University of British Columbia

MECH 325 - Mechanical Design I

Assignment 1

Gear Train Design

GROUP C2

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Velocity = 5.9093 mm/sec
Cost = $256.69
Performance Metric = 0.0230 mm/$s

1 Summary

1.1 Introduction

This report demonstrates the design process of a gear train to operate a 2-stage worm gear and power screw mechanism that raises a mass of 2500 kg over a total stroke of 30 cm. The performance metric is determined by the vertical speed of the power screw divided by the total cost of the system.

1.2 Final Performance Results

Resulting performance metric for our gear train was 0.0230 mm/$s with a final raising speed of 5.9093 mm/sec and a total cost of $256.69 using a gear ratio of 1:5.

1.3 Approach & Methods

Due to the available number of potentially suitable gears and the sheer number of possible permutations, we decided it was best practice to write code in Python to compute the performance metric for any given gear choices. This involved creating a program representation of our system.

Starting from the minimum torque required to raise the power screw and assuming a sub-100% efficiency, we determined the total gear ratio from the motor to the worm gear. Parts were selected from McMaster-Carr with the following major assumptions:

- Spur gear efficiency = 94%.
- Temperature of gear train < 120°C.
- Hardness of carbon steel = 250HB.
- Appropriate shafts, couplers, bearings would be available.
- Motor size does not affect the system.
- Each element in the system rotates with constant angular speed.
- 1020 Carbon steel has the same Poisson's ratio as steel

1.4 Power Screw Results

The cornerstone of our design came from the results obtained from our analysis of the power screw for the raising and lowering of the load. We concluded our analysis with the following results:

Parameters			
Parameter	Power (W)	Torque (Nm)	Vertical Speed (mm/s)
Power Screw (Raising)	492.3	79.55	5.91
Power Screw (Lowering)	305.1	32.41	8.99

1.5 Gear System Overview

Below is an overview diagram of the gear train system we designed for this assignment. The gear attached to the worm gear shaft does not exceed 250mm as required. Since there is a difference between the shaft diameter of the bigger gear and the shaft diameter of the worm gear, a shaft coupler is placed to connect them.

