UIT2201 Computer Science and the IT Revolution: Travelling Salesman Problem

Slide 1: Good morning, Prof and fellow course mates, my name is Donovan and today I will be presenting about a paper on the Travelling Salesman Problem.

Slide 2: But first, what is the Travelling Salesman Problem? In Computer Science, the Travelling Salesman Problem is a real and classic computational optimization problem that has wide ranging applications from vehicle-routing to city logistics planning, drone routing and even DNA sequencing. It is known to be NP-hard, meaning that it cannot be solved in polynomial time, and so it takes very long to solve it.

Slide 3: The Travelling Salesman Problem is the following: "Given a list of cities and the distances between each pair of cities, what is the shortest possible route that a salesman visits each city exactly once and returns to the origin city?"

Slide 4: In this example, given 4 cities and the respective distances between each, we can write out all circuits starting and ending at a particular city, compute the total distance for each, and pick one for which the total is minimal. In this case it is ABCDA or ADCBA. But this becomes very inefficient as the number of cities increase. For just 30 cities, it there would be 4.42 x 10^30 circuits. This would take 1.4 x 10^14 years to compute. Finding an exact solution is possible, but it takes a lot of computation. An approximate solution, guaranteed to be at most 50% longer than the shortest possible route, can be found much more easily. Is it possible to do better than this?

Slide 5: This is what the research paper titled “A Randomized Rounding Approach to the Travelling Salesman Problem” by Amin Saberi and Mohit Singh from Stanford University and Georgia Institute of Technology respectively aimed to solve.

Slide 6: For 45 years, computer scientists depended on approximation algorithms such as Christofides’ algorithm for the best approximate solutions to this problem. Christofides’ algorithm works by first forming a graph with no cycles that connects all the cities in a way that gives a minimum overall sum (in what is known as a Minimum Spanning Tree), and second connecting cities that have odd connections until every city has an even number of connections, in order to produce a round trip.

Slide 7: However, it’s often hard to find the best graph to use for each new connection between cities with odd connections, as connecting cities far apart to form a round trip is very expensive. Hence, the new algorithm uses a random process to generate a graph with no cycles where cities with an odd number of connections tend to have nearby partners. This slightly reduces the cost of connecting cities that have odd connections, producing a round trip that is better than Christofides’ existing method.

Slide 8: The long-standing Travelling Salesman Problem has bewildered mathematicians and computer scientists for decades. For many years, computer scientists depended on Christofides’ algorithm for the best approximate solutions but suspect the existence of an approximation algorithm that outperforms it. Here, after an eighty-page scientific analysis, this new algorithm was accepted by the scientific community to subtract 0.2 billionth of a trillionth of a trillionth of a percent from Christofides’ 50% factor. In scientific and mathematical research, progress is gradual and cumulative. So, this is a major breakthrough that will be improved on in time.

