Figure 1. Caption?

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Abstract.

1. Introduction

$$y = x \tag{1}$$

1.1. Maxwell equations

1.2. Transformation Optics and Metamaterials

2. Methods

2.1. DC Magnetic Fields

Helmholtz coils were powered by a constant DC current to create a uniform magnetic field within their center. A commercially available XXX Hall probe was zeroed by using a MuMetal cannister, and then placed at the center of the Helmholtz coils. A Hall probe relates a measured Hall voltage, V_H, to a surrounding magnetic field, B [**?**] as

$$V_H = \frac{IB}{net}. \tag{2}$$

The probe maintains constant current supply I, and material paramaters n (charge carrier density), e (charge of electron) and t (thickness of probe) meaning a calibrated probe may give accurate readings for magnetic fields.

The magnetic field, B, produced at the center of Helmholtz coils with radius R, seperated by a distance R should follow,

$$B = \frac{8}{5\sqrt{5}} \frac{\mu_0 n I}{R}, \tag{3}$$

where I is the current supplied to the coils and n is the number of turns of wire. This equation follows directly from the Biot-Savart law [**?**] and the relative geometry of the coils as seen in figure **??**. From equation 3 it can be seen that the magnetic field should increase linearly with supplied current. Using the Hall probe we ensured this was the case and found the relationship of current supplied to magnetic field produced for our paticular Helmholtz arrangement.

Now, with the capability to produce known external magnetic fields, the described field concentrating shells may be placed within this field and the Hall probe may be placed within their inner radius to measure concentrated field.

2.2. AC characterization

Initially the Helmholtz arrangement was repeated for exploration of the concentrating shells behaviour in alternating magnetic fields. However, instead of a Hall probe, a small solenoid was used to detect the oscillating field. From Faraday's law, a voltage will be induced in a wire loop due to a time dependent magnetic field. A series of loops constituting a small solenoid will respond to a sinuisoidal magnetic field, $B = B_0 \cos wt$, with the relationship,

$$V = -NAB_0\omega \sin \omega t, \tag{4}$$

where A is the area of one loop and N is the number of loops, ω is the angular frequency of the alternating magnetic field and t is time.
As ω is known and all other parameters except external field are kept constant, the voltage across the solenoid may be measured experimentally to find the relative magnetic field strength.
The solenoid must however be characterized in order to find the absolute magnetic field values. This was done by measurement of the self inductance, L, of the solenoid as, XXX

$$L = \mu_0\mu_r N^2 A/l \tag{5}$$

XXX
Due to the induced voltage across the inductor being small and background noise being high, a lock-in amplifier was used to select only the desired signal frequency. This substantially reduced noise in our readings allowing higher frequency and lower magnetic field strength experiments.

Use of solenoid, limitations of Helmholtz and pick up. Use of RLC circuitary.

2.3. Power Transfer

Power transfer experiments measure power dropped across a load resistor in a receiving circuit verses power lost in the transmitting circuit's inductor. The receiving circuit, seen in figure **??**, has multiple arrangements to optimise power transfer. The simplest of which is the load resistor in series with the receiving inductor. In this case the optimal load resistance is $R = \omega * L$, where ω is the angular frequency of the oscillating magnetic field and L is the circuit inductance.
To maximise power transfer an RLC circuit is constructed on the receiving circuit. Ideally in an RLC circuit the complex impedance of the inductance and capacitance cancel leaving only the load resistance. Our inductance is set by the solenoid we chose to use and so for exploring power transfer at various frequencies, a capacitance can be found to satisfy the resonance condition. If inductance, L, does not vary then capacitance, C, is easily found by,

$$C = \frac{1}{w^2 L}. \tag{6}$$

However, we need not assume that inductance is constant. A series RLC circuit can be constructed as seen in figure **??** to ensure resonance is met. Resonance occurs in this circuit when V_A is exactly in phase with V_B. In this case any imaginary impedances are cancelled and only real resistance remains. For maximal power transfer a parallel RLC circuit is preferable however due to the restraints of the experiement a non-ideal version must be made as seen in figure **??**. The resonant condition found by the series RLC is a close approximation to this more complicated parallel circuit.

To maximise power transfer in the series RLC case, a familiar idea of impedance matching occurs, i.e. Power is maximised when the load resistance is equal to any internal resistances of the components [**?**]. As internal resistances are difficult to measure and may depend on current XXX, this was found experimentally by measuring voltage and current over the load resistance whilst varying load resistance.
For the parallel RLC case

Figure 2.

Figure 3.

2.4. COMSOL

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3. Results

3.1. DC Magnetic Fields

Using the DC Helmholtz set up as described in Methods, we observed constant concentration factors for different shell constructions in an external magnetic field ranging from 1 to $22G$. No shell, 18 MuMetal, 36 MuMetal, 18 Copper and 18 MuMetal + 18 Copper shells were used and their behaviour with field may be seen in figure 2.
It was found that the shell contruction of 36 MuMetal thin sheets gave the optimum concentration of $C = 2.38$ with minimal error (0.1%) at higher field strengths and a maximum error of 4.0% at an external field of 1.4 G. This increase in error at low magnetic fields is due to limited sensitivity of our Hall probe and current measurements over the Helmholtz coils.
Similar error relationships are observed for the other constructions. It should be noted that we assume the dipole has been placed in the same position and orientation in all experiments and so errors due to placement are excluded here.
The copper only shell showed no concentration of internal field as expected. This is due to copper having a relative permeability similar to air, $\mu_r = 1.0$, and so negligible field guiding properties. Furthermore, in the DC regime copper will not shield XXangularXX fields as required by the optimal TO concentrator.

