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Problem Sheet 1 - Submission Deadline: 16 October at 6pm (GMT) on Blackboard

1) **[5pts]** Formulate the following problem as a linear programming problem. Write the decision variables, constraints and objective function.

Biocare makes liquid plant food for fruit and vegetables. It makes two types: Growrite (G), a high nitrogen fertiliser for green vegetables, and Tomfood (T), with a high potassium content for tomatoes, cucumbers and so on. Both types need the same basic ingredients - Ammonium Nitrate for Nitrogen (N), Phosphorus Pentoxide for Phosphorus (P), and Potassium Dioxide for Potassium (K) - but in different amounts. One litre of G requires 0.11kg of N, 0.06kg P and 0.02kg of K. One litre of T requires 0.08kg of N, 0.03kg P and 0.08kg of K. There are available each day 600kg of N, 300kg of P and 330kg of K. The selling prices per litre are £2.80 for Growrite and £3.00 for Tomfood. At these prices, Biocare can sell all it produces. Biocare wishes to maximise its daily income. How should it do so?

2) Consider the following linear programming problem.

$$
\begin{array}{rc}
max & 35x + 20y \\
s.t. & 23x + 11y \leq 176 \\
 & 4x + 4y \leq 51 \\
 & x, y \geq 0
\end{array}
$$

a) **[4 pts]** Draw (by hand or using some software) the feasible set. Then determine graphically an optimal solution and indicate which constraints are active.
b) **[4 pts]** Determine an approximate optimal solution using Matlab's function linprog. You can either write the commands by hand or include your m-file in your submission.

3) **[5 pts]** Write the following linear programming problem in standard form.

$$
\begin{array}{rc}
min & 30x + 21y + 18z \\
s.t. & 3x - 7z \leq 176 \\
 & 8z - 2y + x - 6 \geq 12 \\
 & 4x + 3y = 19 \\
 & x, y, z \in \mathbb{R}
\end{array}
$$

4) **[5 pts]** Derive the dual problem of the following linear programming problem, identifying clearly the Lagrangian and the dual function.

$$
\begin{array}{rc}
min & x + 4y - 9z \\
s.t. & x - z = 7 \\
 & x + y + z = 2 \\
 & 4x + 3y = 19 \\
 & x, y, z \geq 0
\end{array}
$$

5) **[5 Marks]** Formulate the following optimization problem as a linear programming problem.

$$\max x_1 + x_2 \;\; s.t. \|x\|_1^2 \leq 4, x \in \mathbb{R}^2$$

6) **[5 Marks]** Let $m, n \in \mathbb{N}$ be such that $m < n$. Let $A \in \mathbb{R}^{m,n}$ have full rank, and let $y \in \mathbb{R}^m$ be in the column space of A. Let $x^* := (w^*, z^*) \in \mathbb{R}^{2n}$ be the optimal solution to the linear programming problem

$$\begin{array}{ll} min & z_1 + z_2 + \cdots + z_n \\ s.t. & -z \leq w \leq z \\ & Aw = y \\ & w, z \in \mathbb{R}^n \end{array} .$$

Prove that $z^* = |w^*|$.

7) **[5 pts]** Let $c = (\cos(\alpha), \sin(\alpha))^T$. For which values of $\alpha \in [0, 2\pi)$ is $x = (1,1)^T$ an optimal solution to the following linear programming problem?

$$\begin{array}{ll} max & c^T x \\ s.t. & x + 2y \leq 3 \\ & 3x + y \leq 4 \\ & x, y \geq 0 \end{array}$$

8) **[6 Marks]** Write the following linear programming problem in standard form. Then, show that this problem is infeasible if and only if there is an $x \geq 0$ such that $Ax = 0$ and $c^T x < 0$.

$$\begin{array}{ll} max & b^T y \\ s.t. & c - A^T y = s \\ & s \geq 0, \; y \in \mathbb{R}^m \end{array}$$

9) **[6 Marks]** Solve the following problem by use of the branch-and-bound method.

$$\begin{array}{ll} min & x + y \\ s.t. & 2x + 2y \geq 5 \\ & 12x + 5y \leq 30 \\ & x, y \geq 0, \quad x, y \in \mathbb{N} \end{array}$$