



Common Lines Implied Clustering

Donovan Webb

eBIC/University of Bath



Single Lines



Finding Common Lines



Dimensional Reduction



Clustering



Single Lines

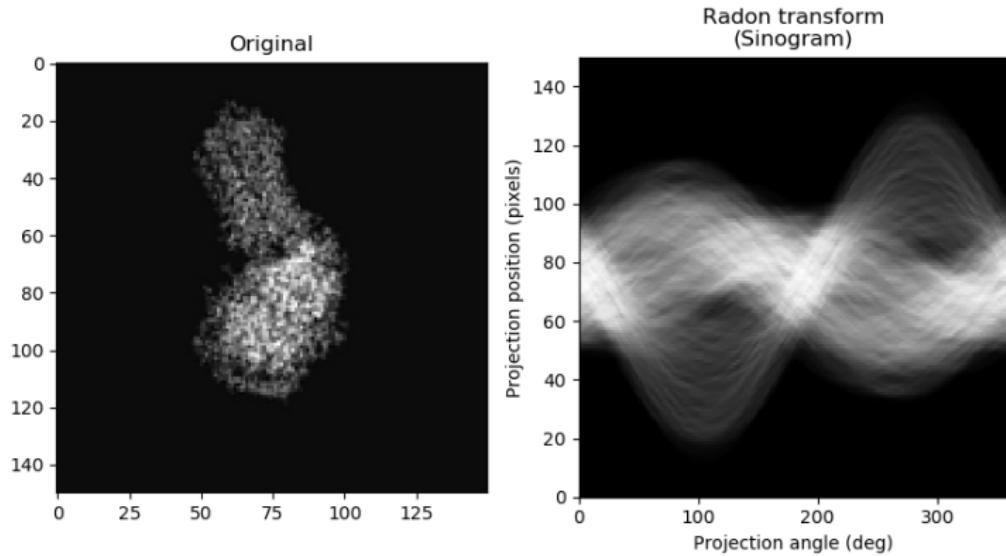
Single Lines
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Finding Common Lines
○○○

Dimensional Reduction
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Clustering
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The Radon Transform



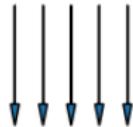
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Finding Common Lines
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Dimensional Reduction
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Clustering
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The Radon Transform



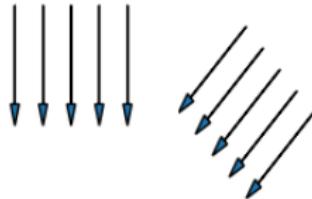
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Clustering
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The Radon Transform



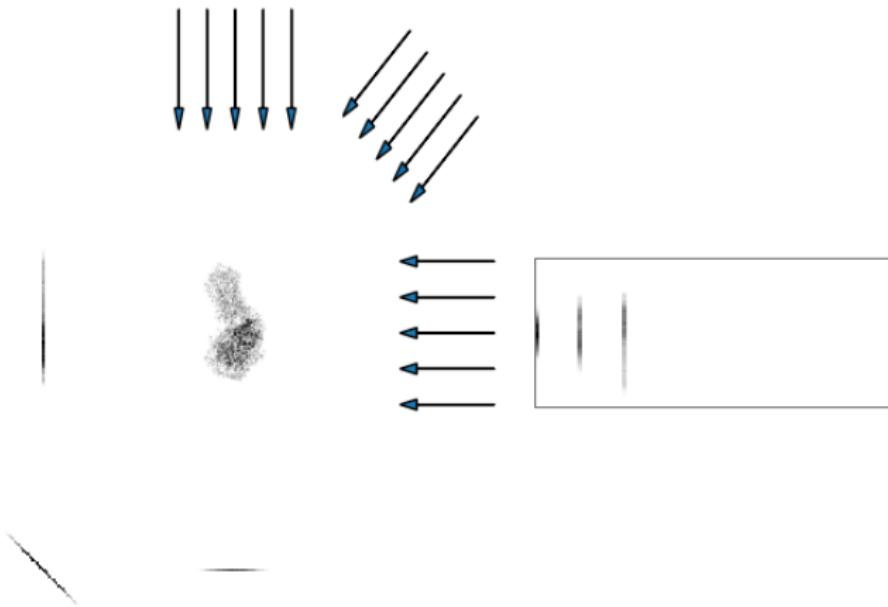
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Clustering
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The Radon Transform



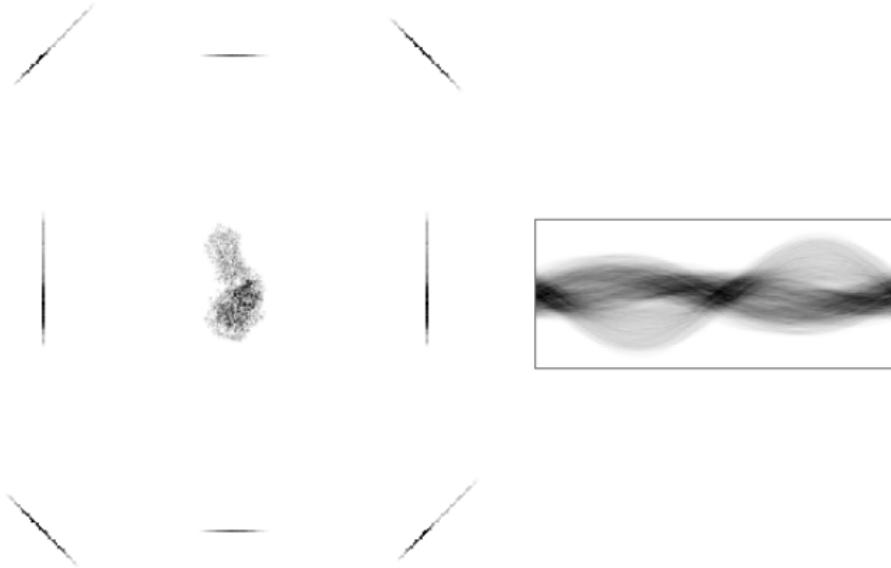
Single Lines
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Finding Common Lines
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Dimensional Reduction
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Clustering
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The Radon Transform



