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Abstract.

XXPREVIOUSXX A device capable of uniformly concentrating a magnetic fields inside of a free space cavity will increase the efficiency of many magnetic devices and sensors. This project shall look at a proposed design for a magnetic field concentrator informed by the transformation optic technique. A metamaterial shell comprised of high and low permeability sections alternating in the angular direction has been shown to approximate the designed concentrator[?]. The ability of the shell acting as a concentrator will be explored in various regimes with a specific focus on improving efficiency of wireless power transmission.

1. Introduction

The manipulation of magnetic fields is a critical tool for many modern technologies. Magnetic devices often have efficiencies dependent on the strength of interaction with an external magnetic field. Examples include energy harvesting from magnetic fields [?] to brain activity scans by locating small magnetic gradients [?]. These devices may have increased efficiency by concentrating the desired magnetic field within the area of sensing or harvesting.

Magnetic fields may be described by Maxwell's equations [?] and are guided by materials due to their optical properties, such as permittivity and permeability [?]. Fermat's principle of least time allowed the design of many optical devices using geometrical lenses [?], however, with the maturation of fabrication techniques, many materials may be produced with exotic anisotropic optical properties [?] prompting the development of transformation optics (TO) — a modern approach to optical device design.

Here we describe one such TO designed device; The magnetic concentrator [?], with a particular focus on its efficacy in wireless power transmission.

1.1. Transformation Optics and Metamaterials

TO informed the design of many new devices such as perfect lenses [], magenetic-hoses [], -cloaks [], -rotators [], -blackholes [], and -concentrators []. It can be shown that due to the form invariance of Maxwell's equations, a spatial coordinate transform is equivalent to the insertion of a material with specific permeabilities and permitivities. This is shown conceptually by the three steps of the schematic shown in figure 1. First a ray is considered in free cartesian space in panel *a*, which due to Fermat's principle of least time will be following the horizontal spatial grid lines. The space is then transformed arbitrarily in panel *b* so that the ray adopts the desired path for the final device. The transformation required, A , to morph the ray now informs the optical properties for the specific inserted material of panel *c*, which is once again located in untransformed space.

The form invariance of Faraday's law is described by the equivalent expressions

$$\nabla' \times \mathbf{E}' = -jw[\mu_0]\mathbf{H}' \quad \text{and} \quad \nabla \times \mathbf{E} = -jw[\mu']\mathbf{H}, \quad (1)$$

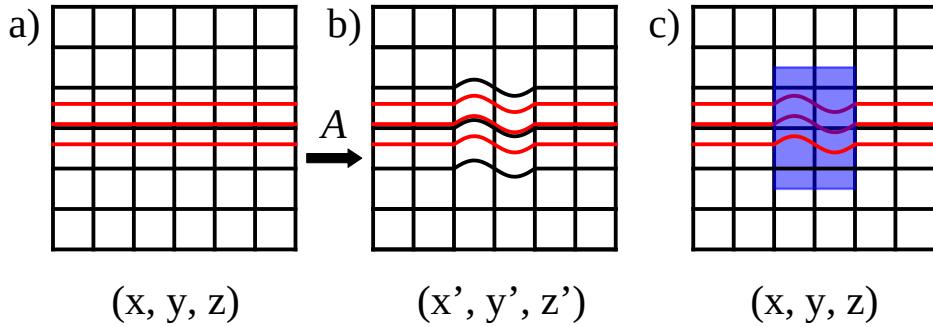


Figure 1. The steps of Transformation Optics. a) A ray (red) travelling in untransformed spatial coordinates (black grid) follows the path of least time. b) A spatial coordinate transformation A is applied to guide the ray along a desired path. c) A material (blue) is inserted into the untransformed space with corresponding permeability and permittivity which mimics the spatial coordinate transform A for the ray.

where the first is expressed in transformed coordinate space $x'(x, y, z), y'(x, y, z), z'(x, y, z)$ and free space permeability $[\mu_0]$, whilst the second is expressed in untransformed cartesian space x, y, z but with some space dependent permeability $[\mu']$. Equivalent expressions exist for the remaining Maxwell equations and similarly some non-free permittivity $[\epsilon']$ is required. The space dependant permeabilities and permittivities are found by,

$$\mu' = \frac{A\mu_0 A^T}{|A|} \quad \text{and} \quad \epsilon' = \frac{A\epsilon_0 A^T}{|A|} \quad (2)$$

where A is the Jacobian matrix describing the transformation of coordinate systems (e.g. between panel *a* and panel *b* in figure 1).

