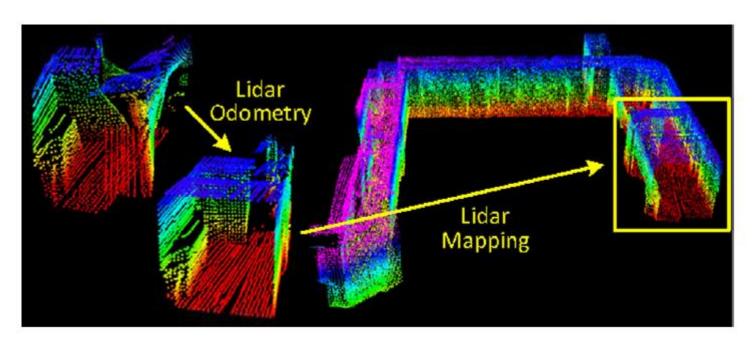
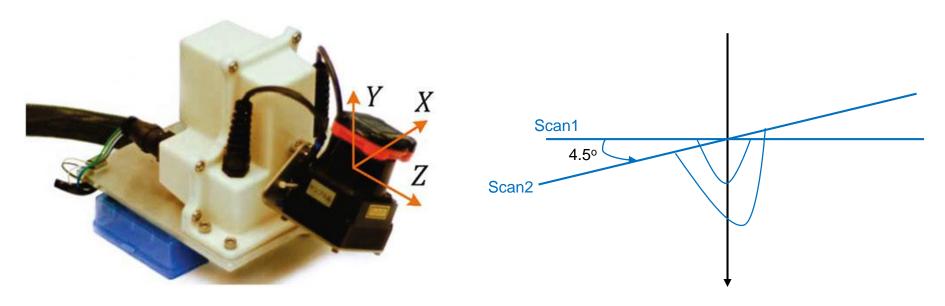
Lidar Odometry And Mapping

- Lidar Odometry
- □ Lidar Mapping





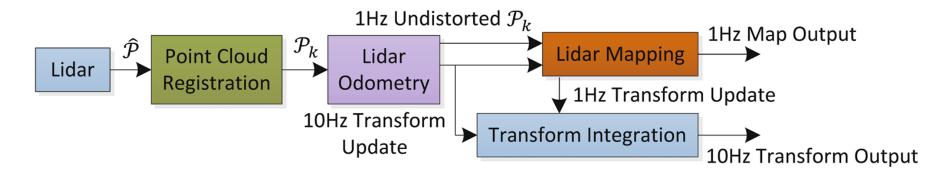
- 1) Motor- 1time/s, from -90° to 90°---One Sweep.
- 2) Lidar- has 180° field of view with 0.25° resolution and 40 lines/s scanning rate.
- 3) One sweep contains 40 scans.---40 scans== 40lines
- 4) Two coordinate system: lidar coordinate system $\{L\}$ world coordinate system $\{W\}$

Signs' meaning:

- 1) $\hat{\mathcal{P}}$ the points received in a laser scan.
- 2) \mathcal{P}_k the combined point cloud during sweep k
- 3) $X_{(k,i)}^{L}$ a point *i* received during sweep k
- 4) $m{T}_k^L(t)$ the transform projecting a point received at time t to the beginning of the sweep k

Problem Given a sequence of lidar cloud \mathcal{P}_k , $k \in \mathbb{Z}^+$, compute ego-motion of the lidar in the world, $\mathbf{T}_k^W(t)$, and build a map with \mathcal{P}_k for the traversed environment.

Block diagram of the lidar odometry and mapping software system



- 1) Transform?
- 2) 1HZ And 10HZ?

Feature point extraction

2D Example- focus on a scan plane

$$c = \frac{1}{|\mathcal{S}| \cdot ||X_{(k,i)}^L||} \left\| \sum_{j \in \mathcal{S}, j \neq i} (X_{(k,i)}^L - X_{(k,j)}^L) \right\|. \tag{1}$$

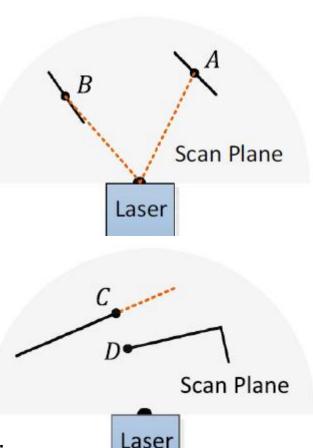
 $\mathcal S$ -be the set of consecutive points of i returned by the laser scanner in the same scan.

