PHYS 403 Formula Sheet

Thermodynamics

Potential	Function	Differential
Internal	U	$dU = T\,dS - p\,dV$
Enthalpy	$H = U + pV$	$dH = T\,dS + V\,dp$
Helmoltz	$F = U - TS$	$dF = -S\,dT$
Gibbs	$G = U - TS + pV$	$dG = -S\,dT + V\,dp$

Internal Energy:
$$\Delta U = Q + W$$
First Law of Thermodynamics:
$$dU = \delta Q - p\,dV$$
Reversible Process:
$$dU = T\,dS$$
$$dS = \frac{\delta Q}{T}$$
Heat Capacities:
$$C_V = \left(\frac{\partial U}{\partial T}\right)_V \qquad C_p = \left(\frac{\partial H}{\partial T}\right)_p$$
Entropy:
$$S = k_B \ln(W)$$

The Canonical Distribution

Boltzmann Factor:
$$e^{-\beta E_i}$$
The Canonical Distribution:
$$P_i = Z^{-1} e^{-\beta E_i}$$
Partition Function with degeneracy g_j:
$$Z = \sum_i g_i e^{-\beta E_i}$$
Continuous Partition function for degeneracy per unit volume $g(E)$:
$$Z = V \int_0^\infty g(E) e^{-\beta E}\, dE$$
Mean energy of a microsystem:
$$\langle E \rangle = \sum_i P_i E_i = -\frac{1}{Z}\frac{\partial Z}{\partial \beta}$$
Mean energy of a macrosystem with N identical and weakly interacting microsystems:
$$U = N\langle E \rangle$$
Many-particle partition function:
$$Z_N = Z_1^N$$
Free Energy:
$$Z_N = e^{-\beta F}$$
Entropy for probability $P_i = Z_N^{-1} e^{-\beta E_i}$ that the system is in the i-th macrostate:
$$S = -k_B \sum_i P_i \ln(P_i)$$

N Distinguishable Particles in a Box

Degeneracies for spin s:
$$g_{1D}(E) = (2s+1)\frac{2}{\hbar}\sqrt{\frac{m}{2E}}$$
$$g_{2D}(E) = (2s+1)\frac{m}{2\pi\hbar^2}$$
$$g_{3D}(E) = (2s+1)\frac{1}{4\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}\sqrt{E}$$

The Grand Canonical Distribution

Chemical Potential:
$$\mu = -T\left(\frac{\partial S}{\partial N}\right)_{U,V} = \left(\frac{\partial F}{\partial N}\right)_{T,V}$$
Gibbs Factor:
$$e^{\beta(n\mu - E_i)}$$
The Grand Canonical Distribution:
$$P(n, E_i) = \Xi^{-1} e^{\beta(n\mu - E_i)}$$
Grand Partition Function:
$$\Xi = \sum_{i,n} e^{\beta(n\mu - E_i)}$$
Absolute Activity:
$$\alpha = e^{\beta\mu}$$
Mean number of particles:
$$\langle n \rangle = \Xi^{-1}\sum_{i,n} n e^{\beta(n\mu - E_i)} = \frac{1}{\beta\Xi}\left(\frac{\partial \Xi}{\partial \mu}\right)_T = \frac{\alpha}{\Xi}\left(\frac{\partial \Xi}{\partial \alpha}\right)_T$$
Mean energy:
$$\langle E \rangle = -\frac{1}{\Xi}\left(\frac{\partial \Xi}{\partial \beta}\right)_T + \mu\langle n \rangle$$

Quantum and Classical Gasses

One-particle distribution function over energy:
$$f(E) = \langle n(E) \rangle$$
Fermi-Dirac Distribution:
$$f(E) = \frac{1}{e^{\beta(E-\mu)} - 1}$$
Total number of particles:
$$N = \int_0^\infty V g(E) f(E)\, dE$$
Fermi Energy ($\mu \approx E_F$ at very low temperatures):
$$n = \int_0^{E_F} g(E)\, dE$$
Bose-Einstein Distribution:
$$f(E) = \frac{1}{e^{\beta(E-\mu)} - 1} = \frac{1}{\alpha^{-1} e^{\beta E} - 1}$$
Maxwell-Boltzmann Distribution ($f(E) \ll 1$):
$$f(E) = e^{\beta(\mu - E)}$$

Updated April 26, 2021
https://github.com/DonneyF/formula-sheets