# PHYS 304 Formula Sheet

## The Wave Function

Time dependent Schrodinger Equation:

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + V\Psi$$

Standard Deviation:

$$\sigma = \sqrt{\langle j^2\rangle - \langle j\rangle^2}$$

Momentum:

$$\langle p\rangle = m\frac{d\langle x\rangle}{dt} = \int \Psi^*\left(\frac{\hbar}{i}\frac{\partial}{\partial x}\right)\Psi\,dx$$

## Infinite Square Well

Time Independent Schrodinger Equation:

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V\psi = E\psi$$

Eigenstate Expansion:

$$\Psi(x,t) = \sum_{n=1}^{\infty} c_n\psi_n(x)e^{-iE_nt/\hbar} = \sum_{n=1}^{\infty} c_n\Psi_n(x,t)$$

Energy In Infinite Square Well:

$$E_n = \frac{n^2\pi^2\hbar^2}{2ma^2}$$

Stationary States:

$$\psi_n(x) = \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi}{a}x\right)$$

Determining Coefficients:

$$c_n = \sqrt{\frac{2}{a}}\int_0^a \sin\left(\frac{n\pi}{a}x\right)\Psi(x,0)\,dx$$

Expectation Value of Energy:

$$\langle H\rangle = \sum_{n=1}^{\infty}|c_n|^2E_n$$

## Harmonic Oscillator

$$k = \omega^2 m$$

Ladder Operators:

$$a_\pm = \frac{1}{\sqrt{2\hbar m\omega}}(\mp ip + m\omega x)$$

$$a_+\psi_n = \sqrt{n+1}\,\psi_{n+1}$$

$$a_-\psi_n = \sqrt{n}\,\psi_{n-1}$$

Operators:

$$x = \sqrt{\frac{\hbar}{2m\omega}}(a_+ + a_-)$$

$$p = i\sqrt{\frac{\hbar m\omega}{2}}(a_+ - a_-)$$

$$x^2 = \frac{\hbar}{2m\omega}\left[(a_+)^2 + (a_+a_-) + (a_-a_+) + (a_-)^2\right]$$

Commutation:

$$[x,p] = i\hbar$$

$$[a_-,a_+] = 1$$

Hamiltonian:

$$H = \hbar\omega\left(a_+a_- + \frac{1}{2}\right)$$

$$a_+a_- + a_-a_+ = 2\left(\frac{H}{\hbar\omega}\right)$$

States:

$$\psi_0(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{-\frac{m\omega}{2\hbar}x^2}$$

$$\psi_n = \frac{1}{\sqrt{n!}}(a_+)^n\psi_0$$

Energy:

$$E_n = \left(n + \tfrac{1}{2}\right)\hbar\omega$$

## Generalized Statistical Interpretation

Momentum Expansion:

$$\Phi(p,t) = \frac{1}{\sqrt{2\pi\hbar}}\int_{-\infty}^{\infty} e^{-ipx/\hbar}\,\Psi(x,t)\,dx$$

$$\Psi(x,t) = \frac{1}{\sqrt{2\pi\hbar}}\int_{-\infty}^{\infty} e^{ipx/\hbar}\,\Phi(p,t)\,dp$$

## Angular Momentum

$$L_x = yp_z - zp_y \qquad L_y = zp_x - xp_z \qquad L_z = xp_y - yp_x$$

Commutators:

$$[L_x,L_y] = i\hbar L_z \qquad [L_y,L_z] = i\hbar L_x \qquad [L_z,L_x] = i\hbar L_y$$

$$[L_z,x] = i\hbar y \qquad [L_z,y] = -i\hbar x \qquad [L_z,z] = 0$$

$$[L_z,p_x] = i\hbar p_y \qquad [L_z,p_y] = -i\hbar p_x \qquad [L_z,p_z] = 0$$

Square of Angular Momentum:

$$L^2 \equiv L_x^2 + L_y^2 + L_z^2 \qquad [L^2,\mathbf{L}] = 0$$

Ladder Operators:

