



Common Lines Implied Clustering

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eBIC/University of Bath



Common Lines
●○○○○

Finding Common Lines
○○○

Dimensional Reduction
○○○○○○○○

Clustering
○○○○○○

Common Lines

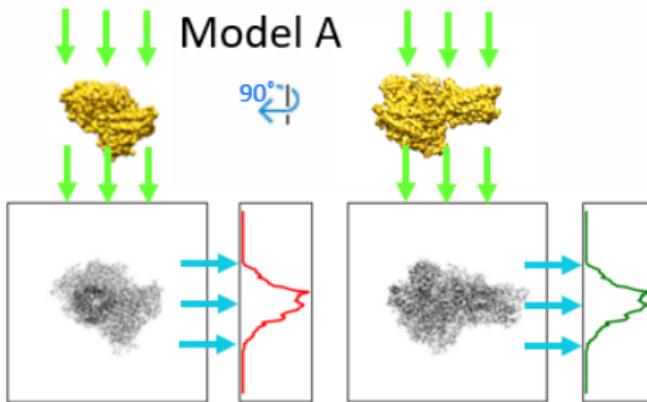
Common Lines
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Dimensional Reduction
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Clustering
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A Shared Single Line



Common Lines
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Finding Common Lines
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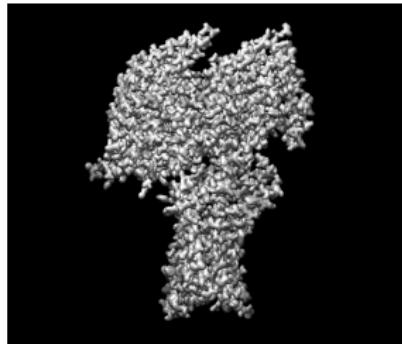
Dimensional Reduction
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Clustering
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Common Lines



Two projections of the same 3D volume share at least one common line in the Radon transform



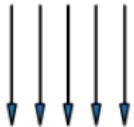
Common Lines
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Dimensional Reduction
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Clustering
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The Radon Transform



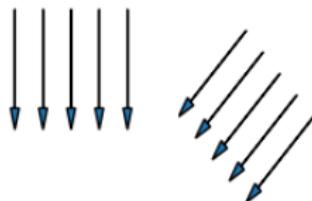
Common Lines
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Dimensional Reduction
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Clustering
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The Radon Transform



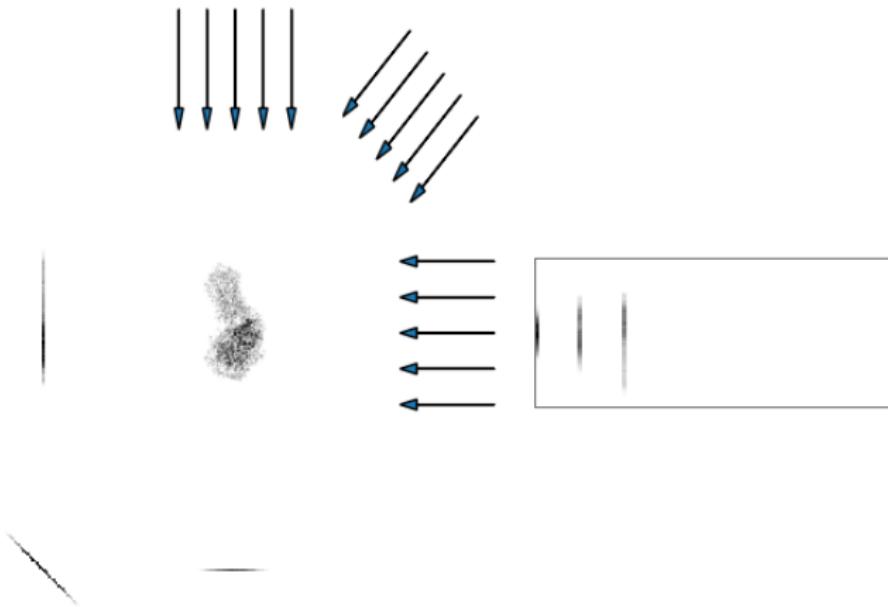
Common Lines
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Dimensional Reduction
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Clustering
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The Radon Transform



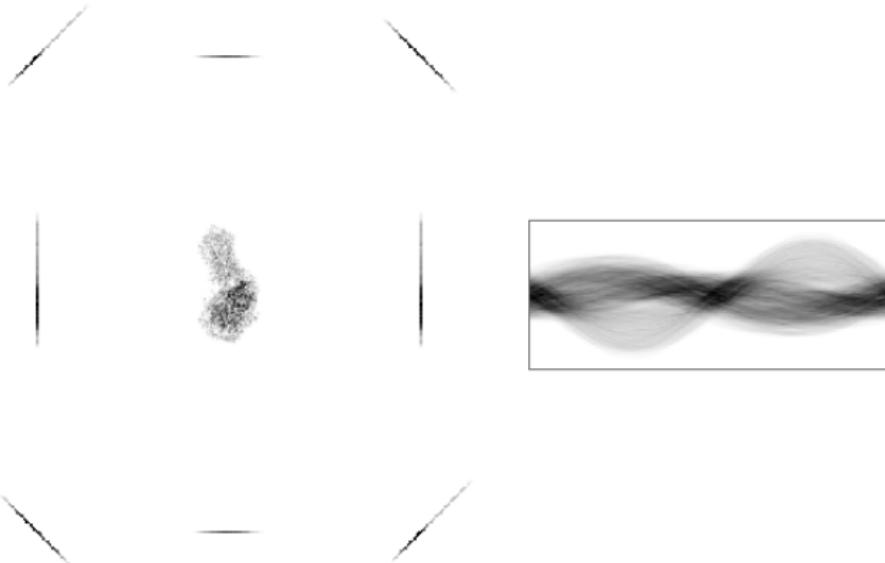
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Dimensional Reduction
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Clustering
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The Radon Transform



