Coursework 1
Deadline: Nov 14th

Exercise 1 (10 marks). *(1) Let* $\{a_n\}$ *and* $\{b_n\}$ *be two bounded sequences. Prove that there exists a sequence of natural numbers* $\{n_k\}$ *as indices, such that both* $\{a_{n_k}\}$ *and* $\{b_{n_k}\}$ *are convergent.*

(2) Prove that $f(x)$ *is uniformly continuous on a bounded interval* I *if and only if for all Cauchy sequences* $\{a_n\}$, $\{f(a_n)\}$ *is a Cauchy sequence.*

Exercise 2 (5 marks). *True or False: There exists a function* f *defined in a neighborhood of* x_0 *such that*
(1) f *is continuous at the point* x_0*;*
(2) f *is not continuous elsewhere.*

If you think the proposition is true, give an example; if you think it is false, give a proof.

Exercise 3 (5 marks). *Let* $A > 0$, $0 < b_0 < \frac{1}{A}$, $b_{n+1} = b_n(2 - Ab_n)$ *for* $n \in \mathbb{N}$. *Prove that* $\lim_{n\to\infty} b_n$ *exists and evaluate its value.*