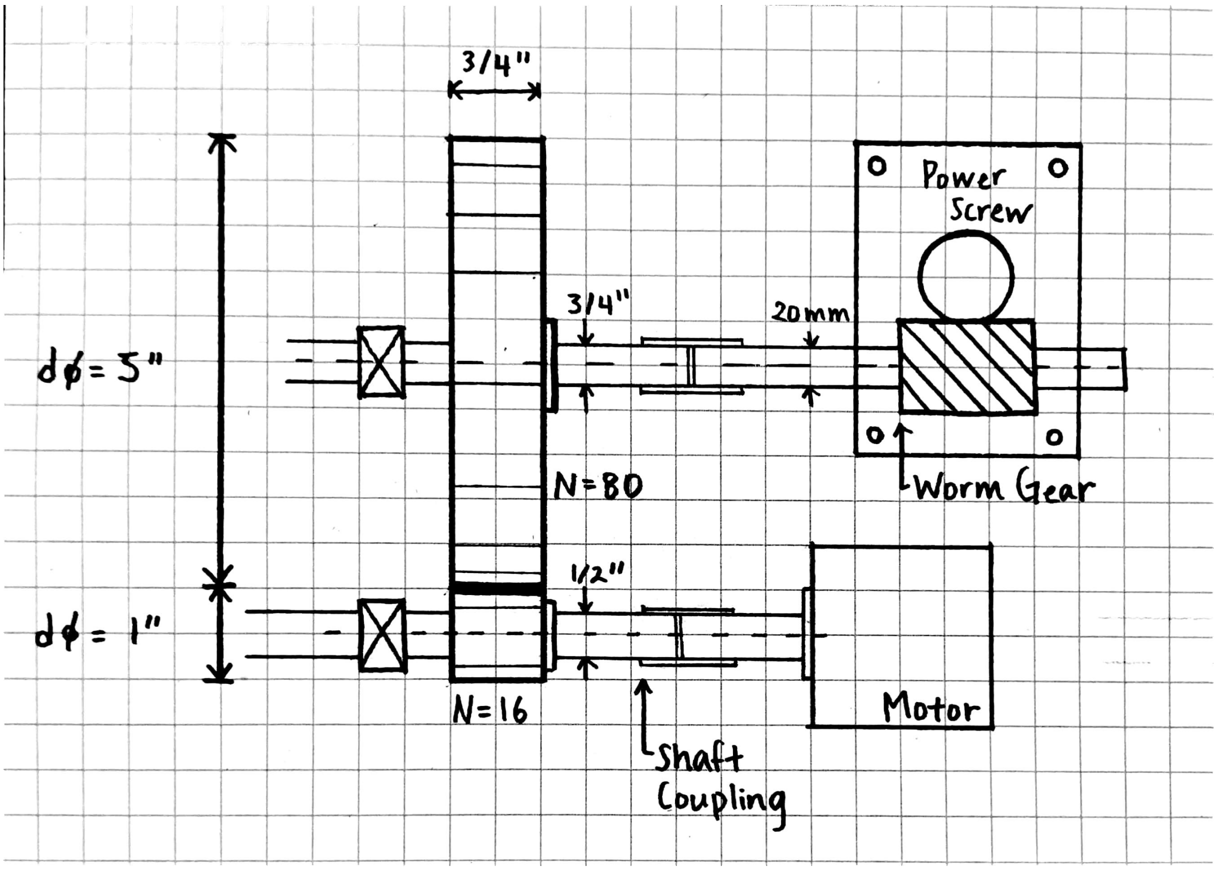

Figure 1: Gearbox and power screw apparatus

Gear Information (McMaster-Carr)			
Gear Type	Teeth Number	Price	Reference Number
Pinion	16	23.1	5172T21
Gear	80	93.37	5172T25

1.6 Optimization Script

To find the most optimal outcome regarding the proposed performance metric, our team decided to create a script which will loop through all permutations of gear combinations and calculate the performance metric for each pair of pinion and gear. The script can be found at the GitHub repository below:

https://github.com/DonneyF/MECH-325-Assignments

2 Appendix

2.1 Power Screw Analysis

The objective of this section is to verify the power screw is self-locking and find the minimum required torque and rotational speed needed to lift the 2500 kg load at 4 mm/sec. Self-locking is ensured when the coefficient of thread friction is greater than a threshold defined as follows:

$$f > \tan(\lambda) \tag{1}$$

$$\tan\lambda = \frac{l}{\pi d_m} = 0.0335$$

Since $f = 0.08 > \tan\lambda = 0.03$, the current design is self-locking. The torque required to lift the given load with gravitational force F is:

$$\tau_{\text{raise}} = \frac{F d_m}{2}\left(\frac{l + \pi f d_m}{\pi d_m - f l}\right) \tag{2}$$

The torque required to lower the given load with gravitational force F is:

$$\tau_{\text{lower}} = \frac{F d_m}{2}\left(\frac{\pi f d_m - l}{\pi d_m + f l}\right) \tag{3}$$

Parameters			
Symbol	Value	Units	Description
F	2500×9.81	N	Axial compressive force
d	60	mm	Major diameter
d_m	57	mm	Mean diameter, $d - \frac{l}{2}$
l	6	mm	Pitch
f	0.08	N/A	Friction Coefficient

A torque of 79.5 Nm is required to lift the load and 32.4 Nm is required to lower the load, where efficiency losses in the power screw is accounted for by the friction coefficient, f.

2.2 Worm Gear Analysis

Worm gears are used in high torque applications but they are subject to efficiency losses during operation. Calculating this value allows us to find the gear train required to raise the load.

$$\eta = \frac{\cos\phi_n - f\tan\lambda}{\cos\phi_n + f\cot\lambda} \tag{4}$$

The lead angle component is as follows:

$$\tan\lambda = \frac{p_x}{\pi d_p} = 0.4074366 \tag{5}$$

We find the worm gear is therefore 80.34% efficient.

Parameters			
Symbol	Value	Units	Description
N_G	18	N/A	Number of teeth on worm drive gear
N_w	2	N/A	2-thread worm
ϕ_n	14.5	degrees	Pressure angle
l	16	mm	pitch
p_x	32	mm	Axial pitch
d_p	25	mm	Worm pitch diameter
d_s	20	mm	Worm shaft diameter

The 2-threaded worm gear engages with an 18 teeth worm drive nut. For each 9 revolution of the worm gear, the nut completes 1 revolution, resulting in a 9 fold torque increase.

$$\frac{\tau}{9\eta_{\text{worm}}} = \frac{79.5}{9(0.8034)} = 11.0 \text{ Nm}$$

Therefore, the system requires 11.0 Nm to reach the worm gear.

2.3 Motor Torque and Gear Reduction

The motor provided has the following torque-speed curve. The maximum power output occurs when the motor operates at 2500 rpm with 2.5 Nm of torque.

