Efficient, General Point Cloud Registration With Kernel Feature Maps

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提纲

- 一、研究背景
- 二、主要方法
- 三、实验对比
- 四、讨论

一、研究背景

$$M_1 = \{x_i^{(1)}\}_{i=1}^{l_1}$$

$$M_2 = \{x_i^{(2)}\}_{i=1}^{l_2}$$

不同变种下的ICP，
$\omega_{i,j}$的表现形式不同；

$$\{\mathbf{R}^*, \mathbf{b}^*\} = \arg\min_{R,b} \sum_{i=1}^{l_1} \sum_{j=1}^{l_2} (\mathbf{R}x_i^{(1)} + \mathbf{b} - x_j^{(2)})^2 \boxed{\omega_{i,j}}$$

- Point correspondence and then???
  - ✓ Euclidean distance
- Simpler
- Cheaper
- ...

一、研究背景

论文方法的核心思想：

This method **first maps all points to a higher dimensional (reproducing kernel Hilbert) feature space using kernel methods.**

Problems:

1) **What is a higher dimensional feature space?**
2) **How to get this higher dimensional feature space with kernel methods?**
3) **How to do the registration in the higher dimensional feature space?**
    a) **Advantages and Disadvantages?**
    b) **Theory of this method**

一、研究背景

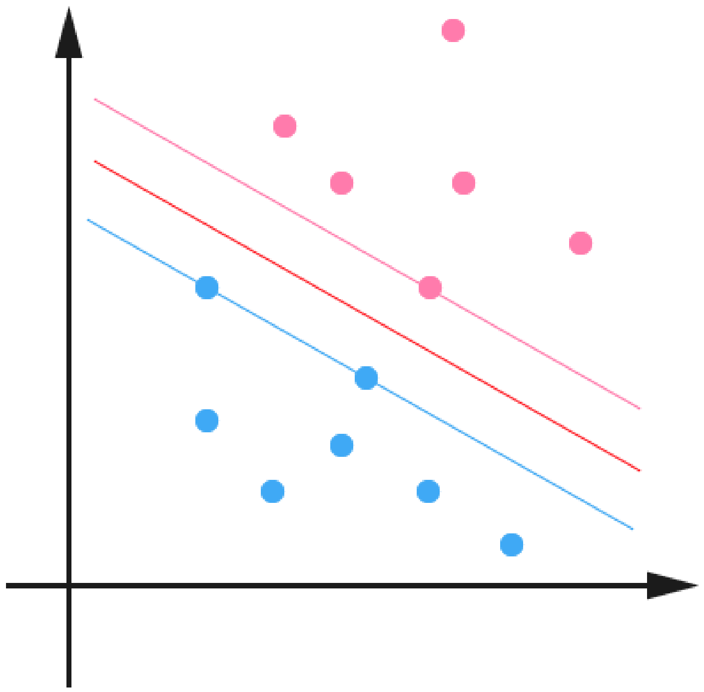
Problems:

1) **What is a higher dimensional feature space-Hilbert Feature Space ?**

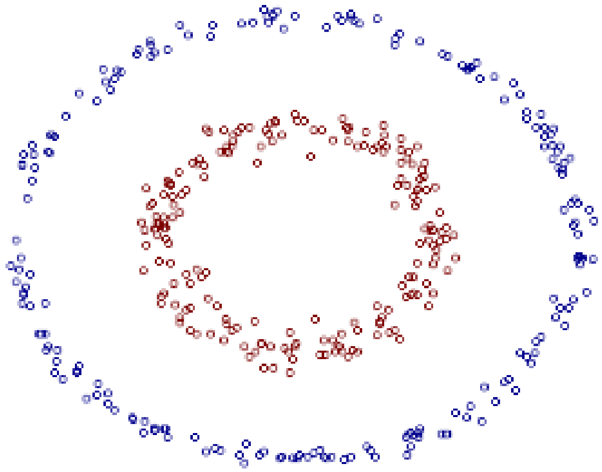
2) **How to get this higher dimensional feature space with kernel methods?**

We explain these using a machine learning method-support vector machine.