Single Lines
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Finding Common Lines
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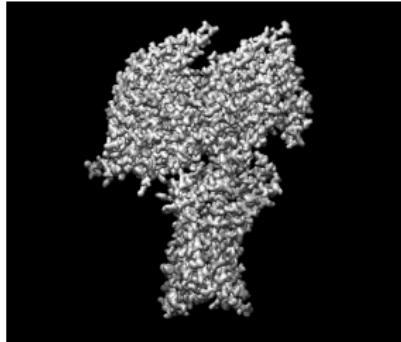
Dimensional Reduction
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Clustering
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Common Lines



Two projections of the same 3D volume share at least one common line in the Radon transform



Single Lines
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Finding Common Lines
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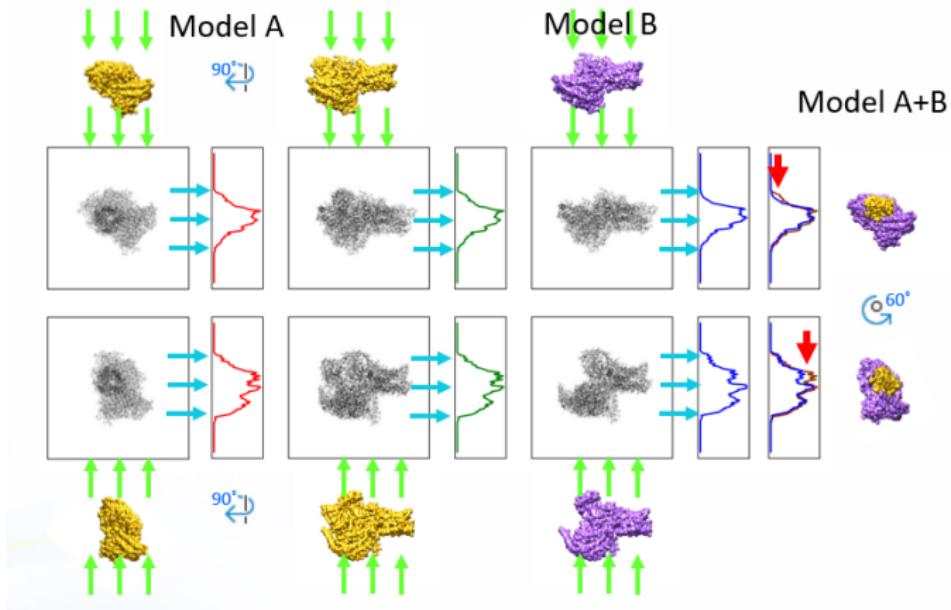
Dimensional Reduction
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Clustering
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Common Lines



What about two different 3D volumes?



Single Lines
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Finding Common Lines
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Dimensional Reduction
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Clustering
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Finding Common Lines

Single Lines
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Finding Common Lines
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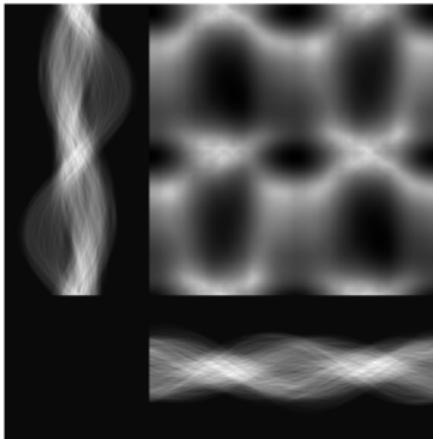
Dimensional Reduction
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Clustering
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Sinogram Cross Correlation



Finding the common line between two sinograms



Single Lines
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Finding Common Lines
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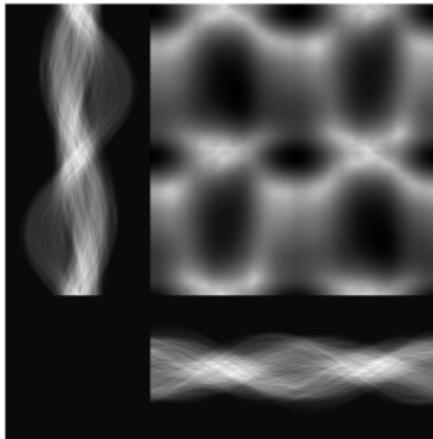
Dimensional Reduction
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Clustering
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Sinogram Cross Correlation



Finding the common line between two sinograms



But what about N sinograms?

Single Lines
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Finding Common Lines
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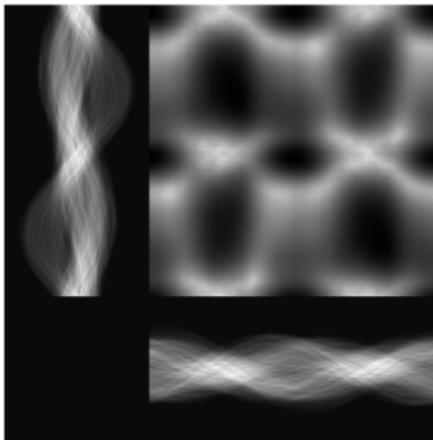
Dimensional Reduction
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Clustering
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Sinogram Cross Correlation



Finding the common line between two sinograms



But what about N sinograms?

What about N sinograms from a heterogenous dataset?