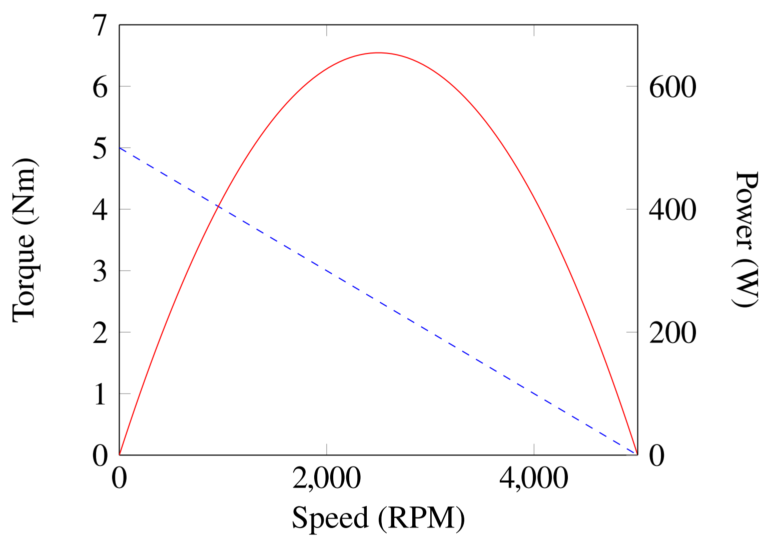

Figure 2: Torque and power curve for chosen motor. The dotted line shows the torque and continuous line shows the power output. Power is maximized at 2500 RPM.

The relationship to determine train value given torque specifications, e is:

$$e = \frac{T_{\text{in}} f_g}{T_{\text{out}}} = \frac{2.5}{11} f_g = 0.227 f_g \tag{6}$$

where T_{out} = torque output, T_{in} = torque input, and f_g = gear efficiency.
Spur gears are selected for their efficiency, between 94 - 98 %, and simplicity. We assume the lower bound of efficiency.

$$\text{(simple two-gear system)} \qquad e = (0.227)(0.94) = 0.2136 \tag{7}$$

$$\text{(compound four-gear system)} \qquad e = (0.227)(0.94)^2 = 0.2 \tag{8}$$

If only two gears are used, select gear ratio to be at least 1:4.68. If a compound four gear system is appropriate, select gear ratio to be at least 1:5.

Because we chose a two-gear system, the total efficiency of our gearbox is:

$$\eta_{\text{gearbox}} = \eta_{\text{worm}} \cdot f_g = 75.5\%$$

2.4 Motor Speed Calculation

The raising and lowering torques for the load are used to compute torque at the motor which in turn gives the motor revolution per minute (rpm) as seen in figure 1.

$$\tau_{\text{motor, raise}} = 79.5(e)(\text{worm-power screw ratio})/\eta_{\text{gearbox}} = 2.34 \text{ Nm}$$

$$\tau_{\text{motor, lower}} = 32.41(e)(\text{worm-power screw ratio})/\eta_{\text{gearbox}} = 0.95 \text{ Nm}$$

The worm-power screw ratio is 1/9. From the motor torque-speed curve,

$$\text{torque} = -\text{speed}/1000.0 + 5$$

The raising RPM is 2659 and lowering RPM is 4050 RPM.

2.5 Power Screw Power/Speed Calculation

The raising and lowering speed of power screw is dependent on motor rpm. Given gear train value, $e = 1/5$,

$$\omega_{\text{motor}} = \text{rpm}(2\pi/60)$$

$$\omega_{\text{worm}} = e\omega_{\text{motor}}$$

$$\omega_{\text{screw}} = \omega_{\text{worm}}(1/9)$$

$$\text{screw speed} = \omega_{\text{screw}}(6mm)/(2\pi)$$

The worm-power screw ratio is $1/9$, power screw pitch is $6mm$, and $2\pi/60$ converts rpm to radians per second. Given the rpm found in section 2.4, we obtain raising speed of $5.91mm/s$ and lowering speed of $8.99mm/s$.

We then calculate the power required to raise and lower the load by the following formula:

$$P = T\omega = \frac{2\pi \times \text{vertical velocity} \times T}{\text{Pitch}} \tag{9}$$

The raising power was found to be 492.3 W, and the lowering power is 305.1 W.

