



Common Lines Implied Clustering

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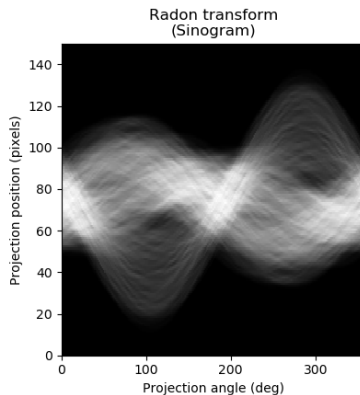
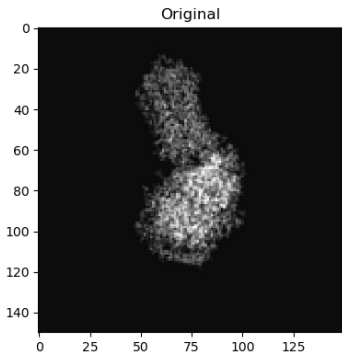
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Single Lines

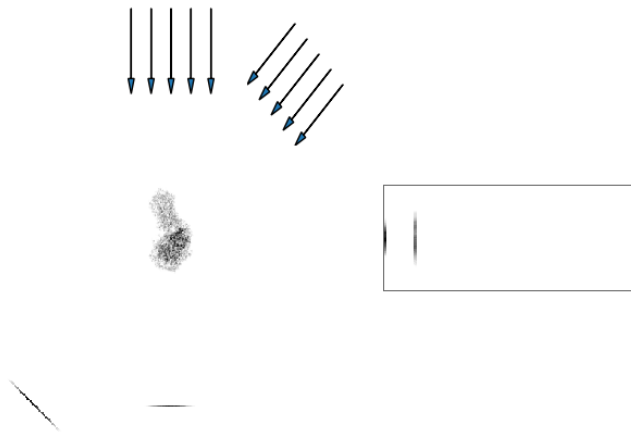
The Radon Transform



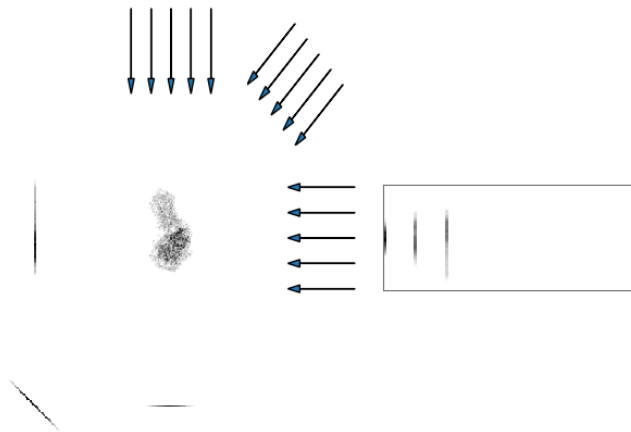
The Radon Transform



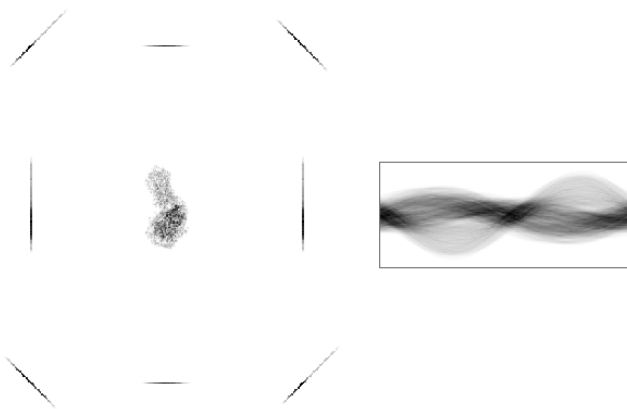
The Radon Transform



The Radon Transform



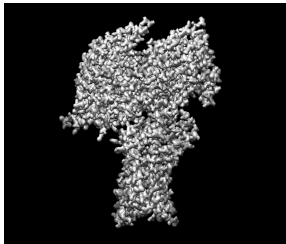
The Radon Transform



Common Lines



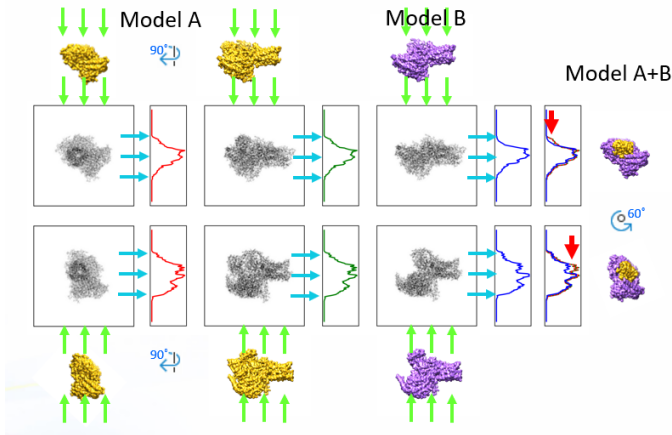
Two projections of the same 3D volume share at least one common line in the Radon transform



