**Coursework 1**

**Deadline: Dec 15th**

**Exercise 1** (5 marks)**.** *Let $f \in C(\mathbb{R})$ satisfying $f(f(x)) = x$ for all $x \in \mathbb{R}$. Prove that $\exists \eta \in \mathbb{R}$ such that $f(\eta) = \eta$.*

**Exercise 2** (5 marks)**.** *Prove the generalized Rolle's theorem: Let $f \in C[a, +\infty) \cap D(a, +\infty)$ satisfying*

$$\lim_{x \to +\infty} f(x) = f(a)$$

*then $\exists \eta > a$ such that $f'(\eta) = 0$.*

**Exercise 3** (10 marks)**.** ***True or False:*** *(1) If $f(x)$ is differentiable and unbounded on $(a, b)$, then $f'(x)$ is unbounded on $(a, b)$.*

***True or False:*** *(2) If $f(x)$ is differentiable and $f'(x)$ is unbounded on $(a, b)$, then $f(x)$ is unbounded on $(a, b)$.*

*If you think the proposition is true, give a proof; if you think it is false, give an example. In the later case, you need also prove that your example works to disprove the proposition.*