

MA2252 Introduction to Computing

Lecture 14 Interpolation

Sharad Kumar Keshari

School of Computing and Mathematical Sciences

University of Leicester

At the end of lecture, students will be able to

- understand interpolation problem
- understand theory of interpolation methods
- implement interpolation methods in MATLAB

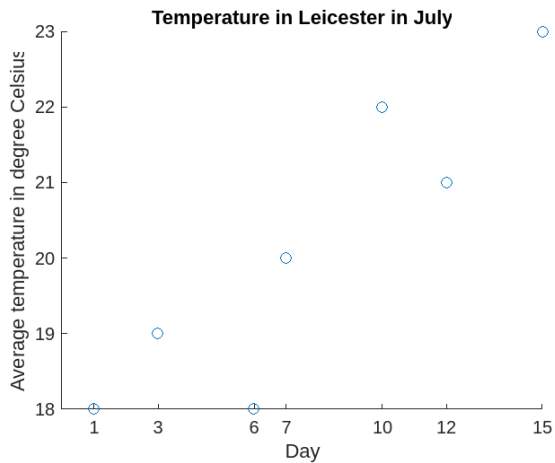
In mathematics, **interpolation** means to estimate the value of a function $f(x)$ in a given interval $x \in [a, b]$ based on some known values of the function inside and at end points of this interval.

Introduction (contd.)

Example: The table below shows the average temperatures in Leicester in July. Can we predict temperature on other days between 1st and 15th July?

Day	Average Temperature °C
1	18
3	19
6	18
7	20
10	22
12	21
15	23

Introduction (contd.)



Interpolation vs Regression

- Both techniques are used to describe the given data set as good as possible.
- Unlike regression, interpolation requires the estimation function to pass through all data points.

Interpolation Problem Statement

Suppose we have a data set containing n data points $(x_i, y_i), i = 1, 2, \dots, n$.

Goal: To find an estimation function $\hat{y}(x)$ with domain $x \in [x_1, x_n]$ such that $\hat{y}(x_i) = y_i$.

The function $\hat{y}(x)$ is called **interpolation function**.

Note: The choice of interpolation function depends on other factors such as accuracy, underlying physics etc. Depending on this choice, there are different interpolation methods.

Some Interpolation methods

- Linear Interpolation
- Cubic Spline Interpolation
- Lagrange Polynomial Interpolation

Linear interpolation

Here, the interpolation function $\hat{y}(x)$ is defined piecewise by linear polynomials (or straight lines). So,

$$\hat{y}_i(x) = y_i + \frac{(y_{i+1} - y_i)(x - x_i)}{x_{i+1} - x_i}, \quad x_i \leq x \leq x_{i+1} \quad (i = 1, 2, \dots, n-1) \quad (1)$$

MATLAB's `interp1()` function can be used to make life easier.

Linear interpolation (contd.)

Example: Use linear interpolation to find the average temperature in Leicester on 8th July.

Day	Average Temperature °C
1	18
3	19
6	18
7	20
10	22
12	21
15	23

Demo

Cubic spline interpolation

A cubic spline is a function defined piecewise by cubic polynomials.

In cubic spline interpolation, the interpolating function is a cubic spline defined as

$$S_i(x) = a_i x^3 + b_i x^2 + c_i x + d_i, \quad x_i \leq x \leq x_{i+1} \quad (i = 1, 2, \dots, n) \quad (2)$$

Again, MATLAB's `interp1` function can be used by giving 'cubic' as argument.

Cubic spline interpolation (contd.)

Example: (from book)

Demo

Cubic spline interpolation (contd.)

The unknown parameters a_i , b_i , c_i and d_i are found using these conditions:

$$S_i(x_i) = y_i \quad (i = 1, 2, \dots, n-1) \quad (3a)$$

$$S_i(x_{i+1}) = y_{i+1} \quad (i = 1, 2, \dots, n-1) \quad (3b)$$

$$S'_i(x_{i+1}) = S'_{i+1}(x_{i+1}) \quad (i = 1, 2, \dots, n-2) \quad (3c)$$

$$S''_i(x_{i+1}) = S''_{i+1}(x_{i+1}) \quad (i = 1, 2, \dots, n-2) \quad (3d)$$

$$S''_1(x_1) = 0 \quad (3e)$$

$$S''_{n-1}(x_n) = 0 \quad (3f)$$

Cubic spline interpolation (contd.)

Derivation of parameters: Please refer book and lecture recording.

Lagrange Polynomial Interpolation

Here, the interpolation function is a **Lagrange polynomial** defined as

$$L(x) = \sum_{i=1}^n y_i P_i(x) \quad (4)$$

where

$$P_i(x) = \prod_{j=1, j \neq i}^n \frac{x - x_j}{x_i - x_j}. \quad (5)$$

The Lagrange polynomial $L(x)$ of degree $n - 1$ passes through n data points i.e. it satisfies $L(x_i) = y_i$ ($i = 1, 2, \dots, n$).

Lagrange Polynomial Interpolation (contd.)

Example: (from book)

End of Lecture 14

Please provide your feedback [▶ here](#)