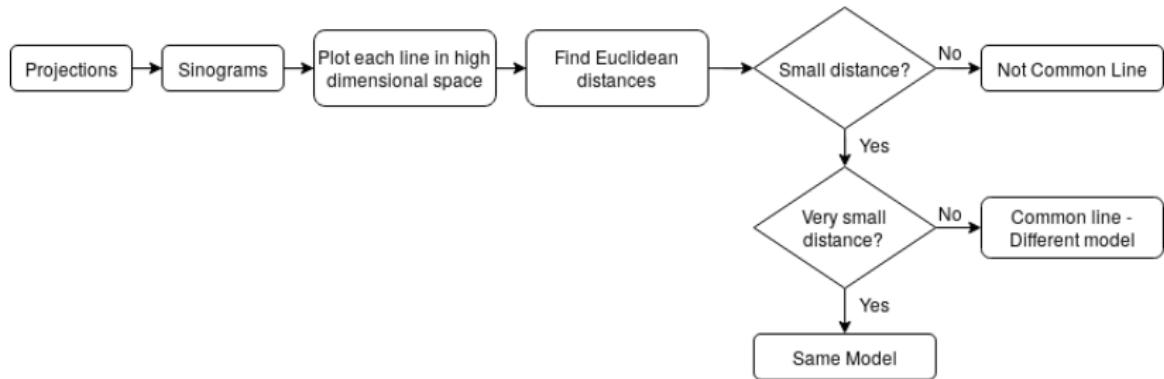
Single Lines
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Finding Common Lines
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Dimensional Reduction
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Clustering
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Concept's flowchart



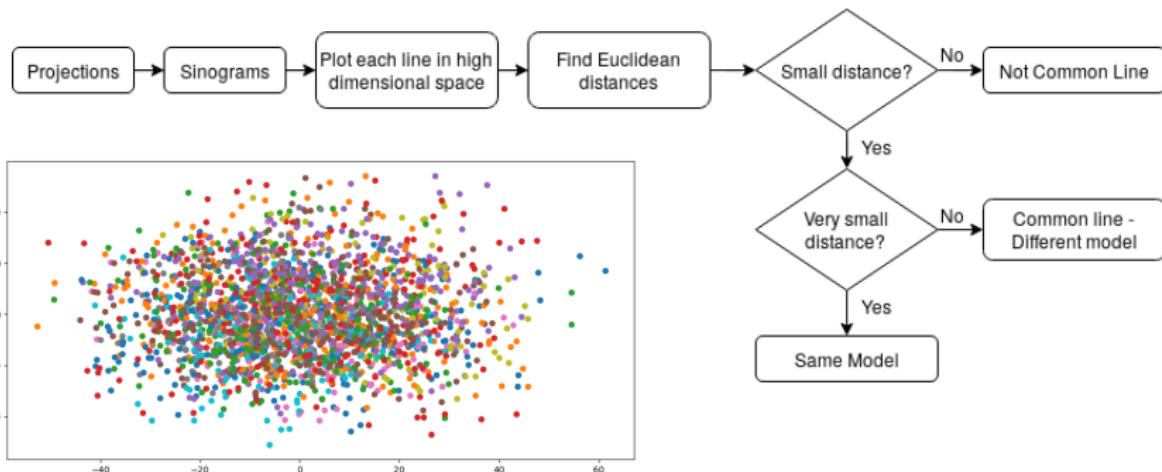
Single Lines
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Finding Common Lines
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Dimensional Reduction
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Clustering
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Concept's flowchart



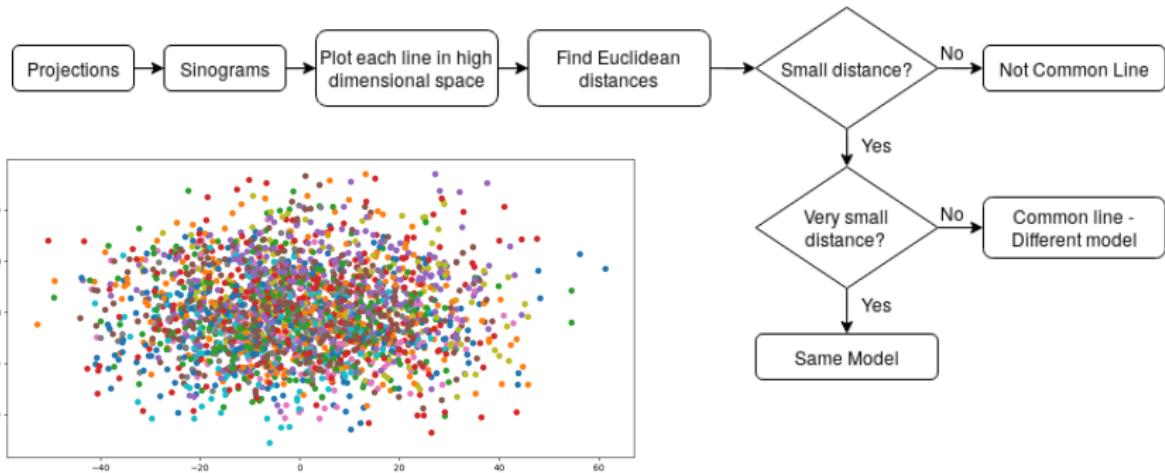
Single Lines
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Finding Common Lines
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Dimensional Reduction
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Clustering
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Concept's flowchart



Curse of Dimensionality.

Single Lines
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Finding Common Lines
○○○

Dimensional Reduction
●○○○○○○○

Clustering
○○○○○

Dimensional Reduction

Single Lines
○○○○○

Finding Common Lines
○○○

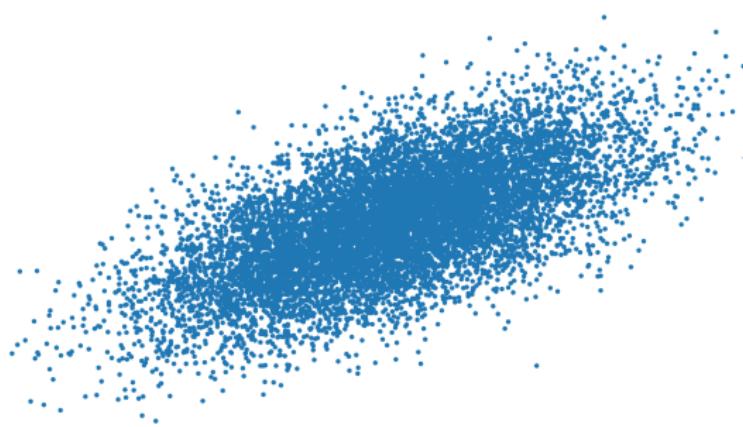
Dimensional Reduction
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Clustering
○○○○○

Dimensional Reduction - Linear (PCA)



How to display high dimensional data in lower dimensional space?