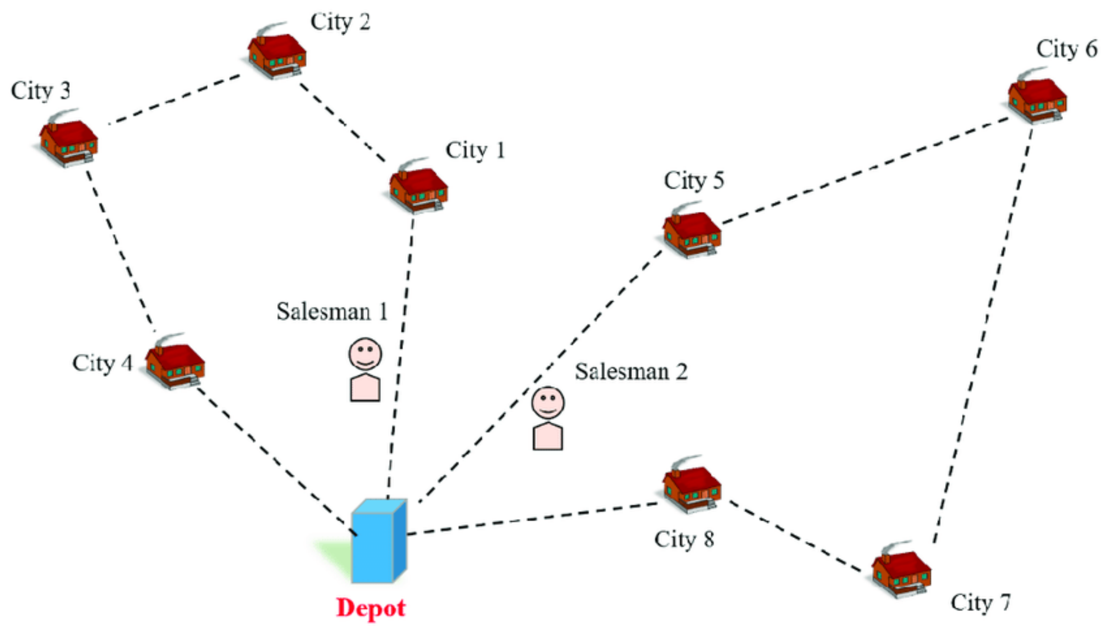
Slide 9: I enjoyed the paper because the idea is novel and the result record-breaking. However, it took me a while to understand and appreciate it due to the lack of intuitive visuals presented. Hence, the only critique I have is to include more intuitive visuals while presenting complex ideas and that is something I would do in future problems.

Slide 10: I believe the concepts of creativity, simplicity and problem-solving are essential soft-skill takeaways that I have learnt from the scientists solving the Travelling Salesman Problem, which I can apply to my problems in future. Here, being able to improve on the approximation for the Travelling Salesman Problem is a promising step in the right direction that can potentially benefit millions of people that rely on applications that depend on its optimization.

Slide 11: Thank you!

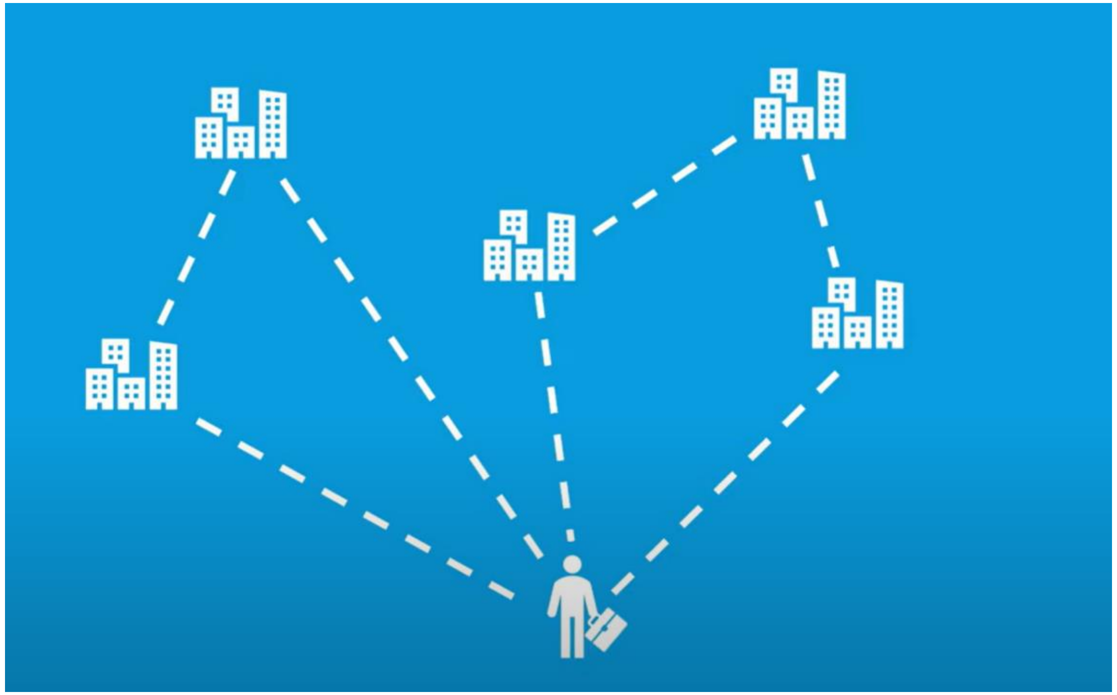
TRAVELLING SALESMAN PROBLEM

PRESENTED BY: DONOVAN, UIT2201



WHAT IS TRAVELLING SALESMAN PROBLEM?