Common Lines



What about two different 3D volumes?

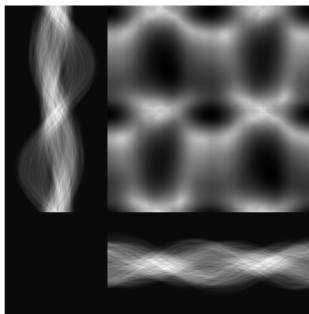


Finding Common Lines

Sinogram Cross Correlation



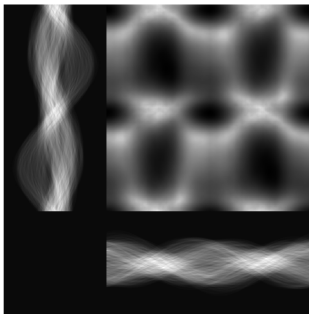
Finding the common line between two sinograms



Sinogram Cross Correlation



Finding the common line between two sinograms

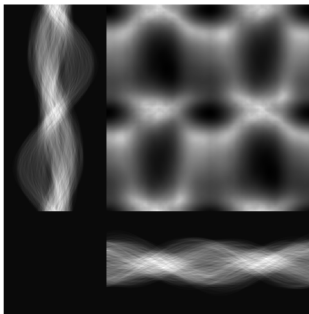


But what about N sinograms?

Sinogram Cross Correlation

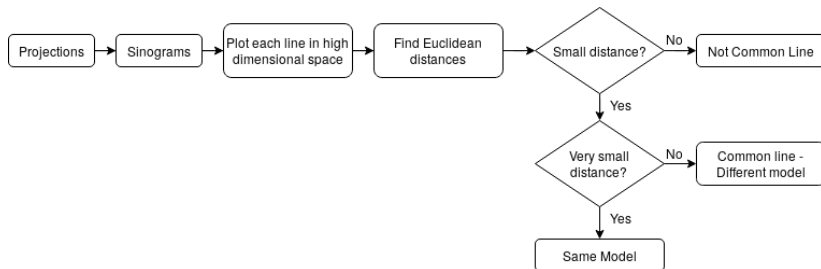


Finding the common line between two sinograms

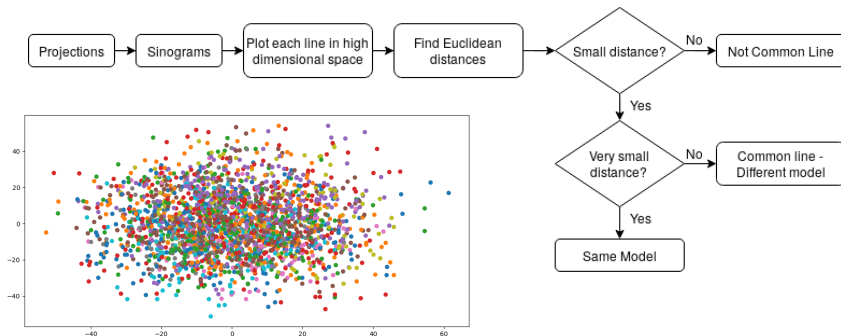


But what about N sinograms?
What about N sinograms from a heterogenous dataset?