3.2. AC characterization

3.3. Helmholtz

The Helmholtz coils were supplied with an alternating current in order to create an alternating magnetic field. Now using the voltage induced across solenoid to detect alternating magnetic field strength as decribed in Methods, the concentration of various shell arrangements were explored.

The concentration factors between 0.5 and 30 kHz can be seen in figure 3. It was found that a mixed shell of alternating 18 copper and 18 MuMetal sheets had the optimum concentration factor of $C = 3.12$ at 5 kHz.
Here we can see that the copper sheets now have a modest concentrating effect as frequency increases. This behaviour is expected as copper will shield perpendicular alternating magnetic fields which is desired for the optimal TO concentrator. However, it is suprising that this shielding occurs at such low frequencies, i.e. Much less than skin depth of copper.XXX We found that the copper shell increases in efficacy from $C = 1.0$ at 50 Hz until $C = 1.3$ (2SF) at 10 kHz and does not increase substantially more as frequency is increased. This suggests that the shielding effect of the copper depends on frequency but saturates early.
Supporting work done on COMSOL (see Methods) suggests a similar observed effect where 36 copper sheets increase rapidly in concentration factor from $C = 1$ at 0 Hz to $C = 1.5$ (2SF) at 10 kHz.

The strong linear decay of field concentration after $5 - 10$ kHz for all but the copper only shell is also a suprising result which, although could be explained by the MuMetal permeability frequency

response, also appears to occur at too low a frequency.
COMSOL work does not show this relationship and so the source may be either not modelled appropriately within COMSOL or be a fault in this experimental design.

Apart from instrument and measurement reading errors which constitute only a small error (XXX%), we observed errors due to high pick-up in cables connecting the solenoid to the lock-in amplifier. This source of noise was at the same frequency as our desired signal and so is difficult to remove other than careful cable placement and using shielding. We believe this pick-up was worsened by the fact the Helmholtz coils must be driven with high voltage and current to create a useful magnetic field and that the magnetic field was not localised to just our solenoid and shell but also was subject to the cabling and any nearby detectors. This prompted a decision to focus on two dipole coupling experiments as this pick-up error can be greatly reduced.

3.4. Power transfer

First we show the simplest power transfer experiment where the load resistance is in series with the receiving inductor. Figure **??** shows the relative PTE versus load resistance for various frequencies. The peaks of these curves confirm the expected optimum load resistance of $R = \omega L$.
An Oscillating magnetic field, B, produced from a solenoid and concentrated by a shell follows,

$$B = CI\mu_0 n \cos \omega t,$$

where C is the concentration factor, I is the current through the solenoid and n is the number of turns of the solenoid. If a second solenoid is placed within the field of the first, as shown in figure **??**, then voltage will be induced across it according to Faraday's law,

$$V = -NA\frac{dB}{dt},$$

$$V = C\omega INA\mu_0 n \sin \omega t,$$

where N is the number of turns of the solenoid and A is the area of one turn. The power dropped across a resistor with magnitude ωL in series with this inductor will then be described by,

$$P = V^2/R$$

$$P = \frac{C^2 w I^2 k}{L} \sin \omega t^2$$

where k is the collection of constant coefficients that will remain constant between different shells. Max power received in the second circuit is therefore proportional to w, $\frac{C^2}{L}$ and I^2. Plotting $\frac{P}{I^2}$ against angular frequency w therefore gives $\frac{C^2 k}{L}$ as shown in XX figure **??**. Assuming the coefficients in k remain constant, comparisons of this gradient between a concentrating shell with inductance L_s and no shell with inductance L_0 and $C_0 = 1$ yields $\kappa = \frac{C^2 L_0}{L_s}$. Figure XXX **??** shows how κ depends on frequency for various shell configurations.
As observed in the Helmholtz driven field case, we see that the copper sheets begin to have a concentrating effect between 0 and 10 kHz. The Copper only shell increases to around $\kappa = 2$ which, if it is assumed that $L_0 = L_s$, corresponds to a power transfer increase of 2x or a corresponding field concentration of $\sqrt{2}$ within the shell's cavity. It can be seen that using only MuMetal sheets gives a power transfer increase of 6x and a concentration of field that is independent of field oscillation frequency for the range $0 - 30$ kHz. This differs from the previous Helmholtz result where a steady drop off of concentration factor was observed as frequency increased past 10 kHz.
The mixed shell of 18 MuMetal sheets and 18 Copper sheets was found to have the best power transfer increase of 9x after the copper sheet effectively shields the angular field at 10 kHz. This power

increase corresponds to a magentic field concentration of 3 within the shells cavity.

Figure **??** XXX shows the optimal PTE for a range of frequencies with different shell constructions around the receiving inductor. Assuming the inductance value remains constant with different shell configurations (an exploration of this assumption is considered in DiscussionXXX), absolute power transfer can be calculated as described in Methods. Table **??** gives the maximal power transfer for different arrangements of shells.

3.5. Other COMSOL

4. Discussion

5. Conclusions

6. References