Exercise 1. *Prove that* $\forall x, y \in \mathbb{R}$,

(1)

$$||x| - |y|| \leq |x - y| \leq |x| + |y|$$

(2) For $p > 0$,

$$|x + y|^p \leq 2^p \max\{|a|^p, |b|^p\}$$

Exercise 2. *By using the definitions of* $\sin$ *and* $\cos$, *prove that* $\forall x, y \in \mathbb{R}$,

(1) $\cos(-x) = \cos(x), \quad \sin(-x) = -\sin(x)$;

(2) $\cos^2 x + \sin^2 x = 1$;

(3) $\cos(x - y) = \cos x \cos y + \sin x \sin y$

Then try to deduce

(4) $\sin(x + y) = \sin x \cos y + \cos x \sin y$;

(5) $|\sin(nx)| \leq n|\sin x|$ *for* $n \in \mathbb{N}$.

Exercise 3 (Hard)**.** *Prove that* $\forall n \in \mathbb{N}$,

$$\left(1 + \frac{1}{n}\right)^n < 3$$