MECH 360 Formula Sheet

Stress & Strain

Average normal stress:
$$\sigma = \frac{P}{A}$$
Average shear stress:
$$\tau = \frac{V}{A}$$
Double shear:
$$\tau = \frac{P}{2A}$$
Bearing stress:
$$\sigma_b = \frac{P}{A} = \frac{P}{td}$$
Stresses on a 2-force member
(θ measured from vertical):
$$\sigma = \frac{P}{A_\perp}\cos^2\theta \qquad \tau = \frac{P}{A_\perp}\sin\theta\cos\theta$$
Factor of safety:
$$\text{Factor of safety} = \frac{\text{Ultimate Load}}{\text{Allowable Load}}$$
Normal strain:
$$\epsilon = \frac{\delta}{L} = \frac{d\delta}{dx}$$
Local shear strain (Change of $\pi/2$):
$$\gamma = \pi/2 - \theta$$

Axial Load

Hooke's Law and Modulus of Elasticity:
$$\sigma = E\epsilon$$
Elastic deformation under axial loading:
$$\delta = \frac{FL}{AE} = \sum_i \frac{F_i L_i}{A_i E_i}$$
Temperature change:
$$\delta_T = L_o \alpha \Delta T$$
Poisson's Ration:
$$\nu = -\frac{\epsilon_{\text{lat}}}{\epsilon_{\text{long}}}$$
Shear Stress-Strain Diagrams:
$$G = \frac{E}{2(1+\nu)}$$
$$\tau = G\gamma \qquad \text{(elastic region)}$$
Elastic Strain Energy:
$$u = \int_0^\sigma \sigma\, d\epsilon = \frac{1}{2}\frac{\sigma^2}{E}$$

Torsion

Polar Moment of Inertia:
$$J = \int r^2\, dA$$
$$J = \frac{\pi c^4}{2} \qquad \text{(full tube)}$$
$$J = \frac{\pi}{2}(c^4 - a^4) \qquad \text{(hollow tube)}$$
Shear Stress:
$$\tau = \frac{T\rho}{J} \qquad \tau_{\max} = \frac{Tc}{J}$$
Power:
$$P = T\omega$$
Angle of Twist:
$$\phi = \frac{TL}{JG} = \int_0^L \frac{T(x)}{J(x)G(x)}\, dx$$
Stress Concentrations:
$$\tau_{\max} = K\frac{Tc}{J}$$

Bending

Distributed Load Intensity at each point:
$$w = \frac{dV}{dx}$$
Shear at each point:
$$V = \frac{dM}{dx}$$
Normal Strain:
$$\epsilon_x = -\frac{y}{p} = -\frac{y}{c}\epsilon_{\max}$$
Normal Stress:
$$\sigma = -\frac{y}{c}\sigma_{\max}$$
$$\sigma = \frac{My}{I} \qquad \sigma_{\max} = \frac{Mc}{I}$$
Second Moment of Inertia:
$$I = \int y^2\, dA$$
$$\text{Circle: } I = \frac{\pi}{4}r^4$$
$$\text{Rectangle: } I = \frac{1}{12}bh^3$$
Neutral Axis:
$$\int y\, dA = 0 \qquad \overline{\text{Y}} = \frac{\sum_i^n \overline{y}A}{\sum_i^n A}$$
Section Modulus:
$$S = I/c$$
Parallel Axis Theorem:
$$I_\parallel = I_G + Md^2$$
Composite Beams:
$$n = \frac{E_2}{E_1} \text{ for } E_2 > E_1$$
$$\sigma_2 = n\sigma_1$$
Product of Inertia:
$$I_{xy} = \int xy\, dA$$
$$I_{x'} = I_x\cos^2\theta + I_y\sin^2\theta - I_{xy}\sin\theta\cos\theta$$
$$I_{y'} = I_x\sin^2\theta + I_y\cos^2\theta - I_{xy}\sin\theta\cos\theta$$
$$I_{x'y'} = I_x\sin\theta\cos\theta - I_y\sin\theta\cos\theta + I_{xy}(\cos^2\theta - \sin^2\theta)$$
Asymmetric Bending:
$$\sigma_x = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$$
$$\tan\phi = \frac{y}{z} = \frac{I_z}{I_y}\tan\theta$$

Stress Transformations

$$\sigma'_x = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$
$$\sigma'_y = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$
$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$
Principal Stresses:
$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$
Maximum In-Plane Shear Stress:
$$\tau_{\max} = R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$
Angle of Principal In-Plane Stresses:
$$\tan 2\theta_p = \frac{2\tau_{xy}}{(\sigma_x - \sigma_y)}$$
Angle of Maximum In-Plane Stresses:
$$\tan 2\theta_s = -\frac{(\sigma_x - \sigma_y)}{2\tau_{xy}}$$
3D Analysis:
$$\tau_{\max} = \tfrac{1}{2}|\sigma_{\max} - \sigma_{\min}|$$

Theories of Failure

Maximum-Shearing-Stress Criterion for a yield strength σ_Y and principal stresses σ_a, σ_b:

Same sign: $|\sigma_a| < \sigma_Y$ and $|\sigma_b| < \sigma_Y$

Opposite sign: $|\sigma_a - \sigma_b| < \sigma_Y$

Distortion Energy per unit volume:
$$u_d = \frac{1}{6G}(\sigma_a^2 - \sigma_a\sigma_b + \sigma_b^2)$$
Maximum Distortion Energy Criterion:
$$\sigma_a^2 - \sigma_a\sigma_b + \sigma_b^2 < \sigma_Y^2$$

Thin-Walled Pressure Vessel Stress

Cylindrical

Hoop Stress:
$$\sigma_1 = \frac{pr}{t}$$
Longitudinal Stress:
$$\sigma_2 = \frac{pr}{2t}$$
Maximum In-Plane Shearing Stress:
$$\tau_{\max} = \frac{1}{2}\sigma_2 = \frac{pr}{4t}$$
Maximum Out-of-Plane Shearing Stress (45°rotation around a longitudinal axis):
$$\tau_{\max} = \frac{pr}{2t}$$

Circle

$$\sigma_1 = \sigma_2\frac{pr}{2t}$$
Maximum In-Plane Shearing Stress:
$$\tau_{\max} = 0 \text{ (reduces to a point)}$$
Maximum Out-of-Plane Shearing Stress (45°rotation around a longitudinal axis):
$$\tau_{\max} = \frac{1}{2}\sigma_1 = \frac{pr}{4t}$$

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https://github.com/DonneyF/formula-sheets