# ELEC 404 Formula Sheet

## Basic Concepts in RF Design

Signal power in dBm:

$$P_{sig|\mathrm{dBm}} = 10\log\left(\frac{P_{sig}}{1\text{ mW}}\right)$$

### Non-Linearity

Peak 1-dB compression point:

$$P_{1\mathrm{dB}} = \sqrt{0.145\left|\frac{\alpha_1}{\alpha_3}\right|}$$

Input Third Intercept Point:

$$A_{IIP3} = \sqrt{\frac{4}{3}\left|\frac{\alpha_1}{\alpha_3}\right|}$$

$$\frac{A_{IIP3}}{P_{1\mathrm{dB}}} \approx 9.6\mathrm{dB}$$

Cascaded Non-Linear Stages:

$$A_{IIP3} = \sqrt{\frac{4}{3}\left|\frac{\alpha_1\beta_1}{\alpha_3\beta_1 + 2\alpha_1\alpha_2\beta_2 + \alpha_1^3\beta_3}\right|}$$

$$\frac{1}{A_{IIP3}^2} = \left|\frac{1}{A_{IIP3,1}^2} + \frac{3\alpha_2\beta_2}{2\beta_1} + \frac{\alpha_1^2}{A_{IIP3,2}^2}\right|$$

$$\frac{1}{A_{IIP3}^2} \approx \frac{1}{A_{IIP3,1}^2} + \frac{\alpha_1^2}{A_{IIP3,2}^2} + \frac{\alpha_1^2\beta_1^2}{A_{IIP3,3}^2} + \cdots$$

### Noise

Output Spectrum of a linear, time invariant system:

$$S_y(f) = S_x(f)|H(f)|^2$$

Resistor Noise Power Spectral Density:

$$\overline{V_n^2} = 4kTR \qquad \overline{I_n^2} = 4kTR$$

MOSFET Noise Power Spectral Density ($\gamma \approx 2/3$):

$$\overline{V_n^2} = 4kT\gamma/g_m \qquad \overline{V_n^2} = 4kT\gamma g_m$$

MOSFET $1/f$ noise corner frequency for some process constant $K$:

$$f_c = \frac{K}{WLC_{ox}}\frac{g_m}{4kT\gamma}$$

Noise Figure:

$$\mathrm{NF} = \frac{SNR_{in}}{SNR_{out}} \qquad \mathrm{NF}|_{\mathrm{dB}} = 10\log\left(\frac{SNR_{in}}{SNR_{out}}\right)$$

Attenuation Factor:

$$\alpha = \frac{Z_{in}}{Z_{in} + R_S}$$

Noise figure as the total noise at the output divided by the noise at the output due to the source impedance ($\overline{V_n^2}$ is the output noise of device):

$$\mathrm{NF} = \frac{1}{\overline{V_{RS}^2}} \cdot \frac{\overline{V_{RS}^2}|\alpha|^2A_v^2 + \overline{V_n^2}}{|\alpha|^2A_v^2} = 1 + \frac{\overline{V_n^2}}{|\alpha|^2A_v^2} \cdot \frac{1}{\overline{V_{RS}^2}}$$

Available power gain:

$$A_P = \frac{\text{Power delivered to a matched load by stage}}{\text{Power delivered to a matched load by source}}$$

Noise figure of cascaded stages (Friis' Equation):

$$\mathrm{NF}_{tot} = 1 + (\mathrm{NF}_1 - 1) + \frac{\mathrm{NF}_2 - 1}{A_{P1}} + \cdots + \frac{\mathrm{NF}_m - 1}{A_{P1}\cdots A_{P(m-1)}}$$

### Passive Impedance Transformation

Capacitor Quality Factor:

$$Q_S = \frac{1}{R_SC\omega} \qquad Q_P = \frac{R_PC\omega}{1}$$

Inductor Quality Factor:

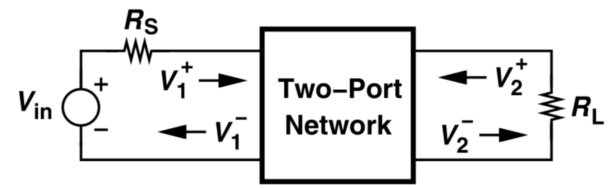
$$Q_S = \frac{L\omega}{R_S} \qquad Q_P = \frac{R_P}{L\omega}$$

Series to Parallel Conversion ($Q_P = Q_S$):

$$R_P = (Q_S^2 + 1)R_S$$

$$C_P = \left(\frac{Q_S^2}{Q_S^2 + 1}\right)C_S \qquad L_P = \left(\frac{Q_S^2 + 1}{Q_S^2}\right)L_S$$

### Scattering Parameters




$$V_1^- = S_{11}V_1^+ + S_{12}V_2^+$$
$$V_2^- = S_{21}V_1^+ + S_{22}V_2^+$$

Accuracy of input matching:

$$S_{11} = \frac{V_1^-}{V_1^+}|_{V_2^+=0} \approx \frac{Z_{in} - R_S}{Z_{in} + R_S}$$

Reverse isolation:

$$S_{12} = \frac{V_1^-}{V_2^+}|_{V_1^+=0}$$

Accuracy of output matching:

$$S_{22} = \frac{V_2^-}{V_2^+}|_{V_1^+=0} \approx \frac{Z_{out} - R_S}{Z_{out} + R_S}$$

Circuit gain:

$$S_{21} = \frac{V_2^-}{V_2^+}|_{V_2^+=0}$$

Scattering parameters in dB:

$$S_{mn|\mathrm{dB}} = 20\log|S_{mn}|$$

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Updated April 21, 2021
https://github.com/DonneyF/formula-sheets