- 1) separate a scan into four identical subregions
- 2) Each subregion can provide maximally 2 edge points and 4 planar points
- 3) A point *i* can be selected as an edge or a planar point only if its *c* value is larger or smaller than a threshold-5*10⁻³

Feature point extraction

2D Example- focus on a scan plane-Trick

able. To avoid the aforementioned points to be selected, we find again the set of points S. A point i can be selected only if S does not form a surface patch whose normal is within 10° to the laser beam, and there is no point in S that is disconnected from i by a gap in the direction of the laser beam and is at the same time closer to the lidar then point i (e.g. point S in Fig. 4b).

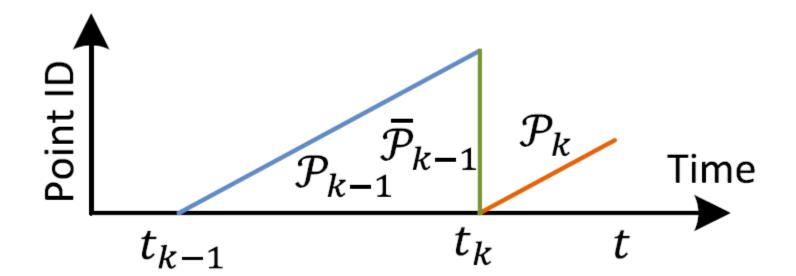


Point B and C are usually considered as unreliable points.

Feature point extraction-summary

- The number of selected edge points or planar points cannot exceed the maximum of the subregion, and
- None of its surrounding point is already selected, and
- It cannot be on a surface patch whose normal is within 10° to the laser beam, or on boundary of an occluded region.

Finding feature point correspondence



Finding feature point correspondence

Signs' meaning:

- 1) \mathcal{E}_k the sets of edge points during sweep k
- 2) \mathcal{H}_k the sets of planar points during sweep k
- 3) \mathcal{E}_k , \mathcal{H}_k projected into $\tilde{\mathcal{E}}_k$, $\tilde{\mathcal{H}}_k$
- 4) combine $ilde{\mathcal{E}}_k$, $ilde{\mathcal{H}}_k$ with the points in $ar{\mathcal{P}}_{k-1}$
- 5) $\bar{\mathcal{P}}_{k-1}$ the projected point cloud. undistorted points modified by the transform.
- 6) Transform algorithm plays the important role.

Finding feature point correspondence-Edge Points

Find the edge line-(two points) as the correspondence of an edge point:

$$i \in \tilde{\mathcal{E}}_k$$

j - the closest neighbor of i in $\bar{\mathcal{P}}_{k-1}$

I - be the closest neighbor of *i* in the preceding and following two scans to the scan of j

verify both *j* and *l* are edge points or not.

Finding feature point correspondence-Planar Points

Find the planar patch-(three points) as the correspondence of a planar point:

$$i \in \tilde{\mathbf{H}}_k$$

j - the closest neighbor of i in $\bar{\mathcal{P}}_{k-1}$

I - be the closest neighbor of i, but in the same scan of j

m – be the closest neighbor of i, but in the preceding and following scans to the scan of j

verify both *j*, *l*, *m* are planar points or not.

Finding feature point correspondence

After finding the correspondence points:

$$d\varepsilon = \frac{\left| (\tilde{X}_{(k,i)}^{L} - \bar{X}_{(k-1,j)}^{L}) \times (\tilde{X}_{(k,i)}^{L} - \bar{X}_{(k-1,l)}^{L}) \right|}{\left| \bar{X}_{(k-1,j)}^{L} - \bar{X}_{(k-1,l)}^{L} \right|}, \tag{2}$$
 Common line

