MECH 360 Formula Sheet

Stress & Strain

Average normal stress:
$$\sigma = \frac{P}{A}$$

Average shear stress:
$$\tau = \frac{V}{A}$$

Double shear:
$$\tau = \frac{P}{2A}$$

Bearing stress:
$$\sigma_b = \frac{P}{A} = \frac{P}{td}$$

Stresses on a 2-force member
(θ measured from vertical):
$$\sigma = \frac{P}{A_\perp} \cos^2 \theta \qquad \tau = \frac{P}{A_\perp} \sin \theta \cos \theta$$

Factor of safety:
$$\text{Factor of safety} = \frac{\text{Ultimate Load}}{\text{Allowable Load}}$$

Normal strain:
$$\epsilon = \frac{\delta}{L} = \frac{d\delta}{dx}$$

Local shear strain (Change of $\pi/2$):
$$\gamma = \pi/2 - \theta$$

Axial Load

Hooke's Law and Modulus of Elasticity:
$$\sigma = E\epsilon$$

Elastic deformation under axial loading:
$$\delta = \frac{FL}{AE} = \sum_i \frac{F_i L_i}{A_i E_i}$$

Temperature change:
$$\delta_T = L_o \alpha \Delta T$$

Poisson's Ration:
$$\nu = -\frac{\epsilon_{\text{lat}}}{\epsilon_{\text{long}}}$$

Shear Stress-Strain Diagrams:
$$G = \frac{E}{2(1 + \nu)}$$
$$\tau = G\gamma \qquad \text{(elastic region)}$$

Elastic Strain Energy:
$$u = \int_0^\sigma \sigma \, d\epsilon = \frac{1}{2} \frac{\sigma^2}{E}$$

Torsion

Polar Moment of Inertia:
$$J = \int r^2 \, dA$$
$$J = \frac{\pi c^4}{2} \qquad \text{(full tube)}$$

$$J = \frac{\pi}{2}(c^4 - a^4) \qquad \text{(hollow tube)}$$

Shear Stress:
$$\tau = \frac{T\rho}{J} \qquad \tau_{\max} = \frac{Tc}{J}$$

Power:
$$P = T\omega$$

Angle of Twist:
$$\phi = \frac{TL}{JG} = \int_0^L \frac{T(x)}{J(x)G(x)} \, dx$$

Stress Concentrations:
$$\tau_{\max} = K \frac{Tc}{J}$$

Bending

Distributed Load Intensity at each point:
$$w = \frac{dV}{dx}$$

Shear at each point:
$$V = \frac{dM}{dx}$$

Normal Strain:
$$\epsilon_x = -\frac{y}{p} = -\frac{y}{c} \epsilon_{\max}$$

Normal Stress:
$$\sigma = -\frac{y}{c} \sigma_{\max}$$
$$\sigma = \frac{My}{I}$$
$$\sigma_{\max} = \frac{Mc}{I}$$

Second Moment of Inertia:
$$I = \int y^2 \, dA$$
Circle: $I = \frac{\pi}{4} r^4$

Rectangle: $I = \frac{1}{12} bh^3$

Neutral Axis:
$$\int y \, dA = 0 \qquad \overline{Y} = \frac{\Sigma_i^n \overline{y} A}{\Sigma_i^n A}$$

Section Modulus:
$$S = I/c$$

Parallel Axis Theorem:
$$I_\parallel = I_G + Md^2$$

Stress Transformations

$$\sigma_x' = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\sigma_y' = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cos 2\theta - \tau_{xy} \sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

Principal Stresses:

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Maximum In-Plane Shear Stress:

$$\tau_{\max} = R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Angle of Principal In-Plane Stresses:
$$\tan 2\theta_p = \frac{2\tau_{xy}}{(\sigma_x - \sigma_y)}$$

Angle of Maximum In-Plane Stresses:
$$\tan 2\theta_s = -\frac{(\sigma_x - \sigma_y)}{2\tau_{xy}}$$

3D Analysis:
$$\tau_{\max} = \frac{1}{2} |\sigma_{\max} - \sigma_{\min}|$$

Theories of Failure

Maximum-Shearing-Stress Criterion for a yield strength σ_Y and principal stresses σ_a, σ_b:
   Same sign: $|\sigma_a| < \sigma_Y$ and $|\sigma_b| < \sigma_Y$
   Opposite sign: $|\sigma_a - \sigma_b| < \sigma_Y$

Distortion Energy per unit volume:
$$u_d = \frac{1}{6G}(\sigma_a^2 - \sigma_a \sigma_b + \sigma_b^2)$$

Maximum Distortion Energy Criterion:
$$\sigma_a^2 - \sigma_a \sigma_b + \sigma_b^2 < \sigma_Y^2$$

Thin-Walled Pressure Vessel Stress

Cylindrical

Hoop Stress:
$$\sigma_1 = \frac{pr}{t}$$

Longitudinal Stress:
$$\sigma_2 = \frac{pr}{2t}$$

Maximum In-Plane Shearing Stress:
$$\tau_{\max} = \frac{1}{2}\sigma_2 = \frac{pr}{4t}$$

Maximum Out-of-Plane Shearing Stress (45°rotation around a longitudinal axis):
$$\tau_{\max} = \frac{pr}{2t}$$

Circle

$$\sigma_1 = \sigma_2 \frac{pr}{2t}$$

Maximum In-Plane Shearing Stress:
$$\tau_{\max} = 0 \text{ (reduces to a point)}$$

Maximum Out-of-Plane Shearing Stress (45°rotation around a longitudinal axis):
$$\tau_{\max} = \frac{1}{2}\sigma_1 = \frac{pr}{4t}$$