MA2252 Introduction to computing

lectures 19-20

Gaussian elimination, LU factorisation, eigenvalue problems

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Simple systems: upper triangular matrix

A matrix is called **upper triangular** if its elements below the diagonal are zero.

Example: Solve the following system of equations

$$\begin{pmatrix} 2 & 3 & 1 \\ 0 & 1 & 6 \\ 0 & 0 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 11 \\ 20 \\ 9 \end{pmatrix}$$

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- This method of successive substitutions is called **back substitution**.

Simple systems: lower triangular matrix

A matrix is called **lower triangular** if its elements above the diagonal are zero.

Example: Solve the following system of equations

$$\begin{pmatrix} 2 & 0 & 0 \\ 4 & 1 & 0 \\ 1 & 8 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 6 \\ 14 \\ 22 \end{pmatrix}$$

Simple systems: lower triangular matrix

A matrix is called **lower triangular** if its elements above the diagonal are zero.
Example: Solve the following system of equations

$$\begin{pmatrix} 2 & 0 & 0 \\ 4 & 1 & 0 \\ 1 & 8 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 6 \\ 14 \\ 22 \end{pmatrix}$$

- This method of successive substitutions is called **forward substitution**.

General systems: Gaussian elimination

See notes by Steven Pav in Blackboard.

LU factorisation

Assume that a matrix A can be reduced to a un upper triangular matrix U by the process of Gaussian elimination with elementary operations E_k

$$A \to E_1 A \to E_2 E_1 A \to \cdots \to E_k \cdots E_1 A = U$$

Then we have

$$A = LU,$$

where $L = (E_k \cdots E_1)^{-1}$ is a lower triangular matrix.

- If A has an LU factorisation, then $Ax = b$ can be solved efficiently using back and forward substitutions.

Eiganvalue problem

Find λ and x such that

$$Ax = \lambda x$$

In MATLAB use `eig(A)`.