Common Lines
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Finding Common Lines
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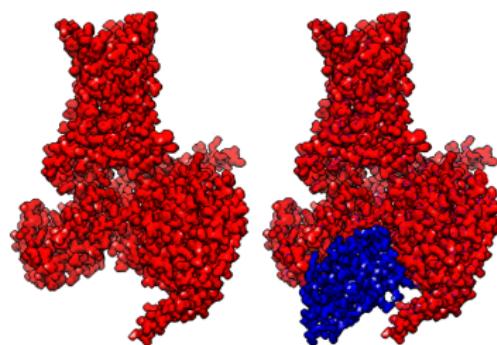
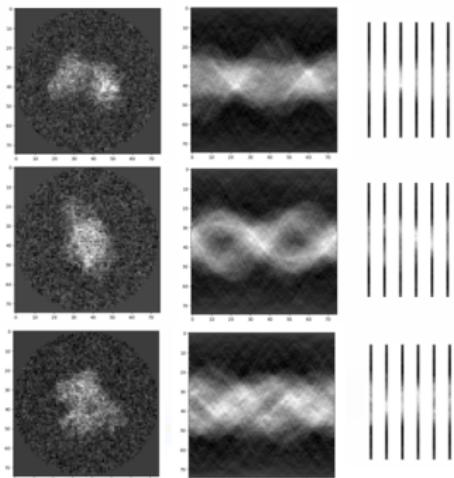
Dimensional Reduction
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Clustering
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The Problem



Can projections of different models be separated based on similarities and differences between common lines?



Common Lines
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Finding Common Lines
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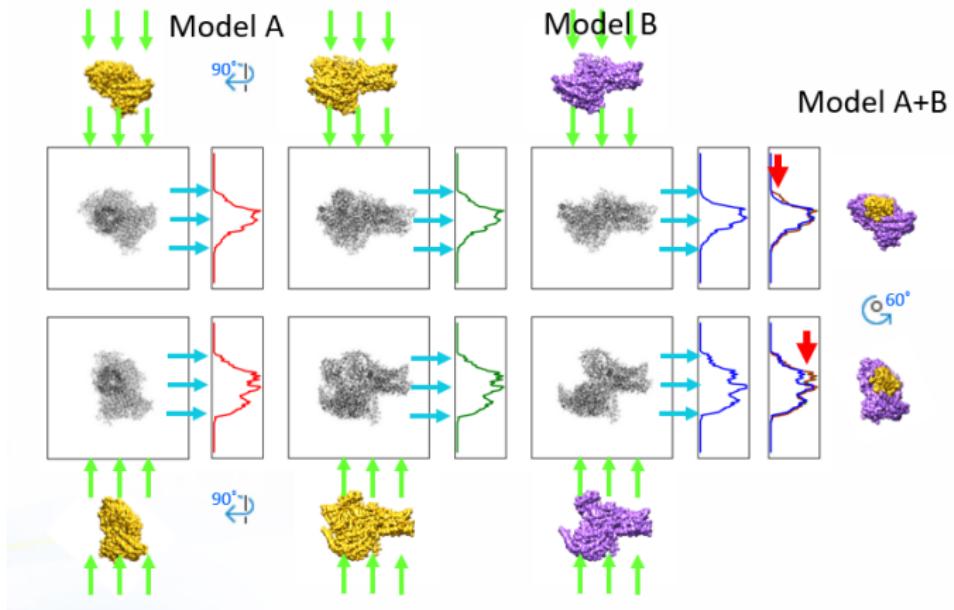
Dimensional Reduction
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Clustering
oooooo

Common Lines Between Different Models



What about two different 3D volumes?



Common Lines
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Finding Common Lines
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Dimensional Reduction
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Clustering
oooooo

Finding Common Lines

Common Lines
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Finding Common Lines
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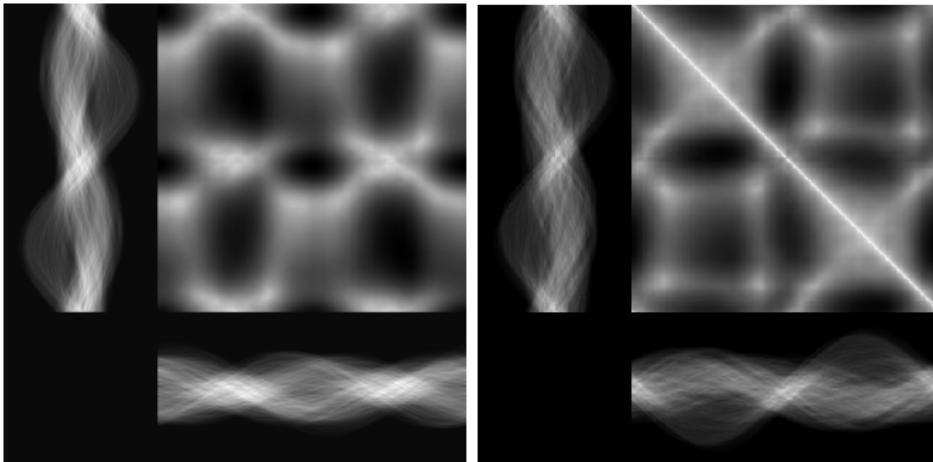
Dimensional Reduction
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Clustering
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Sinogram Cross Correlation



Finding the common line between two sinograms



Common Lines
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Finding Common Lines
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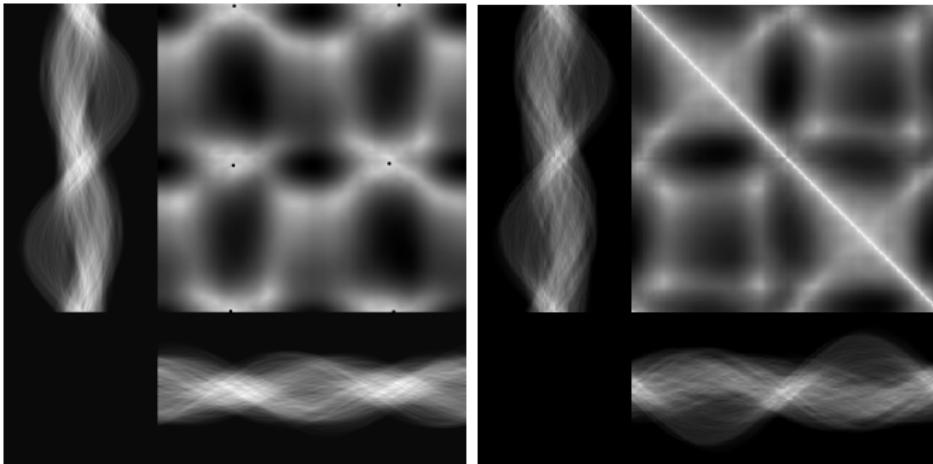
Dimensional Reduction
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Clustering
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Sinogram Cross Correlation



