

Exercise 1. Let $f : (a, b) \rightarrow \mathbb{R}$ be a bounded, monotonic function. Prove that

$$\lim_{x \rightarrow a^+} f(x) \quad \text{and} \quad \lim_{x \rightarrow b^-} f(x)$$

exist.

Exercise 2. Let $f \in C(a, b)$. Assume that

$$\lim_{x \rightarrow a^+} f(x) = A \quad \text{and} \quad \lim_{x \rightarrow b^-} f(x) = B$$

exist, prove that

- (1) f is bounded on (a, b) ;
- (2) f is uniformly continuous on (a, b) ;
- (3) If $AB < 0$, then $\exists \eta \in (a, b)$ such that $f(\eta) = 0$.