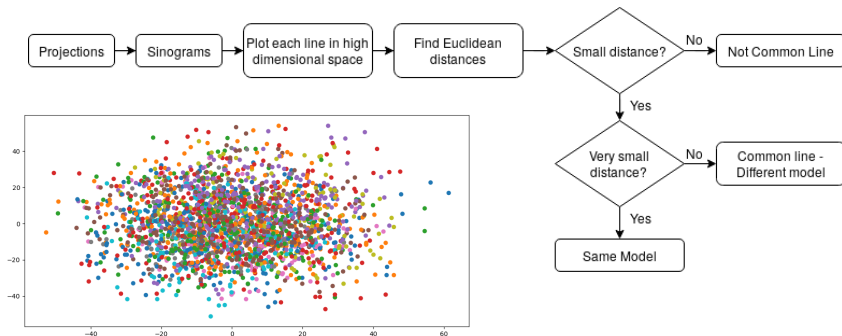
Pipeline



Pipeline

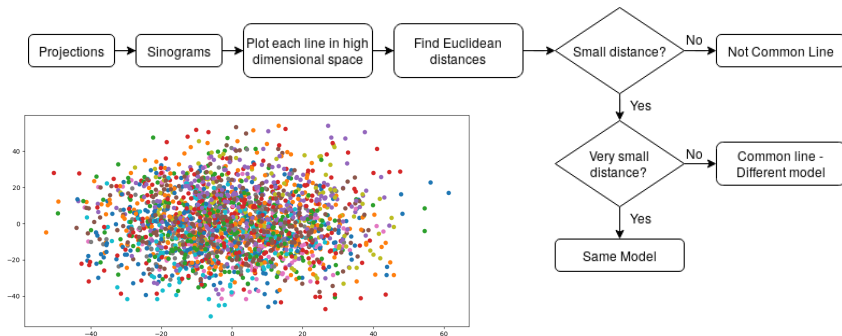


Pipeline



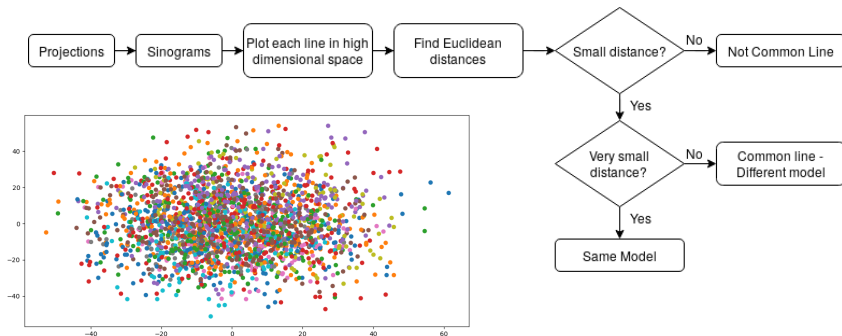
Slow.

Pipeline



Slow. Exhaustive.

Pipeline



Slow. Exhaustive. Doesn't handle noise well.

Dimensional Reduction - Linear



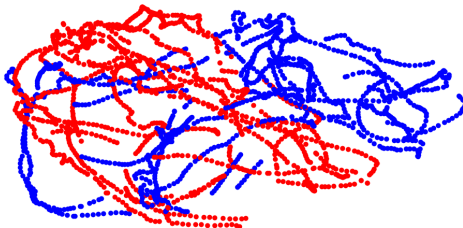
Find features - Reduce noise
Linear (PCA)



Dimensional Reduction - Linear



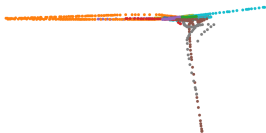
Find features - Reduce noise
Linear (PCA)