Finding the common line between two sinograms



Common Lines
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Finding Common Lines
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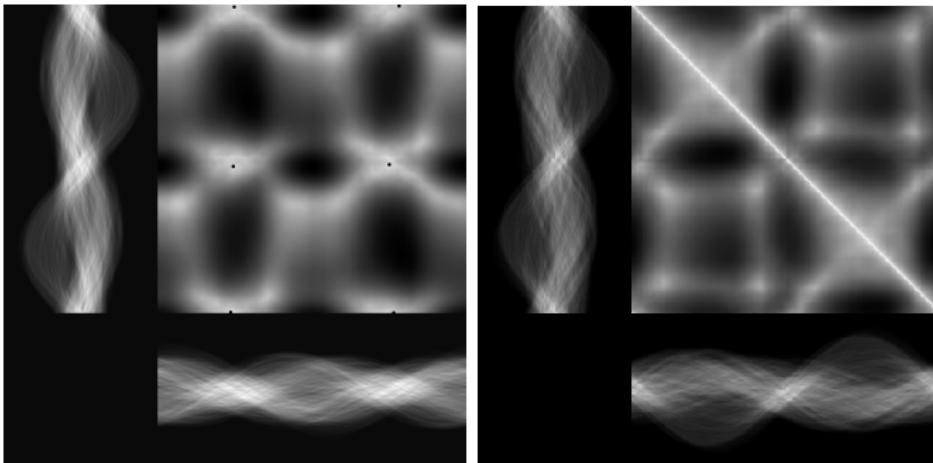
Dimensional Reduction
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Clustering
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Sinogram Cross Correlation



Finding the common line between two sinograms



But what about N sinograms?

Common Lines
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Finding Common Lines
○●○

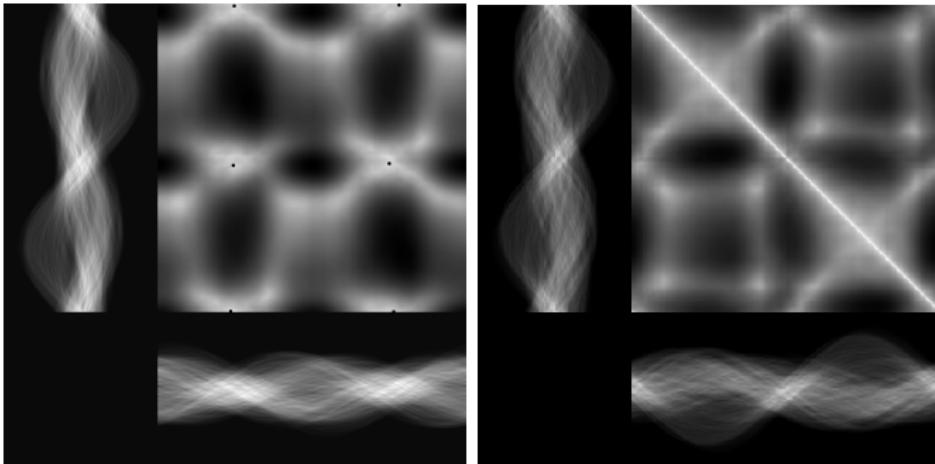
Dimensional Reduction
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Clustering
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Sinogram Cross Correlation



Finding the common line between two sinograms



But what about N sinograms?

What about N sinograms from a heterogenous dataset?

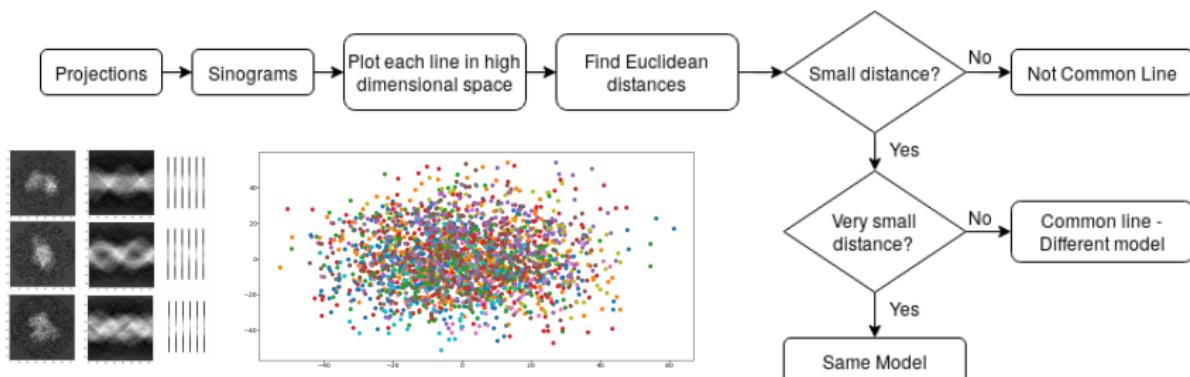
Common Lines
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Finding Common Lines
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Dimensional Reduction
oooooooo

Clustering
oooooo

Concept's flowchart



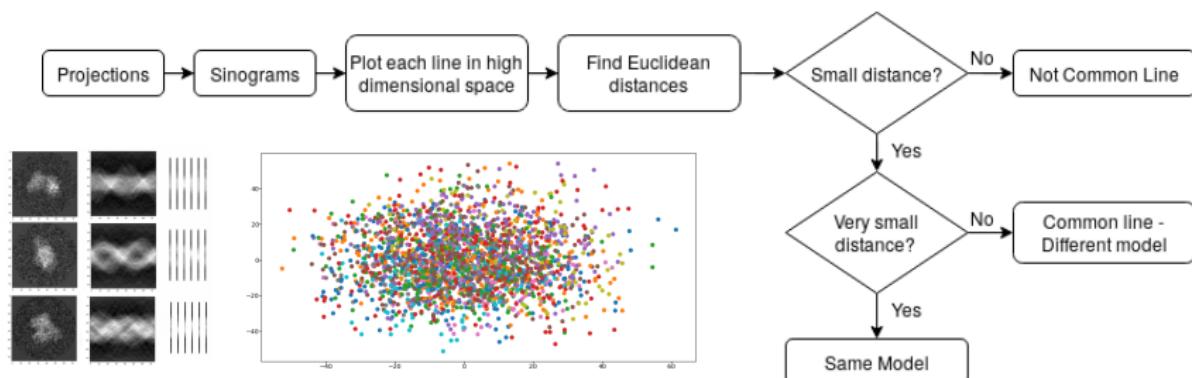
Common Lines
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Finding Common Lines
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Dimensional Reduction
oooooooo

Clustering
oooooo

Concept's flowchart



15 projections - 2 models - 75 dimensional space

Curse of Dimensionality.