2.6 Bending & Contact Stress Analysis

Detailed stress analysis was performed on each gear and pinion considered for the design of the gear train. Major stress factors were determined to come from the bending and contact stresses exerted on the gear/pinion system during meshing. The following equation was used calculate and verify the allowable bending stress:

$$\sigma_{\text{bending}} = W^t K_o K_v K_s \frac{P_d}{F} \frac{K_s K_B}{J} \tag{10}$$

Parameters				
Symbol	Units	Gear Values	Pinion Values	Description
W^t	lbf	41.612	41.612	Tangential transmitted load
K_o	N/A	1.75	1.75	Overload factor
K_v	N/A	1.433	1.433	Dynamic factor
K_s	N/A	0.990	0.980	Size factor
P_d	1/in	16	16	Transverse diametral pitch
F	in	0.75	0.75	Face Width of narrower member
K_m	N/A	1.188	1.188	Load-distribution factor
K_B	N/A	1	1	Rim-thickness factor
J	N/A	0.42	0.27	Geometry factor (bending strength)

The contact stress measurement was taken into account by the following equation, along with the following new parameters:

$$\sigma_{\text{contact}} = C_p \sqrt{W^t K_o K_v K_s \frac{K_m}{d_P F} \frac{C_f}{I}} \tag{11}$$

Parameters				
Symbol	Units	Gear Values	Pinion Values	Description
C_p	$\sqrt{lbf/in^2}$	2290.604	2290.604	Elastic Coefficient
C_f	N/A	1	1	Surface condition factor
d_P	in	5	1	Pitch diameter
I	N/A	0.133	0.133	Geometry Factor (pitting resistance)

From our analysis, we obtain the following:

$$\sigma_{b_{\text{gear}}} = 4454.54 \text{ lbf/in}^2$$
$$\sigma_{c_{\text{gear}}} = 30270.38 \text{ lbf/in}^2$$
$$\sigma_{b_{\text{pinion}}} = 6857.85 \text{ lbf/in}^2$$
$$\sigma_{c_{\text{pinion}}} = 67336.92 \text{ lbf/in}^2$$

Note: Many of the factors are constants that were obtained from reading values off graphs and tables. Other values depend on the material, geometry, and physical properties of the gear and pinion. Furthermore, all the parameters in the stress equations are in U.S. customary units.

Here we will discuss the relevant formulas, equations, and references utilized to obtain parameter values found in the bending and contact stress equations. Note that all references to table and figure numbers are from Shigley's Mechanical Engineering Design Textbook.

- **2.6.1. Transmitted Load (W^t):** this parameter represents the load transmitted onto the teeth of the gear and pinion during their interaction. The transmitted load is calculated by finding the motor torque exerted on the pinion, finding the pitch diameter of the pinion, and then applying torque analysis, namely $T = Fr$ to find the transmitted force.

- **2.6.2. Bending-Strength Geometry Factor (J):** this factor is used to determine the bending strength of the spur gear teeth. Values for J were directly read off of Figure 14-6 in Shigley's, and were determined based on the number of teeth on the gear and pinion.

- **2.6.3. Surface-Strength Geometry Factor (I):** this factor is used to determine the pitting resistance of the spur gear teeth. The surface-strength geometry factor is defined as follows:

$$I = \frac{\cos\phi_t \sin\phi_t}{2m_N} \frac{m_G}{m_G + 1} \tag{12}$$

  Where ϕ_t is the transverse pressure angle, m_N is the load sharing ratio (1 for spur gears), and m_G is the gear ratio.

- **2.6.4. Elastic Coefficient (C_f):** this factor is used to represent elastic properties of the pinion and gear during teeth interaction and meshing. The elastic coefficient is defined as follows:

$$C_p = \left[\frac{1}{\pi \left(\frac{1 - v_P^2}{E_P} + \frac{1 - v_G^2}{E_G} \right)} \right]^2 \tag{13}$$

  Where v_P, v_G are the Poisson's ratio of the pinion and gear respectively, and E_P and E_G are the Modulus of Elasticity of the pinion and gear respectively. Based on Table 14-8 in Shigley's, the Poisson's ratio of the pinion and gear were determined to be at 0.30, and since the material for the pinion of gear is steel, the Modulus of Elasticity is 30×10^6 lbf/in^2 for both.

- **2.6.5. Dynamic Factor (K_v):** this factor is used to account for inaccuracies in the manufacture and meshing of gear teeth in action. The dynamic factor is defined as follows:

$$K_v = \left(\frac{A + \sqrt{V}}{A} \right)^B \tag{14}$$

  Where the constants are defined as:

$$A = 50 + 56(1 - B)$$
$$B = 0.25(12 - Q_v)^{2/3}$$

Where Q_v is the AGMA transmission accuracy number. For our analysis, we decided to use a Q_v value of 5 since 3-7 is the range for commercial use.