一、研究背景



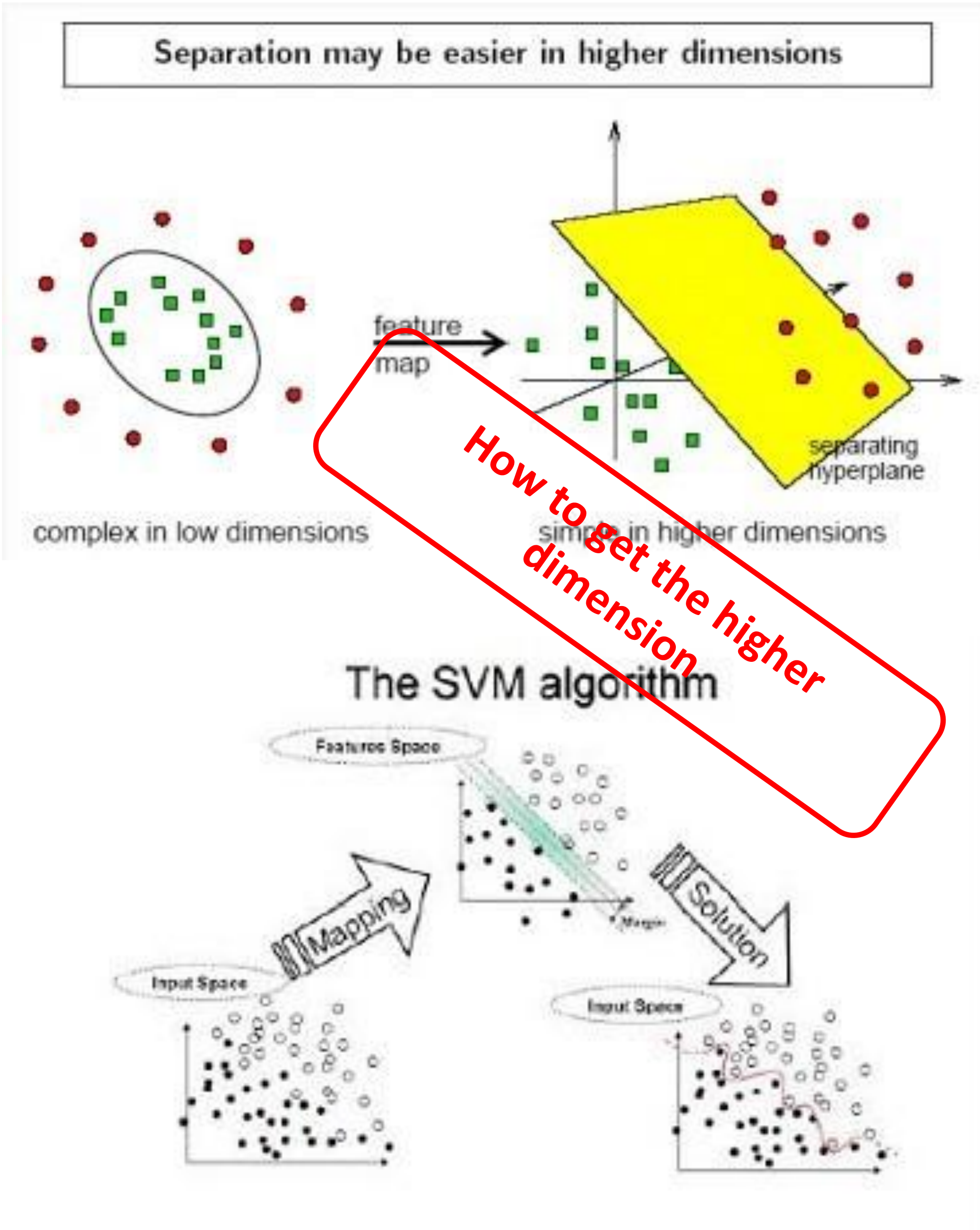
线性可分



线性不可分

SVM如何进行“线性不可分”的？

一、研究背景



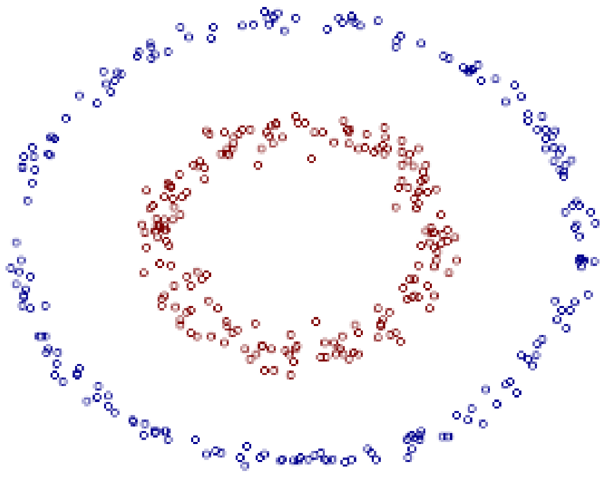
Low dimension-3d space

Feature space-higher dimension

Classification in feature space

Back into lower dimension -3d space

一、研究背景



在R^2空间中的二次曲线：

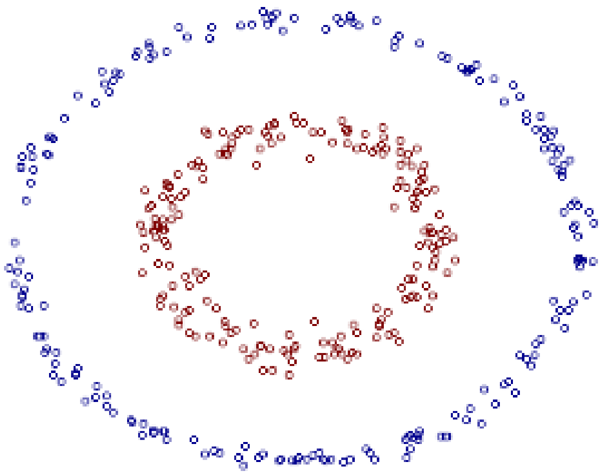
$$a_1X_1 + a_2X_1^{\ 2} + a_3X_2 + a_4X_2^{\ 2} + a_5X_1X_2 + a_6 = 0$$