The resulting calculated optical properties may be anisotropic, have arbitrary magnitude and even be negative [1]. As bulk materials rarely, if ever, show these properties, optical metamaterials are often required [?].

Metamaterials are often comprised of repeating units whose dimensions are much smaller than the wavelength of the interacting radiation [?]. The individual units may have specific geometry, orientation and optical properties to selectively interact with incident waves so that the net effect of the material mimics a bulk substance with different optical properties than its substituent parts.

1.2. Magnetic Concentrator

As described above, a device capable of magnetic field concentration can increase the efficiency of sensors and energy harvesters. Utilising TO we may design an optimal field concentrator which fulfils the criteria: The magnetic field within a region A is concentrated into a cavity B where B is free space only. A possible geometry for this device that has cylindrical symmetry is shown on LHS of figure 2. To satisfy these conditions two coordinate transforms are applied: First the region $\rho < R_2 - \eta$ is radially and linearly compressed to the region $\rho < R_1$; Second, to ensure the continuity of our transformed space, a high (k^{th}) order polynomial radial expansion of $R_2 - \eta < \rho < R_2$ to the region $R_1 < \rho < R_2$ is made. These transformations are described by the coordinate transformations,

$$\begin{aligned} \rho' &= \frac{R_1}{R_2 - \xi} \rho, & \rho' &\in [0, R_2 - \xi] \\ \rho' &= R_2^{1-k} \rho^k & \rho' &\in [R_2 - \xi, R_2]. \end{aligned} \quad (3)$$

By symmetry we see that θ and z remain unchanged through the two transformations. The corresponding Jacobians may be found for these transformations and using equation 2, the

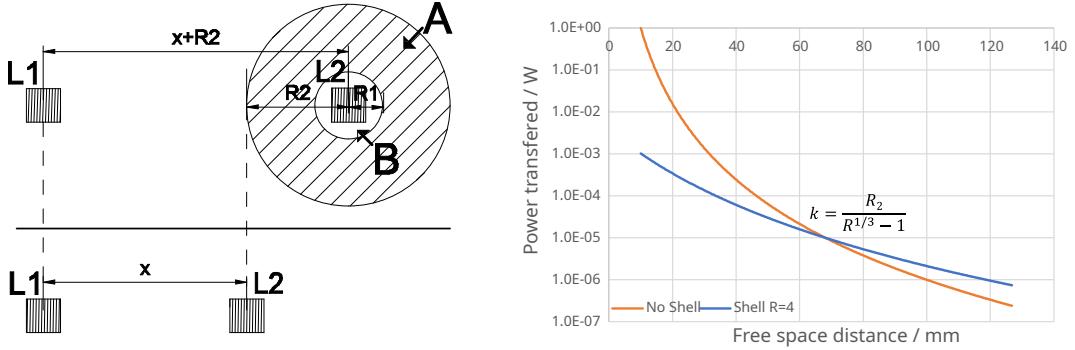


Figure 2. A concentrating shell with inner radius R_1 and outer radius R_2 is useful for increasing the efficiency of wireless power transmission by inductive coupling. PTE is highly sensitive to distance between coils being proportional to r^{-6} . A concentrating shell increases this power transfer by $(R_2/R_1)^2$ but at the cost of less free space, x , between the devices. However, at a critical distance k , the shell provides an improvement over bare dipoles for a given free space distance, x . LHS: The arrangement of two dipoles (boxes) L_1 and L_2 with a fixed free space gap x , either with a shell (Top) or without (Bottom). RHS: Theoretical power transfer plots for these two arrangements when the shell has $R_2 = 40$ mm, $R_1 = 10$ mm.

permeabilities of the required inserted material may be found to be

$$\begin{aligned} \mu' &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & (\frac{R_2-\xi}{R_1})^2 \end{pmatrix} & \rho' \in [0, R_1) \\ \mu' &= \begin{pmatrix} k & 0 & 0 \\ 0 & 1/k & 0 \\ 0 & 0 & \frac{1}{k}(\frac{\rho'}{R_2})^{2/k-2} \end{pmatrix} & \rho' \in [R_1, R_2) \end{aligned} \quad (4)$$