Common Lines
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Finding Common Lines
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Dimensional Reduction
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Clustering
oooooo

Dimensional Reduction

Common Lines
oooooo

Finding Common Lines
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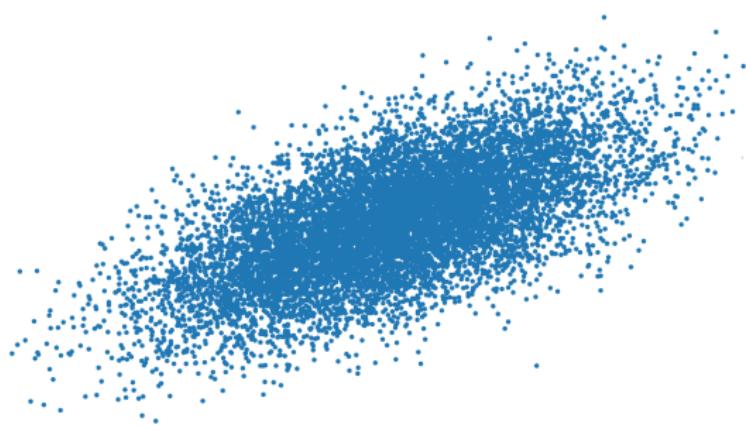
Dimensional Reduction
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Clustering
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Dimensional Reduction - Linear (PCA)



How to display high dimensional data in lower dimensional space?



Common Lines
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Finding Common Lines
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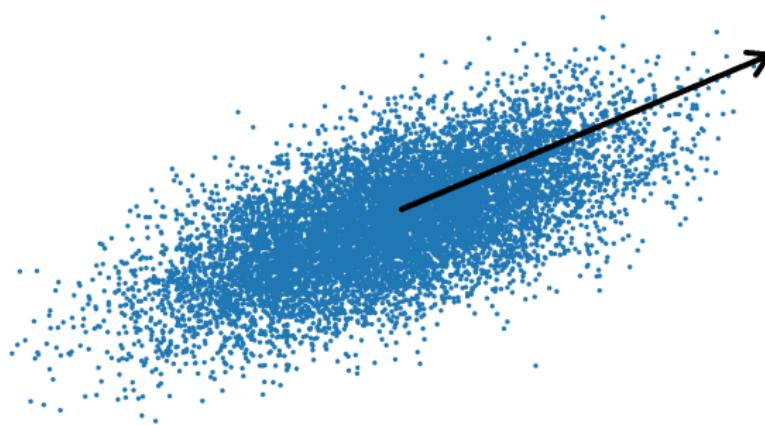
Dimensional Reduction
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Clustering
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Dimensional Reduction - Linear (PCA)



How to display high dimensional data in lower dimensional space?



Common Lines
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Finding Common Lines
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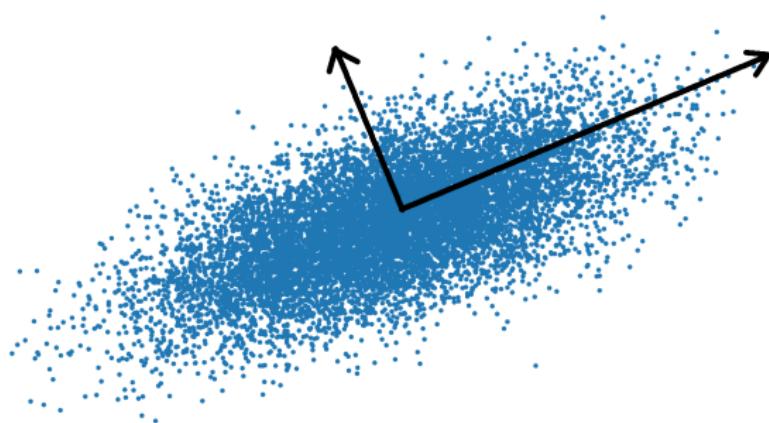
Dimensional Reduction
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Clustering
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Dimensional Reduction - Linear (PCA)



How to display high dimensional data in lower dimensional space?



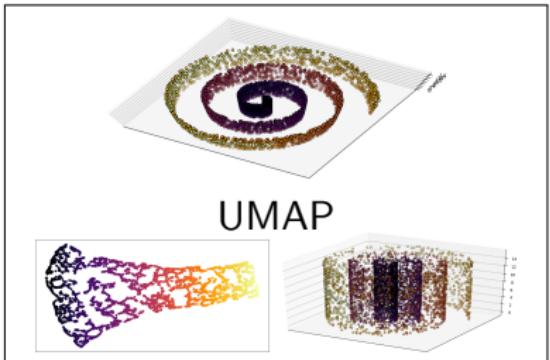
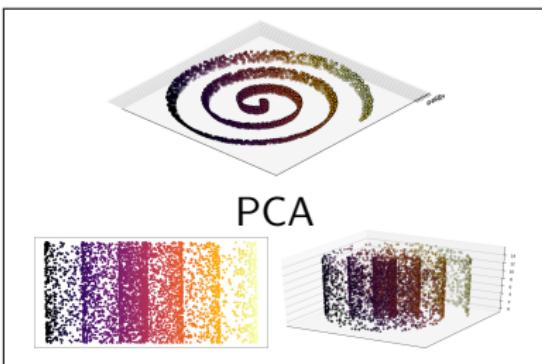
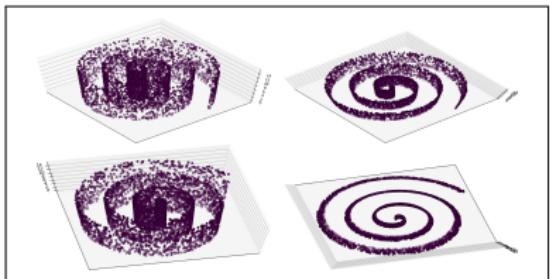
Common Lines
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Finding Common Lines
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Dimensional Reduction
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Clustering
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Linear vs Non-Linear - Swiss Roll



