Exercise 1. *Let $\{a_n\}$ be a real sequence. Prove that*

$$\lim_{n\to\infty} a_n = 0 \quad \Longleftrightarrow \quad \lim_{n\to\infty} |a_n| = 0$$

Exercise 2. *Prove by $\epsilon - N$ definition that*

$$\lim_{nto\infty} \frac{n}{n+1} = 1$$

Exercise 3. *From the Boundedness Property, we know that convergence implies boundedness for a sequence. If $\{a_n\}$ is bounded, can we say it is convergent?*

Exercise 4. *Please review the property of Sign-preserving. Then try to answer*
(1) If $a_n > 0$ and $\lim_{n\to\infty} = a$, can we say $a > 0$?
(2) If $\lim_{n\to\infty} = a$ and $a \geq 0$. What can we say about $\{a_n\}$?

Exercise 5. *Please review the Arithmetic Property. Then try to answer*
(1) Let $\{a_n\}$ be convergent and $\{b_n\}$ be divergent. Prove that $\{a_n + b_n\}$ is divergent.
(2) If both $\{a_n\}$ and $\{b_n\}$ are divergent, what can we say about $\{a_n + b_n\}$?

Exercise 6. *(1) Prove that*

$$\lim_{n\to\infty} \frac{n^2+2n-1}{2n^2-4n-6} = \frac{1}{2}$$

(2) Prove by $\epsilon - N$ definition that

$$\lim_{n\to\infty} \frac{n^2+2n-1}{2n^2-4n-6} = \frac{1}{2}$$

Exercise 7. *Evaluate*

$$\lim_{n\to\infty} \left(\frac{1}{n+\sqrt{1}} + \cdots + \frac{1}{n+\sqrt{n}} \right)$$

Exercise 8. *Let $a > 0$ be a positive real number. Prove that*

$$\lim_{n\to\infty} a^{\frac{1}{n}} = 1$$

Exercise 9. *Let* $\{a_n\}$ *be a sequence satisfying*

$$\lim_{n\to\infty} a_n = a$$

Prove that

$$\lim_{n\to\infty} \frac{a_1 + \cdots a_n}{n} = a$$