UNIVERSITY OF LEICESTER

MA1014
All candidates

Final Resit Test 2022

DO NOT OPEN THE QUESTION PAPER UNTIL INSTRUCTED TO DO SO BY THE CHIEF INVIGILATOR	
School	COMPUTING AND MATHEMATICAL SCIENCES
Module Code	MA1014
Module Title	Calculus and Analysis
Exam Duration	2 hours
CHECK YOU HAVE THE CORRECT QUESTION PAPER	
Number of Pages	3
Number of Questions	4
Instructions to Candidates	This paper contains 4 questions. Full marks are 100 marks. Please attempt all questions.
FOR THIS EXAM YOU ARE ALLOWED TO USE THE FOLLOWING:	
Calculators	No
Books/Statutes provided by the University	No
Are students permitted to bring their own Books/Statutes/Notes?	No
Additional Stationery	Yes

In this exam, you are free to use properties of limit, continuity of elementary functions, the facts

$$\lim_{x\to 0}\frac{\sin x}{x}=1$$

and

$$\lim_{x\to+\infty}(1+\frac{1}{x})^x=e$$

without proof unless explicitly stated.

You may use any results from the course that you state correctly.

1. (a) Compute

$$\int\frac{dx}{x(x+2)}$$

[6 marks]

(b) Compute

$$\int_0^{\frac{\pi}{2}}\sin^5 x\cos x dx$$

[7 marks]

(c) State the definition of $f(x)$ being continuous at a point x_0. **[5 marks]**

(d) Let $g(x)$ be defined on $[0,1]$ such that for all $f(x)$ being continuous on $[0,1]$,

$$\int_0^1 f(x)g(x)dx=0$$

Prove that $\forall x\in[0,1]$, $g(x)=0$. **[7 marks]**

2. (a) Prove that for $x,y\in\mathbb{R}$,

$$2\cos x\cos y=\cos(x+y)-\cos(x-y)$$

[3 marks]

(b) Using (a) above, or otherwise, solve

$$y'+\cos(x+y)=\cos(x-y)$$

You may use the fact

$$\int\sec x\,\mathrm{d}x=\ln|\sec x+\tan x|+C$$

without proof where $\sec x=\frac{1}{\cos x}$. **[7 marks]**

(c) Solve

$$y''-3y'+2y=2e^{-x}$$

with initial conditions

$$y(0)=2 \text{ and } y'(0)=-1$$

[10 marks]

(d) Find an ODE with solutions $y = x$ and $y = x^2$. Check briefly that the functions satisfy your ODE. **[5 marks]**

3. (a) State the comparison test for series. **[3 marks]**

(b) By using the comparison test, or otherwise, determine whether the series is convergent and justify your answer:

$$\sum_{n=1}^{\infty} \frac{n+1}{n^2+2n-1}$$

[5 marks]

(c) Let $\{a_n\}$ be a non-negative sequence such that

$$\sum_{n=1}^{\infty} a_n$$

is convergent. Prove that

$$\sum_{n=1}^{\infty} a_n^2$$

is convergent. **[5 marks]**

(d) State the ratio test for series. **[5 marks]**

(e) Using the ratio test, or otherwises, determine whether the series is convergent and justify your answer:

$$\sum_{n=1}^{\infty} \frac{2^n n!}{n^n}$$

[7 marks]

4. (a) State the definition of $f(x,y)$ being continous at the point (x_0, y_0). **[3 marks]**

(b) Let

$$f(x,y) = \begin{cases} \frac{x^2 y}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0). \end{cases}$$

Compute $\frac{\partial f}{\partial x}(0,0)$ and $\frac{\partial f}{\partial y}(0,0)$, then prove that $f(x,y)$ is continous at the point $(0,0)$. **[7 marks]**

(c) Compute the tangent plane of the surface $z = 2x^4 + 3y^3$ at the point $(2,1,35)$. **[5 marks]**

(d) Let $f(x,y) = x^2 + 2y^2 - 2x - 12y + 6$. Compute all the local maxima and local minima of f on $\mathbb{R}^2$. **[10 marks]**