Dimensional Reduction - Non-Linear



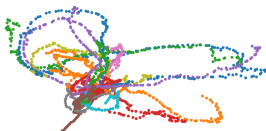
LLE



UMAP



ISOMAP



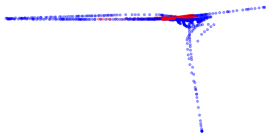
TSNE



Dimensional Reduction - Non-Linear



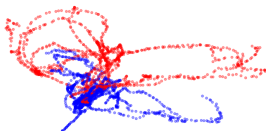
LLE



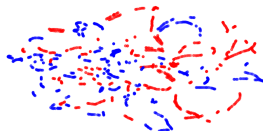
UMAP



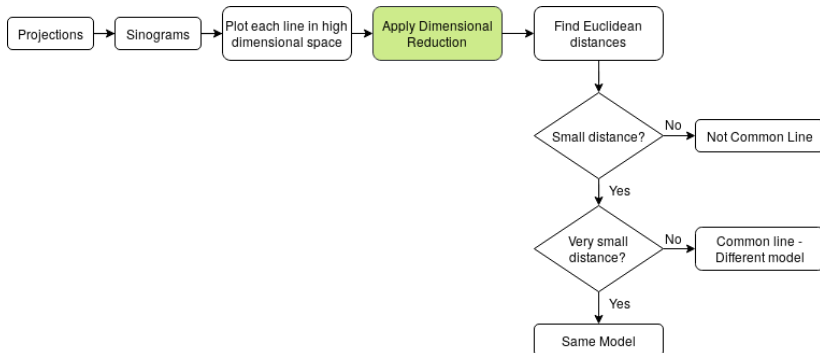
ISOMAP



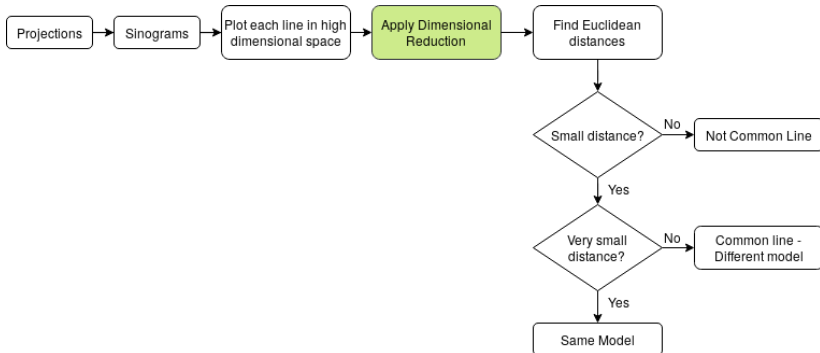
TSNE



Pipeline



Pipeline



But how do we assign clusters?