Taking the limit $\eta \rightarrow 0$ in order to concentrate all of the field within A into B , and matching the boundary conditions at $R_2 - \eta$ and at R_1 , we find that $k \rightarrow \infty$. From this we find that the required permeability within B is satisfied by free space whilst a material with radial permeability $\mu_\rho \rightarrow \infty$ and angular permeability $\mu_\theta \rightarrow 0$ is required for region A . The z components of permeability are ignored as we assume the cylindrical shell is infinitely long so the z component must be invariant. To satisfy these highly anisotropic conditions, an exploration of metamaterials is required. A possible discretized shell construction is proposed [?] where the high radial permeability is provided by ferromagnetic materials whilst the angular permeability is shielded by superconducting material. Materials such as MuMetal, have relative permeabilities of up to 300,000+ [?] and ideal superconductors in their Meissner state will exclude all magnetic fields from within their interior giving a relative permeability of 0 [?]. In an alternating field, a far cheaper option for blocking the angular magnetic field component is the use of copper sheeting. From Lenz's law an oscillating magnetic field perpendicular to a metal sheet will induce Eddy currents which will oppose the changing field. At an appropriate frequency, which is proportional to the copper thickness, the effective angular permeability is reduced to near zero. To approximate the conditions proposed by TO design, these two materials may be arranged in alternating angular sheets, as seen in figure 4.

In an external static uniform magnetic field, the ideal shell will increase the field within region B by a factor of R_2/R_1 . Similarly if a magnetic dipole is surrounded by the concentrating shell, then the field outside of the shell will be increased by a factor of R_2/R_1 .

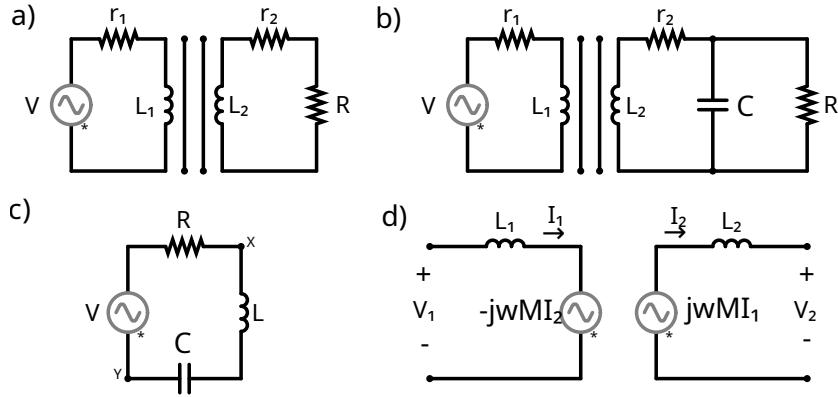


Figure 3. Wireless power transmission by inductive coupling. a) Simple circuit for coupling L_1 and L_2 coils. The useful power transmitted is across load resistance R whilst power lost is internal resistance of coils, r_1 and r_2 . b) The receiving inductor is now part of a resonant RLC circuit. c) A series RLC circuit — when voltage at x and y are in phase, the resonant condition is met. d) An equivalent circuit to the coupled inductor circuits but with explicit induced voltages shown.

1.3. Wireless Power Transfer

The ability to transfer power between devices that are not connected by wires is useful in many settings: for convenience, e.g. mobile phones; for safety, e.g. implanted medical devices; or for practicality, e.g. satellites. We shall focus on near field power transmission by the use of inductive coupling. Inductive coupling transfers power between two coils of wire (two inductors) by an oscillating magnetic field. The transmitting coil is supplied with an alternating current, which by Ampere's law creates an oscillating magnetic field. The receiving coil is placed within this magnetic field so that a current is induced as described by Faraday's law. A schematic of these two coupled circuits can be seen in figure 3 a, b, and d. This strategy for power transfer is highly sensitive to distance as the magnetic field of a dipole drops off with distance cubed, r^{-3} , therefore the power is proportional to r^{-6} if the dipole is driven at a fixed frequency and current. The power transfer efficiency, PTE, also suffers with distance as the non-ideal resistive losses in the primary coil remain (relatively) constant with distance. To somewhat counteract this sharp drop off of efficiency with distance, concentrating shells may be used to magnify the transmitted field and locally concentrate the field around the receiving solenoid. The concentrating shell is larger than the coil by necessity, however the presence of a shell is always advantageous when the free space distance between the power transmitting and power receiving device is greater than k , where

$$k = \frac{R_2}{\sqrt[3]{R} - 1} \quad (5)$$

and R is the ratio R_2/R_1 . This arrangement and the critical distance k is shown in figure 2.