建立映射关系，$Z_1 = X_1, Z_2 = X_1^{\ 2}, Z_3 = X_2, Z_4 = X_2^{\ 2}, Z_5 = X_1X_2$

$R^2 \to R^5$ 得到线性方程：

$$\sum_{i=1}^{5} a_i Z_i + a_6 = 0$$

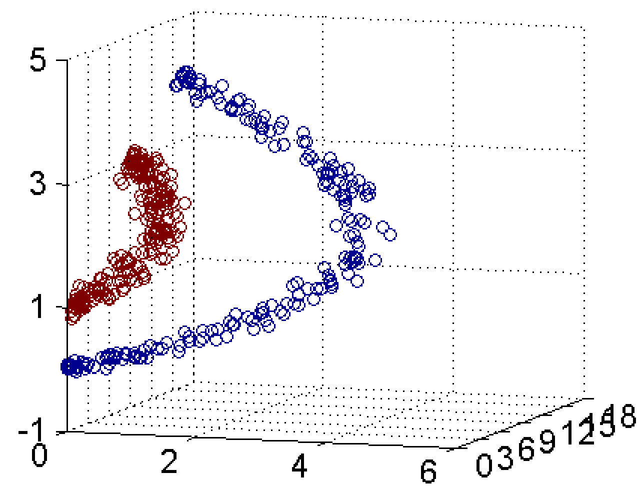
一、研究背景



简化一下：在R^2空间中，是2个不同半径的同心圆：

$$a_1X_1^2 + a_2(X_2 - c)^2 + a_3 = 0$$

建立映射关系，$Z_1 = X_1^2, Z_2 = X_2^2, Z_3 = X_2$



$R^2 \to R^3$ 得到平面方程：

$$\sum_{i=1}^{3} a_i Z_i = 0$$

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二、主要方法

Advantages:

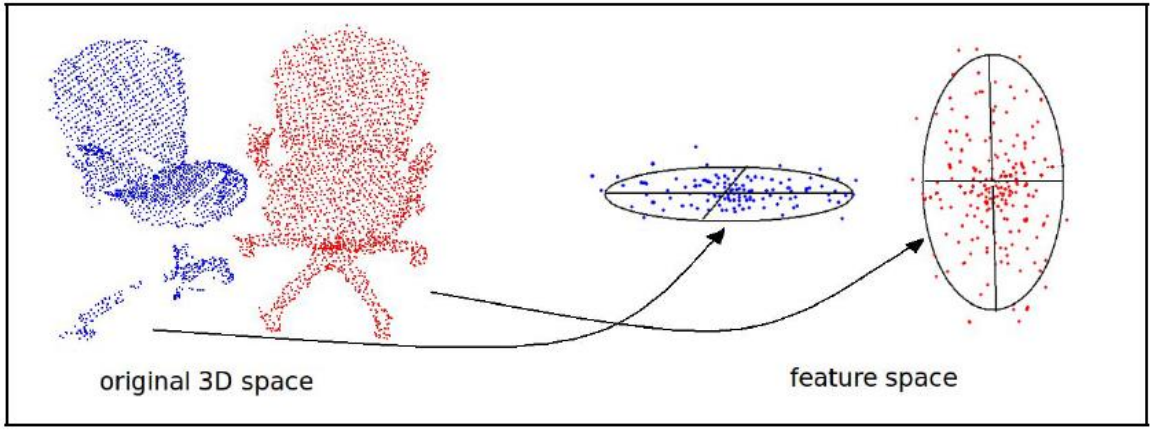

Figure 1. Mapping point clouds from 3D space to an infinite-dimensional Hilbert space, where a single Gaussian is sufficient to model distributions of complex shape.

- ✓ Gaussian in 3D space is too limited to capture the 3D point distribution of real-world objects
- ✓ Mapped to a much higher dimensional Hilbert feature space, **where a single Gaussian can fit well**

二、主要方法

Advantages:

Instead of computing the optimal alignment in 3D space directly, the alignment of two point clouds in feature space corresponds to **aligning two Gaussians.**

$R, b \rightarrow R_H, b_H$

$$\boxed{\begin{aligned} \mathbf{R}_{\mathcal{H}} &= \tilde{\mathbf{U}}_2 \tilde{\mathbf{U}}_1^\top \\ &= \phi(\mathbf{M}_2)^\top \underbrace{\mathbf{I}_2^{\mathbf{E}} \left(\sum_{k=1}^{\mathbf{D}} \tilde{\boldsymbol{\alpha}}_2^k \tilde{\boldsymbol{\alpha}}_1^{k\top} \right) \mathbf{I}_1^{\mathbf{E}}}_{\boldsymbol{\Theta}_{\boldsymbol{\alpha}}} \phi(\mathbf{M}_1) \end{aligned}}$$

Eigenvectors's calculation in feature space.

$$\mathbf{b}_{\mathcal{H}} = \boldsymbol{\mu}_{\mathcal{H}}^{(2)} = \frac{1}{l_2} \phi(\mathbf{M}_2)^\top \mathbf{1}_{l_2}$$

二、主要方法

Theory:

$\phi(\cdot)$ corresponds to an infinite-dimensional feature map.

$$K(\mathbf{x}_i, \mathbf{x}_j) = \langle \phi(\mathbf{x}_i), \phi(\mathbf{x}_j) \rangle$$

核函数：向量内积

$$K(\mathbf{x}_i, \mathbf{x}_j) = \exp \frac{-\|\mathbf{x}_i - \mathbf{x}_j\|^2}{2\sigma^2}$$

核函数-高斯核

$$\boldsymbol{\mu}_{\mathcal{H}} = \frac{1}{l} \sum_{i=1}^{l} \phi(\mathbf{x}_i) = \frac{1}{l} \phi(\mathbf{M})^\top \mathbf{1}_l$$

$$\boldsymbol{\Sigma}_{\mathcal{H}} = \frac{1}{l} \sum_{i=1}^{l} \left(\phi(\mathbf{x}_i) - \boldsymbol{\mu}_{\mathcal{H}}\right) \left(\phi(\mathbf{x}_i) - \boldsymbol{\mu}_{\mathcal{H}}\right)^\top$$

