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Abstract.

1. Introduction

1.1. Maxwell equations

1.2. Transformation Optics and Metamaterials

1.3. Wireless Power Transfer

Abstract.
A device capable of uniformly concentrating a magnetic fields inside of a free space cavity will increase the efficiency of many magnetic devices and sensors. This project shall look at a proposed design for a magnetic field concentrator informed by the transformation optic technique. A metamaterial shell comprised of high and low permeability sections alternating in the angular direction has been shown to approximate the designed concentrator[?]. The ability of the shell acting as a concentrator will be explored in various regimes with a specific focus on improving efficiency of wireless power transmission.

Introduction To Physics

Transformation Optics and Metamaterials

Transformation optics has enabled the design and development of many new devices such as magnetic cloaks [?], rotators [?], hoses [?], perfect lenses [?] and magnetic concentrators [?][?]. By exploiting the form invariance of Maxwell's equations, any spatial coordinate transformation may instead be expressed by the permeability and permittivity of an inserted material [?]. This is shown schematically in figure 1, where a is a ray in free space, b is a ray in a transformed space and c is again in free space but with a specific material inserted.
In the static case, this may be described by rearranging,

$$\nabla' \times \mathbf{E'} = -jw[\mu_0]\mathbf{H'}, \tag{1}$$

where Faraday's law expressed in a transformed coordinate space $x'(x, y, z), y'(x, y, z), z'(x, y, z)$ and $[\mu_0]$ is the permeability of free space, to

$$\nabla \times \mathbf{E} = -jw[\mu']\mathbf{H}, \tag{2}$$

where we are now in Cartesian space x, y, z and $[\mu']$ is a non-free space permeability. A similar method may be used for the other Maxwell equations to find permittivity.

Figure 1. Schematic of the methodology of transformation optics. *a*) Ray in free Cartesian space. *b*) Ray in free transformed coordinate system. *c*) Ray in Cartesian space with material inserted with permeabilities and permittivities found by transformation optics. Figure taken from [**?**].

This process can be simplified as the required permittivity and permeability may be found if the Jacobian describing transformation from free space to the transformed space is known. This is described by

$$\mu' = \frac{A\mu_0 A^T}{|A|} \qquad \textit{and} \qquad \epsilon' = \frac{A\epsilon_0 A^T}{|A|} \tag{3}$$

where A is the Jacobian matrix and ϵ is relative permittivity.

Resulting permeabilities may be anisotropic, negative and have arbitrary magnitude when using this method. As bulk materials often do not have these properties, electromagnetic metamaterials are often employed. Metamaterials are materials engineered to have properties not observed naturally by utilising structures of a scale much smaller than the wavelength of an interacting field. This results in an apparent homogeneous material with unusual refractive indices. Metamaterials with anisotropic and negative [**?**] permeabilities and permittivities have been realized.

The focus of this research will be on magnetic concentrators. It is well known[ref] that ferromagnetic materials concentrate magnetic fields within their bulk. However this has only limited applications due to the concentrated region being within a material. Employing transformation optics we may solve for an optimised field concentrator where all the energy of a chosen region A is confined to a region of free space, B. The proposed geometry of such a device is shown in figure 2 *a*. A cylindrical shell is constructed with inner radius R_1 and outer radius R_2 where A is the area between R_1 and R_2, and B is the inner region confined by R_1. It is assumed that this shell is infinite along the z direction. Employing transformation optics we apply two coordinate transforms: First the region $\rho < R_2 - \xi$ is linearly compressed to region $\rho < R_1$, where ρ is the radial dimension and ξ is a positive constant. To ensure the continuity of our transformed space, a high (kth) order polynomial expansion of a "skin" of width ξ is expanded to fill the now empty region A. The limit of $\xi \to 0$ is taken to concentrate all of region B into A.
This process may be described by the following coordinate transforms where $\theta' = \theta$, $z' = z$, and ρ' is given by

$$\rho' = \frac{R_1}{R_2 - \xi}\rho, \qquad \rho' \in [0,\ R_2 - \xi) \quad \rho' = R_2^{1-k}\rho^k. \qquad \rho' \in [R_2 - \xi,\ R_2) \tag{4}$$

Finding the Jacobians of these transforms and utilising equation 3 we find that the following matrices describe the required relative permeabilities of an inserted material,

$$\begin{aligned} \mu' &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & (\frac{R_2-\xi}{R_1})^2 \end{pmatrix} \qquad && \rho' \in [0,\ R_1) \\ \mu' &= \begin{pmatrix} k & 0 & 0 \\ 0 & 1/k & 0 \\ 0 & 0 & \frac{1}{k}(\frac{\rho'}{R_2})^{2/k-2} \end{pmatrix} \qquad && \rho' \in [R_1,\ R_2) \end{aligned} \tag{5}$$

