

# PHYS 304 Formula Sheet

## The Wave Function

Time dependent Schrodinger Equation:

$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \Psi}{\partial x^2} + V\Psi$$

Standard Deviation:

$$\sigma = \sqrt{\langle j^2 \rangle - \langle j \rangle^2}$$

Momentum:

$$\langle p \rangle = m \frac{d \langle x \rangle}{dt} = \int \Psi^* \left( \frac{\hbar}{i} \frac{\partial}{\partial x} \right) \Psi dx$$

## Infinite Square Well

Time Independent Schrodinger Equation:

$$-\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + V\psi = E\psi$$

Eigenstate Expansion:

$$\Psi(x, t) = \sum_{n=1}^{\infty} c_n \psi_n(x) e^{-iE_n t/\hbar} = \sum_{n=1}^{\infty} c_n \Psi_n(x, t)$$

Energy In Infinite Square Well:

$$E_n = \frac{n^2 \pi^2 \hbar^2}{2ma^2}$$

Stationary States:

$$\psi_n(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi}{a}x\right)$$

Determining Coefficients:

$$c_n = \sqrt{\frac{2}{a}} \int_0^a \sin\left(\frac{n\pi}{a}x\right) \Psi(x, 0) dx$$

Expectation Value of Energy:

$$\langle H \rangle = \sum_{n=1}^{\infty} |c_n|^2 E_n$$

## Harmonic Oscillator

$$k = \omega^2 m$$

Ladder Operators:

$$a_{\pm} = \frac{1}{\sqrt{2\hbar m \omega}} (\mp i p + m \omega x)$$

$$a_+ \psi_n = \sqrt{n+1} \psi_{n+1}$$

$$a_- \psi_n = \sqrt{n} \psi_{n-1}$$

Operators:

$$x = \sqrt{\frac{\hbar}{2m\omega}} (a_+ + a_-)$$

$$p = i\sqrt{\frac{\hbar m \omega}{2}} (a_+ - a_-)$$

$$x^2 = \frac{\hbar}{2m\omega} [(a_+)^2 + (a_+ a_-) + (a_- a_+) + (a_-)^2]$$

Commutation:

$$[x, p] = i\hbar$$

$$[a_-, a_+] = 1$$

Hamiltonian:

$$H = \hbar \omega \left( a_+ a_- + \frac{1}{2} \right)$$

$$a_+ a_- + a_- a_+ = 2 \left( \frac{H}{\hbar \omega} \right)$$

States:

$$\psi_0(x) = \left( \frac{m\omega}{\pi \hbar} \right)^{1/4} e^{-\frac{m\omega}{2\hbar} x^2}$$

$$\psi_n = \frac{1}{\sqrt{n!}} (a_+)^n \psi_0$$

Energy:

$$E_n = \left( n + \frac{1}{2} \right) \hbar \omega$$

## Generalized Statistical Interpretation

Momentum Expansion:

$$\Phi(p, t) = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{\infty} e^{-ipx/\hbar} \Psi(x, t) dx$$

$$\Psi(x, t) = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{\infty} e^{ipx/\hbar} \Phi(p, t) dp$$

## Angular Momentum

$$L_x = yp_z - zp_y \quad L_y = zp_x - xp_z \quad L_z = xp_y - yp_x$$

Commutators:

$$[L_x, L_y] = i\hbar L_z \quad [L_y, L_z] = i\hbar L_x \quad [L_z, L_x] = i\hbar L_y$$

$$[L_z, x] = i\hbar y \quad [L_z, y] = -i\hbar x \quad [L_z, z] = 0$$

$$[L_z, p_x] = i\hbar p_y \quad [L_z, p_y] = -i\hbar p_x \quad [L_z, p_z] = 0$$

Square of Angular Momentum:

$$L^2 \equiv L_x^2 + L_y^2 + L_z^2 \quad [L^2, \mathbf{L}] = 0$$

Ladder Operators:

$$L_{\pm} = L_x \pm iL_y \quad [L_z, L_{\pm}] = \pm i\hbar L_{\pm}$$

$$L^2 = L_+ L_- + L_z^2 + \hbar L_z$$

Eigenvalues:

$$L^2 f_l^m = \hbar^2 l(l+1) f_l^m$$

$$L_z f_l^m = \hbar m f_l^m$$

$$L_{\pm} f_l^m = (A_l^m) f_l^{m\pm 1} \quad A_l^m = \hbar \sqrt{l(l+1) - m(m\pm 1)}$$

Spherical Coordinate Representation:

$$L_x = \frac{\hbar}{i} \left( -\sin \phi \frac{\partial}{\partial \theta} - \cos \phi \cot \theta \frac{\partial}{\partial \phi} \right)$$

$$L_y = \frac{\hbar}{i} \left( \cos \phi \frac{\partial}{\partial \theta} - \sin \phi \cot \theta \frac{\partial}{\partial \phi} \right)$$

$$L_z = \frac{\hbar}{i} \frac{\partial}{\partial \theta}$$

$$L_{\pm} = \pm \hbar e^{\pm i\theta} \left( \frac{\partial}{\partial \theta} \pm i \cot \theta \frac{\partial}{\partial \phi} \right)$$

$$L_+ L_- = -\hbar^2 \left( \frac{\partial^2}{\partial \theta^2} + \cot \theta \frac{\partial}{\partial \theta} + \cot^2 \theta \frac{\partial^2}{\partial \phi^2} + i \frac{\partial}{\partial \phi} \right)$$

$$L^2 = -\hbar^2 \left[ \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial}{\partial \theta} \right) + \frac{1}{\sin^2 \theta} \frac{\partial^2}{\partial \phi^2} \right]$$

## Trig Identities

$$2 \cos \theta \cos \phi = \cos(\theta - \phi) + \cos(\theta + \phi)$$

$$2 \sin \theta \sin \phi = \cos(\theta - \phi) - \cos(\theta + \phi)$$

$$2 \sin \theta \cos \phi = \sin(\theta + \phi) + \sin(\theta - \phi)$$

$$2 \cos \theta \sin \phi = \sin(\theta + \phi) - \sin(\theta - \phi)$$

## Integral Identities

$$\int \sin^2 ax dx = \frac{x}{2} - \frac{1}{4a} \sin 2ax + C$$

$$\int \sin^3 ax dx = \frac{\cos 3ax}{12a} - \frac{3 \cos ax}{4a} + C$$

$$\int \cos^2 ax dx = \frac{x}{2} + \frac{1}{4a} \sin 2ax + C$$

$$\int \cos^3 ax dx = \frac{\sin ax}{a} - \frac{\sin^3 ax}{3a} + C$$

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<https://github.com/DonneyF/formula-sheets>