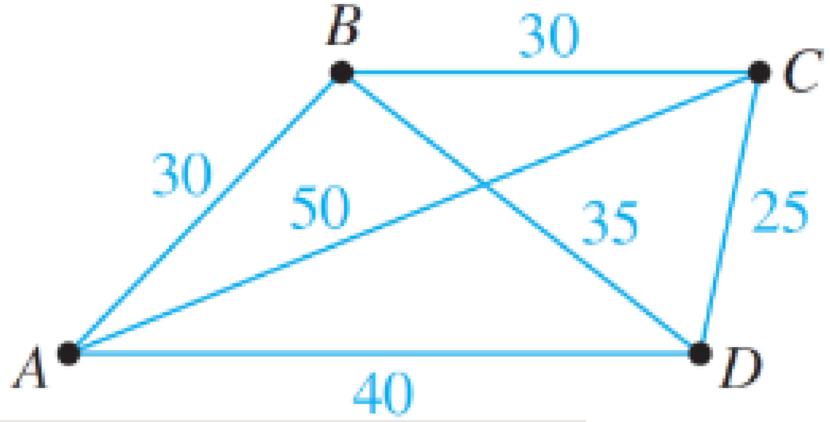
- Classic computational optimization problem
- NP-hard with no polynomial time solution
- Used in vehicle-routing, logistics planning, DNA sequencing, manufacturing microchips



WHAT IS TRAVELLING SALESMAN PROBLEM?

“Given a list of cities and the distances between each pair of cities, what is the shortest possible route that a salesman visits each city exactly once and returns to the origin city?”

WHAT IS TRAVELLING SALESMAN PROBLEM?



Route	Total Distance (In Kilometers)	
ABCDA	$30 + 30 + 25 + 40 = 125$	
ABDCA	$30 + 35 + 25 + 50 = 140$	
ACBDA	$50 + 30 + 35 + 40 = 155$	
ACDBA	140	[*ABDCA* backwards]
ADBCA	155	[*ACBDA* backwards]
ADCBA	125	[*ABCDA* backwards]

- Find: Shortest route visiting all cities and back
- Compute all and get ABCDA or ADCBA
- Inefficient for large number of cities
- For 30 cities, (29!)/2 = 4.42×10^{30} circuits, will take 1.4×10^{14} years to compute

A RANDOMISED ROUNDING APPROACH TO THE TRAVELLING SALESMAN PROBLEM

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(HTTPS://WEB.STANFORD.EDU/~SABERI/TSP.PDF)

A Randomized Rounding Approach to the Traveling Salesman Problem

Shayan Oveis Gharan * Amin Saberi† Mohit Singh ‡

Abstract

For some positive constant ϵ_0, we give a $(\frac{3}{2} - \epsilon_0)$-approximation algorithm for the following problem: given a graph $G_0 = (V, E_0)$, find the shortest tour that visits every vertex at least once. This is a special case of the metric traveling salesman problem when the underlying metric is defined by shortest path distances in G_0. The result improves on the $\frac{3}{2}$-approximation algorithm due to Christofides [13] for this special case.

Similar to Christofides, our algorithm finds a spanning tree whose cost is upper bounded by the optimum, then it finds the minimum cost Eulerian augmentation (or T-join) of that tree. The main difference is in the selection of the spanning tree. Except in certain cases where the solution of LP is nearly integral, we select the spanning tree randomly by sampling from a maximum entropy distribution defined by the linear programming relaxation.