Single Lines
○○○○○

Finding Common Lines
○○○

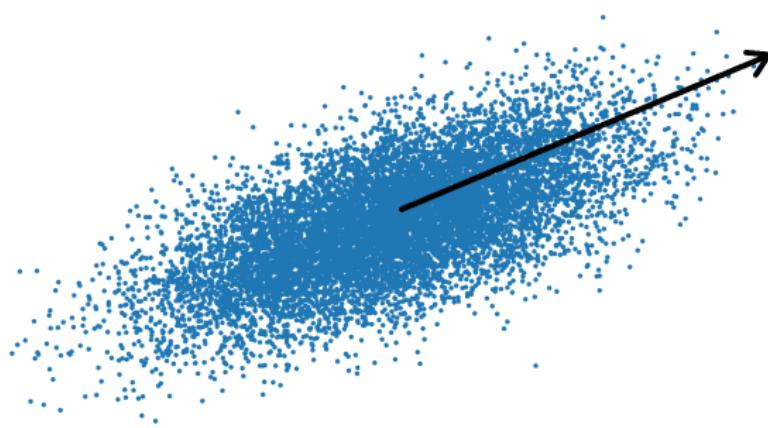
Dimensional Reduction
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Clustering
○○○○○

Dimensional Reduction - Linear (PCA)



How to display high dimensional data in lower dimensional space?



Single Lines
○○○○○

Finding Common Lines
○○○

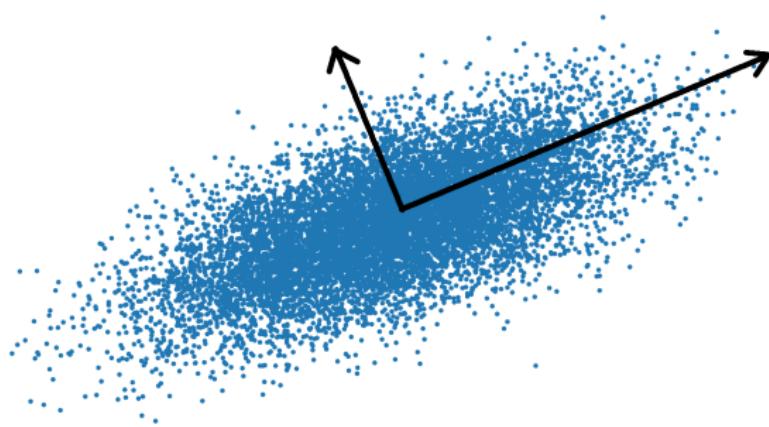
Dimensional Reduction
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Clustering
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Dimensional Reduction - Linear (PCA)



How to display high dimensional data in lower dimensional space?



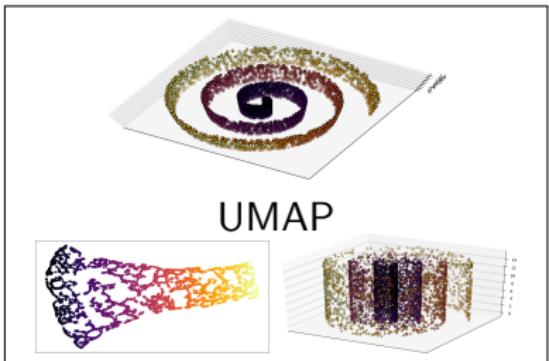
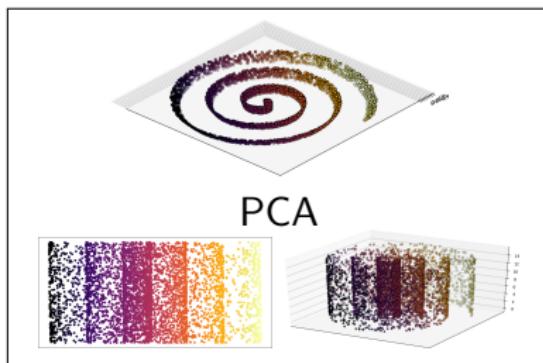
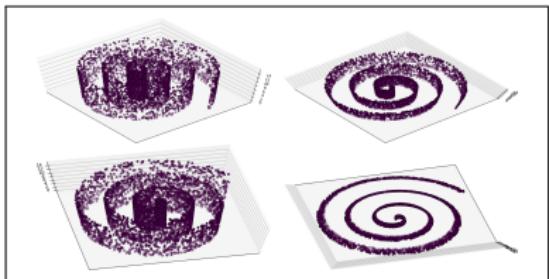
Single Lines
ooooo

Finding Common Lines
ooo

Dimensional Reduction
oo●oooo

Clustering
ooooo

Linear vs Non-Linear - Swiss Roll



UMAP finds manifold, PCA does not

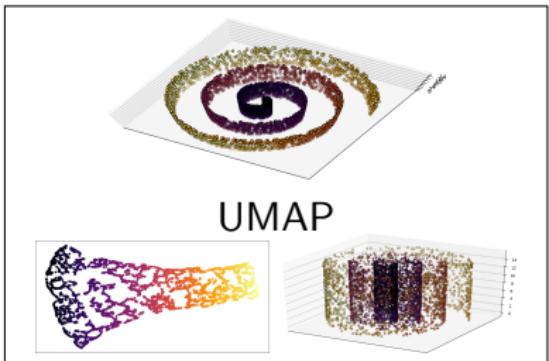
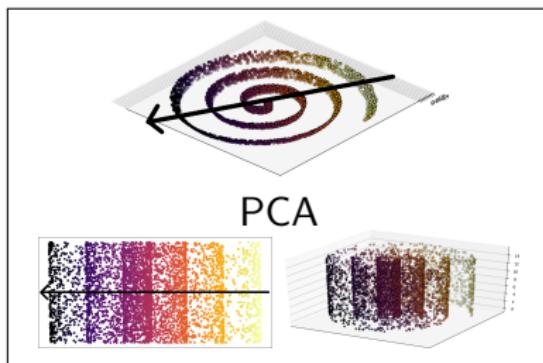
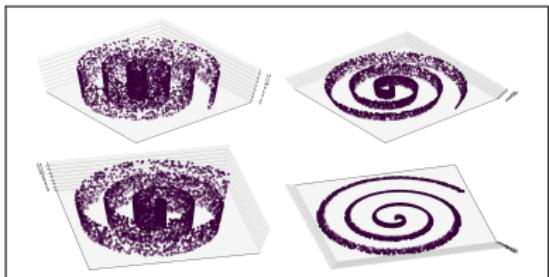
Single Lines
ooooo

