MA2252 Introduction to Computing

Lecture 13

Least Squares Regression

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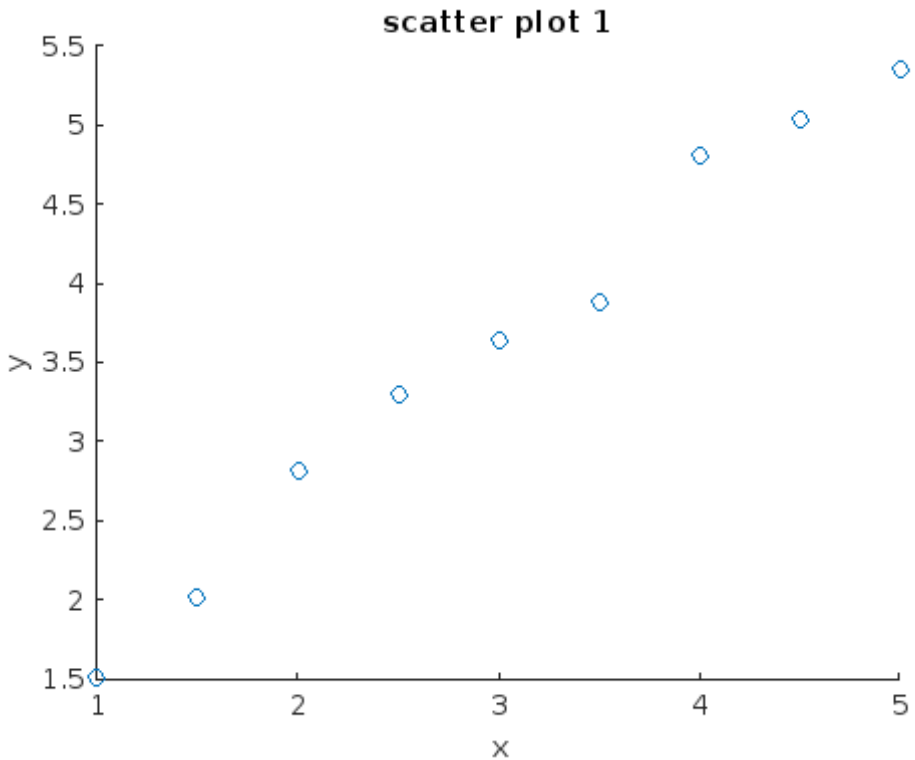
Learning outcomes

At the end of lecture, students will be able to

- understand scatter plot and regression
- understand theory of Least Squares Regression
- solve basic regression problems in MATLAB

Scatter plot

A **scatter plot** plots two different sets of data using dots. Unlike line plot, the dots are not connected by a curve.



Scatter plot (contd.)

MATLAB's scatter() function can be used to create a scatter plot.

Example: The code below creates the scatter plot shown earlier.

```
x=1:0.5:5; %create data for vector x
s=size(x); %find size of vector x
y=x+rand(s); %create a vector y=x+'some random values'
scatter(x,y) %create scatter plot
title('scatter plot 1')
xlabel('x')
ylabel('y')
```

Scatter plot (contd.)

Demo

Regression

Regression is a statistical technique used to find the 'best-fit curve' that describes a scatter plot.

Depending on the data trend in a scatter plot, one may use

- Linear Regression curve
- Non-linear Regression curve

Regression (contd.)

