PHYS 304 Formula Sheet

The Wave Function

Time dependent Schrodinger Equation:

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + V\Psi$$

Standard Deviation:

$$\sigma = \sqrt{\langle j^2\rangle - \langle j\rangle^2}$$

Momentum:

$$\langle p\rangle = m\frac{d\langle x\rangle}{dt} = \int \Psi^*\left(\frac{\hbar}{i}\frac{\partial}{\partial x}\right)\Psi\,dx$$

Infinite Square Well

Time Independent Schrodinger Equation:

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V\psi = E\psi$$

Eigenstate Expansion:

$$\Psi(x,t) = \sum_{n=1}^{\infty} c_n\psi_n(x)e^{-iE_nt/\hbar} = \sum_{n=1}^{\infty} c_n\Psi_n(x,t)$$

Energy In Infinite Square Well:

$$E_n = \frac{n^2\pi^2\hbar^2}{2ma^2}$$

Stationary States:

$$\psi_n(x) = \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi}{a}x\right)$$

Determining Coefficients:

$$c_n = \sqrt{\frac{2}{a}}\int_0^a \sin\left(\frac{n\pi}{a}x\right)\Psi(x,0)\,dx$$

Expectation Value of Energy:

$$\langle H\rangle = \sum_{n=1}^{\infty}|c_n|^2E_n$$

Harmonic Oscillator

$$k = \omega^2 m$$

Ladder Operators:

$$a_\pm = \frac{1}{\sqrt{2\hbar m\omega}}(\mp ip + m\omega x)$$

$$a_+\psi_n = \sqrt{n+1}\,\psi_{n+1}$$

$$a_-\psi_n = \sqrt{n}\,\psi_{n-1}$$

Operators:

$$x = \sqrt{\frac{\hbar}{2m\omega}}(a_+ + a_-)$$

$$p = i\sqrt{\frac{\hbar m\omega}{2}}(a_+ - a_-)$$

$$x^2 = \frac{\hbar}{2m\omega}\left[(a_+)^2 + (a_+a_-) + (a_-a_+) + (a_-)^2\right]$$

Commutation:

$$[x,p] = i\hbar$$

$$[a_-,a_+] = 1$$

Hamiltonian:

$$H = \hbar\omega\left(a_+a_- + \frac{1}{2}\right)$$

$$a_+a_- + a_-a_+ = 2\left(\frac{H}{\hbar\omega}\right)$$

States:

$$\psi_0(x) = \left(\frac{m\omega}{\pi\hbar}\right)^{1/4}e^{-\frac{m\omega}{2\hbar}x^2}$$

$$\psi_n = \frac{1}{\sqrt{n!}}(a_+)^n\psi_0$$

Energy:

$$E_n = \left(n+\tfrac{1}{2}\right)\hbar\omega$$

Generalized Statistical Interpretation

Momentum Expansion:

$$\Phi(p,t) = \frac{1}{\sqrt{2\pi\hbar}}\int_{-\infty}^{\infty} e^{-ipx/\hbar}\,\Psi(x,t)\,dx$$

$$\Psi(x,t) = \frac{1}{\sqrt{2\pi\hbar}}\int_{-\infty}^{\infty} e^{ipx/\hbar}\,\Phi(p,t)\,dp$$

Trig Identities

$$2\cos\theta\cos\phi = \cos(\theta-\phi) + \cos(\theta+\phi)$$

$$2\sin\theta\sin\phi = \cos(\theta-\phi) - \cos(\theta+\phi)$$

$$2\sin\theta\cos\phi = \sin(\theta+\phi) + \sin(\theta-\phi)$$

$$2\cos\theta\sin\phi = \sin(\theta+\phi) - \sin(\theta-\phi)$$

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https://github.com/DonneyF/formula-sheets