Exercise 1. *Prove that*

$$\frac{\sin 2x}{2(2+\sin 2x)} + \frac{\sin 3x}{3(3+\sin 3x)} + \cdots + \frac{\sin nx}{n(n+\sin nx)}$$

is a Cauchy sequence.

Exercise 2. *Prove by* $\epsilon - \delta$ *that*

$$\lim_{x\to 2} x^3 = 8$$

Exercise 3. *Prove that*

$$D(x) = \begin{cases} 1, \ x \in \mathbb{Q} \\ 0, \ x \notin \mathbb{Q} \end{cases}$$

is divergent everywhere.

Exercise 4. *We define*

$$\lim_{x\to\infty} f(x) = +\infty$$

as

$$\forall G > 0, \exists X > 0, \forall x > X, f(x) > G$$

(1) Prove that

$$\lim_{x\to\infty} f(x) = +\infty$$

if and only if

$$\forall \{x_n\} \lim_{n\to\infty} x_n = +\infty \textit{ implies } \lim_{n\to\infty} f(x_n) = +\infty$$

(2) [Harder] Prove that

$$\lim_{x\to\infty} f(x) = +\infty$$

if and only if any sequence $\{x_n\}$ *satisfying* $\lim_{n\to\infty} x_n = +\infty$ *and* $\{x_n\}$ ***is strictly increasing****, we have*

$$\lim_{n\to\infty} f(x_n) = +\infty$$