By matching boundary conditions at $R_2 - \xi$ and taking the limit $\xi \to 0$ we find $k \to \infty$. The z reliance on μ' may be ignored as we assume the infinite tube to be invariant in z. This results in the permeability within B being satisfied by free space and a material with $\mu_\rho \to \infty$, $\mu_\theta \to 0$ satisfies the requirements for region A.
This is where transformation optics ends and the exploration of metamaterials satisfying these equations begins. It was proposed [**?**] that the guiding of field lines by ferromagnetic material

Figure 2. *a*) The geometry of the described problem. All of magnetic energy in region A will be concentrated into region B. *b*) An approximation to the required permeabilities described in equation 5. MuMetal provides large radial permeability whilst superconducting sheets (SC) restrict angular permeability.

($\mu >> 1$) and exclusion of field lines by SC material ($\mu \to 0$) can be arranged as shown in figure 2 *b* to approximate such a material with high radial and near zero angular permeability.
In a uniform external field the theoretical shell should increase field within region B by a factor of R_2/R_1, however the approximated shell, shown in figure 2, was shown experimentally to only yield a concentration factor of 2.7 ($R_2/R_1 = 4$). This suggests that the approximation is far from optimised.
If an internal magnetic field is considered using a dipole, all flux within A is concentrated to the outside of the shell. The magnetic moment of the dipole appears to be increased by a factor of R_2/R_1 to an observer at $\rho > R_2$.

Magnetic Concentrators

Coupling — Wireless charging

Wireless power transmission is needed for powering devices which are inconvenient or dangerous to power with wires. Examples include mobile phones and implanted medical devices.
It was suggested **??** that two of the above described shells may be utilised for power transfer due to the ability to increase coupling of two solenoids within the inner radii of the shells. One solenoid is supplied with an alternating current to create an alternating magnetic field. This field is then concentrated by the second shell and a current is induced in the second solenoid.
Two assumptions must be made: first, the metamaterials act appropriately at the required frequency in an AC field and second, that the transformation optic approach is still relevant in non-static fields.

In the case of wireless power transfer there may be a set distance, d, where no material may exist. Shells take up physical space and so it is worth considering if such shells still offer greater coupling in this regime compared to bringing the dipoles d apart. As the shell increases the dipoles apparent magnetic moment by R_2/R_1, the shell at a distance $d + R_2$ is shown to provide an advantage over bare dipoles when $\frac{R_2}{R_1} > 1 + \frac{R_2}{d}$. For a given R_2 this means that R_1 may always be reduced to satisfy this inequality.

2. Methods

2.1. DC Magnetic Fields

Helmholtz coils were powered by a constant DC current to create a uniform magnetic field within their center. A commercially available XXX Hall probe was zeroed by using a MuMetal cannister, and then placed at the center of the Helmholtz coils. A Hall probe relates a measured Hall voltage, V_H, to a surrounding magnetic field, B [**?**] as

$$V_H = \frac{IB}{net}. \tag{6}$$

The probe maintains constant current supply I, and material paramaters n (charge carrier density), e (charge of electron) and t (thickness of probe) meaning a calibrated probe may give accurate readings for magnetic fields.

The magnetic field, B, produced at the center of Helmholtz coils with radius R, seperated by a distance R should follow,

$$B = \frac{8}{5\sqrt{5}} \frac{\mu_0 n I}{R}, \tag{7}$$

where I is the current supplied to the coils and n is the number of turns of wire. This equation follows directly from the Biot-Savart law [?] and the relative geometry of the coils as seen in figure **??**. From equation 7 it can be seen that the magnetic field should increase linearly with supplied current. Using the Hall probe we ensured this was the case and found the relationship of current supplied to magnetic field produced for our paticular Helmholtz arrangement.
Now, with the capability to produce known external magnetic fields, the described field concentrating shells may be placed within this field and the Hall probe may be placed within their inner radius to measure concentrated field.

2.2. AC characterization

Initially the Helmholtz arrangement was repeated for exploration of the concentrating shells behaviour in alternating magnetic fields. However, instead of a Hall probe, a small solenoid was used to detect the oscillating field. From Faraday's law, a voltage will be induced in a wire loop due to a time dependent magnetic field. A series of loops constituting a small solenoid will respond to a sinuisoidal magnetic field, $B = B_0 \cos wt$, with the relationship,

$$V = -NAB_0\omega \sin \omega t, \tag{8}$$

where A is the area of one loop and N is the number of loops, ω is the angular frequency of the alternating magnetic field and t is time.
As ω is known and all other parameters except external field are kept constant, the voltage across the solenoid may be measured experimentally to find the relative magnetic field strength.
The solenoid must however be characterized in order to find the absolute magnetic field values. This was done by measurement of the self inductance, L, of the solenoid as, XXX

$$L = \mu_0 \mu_r N^2 A/l \tag{9}$$

XXX

Due to the induced voltage across the inductor being small and background noise being high, a lock-in amplifier was used to select only the desired signal frequency. This substantially reduced noise in our readings allowing higher frequency and lower magnetic field strength experiments.