- **2.6.6. Overload Factor (K_o):** this factor is used to make allowance for all externally applied loads in excess of the nominal tangential load W^t. The value for K_o was determined using Figures 14-17 and 14-18, from which a value of 1.75 was obtained since we are assuming there will be an initial shock with uniform motion for the raising and lowering of the power screw system.

- **2.6.7. Surface Condition Factor (C_f):** this factor is used to represent any defects in the gear surface, which could have been caused by residual stress, poor surface finishing, or plastic effects. Since we assume the new gears and pinions purchased to be in good condition, we assign a value of 1 to C_f.

- **2.6.8. Size Factor (K_s):** this factor reflects the non-uniformity of material properties due to size. The size factor is defined as follows:

$$K_s = 1.192 \left(\frac{F\sqrt{Y}}{P} \right)^{0.0535} \tag{15}$$

  Where F is the face width of the narrowest member, Y is the Lewis Form Factor of the gear/pinion, and P is the diametral pitch. Values for Y was determined using Table 14-2.

- **2.6.9. Load-Distribution Factor (K_m):** this factor reflects nonuniform distribution of load across the line of contact. The load-distribution factor is defined as follows:

$$K_m = 1 + C_{mc}(C_{pf}C_{pm} + C_{ma}C_e) \tag{16}$$

  Where $C_{mc} = 1$ since we are using gears/pinions with uncrowned teeth, $C_{pm} = 1$ since the ratio between then pinion offset from center span, S_1, and bearing span, S, is less than 0.175, and $C_e = 1$ since our pinions/gears are not adjusted at assembly.

  C_{pf} is defined as follows:

$$C_{pf} = \begin{cases} \frac{F}{10d} - 0.025 & F \leq 1 \text{ in} \\ \frac{F}{10d} - 0.0375 + 0.0125F & 1 < F \leq 17 \text{ in} \end{cases} \tag{17}$$

  Where F is the minimum face width of the narrowest member and d is the pitch diameter.

  Next, we calculate C_{ma} as follows:

$$C_{ma} = A + BF + CF^2$$

  Where A, B, and C are constants obtained from Table 14-9. We decided to use commercial-grade conditioning for our gears/pinions, and set A = 0.127, B = 0.0158, and C = -0.930(10^{-4}).

- **2.6.10. Rim-Thickness Factor (K_B):** this factor represents is the ratio of rim thickness t_R and tooth height h_t. We downloaded the CAD model and measured the respective distances to obtain the backup ratio:

$$m_B = \frac{t_R}{h_t}$$

We obtain the following:

$$m_{B_{\text{pinion}}} = \frac{0.421875}{0.140625} = 3,\ m_{B_{\text{gear}}} = \frac{0.421875}{0.140625} = 9.5$$

Since all $m_B \geq 1.2$, $K_B = 1$.

2.7 Safety Factor Analysis

To ensure the gear train system maintains an acceptable level of safety (2.2 minimum), bending and contact safety factors were calculated to validate design specifications.

$$S_{F(\text{bending})} = \frac{S_t}{\sigma_b}\frac{Y_N}{K_T K_R} \tag{18}$$

$$S_{H(\text{contact})} = \frac{S_c}{\sigma_c}\frac{Z_N C_H}{K_T K_R} \tag{19}$$

Parameters				
Symbol	Units	Gear Values	Pinion Values	Description
S_t	bf/in^2	32125	32125	AGMA bending strength
S_c	lbf/in^2	109600	109600	AGMA surface endurance strength
Y_N	N/A	1.890	1.890	Stress cycle (bending strength)
Z_N	N/A	1.42	1.42	Stress cycle (pitting resistance)
C_H	N/A	1	1	Hardness-ratio factor
σ_b	lbf/in^2	4454.54	6857.85	Bending stress
σ_c	lbf/in^2	30270.38	67336.92	Contact stress
K_T	N/A	1	1	Temperature factor
K_R	N/A	0.955	0.955	Reliability factor

From our analysis, we obtain the following:

$$S_{F_{\text{gear}}} = 14.27$$
$$S_{H_{\text{gear}}} = 5.37$$
$$S_{F_{\text{pinion}}} = 9.27$$
$$S_{H_{\text{pinion}}} = 2.414$$

Here we will discuss the relevant formulas, equations, and references utilized to obtain parameter values found in the safety factor equations. Note that all references to table and figure numbers are from Shigley's Mechanical Engineering Design Textbook.

