MECH 466 Formula Sheet

Continuous Time Signals

Linearity:
$\mathcal{S}[\alpha x(t) + \beta y(t)] = \alpha \mathcal{S}[x(t)] + \beta \mathcal{S}[y(t)]$
Time Invariance:
If $\mathcal{S}[x(t)] = y(t)$ then $\mathcal{S}[x(t \pm \tau)] = y(t \pm \tau)$.

Laplace Transform

Initial Value Theorem:
$f(0+) \Leftrightarrow \lim_{s\to\infty} sF(s)$
Final Value Theorem:
$\lim_{t\to\infty} f(t) \Leftrightarrow \lim_{s\to 0} sF(s)$

Stability

- BIBO stability: any bounded input provides a bounded output
- Asymptotic stability: Initial conditions generates $y(t)$ converges to zero.

Characteristic Equation:
For $G(s) = N(s)/D(s)$, the characteristic equation is $D(s) = 0$.
Stability Condition in s-Domain:
All poles in open left hand plane $\iff$ System is BIBO and asymptotically stable
Marginal Stability:

- $G(s)$ has no pole in the open RHP
- $G(s)$ has at least one simple pole on the imaginary axis
- $G(s)$ has no repeated pole on the imaginary axis
- BIBO stable except for sinusoidal inputs.
- For any non-zero initial condition, the output neither converges to zero nor diverge.

Polynomials:

- For 1st and 2nd order polynomials, all roots are in LHP $\iff$ coefficients have the same sign
- For 3rd and higher orders, all roots are in LHP $\implies$ coefficients have the same sign

Routh-Hurwitz Criterion

- The number of roots in the open right half-plane is equal to the number of sign changes in the first column of Routh array.
- If zero row appears in the Routh array, roots are in imaginary axis or RHP.
- If zero row appears, replace the zero with the coefficients of derivative of auxiliary polynomial.
- Auxiliary polynomial: The polynomial above the zero row.

Steady State Error

Step Function:
$$r(t) = Ru(t) \implies e_{ss} = \frac{R}{1+K_p} = \frac{R}{1+L(0)}$$
Ramp Function:
$$r(t) = Rtu(t) \implies e_{ss} = \frac{R}{K_v} = \frac{R}{\lim_{s\to 0} sL(s)}$$
Parabolic Function:
$$r(t) = \frac{Rt}{2}u(t) \implies e_{ss} = \frac{R}{K_a} = \frac{R}{\lim_{s\to 0} s^2L(s)}$$

First Order System

$$G(s) = \frac{K}{Ts+1}$$
Step Response:
$y(t) = K(1 - e^{t/T})u(t)$
Time Constant:
$$T = \frac{1}{|\text{Real part of pole}|}$$
Response is slower the closer the pole is to the imaginary axis
Settling Time:
5%: $\approx 3T$ 2%: $\approx 4T$

Second Order System

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$
Underdamped Step Response:
$$y(t) = 1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}} \sin(\omega_d t + \arccos\zeta)$$
$\omega_d = \omega_n\sqrt{1-\zeta^2}$
Poles:
$s = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$
Settling Time:
5%: $\approx 3/\zeta\omega_n$ 2%: $\approx 4/\zeta\omega_n$
Peak Time:
$T_p = \pi/\omega_d$
Overshoot:
16%: $\zeta = 0.5$ 5%: $\zeta = \sqrt{2}/2$
$y_{\max} = 1 + e^{-\zeta\pi/\sqrt{1-\zeta^2}}$
Percent overshoot = $100e^{-\zeta\pi/\sqrt{1-\zeta^2}}$

Root Locus Method

A graphical method to show how the poles move of a feedback system when K varies from 0 to ∞ in the form
$$H(S) = \frac{KL(s)}{1+KL(s)} \qquad L(s) = \frac{N(s)}{D(s)}$$

- RL includes all points on real axis to the left of an odd number of real poles/zeros
- RL originates from the poles of L and terminates at the zeros of L, including infinity zeros.
- Number of asymptotes: $r = \deg(D(S)) - \deg(N(s))$
- Intersection of asymptotes: $\dfrac{\sum \text{poles} - \sum \text{zeroes}}{r}$
- Angle of asymptotes: $\dfrac{\pi(2k+1)}{r}$ $k = 0, 1, \ldots r-1$
- Breakaway points: Each root of $L'(s^\star) = 0$, where $K = -1/L(s^\star) > 0$
- Angle condition: s_0 is on the root locus $\iff$ $\angle L(s_0) = 180°$

Reserved

Updated February 25, 2020
https://github.com/DonneyF/formula-sheets

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