MA3077 (DLI) Operational Research
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Problem Sheet 1 - Submission Deadline: Sunday, 27 October, 23:59

1) Consider the following linear programming problem.

$$\begin{array}{ll} max & 6x - y \\ s.t. & 4y - x \geq 10 \\ & 2x + y \leq 10 \\ & 2y - x \leq 5 \\ & x, y \in \mathbb{R} \end{array}$$

   a) **[4 Marks]** Draw (by hand or using some software) the feasible set. Then determine graphically an optimal solution and indicate which constraints are active.
   b) **[2 Marks]** Determine an approximate optimal solution using Matlab's function linprog. You can either write the commands by hand or include your m-file in your submission.
   c) **[4 Marks]** Derive the dual problem of this linear programming problem, identifying clearly the Lagrangian and the dual function.

2) **[5 Marks]** Formulate the following optimization problem as a linear programming problem.

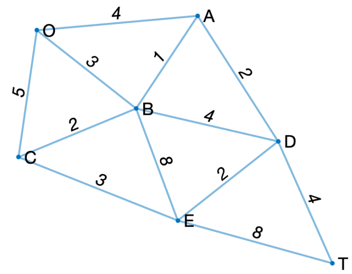
$$\max x \;\; s.t. |x| \in 0 \; \cup [1,2], x \in \mathbb{R}$$

3) **[6 Marks]** Write the following linear programming problem in standard form. Then, show that this problem is infeasible if and only if there is an $x \geq 0$ such that $Ax = 0$ and $c^T x < 0$.

$$\begin{array}{ll} max & b^T y \\ s.t. & c - A^T y = s \\ & s \geq 0, \; y \in \mathbb{R}^m \end{array}$$

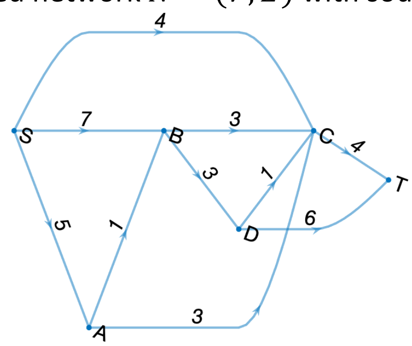
4) **[4 Marks]** In the m-file OR03_compressed_sensing.m (available on Blackboard), the problem (''Problem 1'') $minimise \; \|w\|_1 \; s.t. \; Aw = y, \; w \in \mathbb{R}^{50}$ is solved in the context of a compressed sensing problem. On paper, formulate the problem (''Problem 2'') $minimise \; \|w\|_\infty \; s.t. \; Aw = y, \; w \in \mathbb{R}^{50}$ as a linear programming problem and then modify OR03_compressed_sensing.m to solve Problem 2. Include the figure you obtain with your modified m-file in your submission. Does the solution to Problem 1 or 2 provide a more accurate representation of the original signal? In OR03_compressed_sensing.m, how many nonzero weights w_k does the solution to Problem 1 have? In your modified m-file, how many nonzero weights does the solution to Problem 2 have?

5) Consider the following network.



a) **[4 Marks]** Determine and draw a minimal spanning tree starting at node O using the version of Prim's algorithm presented in class. Show all intermediate steps.
b) **[8 Marks]** Write a Matlab program to compute a minimal spanning tree of the full subgraph induced by $V' = \{O, A, B, C\}$ using Matlab's function intlinprog.

6) Consider the following directed network $N = (V, E)$ with source S and sink T.



Let $f: E \to \mathbb{R}_+$ be defined by $f(e) = \begin{cases} 6, & e = SB, \\ 3, & e \in \{BC, CT, BD, DT\}, \\ 0, & otherwise. \end{cases}$

a) **[2 Marks]** Verify that f is a flow and compute its value.
b) **[2 Marks]** Identify an f-augmenting path and compute its capacity.
c) **[2 Marks]** Provide a nontrivial upper bound for the value of a maximal flow.

7) Consider a two-person zero-sum game with payoff matrix

$$A = \begin{pmatrix} 1 & 0 & 2 & -1 \\ 0 & -1 & 3 & 2 \\ 2 & -2 & -1 & 1 \\ 0 & 3 & 4 & -5 \end{pmatrix}.$$

a) **[3 Marks]** Assuming that players do not employ mixed-strategies, determine their best strategies and explain in simple terms why this game is not stable.
b) **[4 Marks]** Determine the optimal mixed-strategy for player 1.