Use of solenoid, limitations of Helmholtz and pick up. Use of RLC circuitary.

2.3. Power Transfer

Power transfer experiments measure power dropped across a load resistor in a receiving circuit verses power lost in the transmitting circuit's inductor. The receiving circuit, seen in figure **??**, has multiple arrangements to optimise power transfer. The simplest of which is the load resistor in series with the receiving inductor. In this case the optimal load resistance is $R = \omega * L$, where ω is the angular frequency of the oscillating magnetic field and L is the circuit inductance.
To maximise power transfer an RLC circuit is constructed on the receiving circuit. Ideally in an RLC circuit the complex impedance of the inductance and capacitance cancel leaving only the load resistance. Our inductance is set by the solenoid we chose to use and so for exploring power transfer at various frequencies, a capacitance can be found to satisfy the resonance condition. If inductance, L, does not vary then capacitance, C, is easily found by,

$$C = \frac{1}{w^2 L}. \tag{10}$$

However, we need not assume that inductance is constant. A series RLC circuit can be constructed as seen in figure **??** to ensure resonance is met. Resonance occurs in this circuit when V_A is exactly in phase with V_B. In this case any imaginary impedances are cancelled and only real resistance remains.

Figure 3.

Figure 4.

For maximal power transfer a parallel RLC circuit is preferable however due to the restraints of the experiement a non-ideal version must be made as seen in figure **??**. The resonant condition found by the series RLC is a close approximation to this more complicated parallel circuit.

To maximise power transfer in the series RLC case, a familiar idea of impedance matching occurs, i.e. Power is maximised when the load resistance is equal to any internal resistances of the components [**?**]. As internal resistances are difficult to measure and may depend on current XXX, this could also be found experimentally by measuring voltage and current over the load resistance whilst varying load resistance.
For the parallel RLC case, a more complicated expression for optimal load resistance was found which depends non trivially on a combination of internal resistances. A model is proposed below, however, experimentally locating the optimal resistances was chosen as measuring internal resistances proved difficult and time consuming.

2.4. COMSOL

Expleen

3. Results

3.1. DC Magnetic Fields

Using the DC Helmholtz set up as described in Methods, we observed constant concentration factors for different shell constructions in an external magnetic field ranging from 1 to $22G$. No shell, 18 MuMetal, 36 MuMetal, 18 Copper and 18 MuMetal + 18 Copper shells were used and their behaviour with field may be seen in figure 3.
It was found that the shell contruction of 36 MuMetal thin sheets gave the optimum concentration of $C = 2.38$ with minimal error (0.1%) at higher field strengths and a maximum error of 4.0% at an external field of 1.4 G. This increase in error at low magnetic fields is due to limited sensitivity of our Hall probe and current measurements over the Helmholtz coils.
Similar error relationships are observed for the other constructions. It should be noted that we assume the dipole has been placed in the same position and orientation in all experiments and so errors due to placement are excluded here.
The copper only shell showed no concentration of internal field as expected. This is due to copper having a relative permeability similar to air, $\mu_r = 1.0$, and so negligible field guiding properties. Furthermore, in the DC regime copper will not shield XXangularXX fields as required by the optimal TO concentrator.

3.2. AC characterization

3.3. Helmholtz

The Helmholtz coils were supplied with an alternating current in order to create an alternating magnetic field. Now using the voltage induced across solenoid to detect alternating magnetic field strength as decribed in Methods, the concentration of various shell arrangements were explored.

The concentration factors between 0.5 and 30 kHz can be seen in figure 4. It was found that a mixed shell of alternating 18 copper and 18 MuMetal sheets had the optimum concentration factor of $C = 3.12$ at 5 kHz.
Here we can see that the copper sheets now have a modest concentrating effect as frequency increases. This behaviour is expected as copper will shield perpendicular alternating magnetic fields which is desired for the optimal TO concentrator. However, it is suprising that this shielding occurs at such low frequencies, i.e. Much less than skin depth of copper.XXX We found that the copper shell increases in efficacy from $C = 1.0$ at 50 Hz until $C = 1.3$ (2SF) at 10 kHz and does not increase substantially more as frequency is increased. This suggests that the shielding effect of the copper depends on frequency but saturates early.
Supporting work done on COMSOL (see Methods) suggests a similar observed effect where 36 copper sheets increase rapidly in concentration factor from $C = 1$ at 0 Hz to $C = 1.5$ (2SF) at 10 kHz.

