MATH 318 Formula Sheet

Probability Theory

Probability Function:

- $0 \le P \le 1$
- $P(S) = 1$
- $E_1 \cap E_2 = \emptyset \implies P(E_1 \cup E_2) = P(E_1) + P(E_2)$
- $P(E_1) + P(E_2) = P(E_1 \cup E_2) + P(E_1 \cap E_2)$

Conditional Probability:

$$P(E|F) = \frac{P(E \cap F)}{P(F)}$$

Two events are said to be independent if:

$$P(E \cap F) = P(E)P(F)$$

Theorem 1. Let $F_1, F_2 \dots F_n$ be a partition of the sample space S. Assume $F_i \cap F_j = \emptyset$ for any $i \neq j$. Then for any event $E \subset S$,

1. $P(E) = \sum_i^n P(E|F_i)P(F_i)$
2. $P(F_j|E) = \dfrac{P(E|F_j)P(F_j)}{\sum_i^n P(E|F_i)P(F_i)}$ (Bayes' Formula)

Random Variables

Memory-less Property:

$$P(X > m + n|X > n) = P(x > m)$$

Expectation Value:

$$\langle X \rangle = \textstyle\sum_{i=0}^{\infty} x_i p(X = x_i) = \sum_{i=0}^{\infty} x_i p(x_i)$$
$$\langle X \rangle = \textstyle\int_{-\infty}^{\infty} x f(x)\, dx$$

Cumulative Distribution Function:

$$F(x) = \textstyle\int_{-\infty}^{x} f(t), dt$$

Law of the Unconscious Statistician:

$$\langle g(X) \rangle = \textstyle\int_{-\infty}^{\infty} g(x) f(x)\, dx$$

Linearity of Expectation:

$$\langle aX + b \rangle = a \langle X \rangle + b$$

Moments:

$$n\text{-th moment of } X = \begin{cases} \int_{-\infty}^{\infty} x^n f(x)\, dx \\ \sum_i^{\infty} x_i^n p(x_i) \end{cases}$$

Variance:

$$\mathrm{Var}(X) = \left\langle (X - \langle X \rangle)^2 \right\rangle = \left\langle X^2 \right\rangle - \langle X \rangle^2$$

Joint Continuity:

$$P((X,Y) \in C) = \textstyle\iint_C f(x,y)\, dx\, dy$$

Marginal Distribution:

$$P(X \in A) = P(X \in A, Y \in \mathbb{R}) = \textstyle\int_A \int_{-\infty}^{\infty} f(x,y)\, dy\, dx$$

Independence:

If X, Y are independent, then

$$P(X \le a, Y \le b) = P(X \le a)P(Y \le b)$$
$$\langle g(X)h(Y) \rangle = \langle g(X) \rangle \langle h(Y) \rangle$$

Covariance:

$$\mathrm{Cov}(X,Y) = \langle (X - \langle X \rangle)(Y - \langle Y \rangle) \rangle = \langle XY \rangle - \langle X \rangle \langle Y \rangle$$

Correlation Coefficient:

$$\rho(X,Y) = \frac{\mathrm{Cov}(X,Y)}{\sqrt{\mathrm{Var}(X)\mathrm{Var}(Y)}} \in [-1, 1]$$

Cauchy-Swartz Inequality:

$$|\langle XY \rangle|^2 \le \left\langle X^2 \right\rangle \left\langle Y^2 \right\rangle$$

Sum of Random Variables:

$$\mathrm{Var}(X + Y) = \mathrm{Var}(X) + \mathrm{Var}(Y) + 2\mathrm{Cov}(X,Y)$$
$$F_{X+Y}(a) = P(X + Y \le a) =$$
$$\int_{-\infty}^{\infty} \int_{-\infty}^{x+y} f_{X+Y}(x,y)\, dx\, dy = \int_{-\infty}^{\infty} f_x(a-y) f_Y(y)\, dy$$

Characteristic Functions

Extracting Moments: $\dfrac{d^n}{dx^n}|_{t=0}\phi(t) = \langle i^n X^n \rangle$

Inversion Theorem:

$$f(x) = \tfrac{1}{2\pi} \textstyle\int_{-\infty}^{\infty} \phi(t) e^{-itx}\, dt$$

Shifting:

$$\phi_{aX+b}(t) = e^{itb} \phi_X(at)$$

Convergence in Distribution:

$$\lim_{n\to\infty} F_n(x) = F(x) \ \ \forall x \text{ cont.} \iff X_n \xrightarrow{D} X$$

Reserved

Updated February 19, 2020
https://github.com/DonneyF/formula-sheets

Random Variables

Distribution	Mass/Density Function	Mean	Variance	Characteristic Function
Binomial(n, p)	$p(i) = \binom{n}{i} p^i (1-p)^{n-i}$	np	$np(1-p)$	$(1-p+e^{it})^n$
Geometric(p)	$p(k) = (1-p)^{k-1} p$	$1/p$	$\dfrac{1-p}{p^2}$	$\dfrac{pe^{it}}{1-(1-p)e^{it}}$
Poisson(λ)	$p(i) = \dfrac{\lambda^i}{i!} e^{-\lambda}$	λ	λ	$e^{\lambda(e^{it}-1)}$
Uniform(a, b)	$f(x) = \begin{cases} \frac{1}{b-a} & x \in [a, b] \\ 0 & \text{otherwise} \end{cases}$	$\dfrac{a+b}{2}$	$\dfrac{(b-a)^2}{12}$	$\dfrac{e^{ita} - e^{itb}}{it(b-a)}$
Exponential(λ)	$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & x < 0 \end{cases}$	$1/\lambda$	$1/\lambda^2$	$\dfrac{\lambda}{\lambda - it}$
Normal(μ, σ^2)	$f(x) = \dfrac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$	μ	σ^2	$e^{i\mu t - \sigma^2 t^2/2}$