Mock Class Test (model solution)

1. Define an anonymous function `approx_factorial` which will calculate an approximate value of $n!$ using the first three terms of the Stirling series:

$$n! \approx \sqrt{2\pi n}\left(\frac{n}{e}\right)^n\left(1+\frac{1}{12n}+\frac{1}{288n^2}\right)$$

The function should be able to take vector input. Demonstrate that your function works by computing the approximate factorial $n!$ for consecutive integers `n = 1,2,3,4,5,6`.

Solution:

```
>> approx_factorial =
@(n)sqrt(2*pi.*n).*((n/exp(1)).^n).*(1+1./(12*n)+1./(288*n
  .^2))
approx_factorial =
 function_handle with value:
 @(n)sqrt(2*pi.*n).*((n/exp(1)).^n).*(1+1./(12*n)+1./(288*n
   .^2))
>> approx_factorial(1:6)
ans =
 1.0022 2.0006 6.0006 24.0010 120.0025 720.0089
```

2. Write a Matlab function `matDef` that takes as input n and returns the $n \times m$ matrix A with elements

$$A_{ij} = \begin{cases} i+j, & \text{if } i+j \text{ is odd,} \\ ij, & \text{if } i+j \text{ is even,} \end{cases}$$

where $1 \leq i, j \leq n$. Use your function to create A for $n = 7$. What is the complexity of your function?

Solution:

```
function A = matDef(n)
A = ones(n);
for i = 1:n
     for j = 1:n
          if mod(i+j,2) == 0 % i+j is even
             A(i,j) = i+j;
          else % i+j is odd
             A(i,j) = i*j;
     end
end
end
end
```

```
>> A = matDef(7)
A =
 1  1  1  1  1  1  1
 1  2  -1  0  1  2  -1
 1  1  0  0  1  1  0
 1  2  -2  -2  3  4  -4
 1  1  -1  -1  2  2  -2
 1  2  -3  -4  6  8  -10
 1  1  -2  -2  4  4  -6
```

3. The greatest common divisor (gcd) of positive integers a and b, denoted $\gcd(a,b)$, can be calculated recursively as follows:

$$\text{gdc}(a,b) = \begin{cases} a, & \text{if } b = 0, \\ \text{gdc}(b, \text{rem}(a,b)), & \text{if } b > 0, \end{cases}$$

where $\text{rem}(b)$ is the reminder of a divided by b (Matlab built-in function `rem` can be used). Implement the computation of the gcd in a **recursive** Matlab function with header `myGCD(a,b)`. Use your function to calculate the gcd of 315 and 504.

Solution:

```
function d = myGCD(a,b)
if b == 0
    d = a;
else
    d = myGCD(b, rem(a,b));
end
end
```

```
>> myGCD(364,390)
ans =
 26
```

4. Using MATLAB, find a solution (if it exists) to the following system of equations:

a)

$$\begin{aligned} x - 2y + z &= -1 \\ 2x + 2y - 2z &= -1 \\ x + 2y - 2z &= 6 \\ x - 6y + 4z &= -8 \end{aligned}$$

b)

$$\begin{aligned} 2x - y + 2z - w &= 0 \\ x + y - z - w &= 0 \\ x - y - z + w &= 0 \end{aligned}$$

Solution: We analyze the system `Ax = b`:

a)

```
>> A = [1 -2 1; 2 2 -2; 1 2 -2; 1 -6 4]; b = [-1;-1;6;-8];
>> rank(A)

ans =

     3
```

Since `A` is full rank the system has a unique solution:

```
>> x = A\b

x =

   -7.0000
  -12.5000
  -19.0000
```

b) The system is homogeneous so we investigate its null space

```
>> A = [2 -1 2 -1; 1 1 -1 -1; 1 -1 -1 1]
>> P = null(A)

ans =

    0.3162
    0.6325
    0.3162
    0.6325
```

The solutions are of the form `a*P`, for any constant `a`.

5. Create a data set using x=linspace(1,10,100) and y=2*x./(3+x)+rand(size(x)). Write a script file which plots this data set and a regression curve for the estimation function

$$\hat{y}(x) = \frac{\alpha_1 x}{\alpha_2 + x}.$$

Hint: Linearise the estimation function

$$\hat{y}(x)$$

using appropriate transformations

$$\tilde{y} = u(\hat{y}) = \frac{1}{\hat{y}}$$

and
$$\tilde{x} = v(x) = \frac{1}{x}.$$

Solution:

```
%Plot given data set
clc
clear all
x=linspace(1,10,100); %create data for vector x
y=2*x./(3+x)+rand(size(x));
figure
hold on
plot(x,y,'.')
title('Plot of data set with regression curve')
xlabel('x')
ylabel('y')

%Regression
tildex=1./x; %tildex is transformed x
tildey=1./y; %tildey is transformed y
A=[tildex',ones(size(tildex'))]; %create matrix A of basis
   functions
beta=pinv(A)*tildey';
alpha1=1/beta(2);
alpha2=beta(1)/beta(2);
Y=alpha1*x./(alpha2+x);
%plot the regression curve
plot(x,Y)
legend('Data set','Regression curve')
```

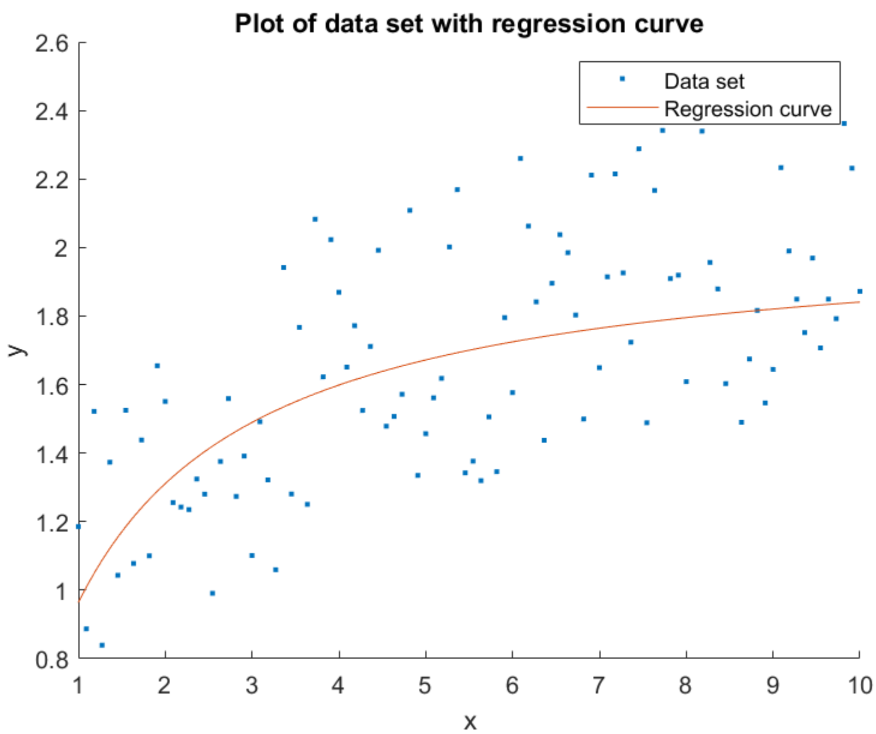
Plot of data set with regression curve
Data set
Regression curve
2.6
2.4
2.2
2
1.8
1.6
1.4
1.2
1
0.8
y
1
2
3
4
5
6
7
8
9
10
x

Figure 1: Q5