2. Methods

2.1. Construction of shell

0.15 mm MuMetal (MSFHP from Thorlabs) and 0.3 mm copper were cut into 21 mm by 30 mm rectangular sheets. Plastic supports for holding 36 sheets of either MuMetal or copper were 3D printed to produce the shells seen in figure 4. The inner radius R_1 and outer radius R_2 were 7 and 21 mm respectively to give an R_2/R_1 ratio of 4. The “Full shell” contains 36 sheets of alternating copper

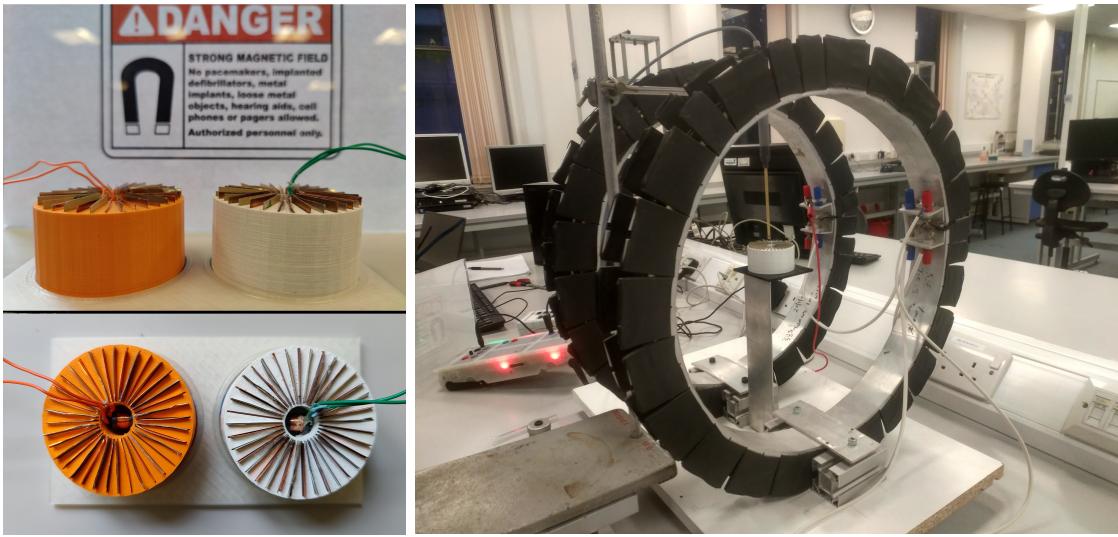


Figure 4. LHS: The constructed shells arranged in the power transfer experiment. RHS: The shell within a pair of Helmholtz coils.

and MuMetal, whilst the “No shell” scenario contains no metal.

2.2. DC Magnetic Fields

Helmholtz coils (figure 4) were powered by a constant DC current to create a uniform magnetic field, B , within their center described by

$$B = \frac{8}{5\sqrt{5}} \frac{\mu_0 n I}{R}, \quad (6)$$

where R is the radius of the coils, I is the current supplied to the coils, and n is the number of turns of wire. A Hall probe placed at the center of the Helmholtz coils will have a Hall voltage induced which is proportional to the magnetic field. Once calibrated the Hall probe can be used to measure the absolute value of the magnetic field present. A concentrating shell may now be placed at the center of the Helmholtz coils and the Hall probe placed within the R_1 section of the shells to explore field concentration.

2.3. Power Transfer

Power transfer experiments measure power dropped across a load resistor in a receiving circuit versus power lost in the transmitting circuit’s inductor. The receiving circuit, seen in figure 3, has multiple arrangements to optimise power transfer, the simplest of which is the load resistor in series with the receiving inductor forming an RL circuit. Assuming the non-ideal real resistances of the coil is $r_2 \ll \omega L$, then the optimal load resistance for power transfer at some frequency ω is found by,

$$V_2 = I_2(j\omega L_2 + R), \quad (7)$$

$$P_R = |I_2|^2 R = \frac{V_2^2 R}{\omega^2 L_2^2 + R^2}, \quad (8)$$

$$\text{and setting } \frac{dP_R}{dR} = 0,$$

$$\Rightarrow R = \omega L_2. \quad (9)$$

The EMF induced in the receiving coil of a coupled inductance system may be found by Faraday’s law,

$$\varepsilon = -\frac{d\phi_{21}}{dt}, \quad \text{or} \quad \varepsilon = -M \frac{dI_1}{dt}, \quad (10)$$

where ϕ_{21} is the magnetic flux through coil 2 due to the current of coil 1 and M is the equivalent mutual inductance between two coils. If the current in coil 1 is alternating sinusoidally, then the

maximum EMF induced in coil 2 will be $MI_1\omega$. Changes in mutual inductance for different shell constructions is a useful metric for field concentration as it is proportional to the magnetic flux present in coil 2. It may also be readily calculated if the current in coil one is known and the voltage across the load resistance in circuit 2 is measured at the optimal load resistance (equation 9). This simple relationship of voltage measurement V_2 to mutual inductance M is described by,

$$V_2 = \frac{MI_1\omega}{\sqrt{2}}. \quad (11)$$

Parallel RLC coupled to coil can be considered as figure ???. The voltage across R_{load} is equal to,

$$V_s = j\omega MI_p - j\omega L_s I_s.$$

I_s can be found by,

$$I_s = j\omega MI_p / z_s$$

where z_s is the total impedance of circuit 2...

$$V_s = \frac{MR}{L_s} I_p. \quad (12)$$

Therefore a plot of V_s/R against I_p will yield a gradient of M/L_s as seen in figure ???. Furthermore, $M = M(\omega)$ for shells with copper present, and so a plot of $(M/L_s)/(M^0/L_s^0)$ gives the relative increase in M/L_s with frequency as seen in figure ???