Linear Regression curve example

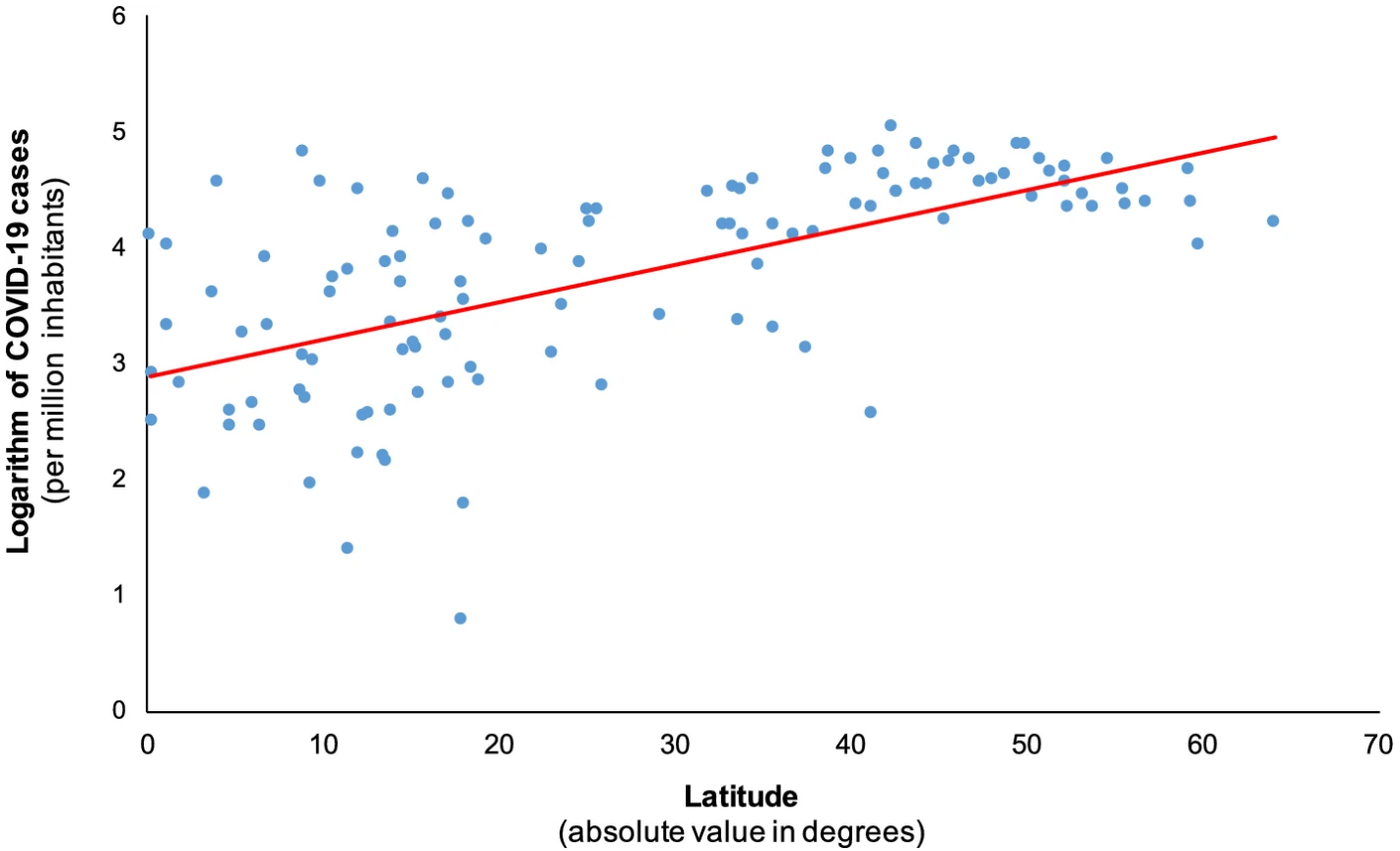


Figure: Scatter plot showing linear trend [1]

Regression (contd.)