UMAP finds manifold, PCA does not

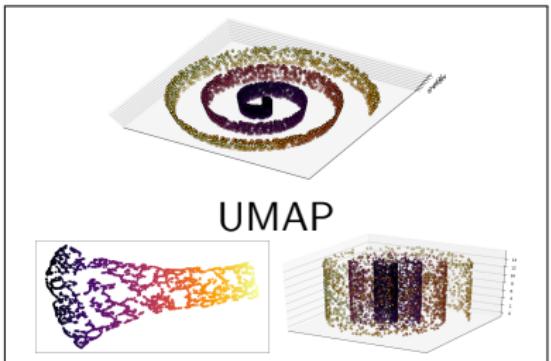
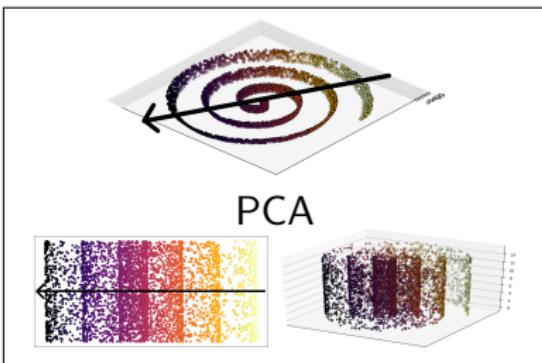
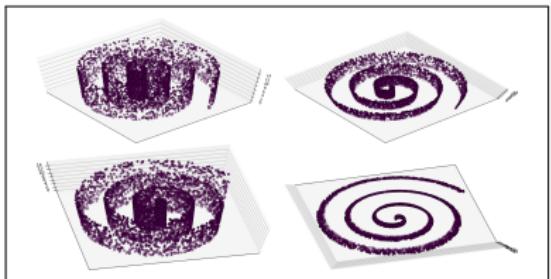
Common Lines
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Finding Common Lines
ooo

Dimensional Reduction
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Clustering
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Linear vs Non-Linear - Swiss Roll



UMAP finds manifold, PCA does not

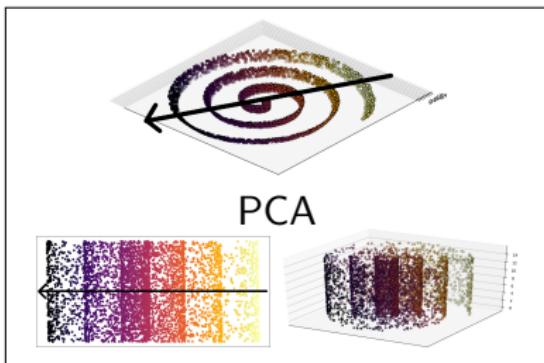
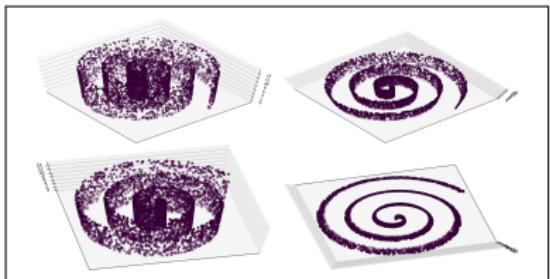
Common Lines
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Finding Common Lines
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Dimensional Reduction
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Clustering
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Linear vs Non-Linear - Swiss Roll



UMAP finds manifold, PCA does not

Common Lines
oooooo

Finding Common Lines
ooo

Dimensional Reduction
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Clustering
oooooo

Dimensional Reduction - Linear



Find features - Reduce noise
Linear (PCA)



Common Lines
oooooo

Finding Common Lines
ooo

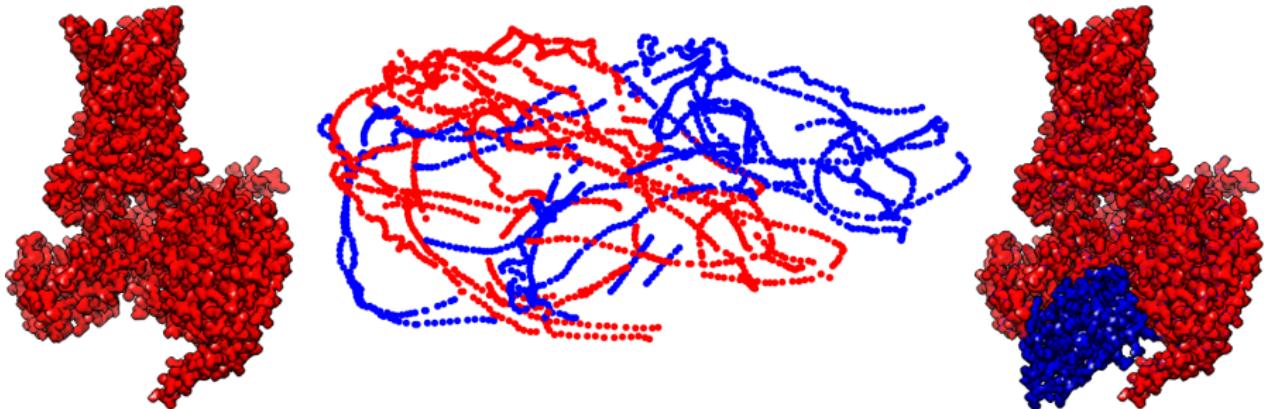
Dimensional Reduction
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Clustering
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Dimensional Reduction - Linear



Find features - Reduce noise
Linear (PCA)



Common Lines
oooooo

Finding Common Lines
ooo

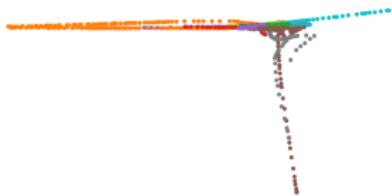
Dimensional Reduction
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Clustering
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Dimensional Reduction - Non-Linear



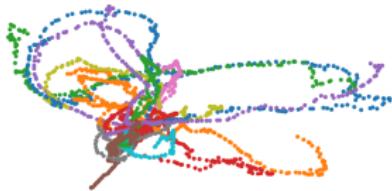
LLE



UMAP



ISOMAP



TSNE



Common Lines
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Finding Common Lines
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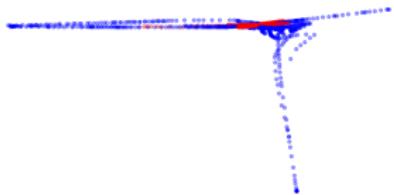
Dimensional Reduction
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Clustering
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Dimensional Reduction - Non-Linear