To maximise power transfer an RLC circuit is constructed on the receiving circuit. Ideally in an RLC circuit the complex impedance of the inductance and capacitance cancel leaving only the load resistance. Our inductance is set by the solenoid we chose to use and so for exploring power transfer at various frequencies, a capacitance can be found to satisfy the resonance condition. If inductance, L , does not vary then capacitance, C , is easily found by,

$$C = \frac{1}{w^2 L}. \quad (13)$$

However, we need not assume that inductance is constant. A series RLC circuit can be constructed as seen in figure 3 c to ensure resonance is met. Resonance occurs in this circuit when V_x is exactly in phase with V_y . In this case any imaginary impedances are cancelled and only real resistance remains. For maximal power transfer a parallel RLC circuit is preferable however due to the restraints of the experiment a non-ideal version must be made as seen in figure 3 b. The resonant condition found by the series RLC is a close approximation to this more complicated parallel circuit.

To maximise power transfer in the series RLC case, a familiar idea of impedance matching occurs, i.e. Power is maximised when the load resistance is equal to any internal resistances of the components [?]. As internal resistances are difficult to measure and may depend on current XXX, this could also be found experimentally by measuring voltage and current over the load resistance whilst varying load resistance.

For the parallel RLC case, a more complicated expression for optimal load resistance was found which depends non trivially on a combination of internal resistances. A model is proposed below, however, experimentally locating the optimal resistances was chosen as measuring internal resistances proved difficult and time consuming.

XXTheory of Rload dependence on power transfer.

3. Results

3.1. DC Magnetic Fields

Using Helmholtz coils powered in DC to create a static external magnetic field (see Methods), we observed constant field concentration factors within the R_1 cavity for various shell constructions.