Non-linear Regression curve example

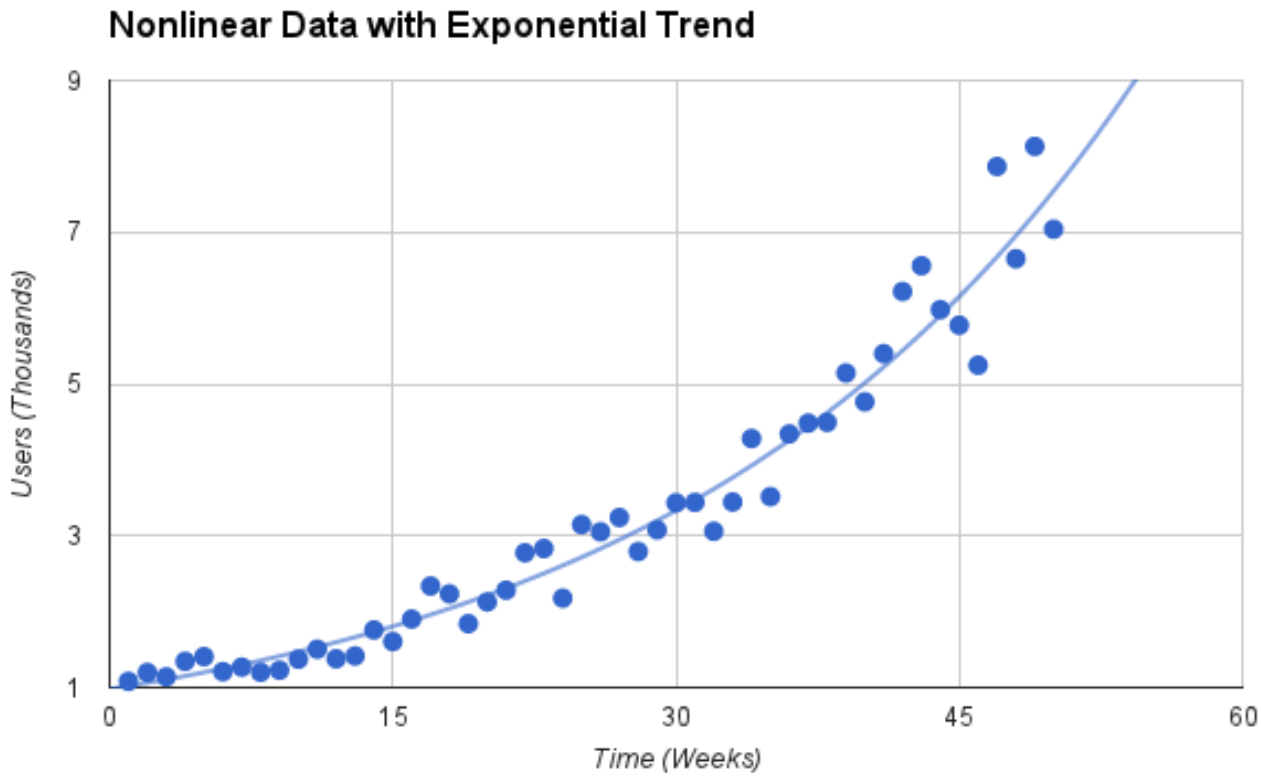


Figure: Data showing number of active users on a website with time [2]

[1] Chen, S., Prettner, K., Kuhn, M. et al. Climate and the spread of COVID-19. Sci Rep 11, 9042 (2021). https://doi.org/10.1038/s41598-021-87692-z

[2] http://sam-koblenski.blogspot.com

Regression model

A regression model provides a function to describe the relationship between one (or more) independent variables and a dependent variable.

A basic regression model is the ‘Least Squares Regression model’.

Least Squares Regression

Here, the relationship between dependent data points $y_i (i = 1, 2, ...m)$ and independent data points x_i is modelled as

$$\hat{y}(x) = \sum_{i=1}^{n} \alpha_i f_i(x) \tag{1}$$

where

- $\hat{y}(x)$ is an estimation function
- α_i are parameters of estimation function
- $f_i(x)$ are linearly independent basis functions

Least Squares Regression (contd.)

The parameters are then found by minimising the total squared error E.

$$E = \sum_{i=1}^{m} (\hat{y} - y_i)^2 \tag{2}$$

Substituting (1) in (2) gives

$$E = \sum_{i=1}^{m} \left(\sum_{j=1}^{n} \alpha_j f_j(x_i) - y_i \right)^2 \tag{3}$$

E is a function of n variables namely $\alpha_j (j = 1, 2, \cdots, n)$.

Least Squares Regression (contd.)

The solution for n parameters α_j which minimise the total squared error E is given as

$$\beta = pinv(A) * Y \tag{4}$$

Here,

- β is a column vector with n entries α_j
- A is a mxn matrix with entries $A(i, j) = f_j(x_i)$
- $pinv(A)$ is the pseudo-inverse of A
- Y is a column vector with m entries y_i

Least Squares Regression (contd.)

Derivation of (4): Please refer book and lecture recording

Least Squares Regression (contd.)

