CLIC

Common Lines Implied Clustering

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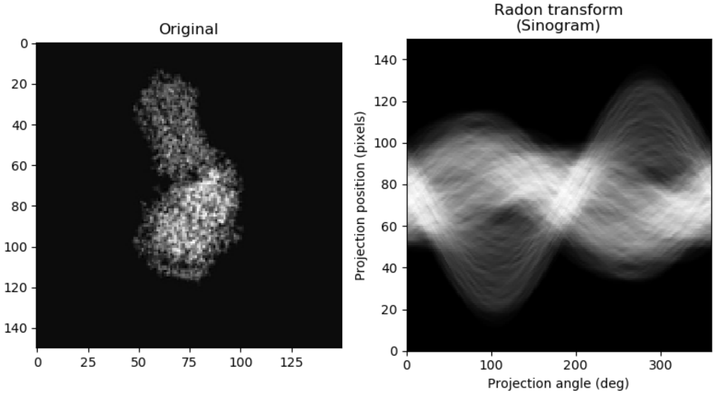
The Radon Transform



The Radon Transform

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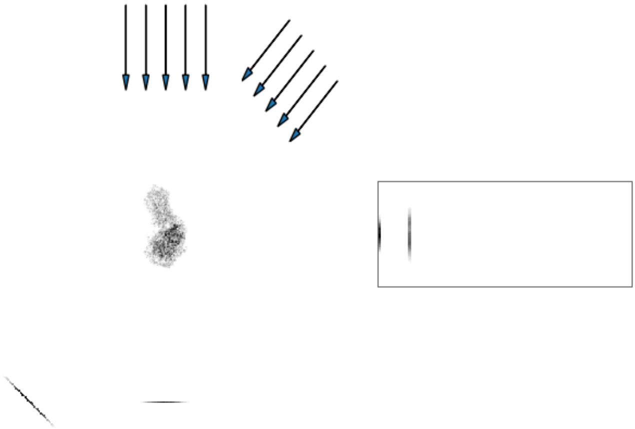

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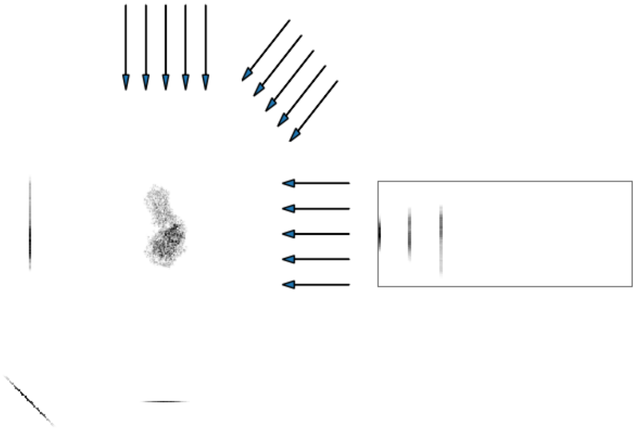

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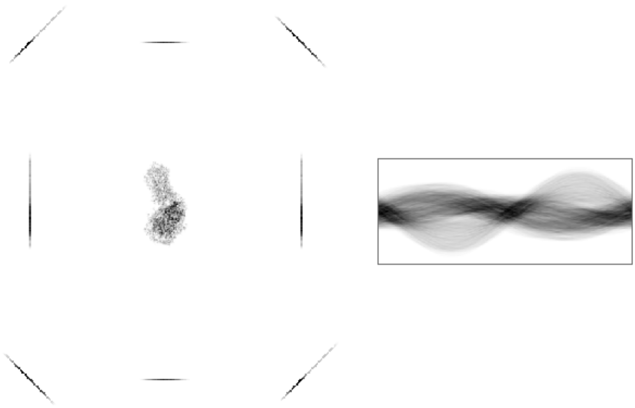
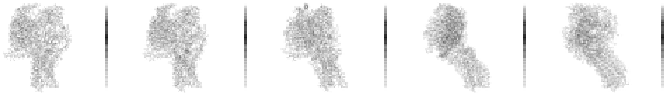
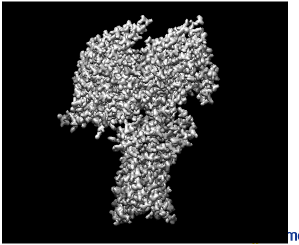


Common Lines

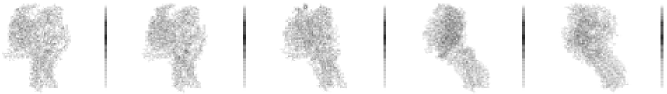
Two projections of the same 3D volume share at least one common line in the Radon transform

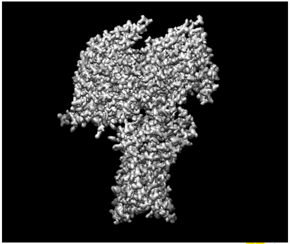




Common Lines

What about two different 3D volumes?