高维空间：

1） 期望；

2） 协方差；

二、主要方法

Theory:

$$\lambda_k \mathbf{u}_k = \boldsymbol{\Sigma}_{\mathcal{H}} \mathbf{u}_k.$$

特征值，特征向量的性质

$$\mathbf{u}_k = \frac{1}{\lambda_k} \boldsymbol{\Sigma}_{\mathcal{H}} \mathbf{u}_k = \sum_{i=1}^{l} \alpha_i^k \phi(\mathbf{x}_i)$$

$$\begin{aligned}
\sum_{j=1}^{l} \phi(\mathbf{x}_j)^\top \lambda_k \mathbf{u}_k &= \sum_{j=1}^{l} \phi(\mathbf{x}_j)^\top \boldsymbol{\Sigma}_{\mathcal{H}} \mathbf{u}_k \\
\Leftrightarrow \quad \lambda_k \sum_{i,j=1}^{l} \alpha_i^k K(\mathbf{x}_i, \mathbf{x}_j) &= \frac{1}{l} \sum_{i,j=1}^{l} \alpha_i^k K(\mathbf{x}_i, \mathbf{x}_j)^2 \\
\Leftrightarrow \quad l \lambda_k \boldsymbol{\alpha}^k &= \mathbf{K} \boldsymbol{\alpha}^k
\end{aligned}$$

和矩阵的数据特征相联系，
非零特征值可能有很多个；
对应的特征向量也随之增多，
为了求解的稳定和高效，进
而需要进行：
PCA-Principal Component
Analysis

$$\tilde{\mathbf{u}}_k = \sum_{i=1}^{l} \tilde{\alpha}_i^k \left(\phi(\mathbf{x}_i) - \boldsymbol{\mu}\right) = \phi(\mathbf{M})^\top \underbrace{\left(\mathbf{I}_l - \frac{1}{l}\mathbf{E}\right)}_{\mathbf{I}^{\mathbf{E}}} \tilde{\boldsymbol{\alpha}}^k$$

特征向量的最终表达式

二、主要方法

Theory:

In this paper, using KPCA-Kernel Principal Component Analysis

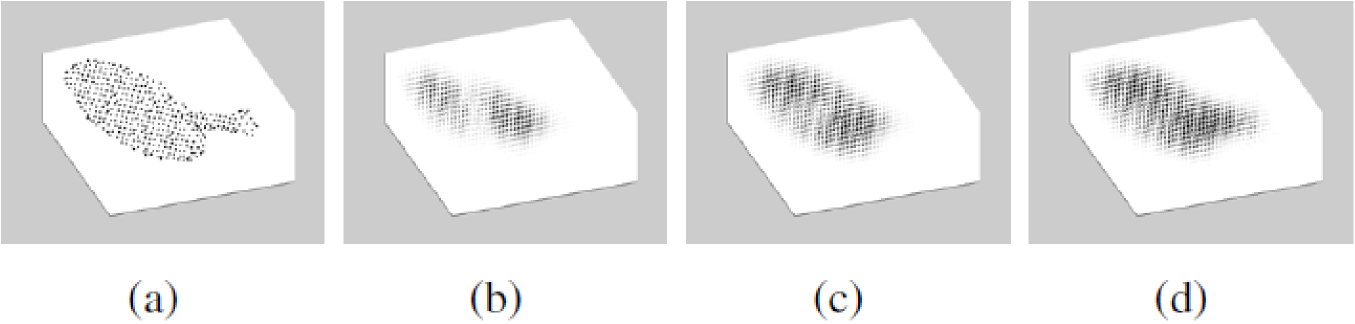

Figure 2. (a) A point cloud of table tennis racket; (b–d) reconstruction using the first 1–3 principal components. For each point in the bounding-box volume, the darkness is proportional to the density of the Gaussian in the feature space $\mathcal{H}$.