Despite the simplicity of the algorithm, the analysis builds on a variety of ideas such as properties of strongly Rayleigh measures from probability theory, graph theoretical results on the structure of near minimum cuts, and the integrality of the T-join polytope from polyhedral theory. Also, as a byproduct of our result, we show new properties of the near minimum cuts of any graph, which may be of independent interest.

1 Introduction

The Traveling Salesman Problem (TSP) is a central and perhaps the most well-known problem in combinatorial optimization. TSP has been a source of inspiration and intrigue. In the words of Schrijver [36, Chapter 58], "it belongs to the most seductive problems in combinatorial optimization, thanks to a blend of complexity, applicability, and appeal to imagination".

In an instance of the TSP, we are given a set of vertices with their pairwise distances and the goal is to find the shortest Hamiltonian cycle which visits every vertex. It is typically assumed that the distance function is a metric.

The best known approximation algorithm for TSP has an approximation factor of $\frac{3}{2}$ and is due to Christofides [13]. Polynomial-time approximation schemes (PTAS) have been found for Euclidean [2], planar [24, 3, 28], or low-genus metrics [16, 15] instances. However, the problem is known to be MAX SNP-hard [33] even when the distances one or two (a.k.a (1, 2)-TSP). It is also

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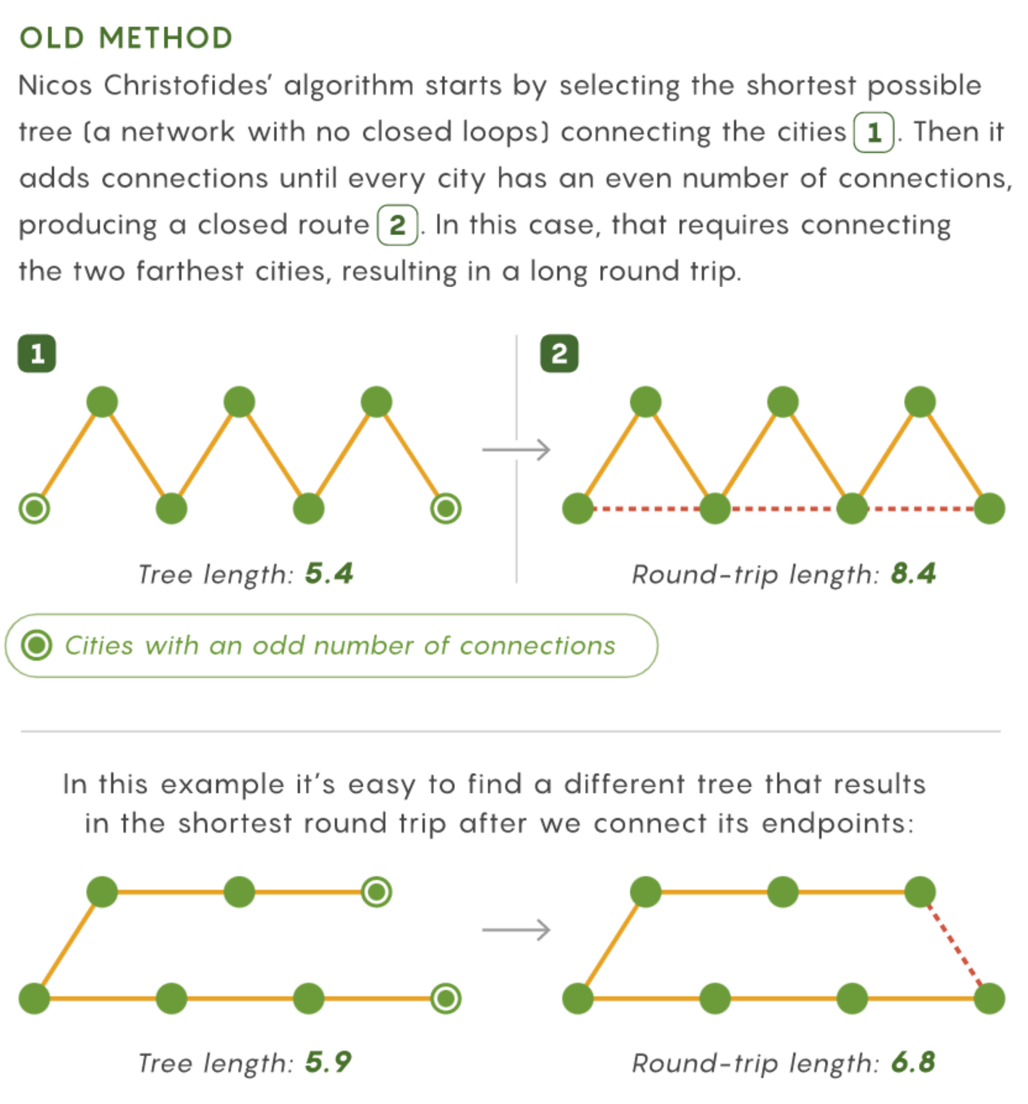
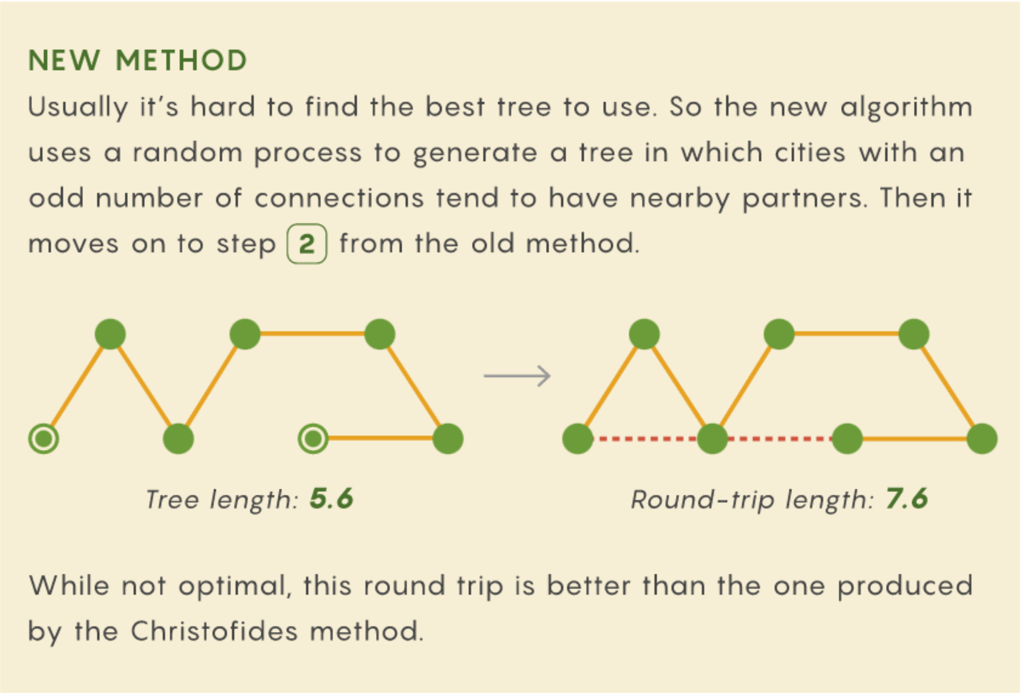


OLD: CHRISTOFIDES' ALGORITHM

- Form a graph with no cycles connecting all cities in a way that **minimises** overall sum
- Add connections to cities with odd connections until all have even connections
- Produces a round trip

NEW: NEW ALGORITHM



NEW METHOD

Usually it's hard to find the best tree to use. So the new algorithm uses a random process to generate a tree in which cities with an odd number of connections tend to have nearby partners. Then it moves on to step 2 from the old method.

While not optimal, this round trip is better than the one produced by the Christofides method.