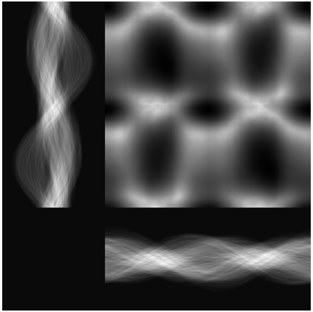




Finding Common Lines

Sinogram Cross Correlation

Finding the common line between two sinograms



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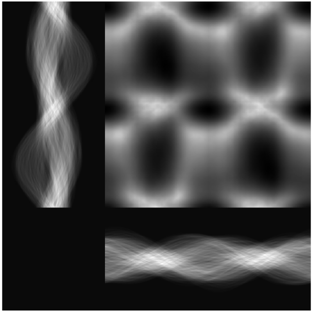

But what about N sinograms?

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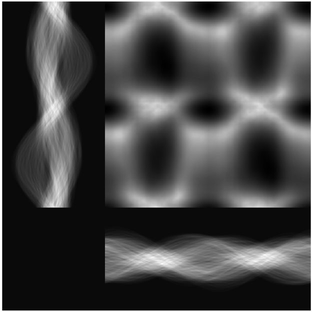

But what about N sinograms?
What about N sinograms from a heterogenous dataset?

Pipeline

- Plot each line in high dimensional space
- Find Euclidean Distances
- Smaller distances mean better agreement
- Best match between two sinograms → Common line!
- Best scoring common lines → From the same model!

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Slow. Exhaustive.

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Slow. Exhaustive. Doesn't handle noise well.

Dimensional Reduction

Find features - Reduce noise

Linear

PCA

Non-Linear

LLE

Isomap

TSNE

UMAP

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But how do we assign clusters?

Clustering

Can heterogenaity be sorted by looking at common lines?

Ground truth: Good seperatation between two classes - but discontinuous

Heirarchical clustering

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- Plot each line in high dimensional space
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- Smaller distances mean better agreement - Create score
- Heirarchical Clustering
- Cut tree to produce clusters

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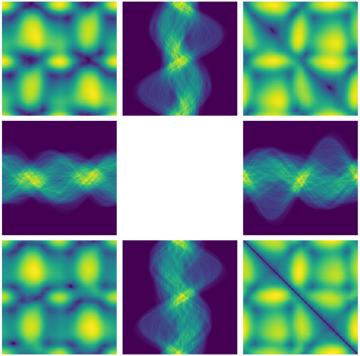
Just a model left!

Reconstruction

Angular recovery from 3 Common lines

Common line gives axis of rotation. Three common lines gives 2 unique solutions for 3D orientation (One mirror of other)



Angular Reconstitution: A Posteriori Assignment of Projection Directions for 3d Reconstruction. Van Heel 1987

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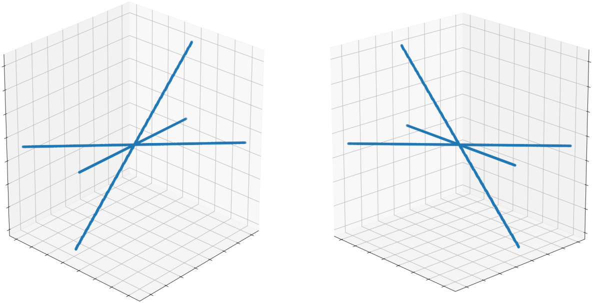

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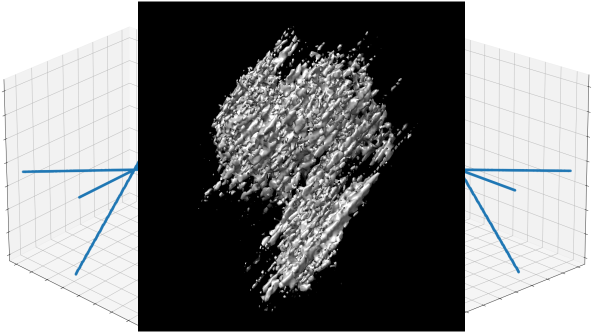

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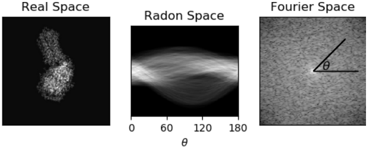
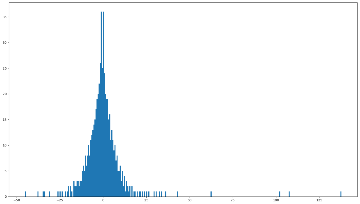
Eigenvector Relaxation

Aim: Given all common lines c for projections P, assign Rotation matrices R for each P to give greatest consensus volume.



$$max \sum_{i \neq j} R_i c_{ij} \cdot R_j c_{ji} \quad (2)$$

$c_{ij} = (cos$



Maths*! Make large ($2N \times 2N$) symmetric matrix S. Can recover R for each P from top 3 eigenvectors of S that maximise (2)!

*Three Dimensional Structure Determination from Common Lines in Cryo-EM by Eigenvectors and Semidefinite Programming, A.Singer and Y.Shkolnisky

Full Pipeline

A full pipeline of the procedure. 2d projs ¿ 2d sins ¿ 1d lines ¿ TSNE ¿ agglo ¿ clusters ¿ split into sep datasets ¿ find common lines ¿ eigenvector relaxation ¿ Models