二、主要方法

Theory:

$$
\begin{tikzcd}
\mathbf{x}_t^{(1)} \arrow[rr, "\phi"] \arrow[d, "\{\mathbf{R},\mathbf{b}\}"'] & & \phi(\mathbf{x}_t^{(1)}) \arrow[d, "\{\mathbf{R}_{\mathcal{H}},\mathbf{b}_{\mathcal{H}}\}"] \\
\mathbf{R}\mathbf{x}_t^{(1)}+\mathbf{b} \arrow[r, "\phi"'] & \underbrace{\phi(\mathbf{R}\mathbf{x}_t^{(1)}+\mathbf{b})}_{\mathbf{\Phi}_t} \sim & \underbrace{\mathbf{R}_{\mathcal{H}}\tilde{\phi}(\mathbf{x}_t^{(1)})+\mathbf{b}_{\mathcal{H}}}_{\mathbf{\Psi}_t}
\end{tikzcd}
$$

Figure 3. The consistency error is defined as the discrepancy between $\phi(\mathbf{R}x_t + \mathbf{b})$ and $\mathbf{R}_{\mathcal{H}}\tilde{\phi}(x_t) + \mathbf{b}_{\mathcal{H}}$.

$$\{\mathbf{R}^*, \mathbf{b}^*\} = \arg\min_{\mathbf{R},\mathbf{b}} \frac{1}{l_1} \sum_{t=1}^{l_1} \|\mathbf{\Psi}_t - \mathbf{\Phi}_t\|^2$$

真正的目标泛函

$$\{\mathbf{R}^*, \mathbf{b}^*\} = \arg\max_{\mathbf{R},\mathbf{b}} \frac{1}{l_1} \sum_{t=1}^{l_1} \mathbf{\Psi}_t^\top \mathbf{\Phi}_t$$

$$\boldsymbol{\rho}_t = \mathbf{\Theta}_\alpha \left(K(\mathbf{x}_t^{(1)}, \mathbf{M}_1) - \frac{1}{l_1}\mathbf{K}_1 \mathbf{1}_{l_1} \right) + \frac{1}{l_2}\mathbf{1}_{l_2}$$

$$= \underbrace{\arg\max_{\mathbf{R},\mathbf{b}} \frac{1}{l_1} \sum_{t=1}^{l_1} K(\mathbf{R}\mathbf{x}_t^{(1)} + \mathbf{b}, \mathbf{M}_2)^\top \boldsymbol{\rho}_t}_{\mathbf{O}}$$

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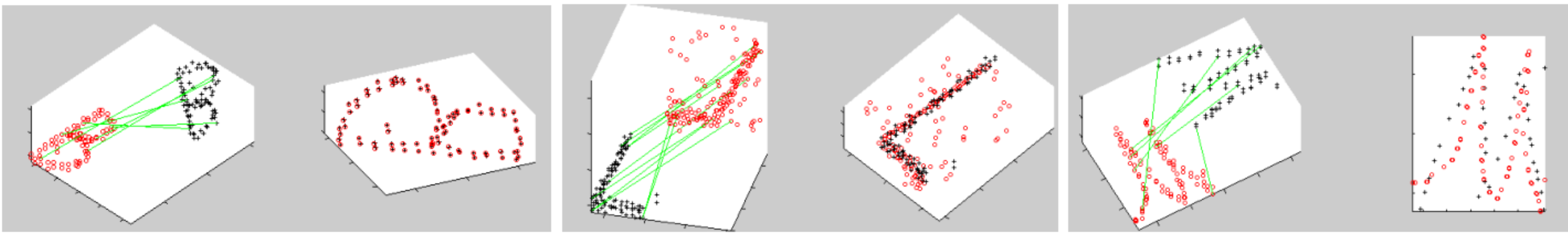
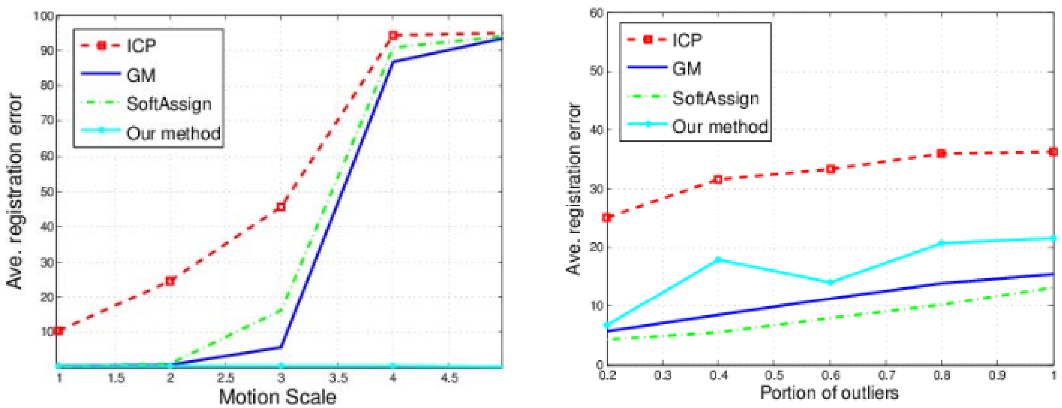
三、实验对比