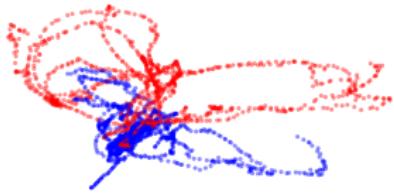
LLE



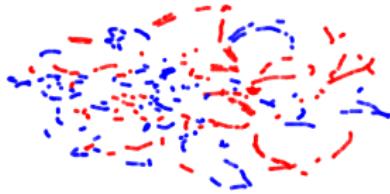
UMAP



ISOMAP



TSNE



Common Lines
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Finding Common Lines
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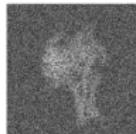
Dimensional Reduction
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Clustering
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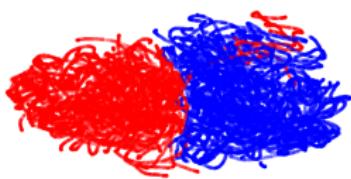
UMAP



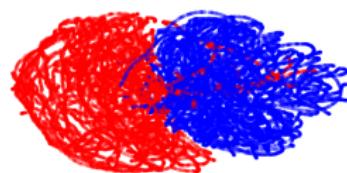
SNR 1



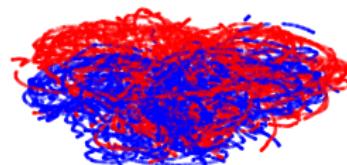
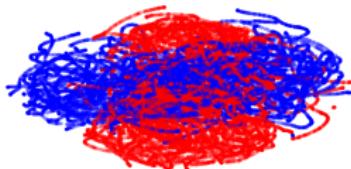
SNR 1/2



SNR 1/3



SNR 1/4



Common Lines
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Finding Common Lines
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Dimensional Reduction
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Clustering
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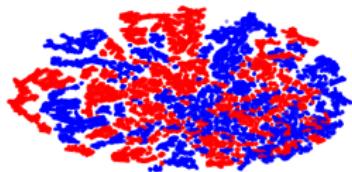
TSNE



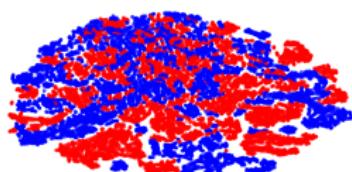
SNR 1



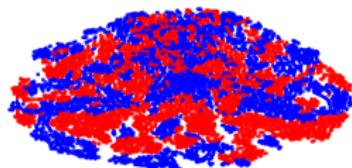
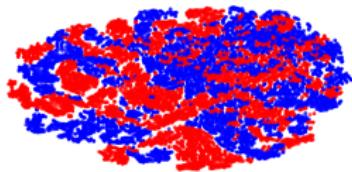
SNR 1/2



SNR 1/3



SNR 1/4



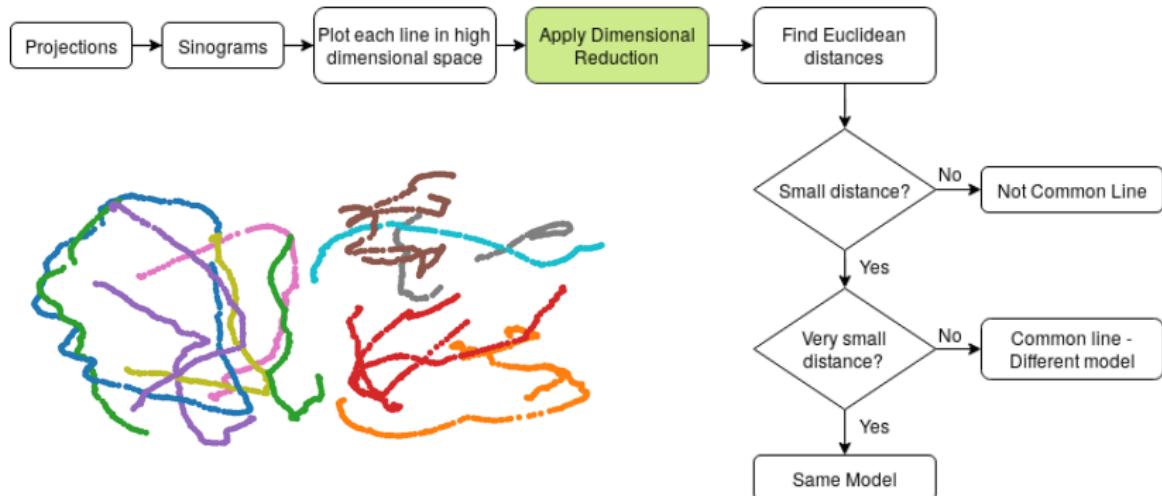
Common Lines
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Finding Common Lines
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Dimensional Reduction
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Clustering
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Flowchart



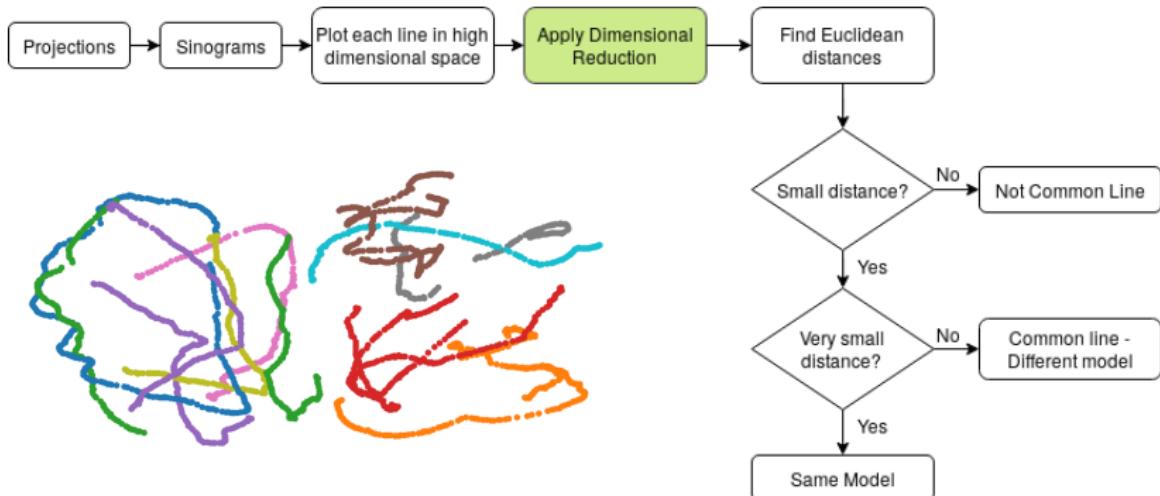
Common Lines
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Finding Common Lines
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Dimensional Reduction
oooooooo●

Clustering
oooooo

Flowchart



But how do we assign clusters?

