + \left[ R(C_1 + C_2) + R_p(C_1 + C_p) \right] s + 1 + g_m R \left[ LC_p s^2 + R_p C_p s + 1 \right] = 0$$

Dividing above equation from both sides with the following term:

$$LR(C_1C_2 + C_1C_p + C_2C_p)s^3 + \left[ L(C_1 + C_p) + RR_p \right] C_p(C_1 + C_2) + C_1C_2 s^2 + \left[ R(C_1 + C_2) + R_p(C_1 + C_p) \right] s$$

gives:

$$\begin{array}{l} \frac{1 + g_m R \left[ LC_p s^2 + R_p C_p s + 1 \right]}{LR(C_1C_2 + C_1C_p + C_2C_p)s^3 + \left[ L(C_1 + C_p) + RR_p \right] C_p(C_1 + C_2) + C_1C_2 s^2 + \left[ R(C_1 + C_2) + R_p(C_1 + C_p) \right] s + 1} = 0 \end{array}$$