Computer Assignment 2

1. Consider the function $f(x) = \frac{\sin x}{x} - \cos x$. What do you observe when you type `f(1e-8)` in `Matlab`? Explain your result. Re-write $f(x)$ for x near 0 so `f(1e-8)` will give you a more satisfactory result.

2. The continued fraction representation of the number 2 is given by

$$2 = 1 + \cfrac{2}{1 + \cfrac{2}{1 + \cfrac{2}{1 + \cfrac{2}{\ddots}}}}. \tag{1}$$

   What is the minimum number of fractions that you need in `MATLAB` to write the number 2 as a continued fraction?

3. Implement a code that read the excel file 'LDIstudents.xls' (in the 'assesments / computer assignments' folder in blackboard) and store the information of each student in a `MATLAB` `structure` that you will call `Student`. For each group, calculate the average mark as well as the standard deviation. Compute the correlation between the marks of all students and the 'Assistance to lectures', 'Assistance to tutorials', and the 'Height'.

4. Write a function that takes as an input a `double` array of positive numbers and outputs its geometric mean. What is the complexity of your algorithm? Using `rand`, `randn`, and `randi`, to corroborate that for a given array a of positive numbers,

$$\text{mean}(a) \geq \text{geometric\_mean}(a).$$

   Find a way to plot your results.

5. Consider the following function which depends on both spatial variables (x and y) and a time variable t.

$$G_\omega(x, y, t) = \frac{e^{ik\sqrt{x^2+y^2}}}{4\pi\sqrt{x^2+y^2}} e^{-i\omega t}.$$

   Here i is the imaginary unit. This function represents a 2D version of point source of an acoustic wave with frequency ω. Using `getfram(gcf)`, create a movie of `real(`$G_\omega(x, y, t)$`)` (the real part of G_ω), for x and y between -2π and 2π, and t starting from 0 to any time of your choice. You are free to choose any value for the frequency ω as well. Consider using `zlim`, `caxis` and an appropriate `shading` to make your movie looks better.

6. Write a recursive function that reverse the order of a 1D input array. Example: `recursiveReverseArray([1 2 3 4 5])` should output `[5 4 3 2 1]`.

7. The Legendre polynomials satisfy the following recurrence relation.

$$(2n + 1)xP_n(x) = (n + 1)P_{n+1}(x) + nP_{n-1}(x),$$

with $P_0(x) = 1$ and $P_1(x) = x$. Show that $P_n(x) = O(x^n)$ for $x \to \infty$.

Hint: use inductive/recursive thinking.

8. Write a function that takes as an input a 1D `double` array and returns the index of the smallest element in the array. What is the complexity of your algorithm?

9. The following is the implementation of the 'Selection Sort' algorithm seen in lectures

```
function out = selectionSort(arr)
n = length(arr);
for i = 1:n
    % Find the minimum element in the unsorted array
    [~,idx]= min(arr(i:end));
    min_idx = idx + i-1;
    % Swap the minimum element with the first
    if min_idx ~= i
        aux = arr(i);
        arr(i) = arr(min_idx);
        arr(min_idx) = aux;
    end
end
out = arr;
end
```

Replace the `min` built-in `MATLAB` function on the code above with your own implementation. What is the complexity of the Selection Sort algorithm using your own `min` function? Plot the average complexity for random inputs of varying length.

10. Re-write the following function to make it more efficient. Showcase your result in a plot.

```
function R = myFunction(M)
        n = size(M,1);
        for i= 1:n
            for j = 1:n
                R(i,j) = M(i,j) + M(j,i);
            end
        end
end
```