The strong linear decay of field concentration after $5 - 10$ kHz for all but the copper only shell is also a suprising result which, although could be explained by the MuMetal permeability frequency response, also appears to occur at too low a frequency.
COMSOL work does not show this relationship and so the source may be either not modelled appropriately within COMSOL or be a fault in this experimental design.

Apart from instrument and measurement reading errors which constitute only a small error (XXX%), we observed errors due to high pick-up in cables connecting the solenoid to the lock-in amplifier. This source of noise was at the same frequency as our desired signal and so is difficult to remove other than careful cable placement and using shielding. We believe this pick-up was worsened by the fact the Helmholtz coils must be driven with high voltage and current to create a useful magnetic field and that the magnetic field was not localised to just our solenoid and shell but also was subject to the cabling and any nearby detectors. This prompted a decision to focus on two dipole coupling experiments as this pick-up error can be greatly reduced.

3.4. Power transfer

First we show the simplest power transfer experiment where the load resistance is in series with the receiving inductor. Figure **??** shows the relative PTE versus load resistance for various frequencies. The peaks of these curves confirm the expected optimum load resistance of $R = \omega L$.
An Oscillating magnetic field, B, produced from a solenoid and concentrated by a shell follows,

$$B = CI\mu_0 n \cos \omega t,$$

where C is the concentration factor, I is the current through the solenoid and n is the number of turns of the solenoid. If a second solenoid is placed within the field of the first, as shown in figure **??**, then voltage will be induced across it according to Faraday's law,

$$V = -NA\frac{dB}{dt},$$

$$V = C\omega INA\mu_0 n \sin \omega t,$$

where N is the number of turns of the solenoid and A is the area of one turn. The power dropped across a resistor with magnitude ωL in series with this inductor will then be described by,

$$P = V^2/R$$

$$P = \frac{C^2 wI^2 k}{L} \sin \omega t^2$$

where k is the collection of constant coefficients that will remain constant between different shells. Max power received in the second circuit is therefore proportional to w, $\frac{C^2}{L}$ and I^2. Plotting $\frac{P}{I^2}$ against angular frequency w therefore gives $\frac{C^2k}{L}$ as shown in XX figure ??. Assuming the coefficients in k remain constant, comparisons of this gradient between a concentrating shell with inductance L_s and no shell with inductance L_0 and $C_0 = 1$ yields $\kappa = \frac{C^2 L_0}{L_s}$. Figure XXX ?? shows how κ depends on frequency for various shell configurations.
As observed in the Helmholtz driven field case, we see that the copper sheets begin to have a concentrating effect between 0 and 10 kHz. The Copper only shell increases to around $\kappa = 2$ which, if it is assumed that $L_0 = L_s$, corresponds to a power transfer increase of 2x or a corresponding field concentration of $\sqrt{2}$ within the shell's cavity. It can be seen that using only MuMetal sheets gives a power transfer increase of 6x and a concentration of field that is independent of field oscillation frequency for the range $0 - 30$ kHz. This differs from the previous Helmholtz result where a steady drop off of concentration factor was observed as frequency increased past 10 kHz.
The mixed shell of 18 MuMetal sheets and 18 Copper sheets was found to have the best power transfer increase of 9x after the copper sheet effectively shields the angular field at 10 kHz. This power increase corresponds to a magentic field concentration of 3 within the shells cavity.

Figure ?? XXX shows the optimal PTE for a range of frequencies with different shell constructions around the receiving inductor. Assuming the inductance value remains constant with different shell configurations (an exploration of this assumption is considered in DiscussionXXX), absolute power transfer can be calculated as described in Methods. Table ?? gives the maximal power transfer for different arrangements of shells.

Parallel RLC circuits are more fitting for optimising power transfer [?]. For the arrangement described in figure ??, a shell comprised of 18 MuMetal and 18 Copper sheets was explored. The optimal load resistance was found by taking voltage measurements across a range of load resistance. An example power versus load resistance curve for 30 kHz can be seen in XXX figure ??.
Optimal load resistances were found for a range of frequencies and PTE were calculated as shown in XXX figure ??. Figure XX ?? shows the ratio of shell present versus no shell present for the range of frequenies. It can be seen that the increase of ratio between 0 and 10 Hz is still present, however due to the high error and few data points, other trends are hard to distinguish. In this arrangement, with the coils seperated by a distance of XXX mm, maximum observed power transfer is XX 0.05%.

To further explore PTE, the distance between the two coils was varied. With a distance of XXX mm and a full shell around the receiving coil, a PTE of XXX% was achieved.

It was expected that a shell around the transmitting coil would further increase the field incident on the receiving coil. Therefore the arrangement described in Methods Figure ?? was constructed and the peak power transfer observed at 30510 kHz was found to be 1.01%.

3.5. Other COMSOL

4. Discussion

5. Conclusions

6. References