Example: Perform a least squares regression for the scatter plot created before using estimation function $\hat{y}(x) = \alpha_1 x + \alpha_2$.

```
openfig('scatter_plot_1.fig') %opens figure scatter_plot_1.fig
a = get(gca,'Children');
x = get(a, 'XData'); %extract x-data points
y = get(a, 'YData'); %extract y-data points
A=[x',ones(size(x'))]; %create the matrix A of basis functions
beta=pinv(A)*y'; %evaluate vector beta containing values of parameters

%plot the regression line
hold on
plot(x,beta(1)*x+beta(2))
```

Least Squares Regression (contd.)

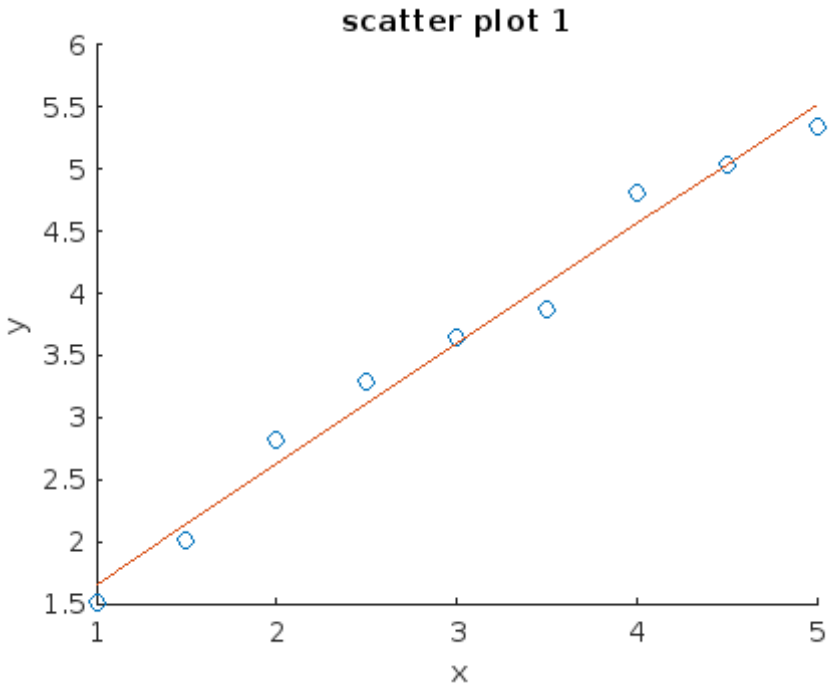


Figure: Regression line

polyfit and polyval functions

When the estimation function $\hat{y}(x)$ is a polynomial, MATLAB's polyfit and polyval functions can be used.

For vectors x and y containing x-data and y-data points respectively,

- p=polyfit(x,y,n) creates a vector p of the coefficients of regression polynomial p(x) of degree n
- polyval(p,x) calculates the values of p(x)

polyfit and polyval functions (contd.)

Example: This code creates the same regression line as was shown before.

```
openfig('scatter_plot_1.fig') %opens figure scatter_plot_1.fig
a = get(gca,'Children');
x = get(a, 'XData'); %extract x-data points
y = get(a, 'YData'); %extract y-data points

p= polyfit(x,y,1); %creates coefficients of regression polynomial of degree 1
Y=polyval(p,x); %evaluates the value of polynomial at x-data points
hold on
plot(x,polyval(p,x)) %plot the regression polynomial
```

polyfit and polyval functions (contd.)

Demo

Nonlinear Estimation Functions

Sometimes, a nonlinear estimation function provides the best fit for a scatter plot. This means we require

$$\hat{y}(x) = g(\alpha_1, \alpha_2, \cdots, \alpha_n, x) \tag{5}$$

where g is some nonlinear function.

In some special cases, a transformation such as

$$\tilde{y}(x) = h(\hat{y}(x)) \tag{6}$$

can linearise the equation (5) into (1).

Nonlinear Estimation Functions (contd.)

Example: Consider the estimation function

$$\hat{y}(x) = \alpha_1 e^{\alpha_2 x} \tag{7}$$

Applying the transformation

$$\tilde{y}(x) = log(\hat{y}(x)) \tag{8}$$

converts (7) into

$$\tilde{y}(x) = \tilde{\alpha_1} + \alpha_2 x \tag{9}$$

where we define $\tilde{\alpha_1} = log(\alpha_1)$. Now, least squares regression can be applied to equation (9).The parameter α_1 can be found using $\alpha_1 = e^{\tilde{\alpha_1}}$.

End of Lecture 13

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