(a) (b) (c)

Figure 6. Test of the proposed algorithm in typical challenging circumstances for registration: (a) large motion; (b) outliers; (c) nonrigid transformation

不同情景下的匹配情况。

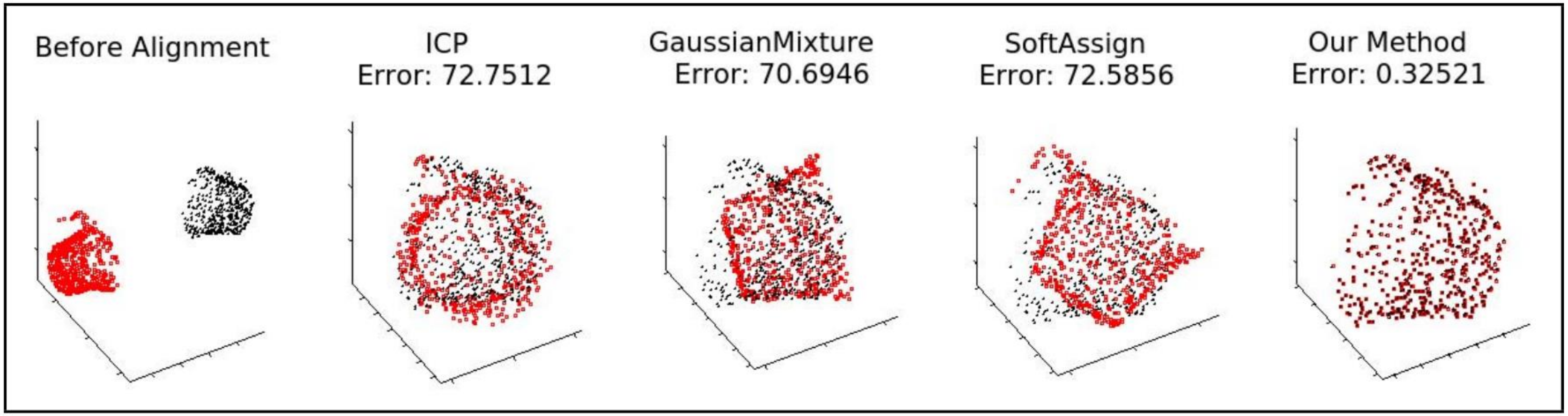


(a) (b)