Common Lines
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Finding Common Lines
ooo

Dimensional Reduction
oooooooo

Clustering
●ooooo

Clustering

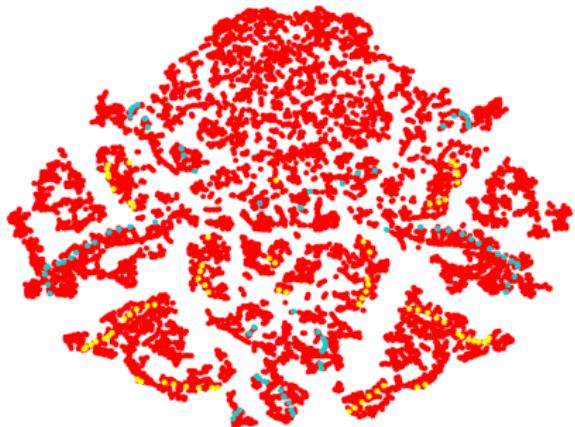
Common Lines
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Finding Common Lines
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Dimensional Reduction
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Clustering
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The data



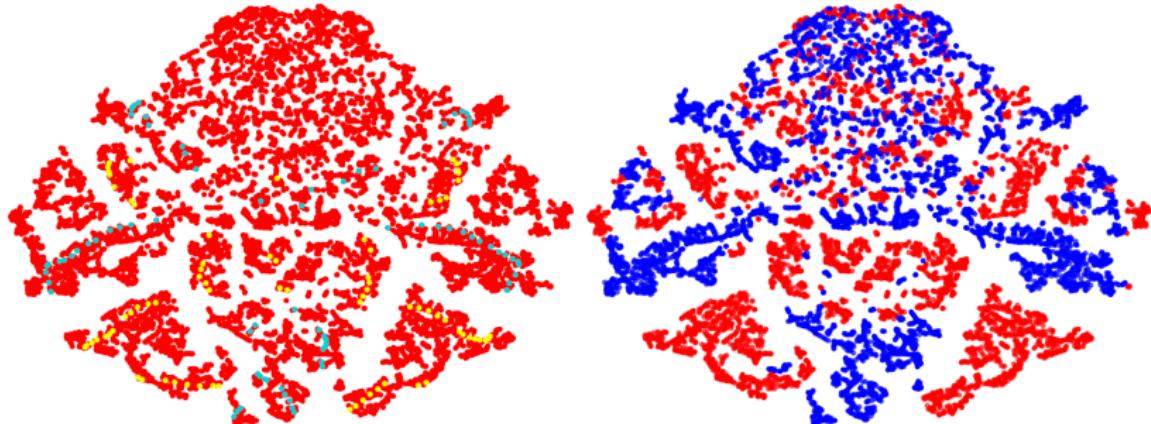
Common Lines
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Finding Common Lines
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Dimensional Reduction
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Clustering
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The data



Ground truth: Good separation between two classes - but discontinuous

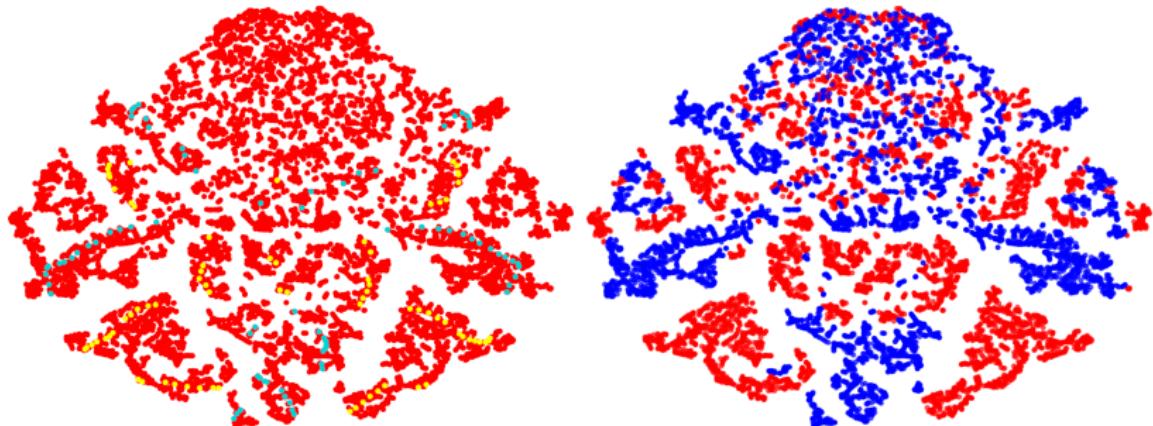
Common Lines
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Finding Common Lines
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Dimensional Reduction
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Clustering
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The data

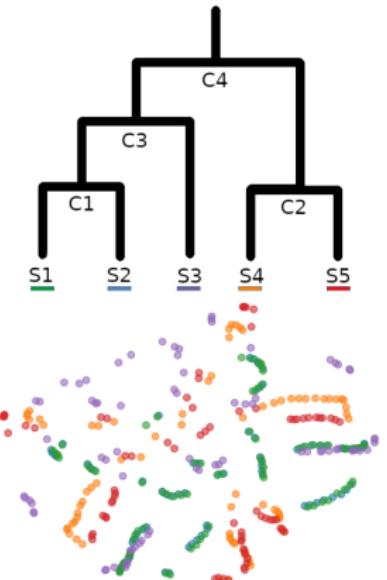


Ground truth: Good separation between two classes - but discontinuous

We know which sinogram each line comes from...

Lines from one sinogram are initial clusters

Heirarchical clustering



- S1 – S5 Are Sinograms.
- S1 and S2 are clustered together into C1 due to good overlap.
- Continue clustering best scoring groups until one cluster remains containing all Sinograms.
- Can cut tree at any point and make models with clustered projections.

