Exercise 1. *Prove that* $f(x) = \cos x$ *is continuous on* $\mathbb{R}$.

Exercise 2. *Let* $f(x)$ *be countinuous on* $[a, +\infty)$, *and*

$$\lim_{x \to +\infty} f(x) = A$$

exists. Prove that $f(x)$ *is bounded on* $[a, +\infty)$.

Exercise 3. *Let* $f(x)$ *be uniformly continuous on* $(a, b]$ *and* $[b, c)$. *Prove that* $f(x)$ *is uniformly continuous on* (a, c).

Exercise 4. *Prove the equation*

$$x + \frac{15}{1 + \cos^2 x} = 5$$

has at least one solution in $\mathbb{R}$.