$$d_{\mathcal{H}} = \frac{\left| \begin{array}{c} (\tilde{X}_{(k,i)}^{L} - \bar{X}_{(k-1,j)}^{L}) \\ ((\bar{X}_{(k-1,j)}^{L} - \bar{X}_{(k-1,l)}^{L}) \times (\bar{X}_{(k-1,j)}^{L} - \bar{X}_{(k-1,m)}^{L})) \\ \hline \left| (\bar{X}_{(k-1,j)}^{L} - \bar{X}_{(k-1,l)}^{L}) \times (\bar{X}_{(k-1,j)}^{L} - \bar{X}_{(k-1,m)}^{L}) \right| \end{array}}{\left| (\bar{X}_{(k-1,j)}^{L} - \bar{X}_{(k-1,l)}^{L}) \times (\bar{X}_{(k-1,j)}^{L} - \bar{X}_{(k-1,m)}^{L}) \right|}.$$
 Common plan (3)

Motion estimation

Formula 2 and 3- optimization problem

Signs' meaning:

- 1) t be the current time stamp
- 2) t_k is the starting time of the current sweep k
- 3) $T_k^L(t)$ be the lidar pose transform between t_k and t

4)
$$T_k^L(t) = [\tau_k^L(t), \theta_k^L(t)]^T$$

5)
$$\tau_k^L(t) = [t_x, t_y, t_z]^T$$

6)
$$\theta_k^L(t) = [\theta_x, \ \theta_y, \ \theta_z]^T$$

Motion estimation

Formula 2 and 3- optimization problem

The projection-transform:

$$\tilde{X}_{(k,i)}^{L} = \mathbf{R}_{(k,i)}^{L} X_{(k,i)}^{L} + \tau_{(k,i)}^{L},$$

$$f_{\mathcal{E}}(X_{(k,i)}^L, T_k^L(t)) = d_{\mathcal{E}}, \quad i \in \mathcal{E}_k.$$

$$f_{\mathcal{H}}(X_{(k,i)}^L, T_k^L(t)) = d_{\mathcal{H}}, \quad i \in \mathcal{H}_k.$$

$$f(T_k^L(t)) = d,$$

 $f_{\mathcal{E}}(X_{(k,i)}^L,T_k^L(t))=d_{\mathcal{E}},\quad i\in\mathcal{E}_k.$ $f(T_k^L(t))=d,$ We solve the lidar motion with the Levenberg-Marquardt method.

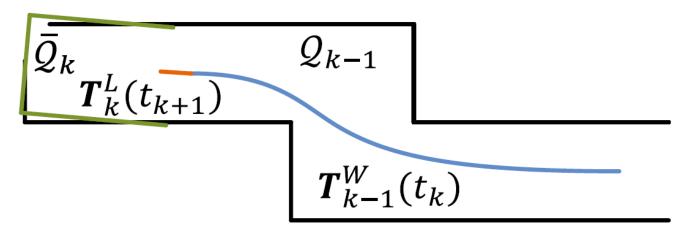
The mapping process

$$ar{\mathcal{Q}}_k egin{array}{c} \mathcal{Q}_{k-1} \ T_k^L(t_{k+1}) \ \end{array}$$

Sign Meaning:

- 1) $\bar{\mathcal{P}}_k$ a undistorted point cloud at the end of sweep k
- 2) $T_k^L(t_{k+1})$ a pose transform at the end of sweep k
- 3) Q_{k-1} the point cloud on the map, accumulated until sweep k -1
- 4) $T_{k-1}^{W}(t_k)$ the pose of the lidar on the map at the end of sweep k -1

The mapping process



 t_k . With the output from lidar odometry, the mapping algorithm extents $\boldsymbol{T}_{k-1}^W(t_k)$ for one sweep from t_k to t_{k+1} , to obtain $\boldsymbol{T}_k^W(t_{k+1})$, and transforms $\bar{\mathcal{P}}_k$ into the world coordinates, $\{W\}$, denoted as $\bar{\mathcal{Q}}_k$. Next, the algorithm matches $\bar{\mathcal{Q}}_k$ with \mathcal{Q}_{k-1} by optimizing the lidar pose $\boldsymbol{T}_k^W(t_{k+1})$.