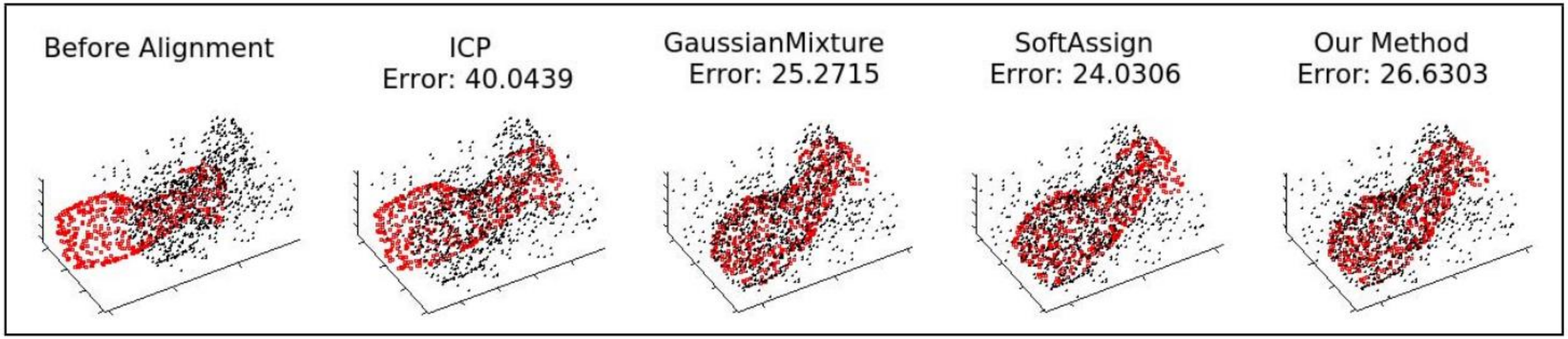
不同方法的性能对比。

Figure 8. Test of four registration algorithm on (a) different scales of motions; (b) different portion of outliers added.

三、实验对比



(a)



(b)

(a) motion scale i = 5; (b) outlier portion= 0.8

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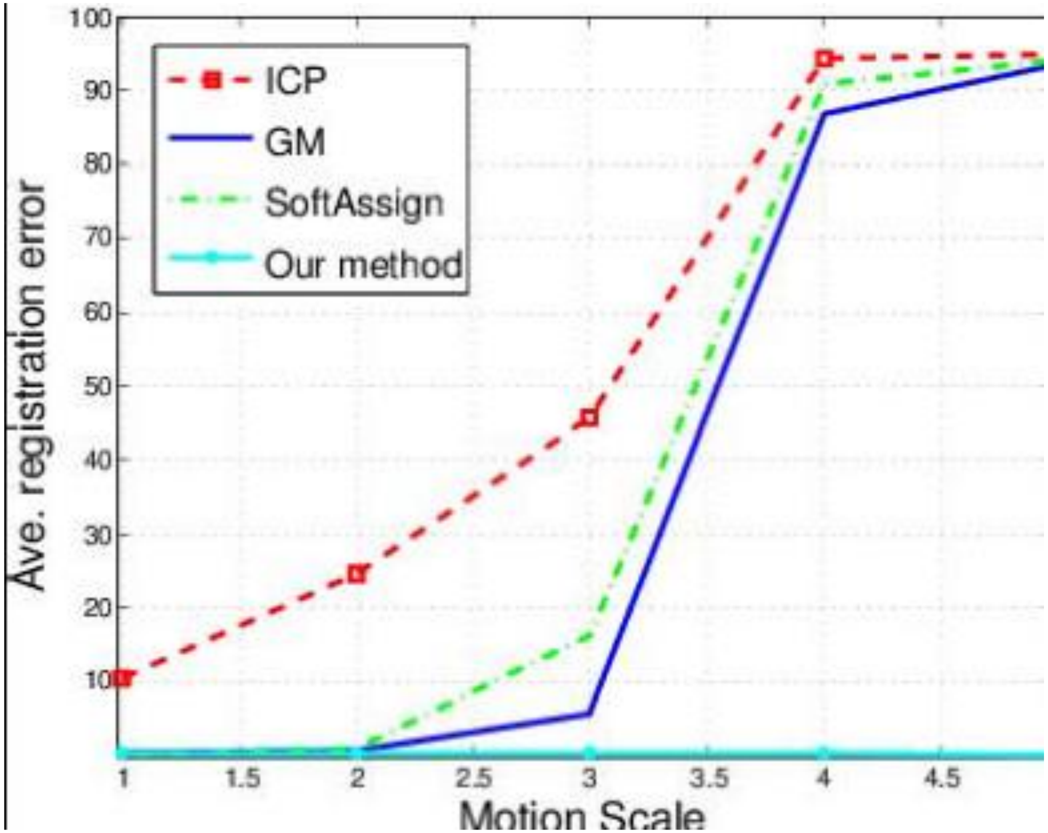
四、讨论

Good performance in large motion scale.



可能适合特定范围内的Registration.

四、讨论

分析和考虑其他核函数的性质：

- 多项式核
- 线性核
- 高斯核-本文所采用的方法
- 高斯核及其变种

Tks