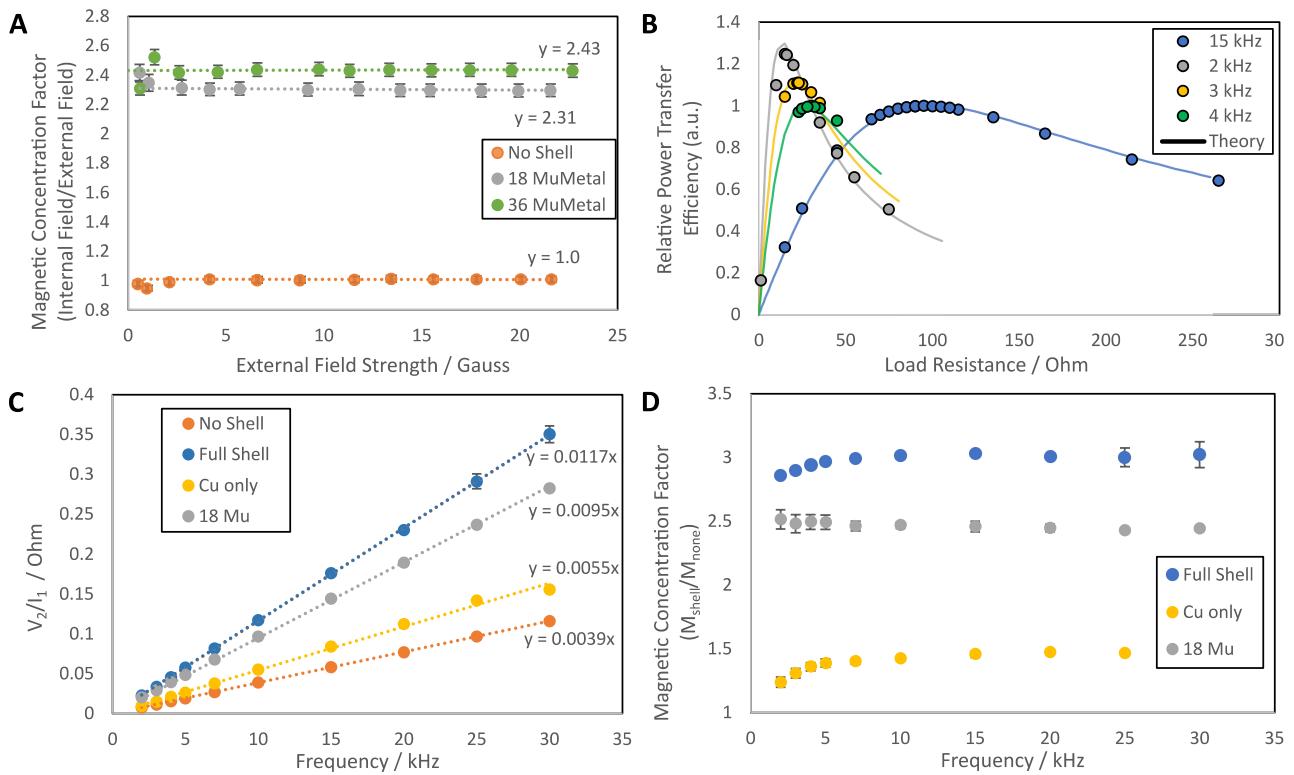


Figure 5. **A** (DC) The concentration of external static fields into the interior cavity of shells comprised of either 18 or 36 MuMetal sheets. **B** (AC) Experimentally found relative power transfer efficiencies dependence on load resistance in a coupled RL inductive circuit for various frequencies. Theory line is found using equation 9. **C** Finding mutual inductance, M , between two coils at a fixed distance for various shell configurations in the RL circuit arrangement. The gradient is equivalent to $M/\sqrt{2}$ (equation 11). **D** The shell's effect on mutual inductance is found by the ratio of (M with shell)/(M bare coil). An increase in mutual inductance corresponds to an increase in magnetic field density within the shell and is shown to be dependant on frequency for shells with copper present.

No shell, 18 MuMetal, and 36 MuMetal shells were used and their concentration factor with field may be seen in figure 5a, where concentration factor, C , is defined as (Magnetic field within shell's cavity)/(external field produced by Helmholtz coils). It was found that the shell construction of 36 MuMetal thin sheets gave the optimum concentration factor of $C = (2.43 \pm 0.04)$ with errors likely due to the sensitivity of dipole orientation within the field. The 18 MuMetal shell gave a magnetic concentration factor of $C = (2.31 \pm 0.04)$, which suggests that the shell behaves more ideally with a greater number of sheets. Simulation work performed on COMSOL further supports this as shown in figureXXX where more sheets corresponded to more ideal behaviour.

3.2. Power transfer

RL

Power is transferred from the primary circuit to a load resistor in the secondary circuit in a coupled inductive arrangement as explained in Methods. This may be simply performed by placing a load resistor in parallel with the receiving inductor L_2 forming an RL circuit as seen in figure 3 a. To maximise power transferred, the optimal load resistance was matched to $R = \omega L$ (see Methods equation 9). Figure 5 b shows the relative PTE versus load resistance for various frequencies confirming the expected optimum load resistance.

Mutual inductances between the coils at a set distance of 59 mm were found by measuring the voltage

across the load resistor in the secondary RL circuit and the current driving the primary circuit and using equation 11. The copper only, MuMetal only, and full copper plus MuMetal shells all showed a concentrating effect by their relative increases in mutual inductance between the two coils as shown in figure 5 *c*. The full shell showed an increase of mutual inductance (magnetic field) by a factor of (3.0 ± 0.1) , which is nearing the theoretical result from TO of $R_2/R_1 = 4$ despite only containing 36 sheets. The frequency dependance on the relative increase in mutual inductance is shown in figure 5 *d*. The two shell configurations containing copper are shown to have concentrating effects dependant on frequency. The concentrating effect rises steeply within the first 10 kHz and then plateaus for high frequencies. This expected behaviour initially motivated the choice for copper sheets in the shells for AC power transfer. At higher frequencies the perpendicular component of alternating magnetic fields produce eddy currents within the copper which, due to Lenz's law, oppose the change of field direction and effectively increases the angular permeability of the shell. Supporting work done on COMSOL (figure ??) shows a similar observed effect where 36 copper sheets increases the concentration factor rapidly from $C = 1$ at 0 Hz to $C = 1.5$ (2SF) by 10 kHz and then only a small further increase in efficacy at higher frequencies.

Apart from errors due to instrument, measurement reading, and coil placement, we observed noise due to stray magnetic fields inducing pick-up within cables connecting the solenoid to the lock-in amplifier. This source of noise is highly frequency dependent and is at the same frequency as our desired signal, so could not be removed by the use of a lock-in amplifier. To suppress this noise careful cable placement and shielding was installed.