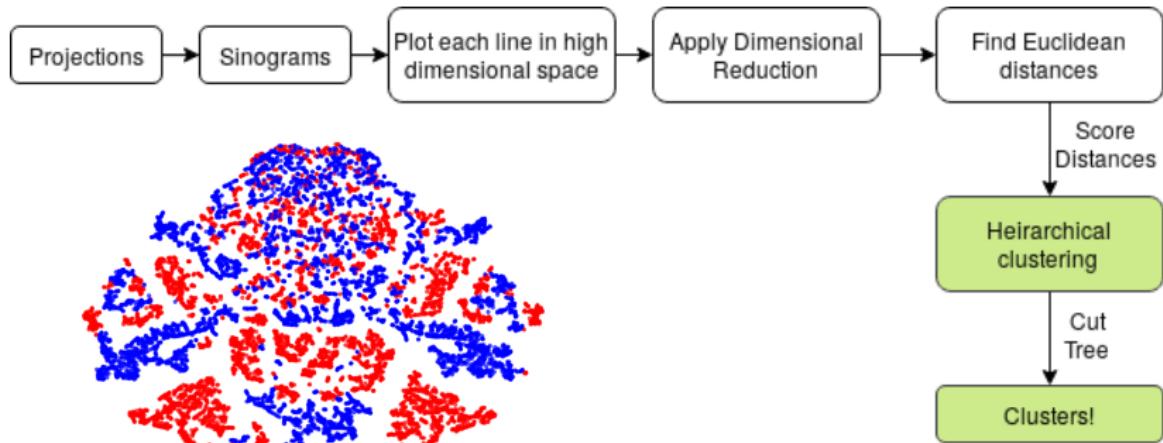
Common Lines
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Finding Common Lines
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Dimensional Reduction
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Clustering
ooo●ooo

Flowchart



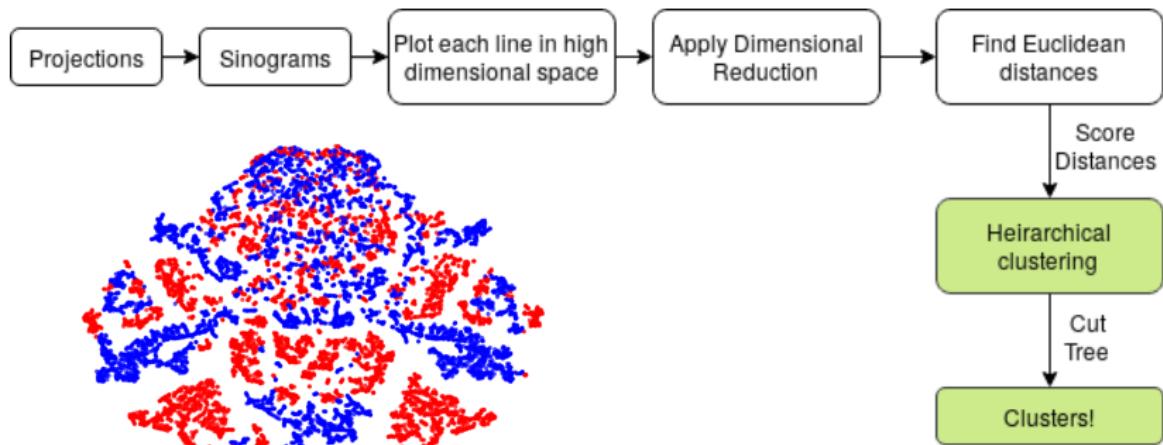
Common Lines
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Finding Common Lines
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Dimensional Reduction
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Clustering
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Flowchart



Just a model left!

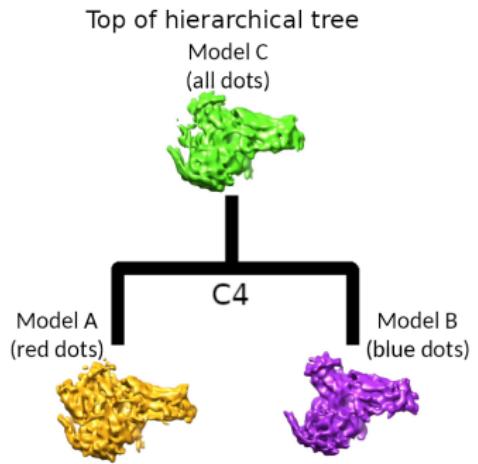
Common Lines
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Finding Common Lines
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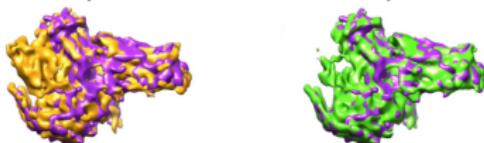
Dimensional Reduction
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Clustering
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Experiments



Model comparison
Overlay B & A Overlay B & C



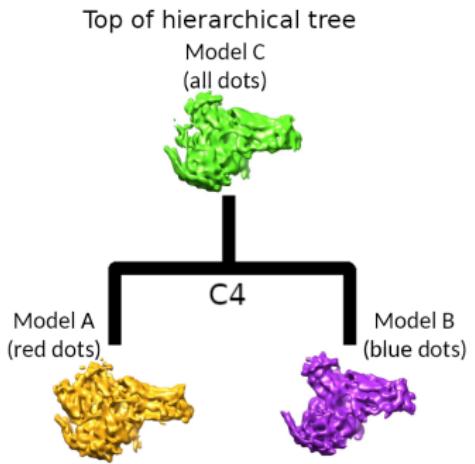
Common Lines
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Finding Common Lines
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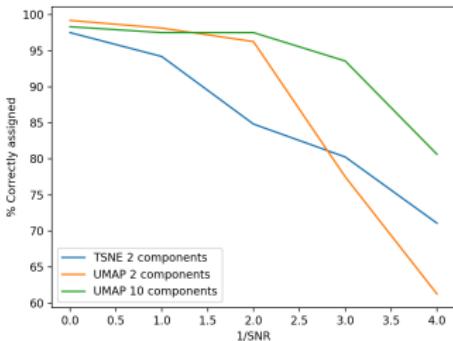
Dimensional Reduction
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Clustering
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Experiments



Model comparison
Overlay B & A Overlay B & C



Common Lines
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Finding Common Lines
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Dimensional Reduction
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Clustering
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Any Questions?



Thanks to eBIC,
Yuriy Chaban - The Concept
&
CCPEM

