Coursework 4

Deadline: May 31st

Exercise 1 (12 marks)**.** *We say that $f : I \to \mathbb{R}$ is an absolutely continuous function on I, if*

$\forall \epsilon > 0, \exists \delta > 0,$ *if we have finitely many open intervals $(x_k, y_k) \subset I$ pairwisely disjoint and*

$$\sum_k |y_k - x_k| < \delta,$$

then we have

$$\sum_k |f(y_k) - f(x_k)| < \epsilon.$$

(1) Prove that if a function f is absolutely continuous on I, then it is uniformly continuous on I. [3 marks]

(2) Give an example that the converse is not true. [9 marks]

Exercise 2 (8 marks)**.** *Let $S \subset \mathbb{R}$ be an open set. Prove that there are (finitely or infinitely) many disjoint open intervals (x_k, y_k) such that*

$$S = \bigcup_k (x_k, y_k)$$