APSC 278 Formula Sheet

Bonding and Properties

Equilibrium at r_0:

$F_{\text{net}} = \frac{dE}{dr} = 0$

Thermal Expansion:

$\Delta L = L_0 \alpha \Delta T$

Mechanical Properties

Hooke's Law:

$\sigma = E\epsilon$

Poisson's Ratio:

$$\nu = -\frac{\epsilon_x}{\epsilon_z} = -\frac{\epsilon_y}{\epsilon_z}$$

Shear Modulus:

$$G = \frac{\tau}{\gamma} = \frac{F}{A_0 \tan\theta}$$

$E = 2G(1+\nu)$

Toughness:

$$\text{Toughness} = \int_0^{\epsilon_f} \sigma \, d\epsilon \approx \frac{\sigma_{\text{YS}} + \sigma_{\text{UTS}}}{2}\epsilon_f$$

Modulus of Resilience:

$$U_r = \int_0^{\epsilon_Y} \sigma \, d\epsilon \approx \frac{\sigma_{\text{YS}}^2}{2E}$$

Work Hardening:

$\sigma_T = K\epsilon_T^n$

True Stress and Strain:

$$\epsilon_{\text{true}} = \int_{L_0}^{L_i} \frac{dL}{L} = \ln(\epsilon_{\text{Eng}} + 1)$$

$\sigma_{\text{true}} = \sigma_{\text{eng}}(\epsilon_{\text{eng}} + 1)$

Crystal Structures

	Lattice Parameter (a)	Atoms per unit cell	APF
BCC	$\frac{4\sqrt{3}}{3}R$	2	0.68
FCC	$2\sqrt{2}R$	4	0.74
HCP	-	6	0.74

Density (g/cm^3):

$$\rho_{\text{Th}} = \frac{NMW_i}{V_c N_A}$$

n = number of atoms per unit cell

MW_i = Atomic weight (g/mol)

V_c = unit cell volume

N_A = Avogadro's Number ($6.023 \cdot 10^{23}$ atoms/mol)

Effect of Temperature on Deformation

Homologous Temperature:

$$T_H = \frac{T_{\text{deformation}}}{T_{\text{melt}}}$$

Homologous Temperature for pure metals:

$$\begin{cases} \text{Cold working} & T_H < 0.4 \\ \text{Hot working} & T_H > 0.4 \end{cases}$$

Homologous Temperature for alloys:

$$\begin{cases} \text{Cold working} & T_H < 0.6 \\ \text{Hot working} & T_H > 0.6 \end{cases}$$

Cold Working (rolling):

$$\%\text{ cold work} = \frac{t_0 - t_f}{t_0} \times 100\%$$

Recrystallization rate:

$$\text{rate} = A\exp\left(-\frac{Q}{RT}\right)$$

$t_{\text{recrx}} = 1/\text{rate}$

Q = thermal activation energy

Grain Growth:

$D(t) = D_0 t^m$

Hall-Petch Equation for grain size d:

$\sigma_{\text{YS}} = \sigma_0 + k_y d^{-1/2}$

Creep rate:

$$\dot{\epsilon}_{c,ss} = K_2 \sigma^n \exp\left(-\frac{Q_c}{RT}\right)$$

Larson Miller parameter ($C \approx 20$):

$m = T(C + \log_{10}(t_r))$

Fracture

Stress Concentration Factor for an elliptical notch:

$$\sigma_m = \sigma_{\text{net}}\left(1 + 2\sqrt{\frac{a}{r_t}}\right) = \sigma_{\text{net}} K_t$$

Griffith Equation for surface energy γ_s:

$$\sigma_{\text{critical}} = \sqrt{\frac{2E\gamma_s}{\pi a_c}}$$

Griffith Equation extension to ductile materials:

$$\sigma_{\text{critical}} = \sqrt{\frac{2EG_c}{\pi a_c}}$$

G_c = critical strain energy release rate

Stress Intensity Factor:

$K = Y\sigma\sqrt{\pi a_c}$

Paris Equation (crack growth in steady state creep):

$$\frac{da}{dN} = C(\Delta K)^N$$

Composites

Isostrain:

$\epsilon_c = \epsilon_m = \epsilon_f$

$F_c = F_f + F_m$

$E_c = E_f f_f + E_m f_m$

$$\frac{F_f}{F_c} = \frac{f_f}{f_f + \frac{E_m}{E_f} f_m}$$

Isostress:

$\sigma_c = \sigma_m = \sigma_f$

$\epsilon_c = \epsilon_m f_m + \epsilon_f f_f$

$$E_c = \frac{E_m E_f}{E_m f_f + E_f f_m}$$

Electrical Properties

Resistance in a straight conductor:

$$R = \frac{\rho l}{A}$$

Transmission Line Sag:

$$\delta \approx \frac{wl^2}{8H}$$

$L = l + \frac{8\delta^2}{3l}$

$L_{\text{total}} = L(1 + \epsilon_\sigma + \epsilon_T)$

Conductivity in metals and semiconductors:

$\sigma = n|e|\mu_e + p|e|\mu_p$

For metals, $p = 0$. For intrinsic semiconductors, $n = p$.

Updated December 6, 2020
https://github.com/DonneyF/formula-sheets