Random process to generate a graph with no cycles, where cities with odd number connections have nearby partners

Connecting cities with odd number connections this way gives better round trip

NOVELTY FROM PAPER

- Travelling Salesman Problem is a long-standing combinatorial optimisation problem
- Current approximation algorithm gives solution at most 50% longer than shortest possible route
- New algorithm reduces that by **0.2 billionth of a trillionth of a trillionth of a %**

CRITIQUE

- Include more intuitive visuals for presenting complex ideas

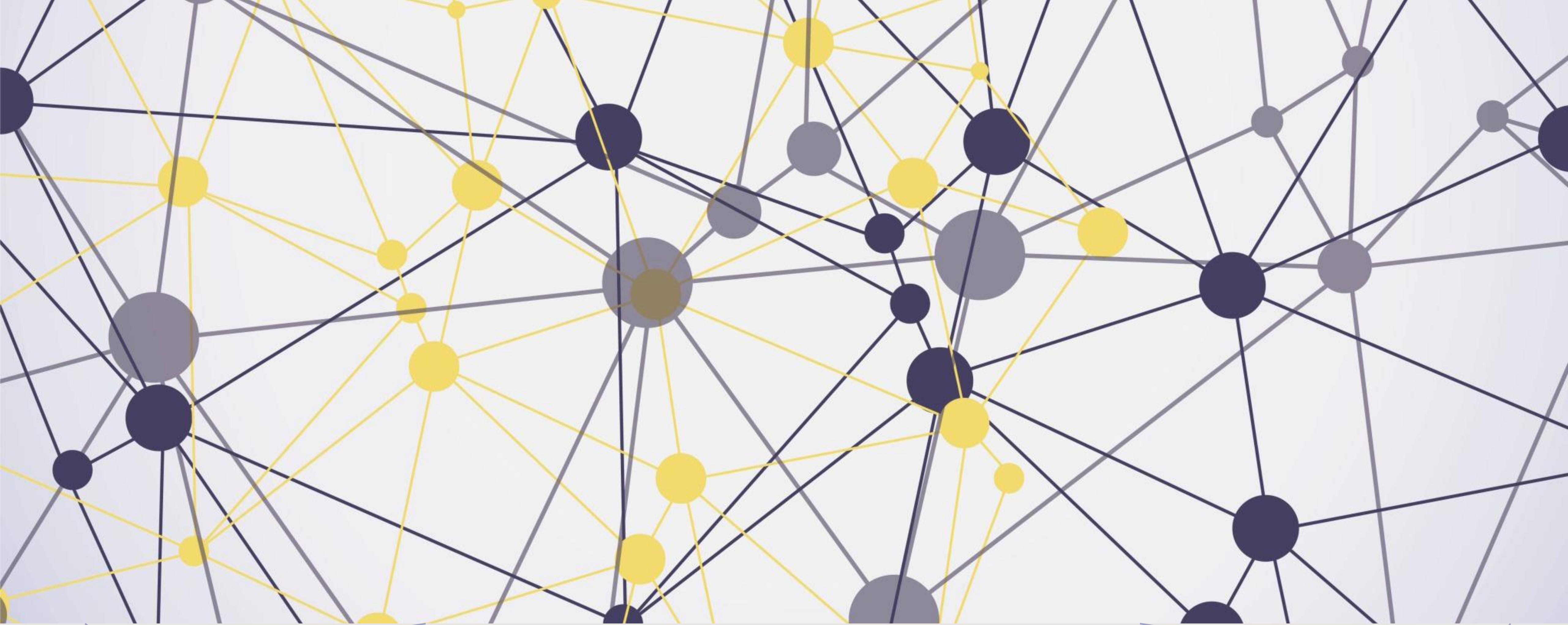
TAKEAWAY AND APPLICATION

Soft-skills:

- Creativity
- Simplicity
- Problem-solving

Overall impact:

Benefit millions of people using applications that depend on Travelling Salesman Problem related optimizations



THANK YOU