Clustering

The data



Ground truth: Good seperation between two classes - but
discontinuous

Single Lines
○○○○○

Finding Common Lines
○○○○○○

Clustering
○○●○

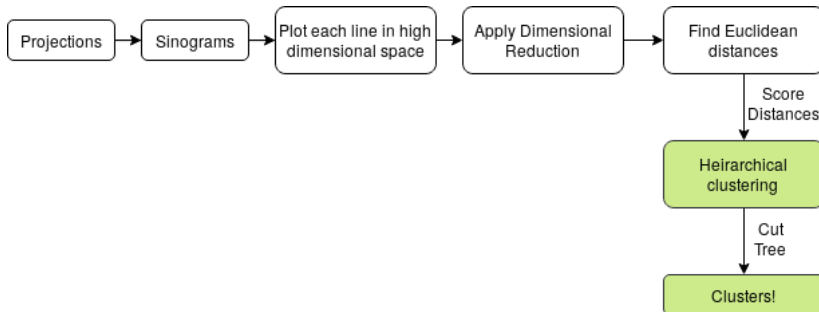
Reconstruction
○○○

Full Pipeline
○○

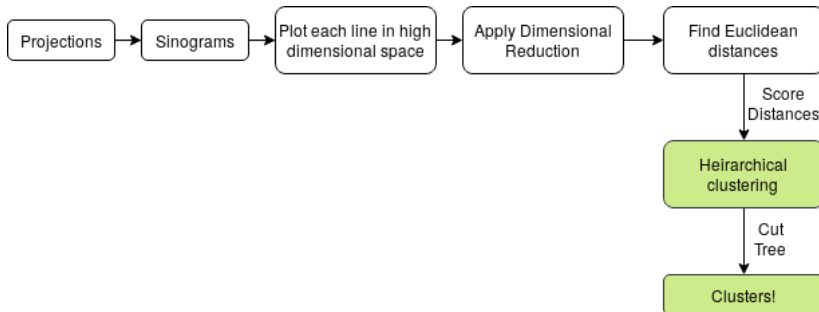
Heirarchical clustering



Pipeline



Pipeline



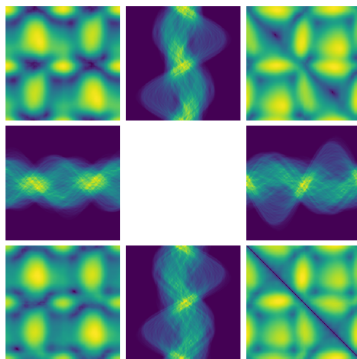
Just a model left!

Reconstruction

Angular recovery from 3 Common lines



Common line gives axis of rotation. Three common lines gives 2 unique solutions for 3D orientation (One mirror of other)

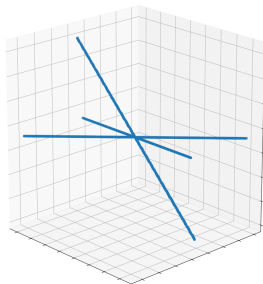
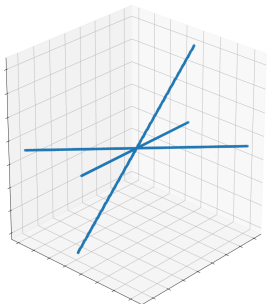


Angular Reconstitution: A Posteriori Assignment of Projection Directions for 3d Reconstruction. Van Heel 1987

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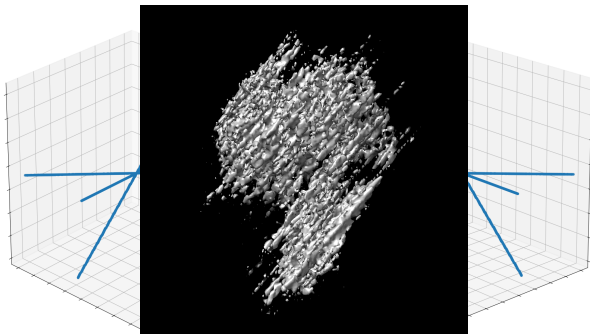


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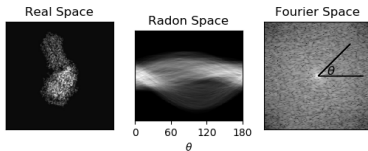


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Eigenvector Relaxation

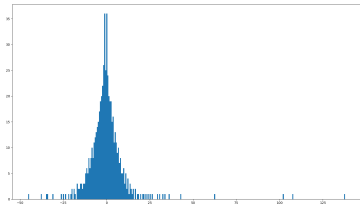


Aim: Given all common lines c for projections P , assign Rotation matrices R for each P to give greatest consensus volume.



$$\max \sum_{i \neq j} R_i c_{ij} \cdot R_j \bar{c}_{ji}^{(2)}$$

Maths*! Make large $(2N \times 2N)$ symmetric matrix S . Can recover R for each P from top 3 eigenvectors of S that maximise (2)!



**Three Dimensional Structure Determination from Common Lines in Cryo-EM by Eigenvectors and Semidefinite Programming. A Singer and Y Shkolnisky*

Full Pipeline

A full pipeline of the procedure. 2d projs \rightarrow 2d sins \rightarrow 1d lines \rightarrow TSNE \rightarrow aggro \rightarrow clusters \rightarrow split into sep datasets \rightarrow find common lines \rightarrow eigenvector relaxation \rightarrow Models