Finding Common Lines
ooo

Dimensional Reduction
oo●oooo

Clustering
ooooo

Linear vs Non-Linear - Swiss Roll



UMAP finds manifold, PCA does not

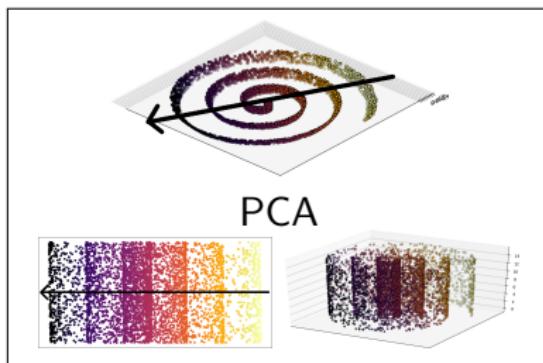
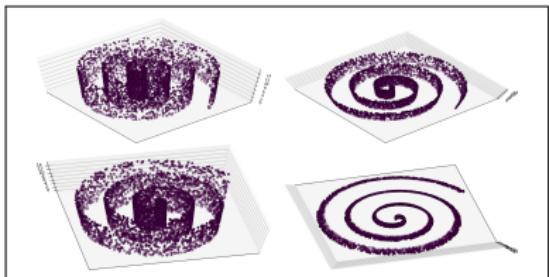
Single Lines
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Finding Common Lines
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Dimensional Reduction
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Clustering
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Linear vs Non-Linear - Swiss Roll



UMAP finds manifold, PCA does not

Single Lines
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Finding Common Lines
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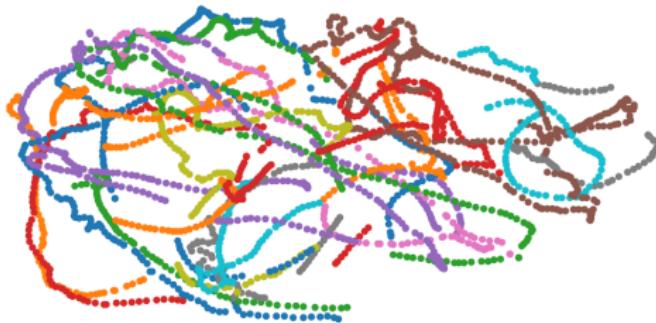
Dimensional Reduction
ooo●ooo

Clustering
ooooo

Dimensional Reduction - Linear



Find features - Reduce noise
Linear (PCA)



Single Lines
ooooo

Finding Common Lines
ooo

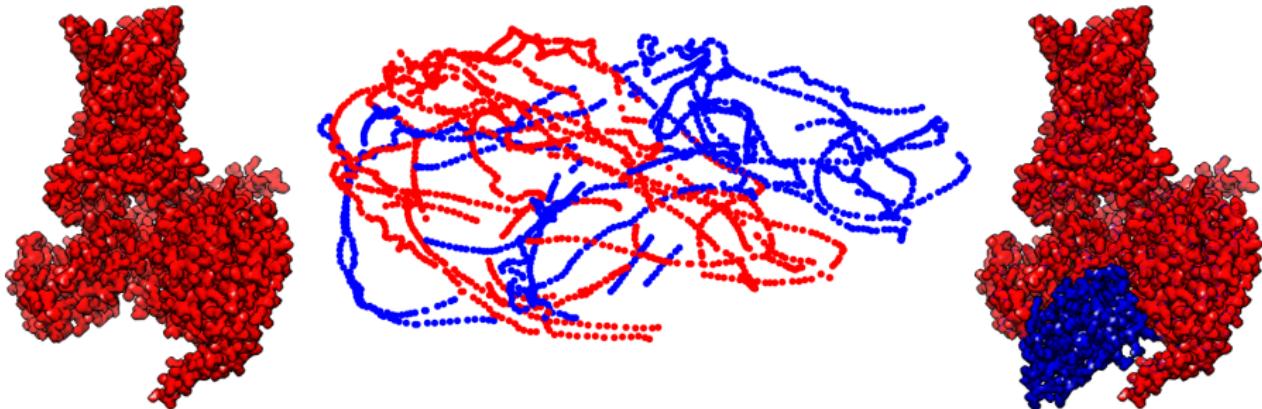
Dimensional Reduction
ooo●ooo

Clustering
ooooo

Dimensional Reduction - Linear



Find features - Reduce noise
Linear (PCA)



Single Lines
○○○○○

Finding Common Lines
○○○

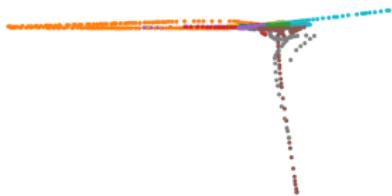
Dimensional Reduction
○○○○●○○○

Clustering
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Dimensional Reduction - Non-Linear



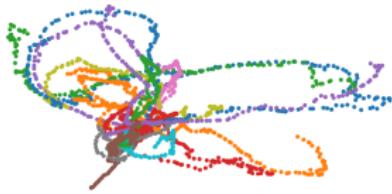
LLE



UMAP



ISOMAP



TSNE



Single Lines
ooooo

Finding Common Lines
ooo

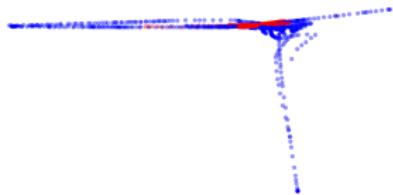
Dimensional Reduction
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Clustering
ooooo