$$L_\pm = L_x \pm iL_y \qquad [L_z,L_\pm] = \pm i\hbar L_\pm$$

$$L^2 = L_\pm L_\mp + L_z^2 \mp \hbar L_z$$

$$L_x = \frac{L_+ + L_-}{2} \qquad L_y = \frac{L_+ - L_-}{2i}$$

$$L_+|l,l\rangle = 0 \qquad L_-|l,-l\rangle = 0$$

Eigenvalues:

$$L^2|l,m\rangle = \hbar^2 l(l+1)|l,m\rangle$$

$$L_z|l,m\rangle = \hbar m|l,m\rangle$$

$$L_\pm|l,m\rangle = A_l^m|l,m+1\rangle$$

$$A_l^m = \hbar\sqrt{l(l+1) - m(m\pm 1)}$$

Expectation Value when $L_x|\psi\rangle = L_y|\psi\rangle$:

$$\langle\psi|L_x^2|\psi\rangle = \langle\psi|L_y^2|\psi\rangle = \frac{1}{2}\langle\psi|L^2 - L_z^2|\psi\rangle$$

Spherical Coordinate Representation:

$$L_x = \frac{\hbar}{i}\left(-\sin\phi\frac{\partial}{\partial\theta} - \cos\phi\cot\theta\frac{\partial}{\partial\phi}\right)$$

$$L_y = \frac{\hbar}{i}\left(+\cos\phi\frac{\partial}{\partial\theta} - \sin\phi\cot\theta\frac{\partial}{\partial\phi}\right)$$

$$L_z = \frac{\hbar}{i}\frac{\partial}{\partial\theta}$$

$$L_\pm = \pm\hbar e^{\pm i\theta}\left(\frac{\partial}{\partial\theta} \pm i\cot\theta\frac{\partial}{\partial\phi}\right)$$

$$L_+L_- = -\hbar^2\left(\frac{\partial^2}{\partial\theta^2} + \cot\theta\frac{\partial}{\partial\theta} + \cot^2\theta\frac{\partial^2}{\partial\phi^2} + i\frac{\partial}{\partial\phi}\right)$$

$$L^2 = -\hbar^2\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\phi^2}\right]$$

## Trig Identities

$$2\cos\theta\cos\phi = \cos(\theta-\phi) + \cos(\theta+\phi)$$

$$2\sin\theta\sin\phi = \cos(\theta-\phi) - \cos(\theta+\phi)$$

$$2\sin\theta\cos\phi = \sin(\theta+\phi) + \sin(\theta-\phi)$$

$$2\cos\theta\sin\phi = \sin(\theta+\phi) - \sin(\theta-\phi)$$

## Integral Identities

$$\int \sin^2 ax\,dx = \frac{x}{2} - \frac{1}{4a}\sin 2ax + C$$

$$\int \sin^3 ax\,dx = \frac{\cos 3ax}{12a} - \frac{3\cos ax}{4a} + C$$

$$\int \cos^2 ax\,dx = \frac{x}{2} + \frac{1}{4a}\sin 2ax + C$$

$$\int \cos^3 ax\,dx = \frac{\sin ax}{a} - \frac{\sin^3 ax}{3a} + C$$

$$\int \cos^n(ax)\sin(ax)\,dx = -\frac{\cos^{n+1}(ax)}{a(n+1)} + C$$

$$\int \sin^n(ax)\cos(ax)\,dx = \frac{\sin^{n+1}(ax)}{a(n+1)} + C$$

$$\int_0^\pi \cos^{2n+1}(ax)\sin(ax)\,dx = 0 \qquad n = 0,1,2\ldots$$

$$\int_0^\pi \cos^{2n}(x)\sin(x)\,dx = \frac{2}{2n+1} \qquad n = 0,1,2\ldots$$

Updated March 9, 2019
https://github.com/DonneyF/formula-sheets