Resonant — RLC

Parallel RLC circuits are more fitting for optimising power transfer [?] as explained in Methods. For the arrangement described in figure 3 *b*, a full shell comprised of 18 MuMetal and 18 copper sheets was explored. The optimal load resistances were found experimentally by measuring power for a sweep of resistances, an example of which is shown in figure 6 *a* for a field frequency of 30 kHz. From equation 12 the voltage measured across the load resistor is proportional to the mutual inductance between the two coils when the optimal load resistor is used and the resonance condition for the RLC circuit is met. Figure 6 *b* confirms this relationship and shows an increase of mutual inductance when the full concentrating shell is present. The gradient was found to be increased by a factor of (2.6 ± 0.2) corresponding to a field concentration of the same factor if the inductance of L_2 is assumed to remain constant.

The frequency dependance on concentration factor is shown in figure 6 *c* and it can be seen that, as with the RL and simulated experiments, the presence of copper in the shell leads to a low initial concentration which increases with frequency until around 10 kHz at which point the concentration factor plateaus.

The PTE for RL and RLC, no shell and full shell arrangements are compared in figure 6 *d*. It can be seen that the shell increases the efficiency in both cases by a similar factor and that the parallel resonant RLC arrangement gives a much greater efficiency than using the simpler RL circuitary.

4. Discussion

For the constructed shells used in these experiments, their R_2/R_1 ratio was (4.0 ± 0.2) . From the theoretical TO work, it is predicted that the field should be concentrated by the perfect shell by R_2/R_1 . We found a range of concentrations for both static, $C = 2.25 - 2.38$ XeX and oscillating, $C = 2 - 3.1$ XeX, external fields whose maximum concentration factors are still well below the expected theoretical concentration factor of 4.0 ± 0.2 . This is likely explained by our approach to approximating the perfect TO designed anisotropic shells with shells comprised of discrete sections of high and low permeability. In both static and oscillating fields, we found that 36 MuMetal sheets