Dimensional Reduction - Non-Linear



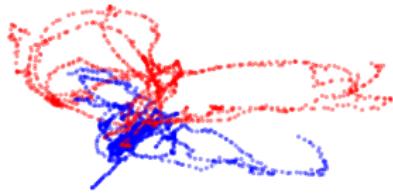
LLE



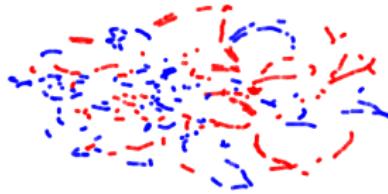
UMAP



ISOMAP



TSNE



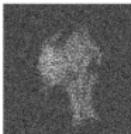
Single Lines
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Finding Common Lines
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Dimensional Reduction
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Clustering
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UMAP



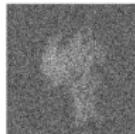
SNR 1



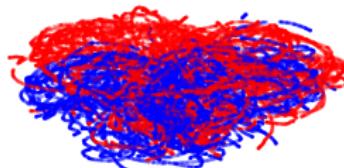
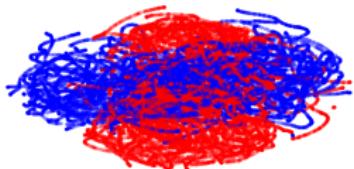
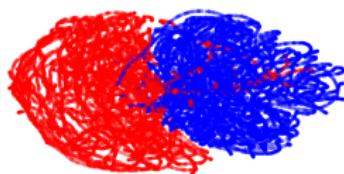
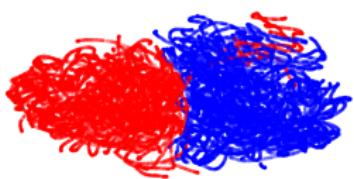
SNR 1/2



SNR 1/3



SNR 1/4



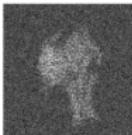
Single Lines
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Finding Common Lines
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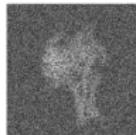
Dimensional Reduction
○○○○○○●○

Clustering
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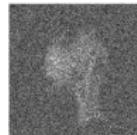
TSNE



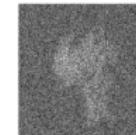
SNR 1



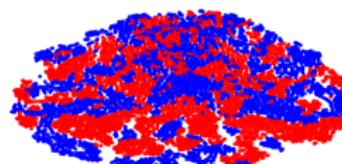
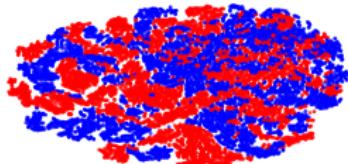
SNR 1/2



SNR 1/3



SNR 1/4



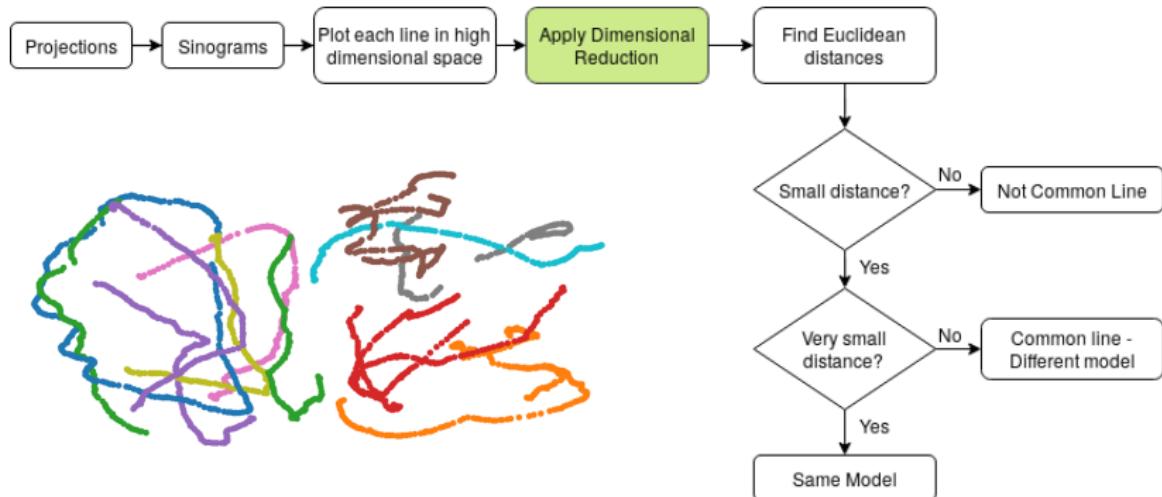
Single Lines
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Finding Common Lines
ooo

Dimensional Reduction
oooooooo●

Clustering
ooooo

Flowchart



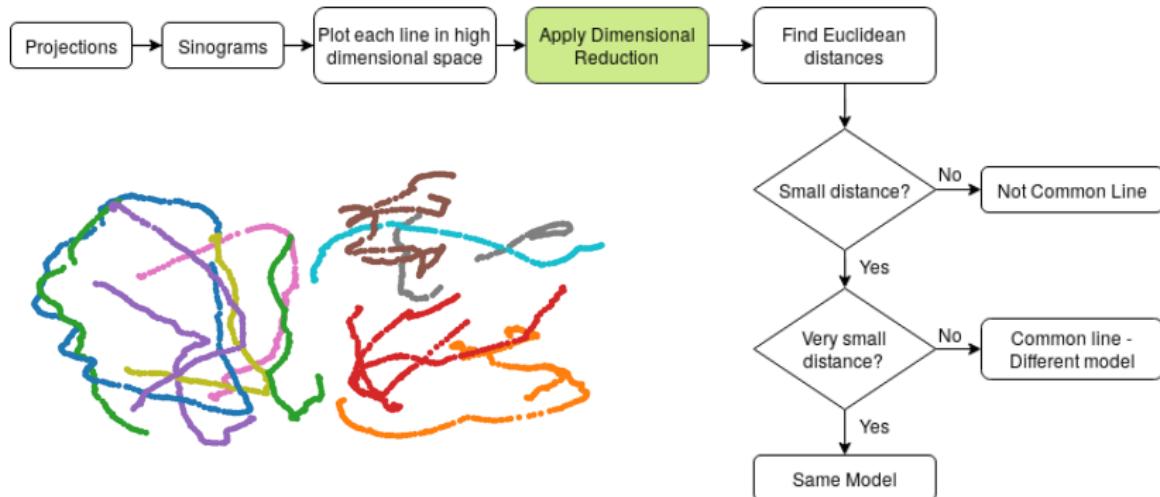
Single Lines
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Finding Common Lines
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Dimensional Reduction
oooooooo●

Clustering
ooooo

Flowchart



But how do we assign clusters?

