

MECH 360 Formula Sheet

Stress & Strain

Average normal stress:

$$\sigma = \frac{P}{A}$$

Average shear stress:

$$\tau = \frac{V}{A}$$

Double shear:

$$\tau = \frac{P}{2A}$$

Bearing stress:

$$\sigma_b = \frac{P}{A} = \frac{P}{td}$$

Stresses on a 2-force member

(θ measured from vertical):

$$\sigma = \frac{P}{A_{\perp}} \cos^2 \theta \quad \tau = \frac{P}{A_{\perp}} \sin \theta \cos \theta$$

Factor of safety:

$$\text{Factor of safety} = \frac{\text{Ultimate Load}}{\text{Allowable Load}}$$

Normal strain:

$$\epsilon = \frac{\delta}{L} = \frac{d\delta}{dx}$$

Local shear strain (Change of $\pi/2$):

$$\gamma = \pi/2 - \theta$$

Axial Load

Hooke's Law and Modulus of Elasticity:

$$\sigma = E\epsilon$$

Elastic deformation under axial loading:

$$\delta = \frac{FL}{AE} = \sum_i \frac{F_i L_i}{A_i E_i}$$

Temperature change:

$$\delta T = L_o \alpha \Delta T$$

Poisson's Ratio:

$$\nu = -\frac{\epsilon_{\text{lat}}}{\epsilon_{\text{long}}}$$

Shear Stress-Strain Diagrams:

$$G = \frac{E}{2(1+\nu)} \\ \tau = G\gamma \quad (\text{elastic region})$$

Elastic Strain Energy:

$$u = \int_0^{\sigma} \sigma d\epsilon = \frac{1}{2} \frac{\sigma^2}{E}$$

Torsion

Polar Moment of Inertia:

$$J = \int r^2 dA$$

$$J = \frac{\pi c^4}{2} \quad (\text{full tube})$$

$$J = \frac{\pi}{2} (c^4 - a^4) \quad (\text{hollow tube})$$

Shear Stress:

$$\tau = \frac{T\rho}{J} \quad \tau_{\text{max}} = \frac{Tc}{J}$$

Power:

$$P = T\omega$$

Angle of Twist:

$$\phi = \frac{TL}{JG} = \int_0^L \frac{T(x)}{J(x)G(x)} dx$$

Stress Concentrations:

$$\tau_{\text{max}} = K \frac{Tc}{J}$$

Bending

Distributed Load Intensity at each point:

$$w = \frac{dV}{dx}$$

Shear at each point:

$$V = \frac{dM}{dx}$$

Normal Strain:

$$\epsilon_x = -\frac{y}{\rho} = -\frac{y}{c} \epsilon_{\text{max}}$$

Normal Stress:

$$\sigma = -\frac{y}{c} \sigma_{\text{max}}$$

$$\sigma = \frac{My}{I} \quad \sigma_{\text{max}} = \frac{Mc}{I}$$

Second Moment of Inertia:

$$I = \int y^2 dA$$

$$\text{Circle: } I = \frac{\pi}{4} r^4$$

$$\text{Rectangle: } I = \frac{1}{12} bh^3$$

Neutral Axis:

$$\int y dA = 0 \quad \bar{Y} = \frac{\sum_i^n \bar{y} A}{\sum_i^n A}$$

Section Modulus:

$$S = I/c$$

Parallel Axis Theorem:

$$I_{\parallel} = I_G + Md^2$$

Composite Beams:

$$n = \frac{E_2}{E_1} \text{ for } E_2 > E_1$$

$$\sigma_2 = n\sigma_1$$

Product of Inertia:

$$I_{xy} = \int xy dA$$

Stress Transformations

$$\sigma'_x = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\theta + \tau_{xy} \sin 2\theta$$

$$\sigma'_y = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cos 2\theta - \tau_{xy} \sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2} \sin 2\theta + \tau_{xy} \cos 2\theta$$

Principal Stresses:

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Maximum In-Plane Shear Stress:

$$\tau_{\text{max}} = R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Angle of Principal In-Plane Stresses:

$$\tan 2\theta_p = \frac{2\tau_{xy}}{(\sigma_x - \sigma_y)}$$

Angle of Maximum In-Plane Stresses:

$$\tan 2\theta_s = -\frac{(\sigma_x - \sigma_y)}{2\tau_{xy}}$$

3D Analysis:

$$\tau_{\text{max}} = \frac{1}{2} |\sigma_{\text{max}} - \sigma_{\text{min}}|$$

Theories of Failure

Maximum-Shearing-Stress Criterion for a yield strength σ_Y and principal stresses σ_a, σ_b :

Same sign: $|\sigma_a| < \sigma_Y$ and $|\sigma_b| < \sigma_Y$

Opposite sign: $|\sigma_a - \sigma_b| < \sigma_Y$

Distortion Energy per unit volume:

$$u_d = \frac{1}{6G} (\sigma_a^2 - \sigma_a \sigma_b + \sigma_b^2)$$

Maximum Distortion Energy Criterion:

$$\sigma_a^2 - \sigma_a \sigma_b + \sigma_b^2 < \sigma_Y^2$$

Thin-Walled Pressure Vessel Stress

Cylindrical

Hoop Stress:

$$\sigma_1 = \frac{pr}{t}$$

Longitudinal Stress:

$$\sigma_2 = \frac{pr}{2t}$$

Maximum In-Plane Shearing Stress:

$$\tau_{\text{max}} = \frac{1}{2} \sigma_2 = \frac{pr}{4t}$$

Maximum Out-of-Plane Shearing Stress (45° rotation around a longitudinal axis):

$$\tau_{\text{max}} = \frac{pr}{2t}$$

Circle

$$\sigma_1 = \sigma_2 = \frac{pr}{2t}$$

Maximum In-Plane Shearing Stress:

$$\tau_{\text{max}} = 0 \text{ (reduces to a point)}$$

Maximum Out-of-Plane Shearing Stress (45° rotation around a longitudinal axis):

$$\tau_{\text{max}} = \frac{1}{2} \sigma_1 = \frac{pr}{4t}$$

Updated October 14, 2018

<https://github.com/DonneyF/formula-sheets>