Exercise 1. *Find an example of a sequence being monotonic but not convergent.*

Exercise 2. *Suppose* $\{a_n\}$ *is unbounded. Prove there exists a subsequence* $\{a_{n_k}\}$ *such that*

$$\lim_{k\to\infty} a_{n_k} = \infty$$

Exercise 3. *Prove that*

(1)

$$\bigcap_{n=1}^{\infty}(-\frac{1}{n}, \frac{1}{n}) = \{0\}$$

(2)

$$\bigcap_{n=1}^{\infty}(0, \frac{1}{n}) = \emptyset$$

Exercise 4. *Prove that* $2 < e < 3$*.*

Exercise 5. *(1) Let* $\{x_n\}$ *be a sequence such that* $x_1 = \sqrt{2}$ *and* $x_{n+1} = \sqrt{2 + x_n}$*.*

(2) Let $\{y_n\}$ *be a sequence such that* $0 < x_1 < 1, x_{n+1} = x_n\,(2 - x_n)$*.*

Prove that $\{x_n\}$ *and* $\{y_n\}$ *are convergent and evaluate their limits.*

Exercise 6. *Prove that*

(1)

$$\lim_{n\to\infty} \frac{a^n}{n!} = 0$$

where $a > 1$*.*

(2)

$$\lim_{n\to\infty} \frac{n!}{n^n} = 0$$