Exercise 1. *Paraphrase the following statements in the form of* $\forall$ *and* $\exists$*:*

(1) There must be a maximum among finitely many real numbers;

(2) The retirement ages vary from different countries.

Exercise 2. *Prove by definition that* $[a, b)$ *is bounded.*

Exercise 3. *Let* $A \subset \mathbb{R}$ *be bounded,* $B \subset A$*. Prove that* B *is bounded.*

Exercise 4. *Let* $S_1, S_2 \subset \mathbb{R}$ *be bounded. Prove that* $S_1 \cup S_2$ *is bounded.*

Exercise 5. *(1) Give the definition of the* ***infimum*** *(the greatest lower bound) of* $S \subset \mathbb{R}$ *in the form of* $\forall$ *and* $\exists$*.*

(2) Prove that if T *has an upper bound, then* $U = \{x : -x \in T\}$ *has an lower bound, and* $\sup T = -\inf U$*.*

(3) Conclude the existence and the uniqueness of the infimum of a set.

Exercise 6. *Let* $S \subset \mathbb{R}$*. We say that* α *is the maximum of* S*, denoted as* $\alpha = \max S$*, if*

(a) $\forall s \in S$*,* $s \leq \alpha$*;*

(b) $\alpha \in S$*.*

Prove that

(1) $\max[-1, 1] = 1$*;*

(2) $S = [-1, 1)$ *has no maximum;*

(3) If $\max S$ *exists, then* $\max S = \sup S$*.*