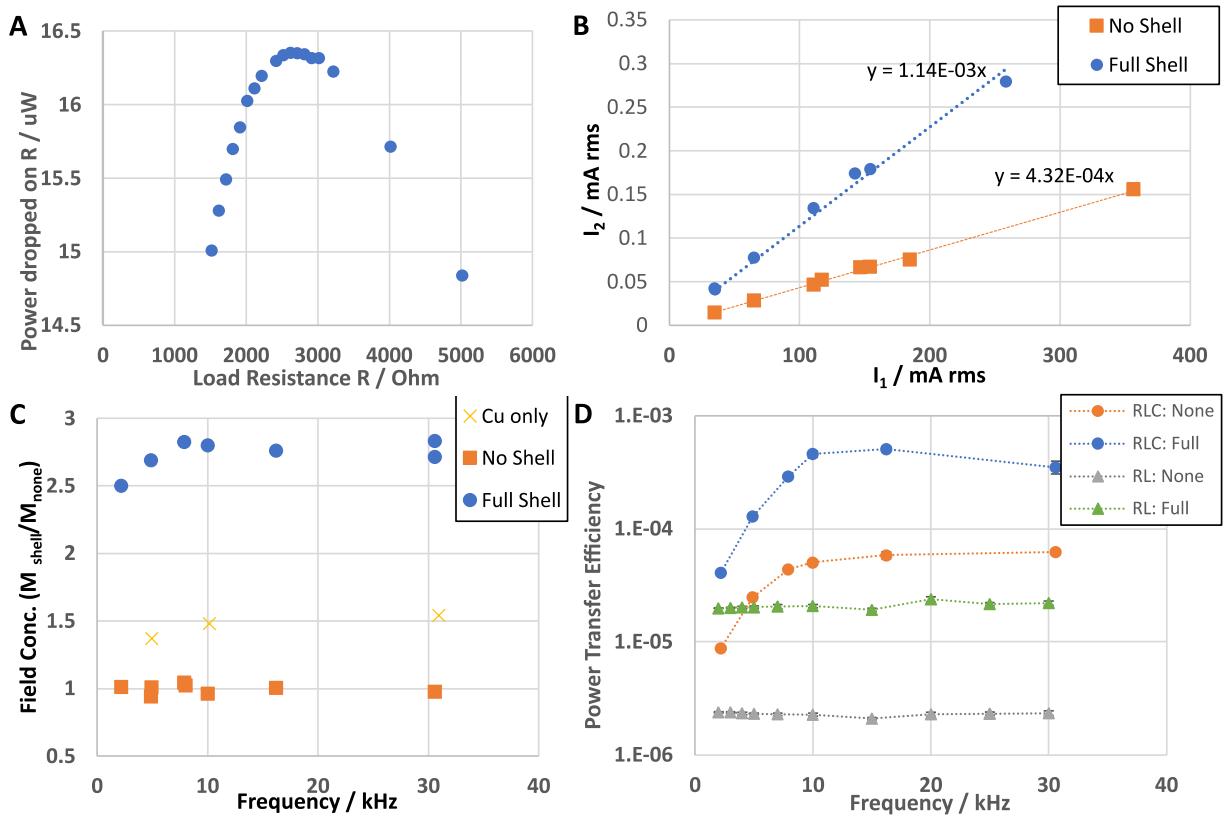


Figure 6. **A)** Locating the optimal load resistance for power transfer in a parallel RLC receiving circuit at an AC frequency of 30 kHz. **B)** The current across the optimal load resistor is proportional to mutual inductance in the resonant parallel RLC circuit. Comparing the gradients of I_2 against I_1 for the case when a shell is present to when one is not presents gives the factor increase of mutual inductance which is equivalent to the concentration of the field within the shell. **C)** The shell's effect on mutual inductance in an RLC circuit is found by the ratio of (M with shell)/(M bare coil). As with the RL circuit, the concentration of field is dependant on frequency for the full shell which has copper present. **D)** A comparison of power transfer efficiencies (PTE) for both a full shell (18 copper and 18 MuMetal sheet) and no shell in either the RL and RLC case. PTE is given as a ratio of power dropped across the load resistor against power lost in the transmitting coil as explained in Methods.

performed better than 18 MuMetal sheets which suggests that finer discretization of the proposed shells gives concentrations closer to the theoretical value. In the static field experiments, only air was used to approximate the zero relative angular permeability. Air has a permeability of $\mu_r = 1$, which although is far less than the MuMetal's $\mu_r = 55,000 - 75,000$, is likely still a poor approximation. This can be seen by the increase in performance in the oscillating external field experiments where copper sheets are introduced in order to effectively screen out angular field density by the production of Eddy currents. Eddy current screening is proportional to frequency of the oscillating field [?], and therefore, it can be seen that the concentration factor for shells with copper present increase with frequency.

Previous work by Pratt et. al.[?] included the exploration of MuMetal and superconducting shells for concentrating static magnetic fields. This work quoted a concentration factor of XXX at T_i/T_c and XXX at T_f/T_c .

So far we have assumed the inductance of the solenoid was independent of the shell surrounding it. Here we shall reexplore some results without this assumption.

The resonant condition of a series RLC circuit (figure 3 c) allows the calculation of effective inductance L using the known capacitance and found resonance frequency. The inductance of the bare coil or when surrounded by a copper only shell was (1.02 ± 0.02) mH whilst the coil surrounded by the full shell was found to be (1.25 ± 0.01) mH. This increase in inductance can be understood by the high permeability MuMetal in the shell being located near to the coil, which increases the relative permeability of space in the vicinity of the coil. A change in inductance will effect the power transfer efficiency as described by XXX equation ???. The use of a resonant parallel RLC setup within the receiving circuit can counteract this loss of power transfer as the optimum load resistance in this case does not depend as strongly on the inductance value, but rather on internal resistances of the components.

Due to the sharp drop off of power transfer with distance, the greatest PTE found was 1.01% XeX at a separation of 48.7 mm with both the transmitting and receiving coil surrounded by a concentrating shell. The reason for this poor efficiency is largely due to the non-ideal real resistive component of the transmitting coil...

DC:

- further explanation on error source in placement

Power transfer

- Generally low PTE due to high loss of inductor. Could have used laminated AC inductor to reduce eddie currents. Could have not used iron core as hysteresis.
- Discussion of other power transfer techniques and coupled inductor papers. Why a shell is better than no shell with distances.
- Why a shell allows a small radii inductor to be used - efficiency of scale Better coupling for given distance. See p.51 pratt.
- Relationship of PTE with distance $-r^{-5.6}$. Effective mutual inductance changed.
- Two shells present.
- Another exploration on error sources and summary of largest sources.

Comparison of RL, series RLC and parallel RLC circuitary:

- models
- expected efficiency

5. Conclusions

EXTENSIONS

0) For further understanding we would need to look at more frequencies and especially at much higher freqs. A selection of different dipoles would also be beneficial to ensure that frequency dependence is not linked to non-ideal dipole.

- 0) Perhaps another circuit could be designed that works more similarly to an RLC? Could propose one however experimentation has not been performed.
- 0) Could a more complicated and directional shell be designed for power transfer?

6. References