Contrast the odometry with the mapping

Odometry

$$P_{k-1} \xrightarrow{transform} \overline{P}_{k-1}$$

$$P_{k} \xrightarrow{extract} \begin{cases} \mathcal{E}_{k} & \text{transform} \\ \rightarrow & \rightarrow \\ H_{k} & \end{cases} \begin{cases} \tilde{\mathcal{E}}_{k} \\ \tilde{H}_{k} \end{cases} form \ an \ optimization \ problem \ under \ the \ transform \ to \ get \ \overline{P}_{k} \ and \ T_{k}^{\ L}(t_{k+1})$$

Mapping

$$\begin{array}{c} \mathcal{G}_{k-1} \\ \overline{P}_{k} \xrightarrow{L \text{ to } W} \overline{\mathcal{G}}_{k} \xrightarrow{extract} \xrightarrow{extract} \xrightarrow{\mathcal{E}_{k}} \begin{cases} \mathcal{E}_{k} \\ \rightarrow \rightarrow \rightarrow \\ H_{k} \end{cases} \text{ form an optimization problem under the transform to get } \mathcal{G}_{k} \text{ and } T_{k}^{W}(t_{k+1}) \end{cases}$$

Contrast the odometry with the mapping

10Hz vs 1Hz

- 1) In mapping, 10 times of feature points are used.
- 2) Eigenvalue decomposition of covariance matrix.
 - a) if edge: to obtain the orientation of the edge line.
 - b) if planar: to obtain the orientation of the planar patch.
- 3) voxel-grid filter.

1.3 Example

1.3 LOAM-Example

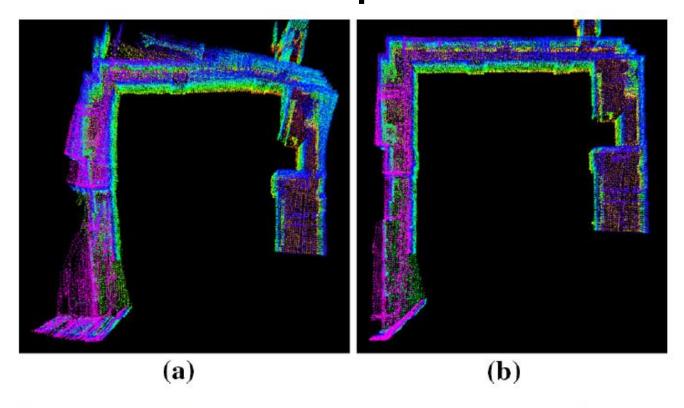
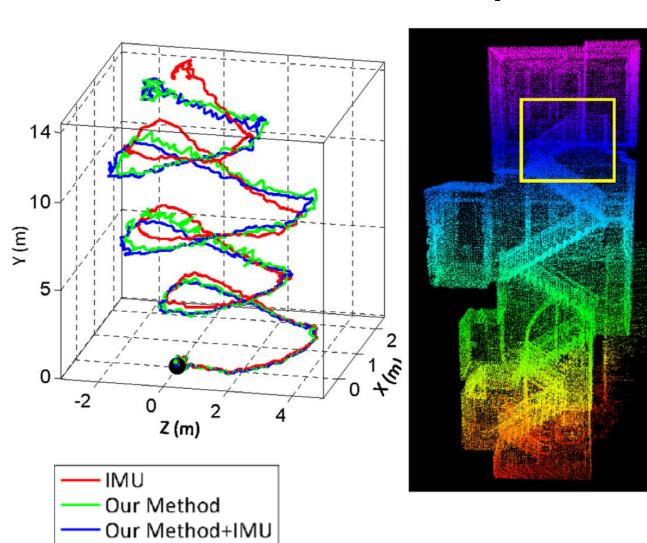
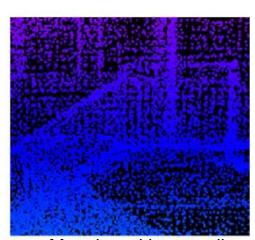


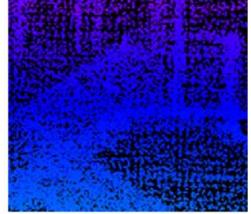
Fig. 12 Comparison between **a** lidar odometry output and **b** final lidar mapping output with the dataset in Fig. 1. The role of lidar odometry is to estimate velocity and remove motion distortion in point clouds. This algorithm has a low fidelity. Lidar mapping further performs careful scan matching to warrant accuracy on the map

1.3 LOAM-Example





Mapping with green line



Mapping with blue line