- **2.7.1. Allowable Bending/Contact Stress Number (S_t, S_c):** this parameter represents AGMA defined stress numbers, obtained from Figures 14-2 and 14-5. Since we are using Grade 1 steel gears/pinions, we are able to calculate the stress numbers using the following:

  $$S_t = 77.3\ H_B + 12800 \text{ psi}$$
  $$S_c = 322\ H_B + 29100 \text{ psi}$$

  Where H_B is the Brinell Hardness of the steel gears/pinions.

- **2.7.2. Stress-Cycle Factors** (Y_N and Z_N): this factor reflects the number of load cycles the gear/pinion can sustain through. From Figures 14-14 and 14-15, we can calculate Y_N and Z_N by the following:

$$Y_N = 6.1514N^{-0.1192}$$
$$Z_N = 2.466N^{-0.056}$$

  Where N is the number of load cycles, which is a minimum of 2×10^4 in this case.

- **2.7.3. Hardness-Ratio Factor (C_H):** this factor is used to account for the effect obtained when a surface-hardened pinion is mated with a through-hardened gear, and adjust the surface strengths for this effect. The hardness-ratio factor is defined as follows:

$$C_H = 1.0 + A'(m_G = 1.0) \tag{20}$$

  Where:

$$A' = 8.98(10^{-3})\left(\frac{H_{BP}}{H_{BG}}\right) - 8.29(10^{-3})\ 1.2 \leq \frac{H_{BP}}{H_{BG}} \leq 1.7$$

  From which we note that:

$$\frac{H_{BP}}{H_{BG}} < 1.2,\ A' = 0$$
$$\frac{H_{BP}}{H_{BG}} > 1.7,\ A' = 0.00698$$

  The terms H_{BP} and H_{BG} are the Brinell hardness of the pinion and gear, respectively. Since we are using carbon steel material, the Brinell hardness of both the pinion and gear was found to be 250 HB.

  Therefore, the ratio of Brinell hardness is 1, from which we obtain that $A' = 0$. Therefore, $C_H = 1$.

- **2.7.4. Reliability Factor (K_R):** this factor accounts for the effect of the statistical distributions of material fatigue failures. The reliability factor is defined as follows:

$$K_R = 0.658 - 0.0759 \ln(1 - R) \tag{21}$$

  Where R is the reliability factor, out of 1. We determined that a safety factor of 0.98 would be sufficient for our gear system, and thus we obtain a K_R value of 0.955.

- **2.7.5. Temperature Factor (K_T):** this factor reflects the limits of temperature values that can be used for proper stress analysis. Since we assume that our gears/pinions never reach a temperature value higher than or equal to 120°C during operation, $K_T = 1$.

2.8 Performance Metric Analysis

- **2.8.1. Total Cost Analysis:** the total cost of the system was calculated as follows:

$$\text{Cost}_{total} = \text{Cost}_{\text{gear}} + \text{Cost}_{\text{pinion}} + \text{Cost}_{\text{motor runtime}}$$

  The cost of the gears were obtained from McMaster-Carr and were as follows:

$$\text{Cost}_{\text{gear}} = \$93.37$$

$$\text{Cost}_{\text{pinion}} = \$23.10$$

  The cost of the motor runtime was calculated as follows:

$$\text{Cost}_{\text{motor runtime}} = \left(\frac{d_{\text{stroke}}}{V_{\text{raising}}} + \frac{d_{\text{stroke}}}{V_{\text{lowering}}}\right)\left(\text{Cost}_{\text{per hour}}\right)\left(\frac{1_{\text{hour}}}{3600_{\text{seconds}}}\right)\left(N_{\text{cycles}}\right)$$

$$\text{Cost}_{\text{motor runtime}} = \$140.22 \tag{22}$$

  Therefore, the total cost of the system was calculated to be \$256.69.

- **2.8.2. Performance Metric Analysis:** the final metric for the system was defined to be:

$$\text{metric} = \frac{V_{\text{raising}}}{\text{Cost}_{\text{total}}} \tag{23}$$

  Where:

  - V_{raising} is the speed with which the power screw raises the load, measured in $\frac{textmm}{s}$,
  - $Cost_{\text{total}}$ is the total cost of the system, measured in \$

The final performance metric of our system was found to be 0.0230 $\frac{\text{mm}}{\$s}$.