Single Lines

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Finding Common Lines

○○○

Dimensional Reduction

○○○○○○○○

Clustering

●○○○○

Clustering

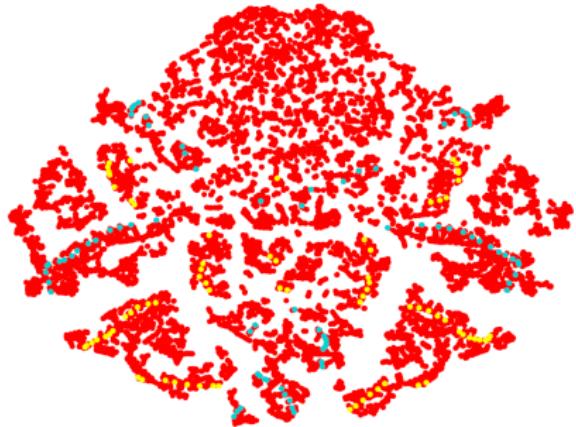
Single Lines
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Finding Common Lines
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Dimensional Reduction
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Clustering
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The data



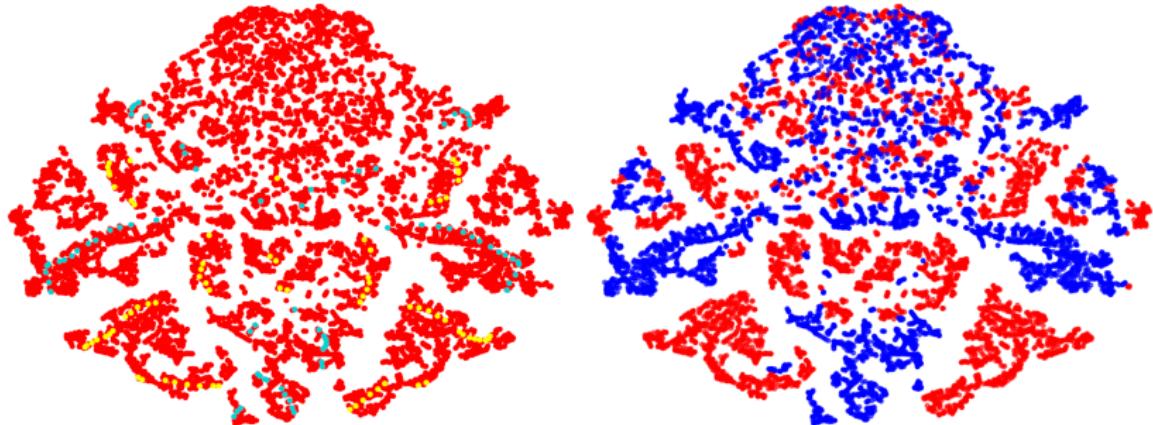
Single Lines
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Finding Common Lines
ooo

Dimensional Reduction
oooooooo

Clustering
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The data



Ground truth: Good separation between two classes - but discontinuous

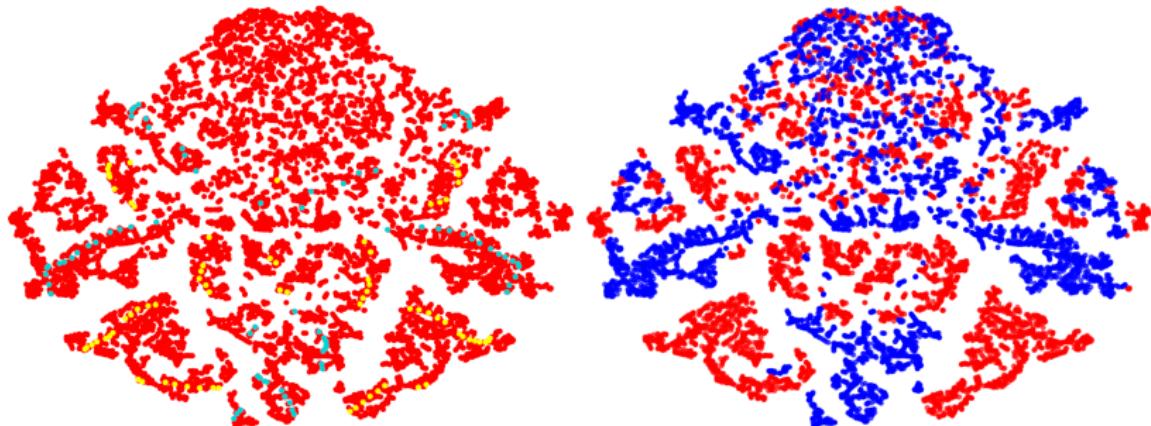
Single Lines
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Finding Common Lines
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Dimensional Reduction
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Clustering
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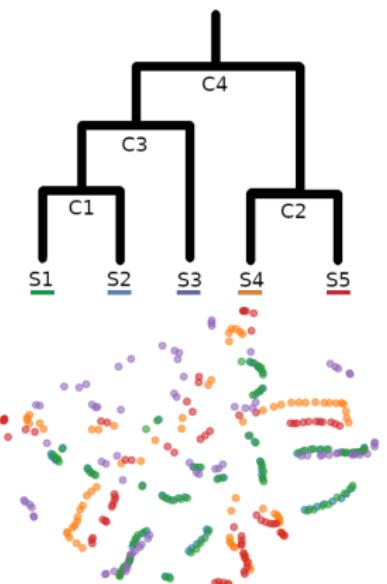
The data



Ground truth: Good separation between two classes - but discontinuous

We know which sinogram each line comes from...
Lines from one sinogram are initial clusters

Heirarchical clustering



- S1 – S5 Are Sinograms.
- S1 and S2 are clustered together into C1 due to good overlap.
- Continue clustering best scoring groups until one cluster remains containing all Sinograms.
- Can cut tree at any point and make models with clustered projections.

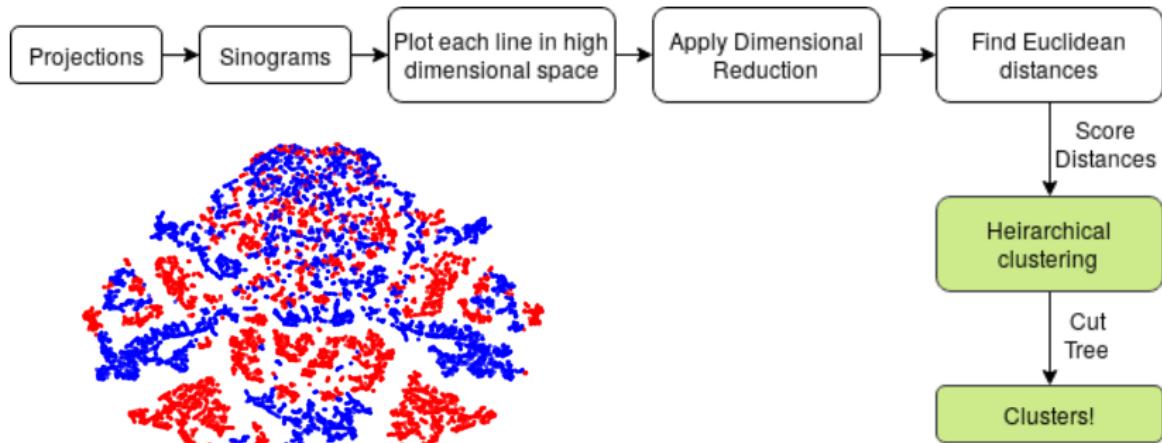
Single Lines
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Finding Common Lines
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Dimensional Reduction
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Clustering
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Flowchart



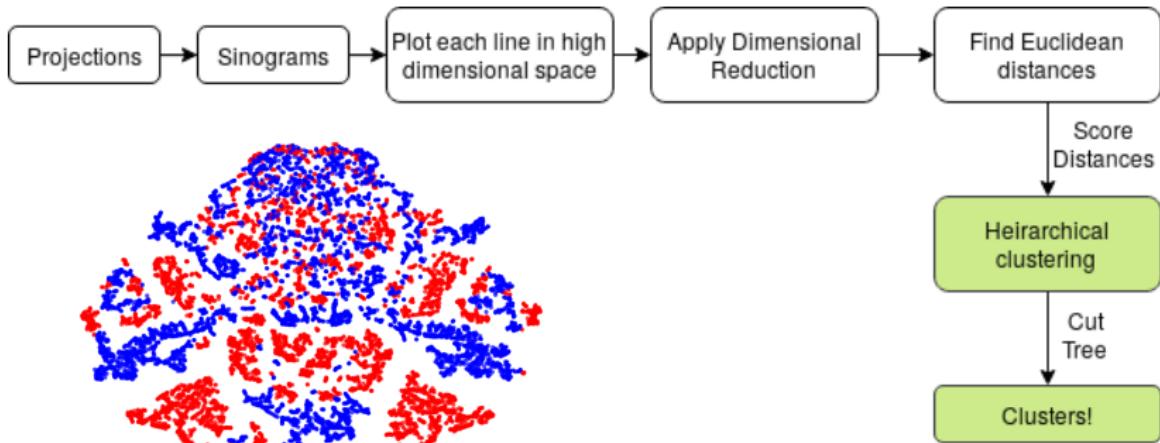
Single Lines
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Finding Common Lines
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Dimensional Reduction
oooooooo

Clustering
oooo●○

Flowchart



Just a model left!

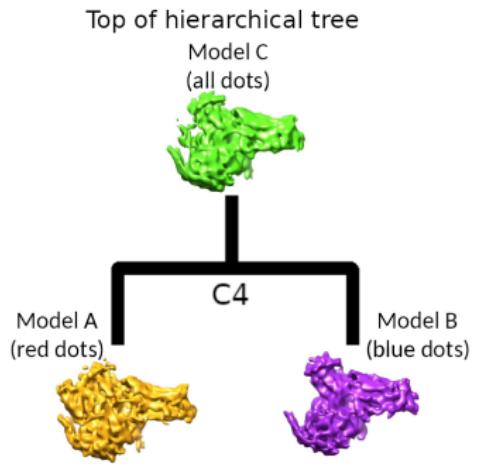
Single Lines
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Dimensional Reduction
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Clustering
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Experiments



Model comparison
Overlay B & A Overlay B & C



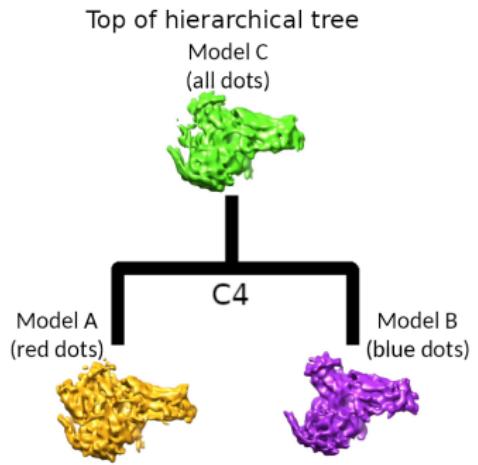
Single Lines
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Dimensional Reduction
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Clustering
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Experiments



Model comparison
Overlay B & A Overlay B & C

