**Optimization Problem**

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**Introduction**

In this assignment, two optimization problems are solved. A transshipment problem and A Quadratic programming model. The project is performed in MS Excel. The transshipment problem is solved by obtaining the optimized route from the by calculating the total cost for shipping. In the second part, the risk of the portfolio is analyzed and the risk is minimized according to the allocations of the investment of Bonds, High tech stocks, Foreign Stocks, Call Options, Put options and Gold.

**Analysis**

**PART-1:**

In this problem, Rockhill Transportation is attempting to reach an agreement with Chimotoxic Chemical Company to get a contract to pick up and send waste products from various factories to different waste sites. Let's begin by reviewing the facts presented to solve the problem.

|  |  |  |  |
| --- | --- | --- | --- |
|  | ***Waste Proposal Site*** | | |
| **Plant:** | ***Orangeburg*** | ***Florence*** | ***Macon*** |
| **Denver** | $12 | $15 | $17 |
| **Morganton** | 14 | 9 | 10 |
| **Morrisville** | 13 | 20 | 11 |
| **Pineville** | 17 | 16 | 19 |
| **Rockhill** | 7 | 14 | 12 |
| **Statesville** | 22 | 16 | 18 |
| Table 1: Shipping costs, per barrel of waste from six plants to three waste disposal sites | | | |

|  |  |
| --- | --- |
| **Plant:** | ***Waste per Week (bbl)*** |
| **Denver** | 45 |
| **Morganton** | 26 |
| **Morrisville** | 42 |
| **Pineville** | 53 |
| **Rockhill** | 29 |
| **Statesville** | 38 |

Table 2: Waste generated by each plant in barrels.

Number of steps are performed to solve this problem.

And then, the direct transport route is compared with the alternate route with plants and disposal sites as intermediate nodes to check which method gives the minimized cost to the company.

***Table-3***

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | **Plant:** | **Orangeburg** | **Florence** | **Macon** | **Sum** |  |  |
|  | **Denver** | 36 | 0 | 9 | 45 | ≥ | 45 |
|  | **Morganton** | 0 | 0 | 26 | 26 | ≥ | 26 |
|  | **Morrisville** | 0 | 0 | 42 | 42 | ≥ | 42 |
|  | **Pineville** | 0 | 53 | 0 | 53 | ≥ | 53 |
|  | **Rockhill** | 29 | 0 | 0 | 29 | ≥ | 29 |
|  | **Statesville** | 0 | 27 | 11 | 38 | ≥ | 38 |
| Shipped To | **Sum** | 65 | 80 | 88 |  |  |  |
|  | Total | ≤ | ≤ | ≤ |  |  | Total |
|  | 250 | 65 | 80 | 105 |  |  | 233 |

The above output from the solver shows that the total number of barrels which can be disposed is 250, while the total number of barrels which can be transported is 233. If we make that comparison, we can see that demand is greater than supply. The table also represents the number of barrels that can be delivered to the disposal location from the plant site.

From Denver, the toxic waste in barrels can be transferred to the Orangeburg (36 bbl) and Macon (9 bbl).

From Morganton, the toxic waste in barrels can be transferred to Macon (26 bbl).

From Morrisville, the toxic waste in barrels can be transferred to Macon (42 bbl).

From Pineville, the toxic waste in barrels can be transferred to Florence (53 bbl).

From Rockhill, the toxic waste in barrels can be transferred to the Orangeburg (29 bbl).

Finally, from Statisville, the toxic waste in barrels can be transferred to Florence (27 bbl) and Macon (11 bbl).

The following model gives the transportation cost of $2,988.

Now, the transshipment model is used to generate the cost of transportation by keeping the plants and disposal sites as the intermediary.

***Table-4***

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **X** | **Orangeburg** | **Florence** | **Macon** | **Denver** | **Morganton** | **Morrisville** | **Pineville** | **Rockhill** | **Statesville** | **Sum** |  |  |  |
| **Denver** | 0 | 0 | 0 | 0 | 45 | 0 | 0 | 0 | 0 | 45 |  | 45 | 45 |
| **Morganton** | 0 | 42 | 29 | 0 | 0 | 0 | 0 | 0 | 0 | 71 |  | 26 | 26 |
| **Morrisville** | 0 | 0 | 59 | 0 | 0 | 0 | 0 | 0 | 0 | 59 |  | 42 | 42 |
| **Pineville** | 0 | 0 | 0 | 0 | 0 | 17 | 0 | 36 | 0 | 53 |  | 53 | 53 |
| **Rockhill** | 65 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 65 |  | 29 | 29 |
| **Statesville** | 0 | 38 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 38 |  | 38 | 38 |
| **Orangeburg** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  | 65 |  |
| **Florence** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  | 80 |  |
| **Macon** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  | 105 |  |
| **Sum** | 65 | 80 | 88 | 0 | 45 | 17 | 0 | 36 | 0 |  |  |  |  |
|  | ≤ | ≤ | ≤ |  |  |  |  |  |  |  |  | Total |  |
|  | 65 | 80 | 105 |  |  |  |  |  |  |  |  | 233 |  |
|  | 65 | 80 | 88 |  |  |  |  |  |  |  |  |  |  |
|  | Total |  |  |  |  |  |  |  |  |  |  |  |  |
|  | 250 |  |  |  |  |  |  |  |  |  |  |  |  |

This allows us to evaluate the minimum shipping cost as well as the total number of barrels that can be transported to the specified location via an intermediate node. We can see that the number of barrels transported is 233 and the total number of barrels which can be disposed is 250.

The following model gives the transportation cost of $2,674.

By comparing the two models, it can be observed that both the models can be used to transport equal number of barrels i.e. 223, but the total cost involved is less when compared to the model where the barrels are directly transported form the plants to the disposal sites.

**PART-2:**

An investor has selected the following asset types in his portfolio. The expected return for each asset type has been estimated by using the historical data:

***Table-5***

|  |  |
| --- | --- |
|  | **Expected Returns** |
| **Bonds** | **7%** |
| **High tech stocks** | **12%** |
| **Foreign stocks** | **11%** |
| **Call options** | **14%** |
| **Put options** | **14%** |
| **Gold** | **9%** |
|  |  |

The following table indicates the covariance matrix of the assets’ returns. Each diagonal entry is the variance of an asset and non-diagonal entries are the covariances between any pairs of assets.

***Table-5***

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | **Bonds** | **High tech stocks** | **Foreign stocks** | **Call options** | **Put options** | **Gold** |
|  |  |  |  |  |  |  |
| **Bonds** | **0.001** | **0.0003** | **-0.0003** | **0.00035** | **-0.00035** | **0.0004** |
| **High tech stocks** | **0.0003** | **0.009** | **0.0004** | **0.0016** | **-0.0016** | **0.0006** |
| **Foreign stocks** | **-0.0003** | **0.0004** | **0.008** | **0.0015** | **-0.0055** | **-0.0007** |
| **Call options** | **0.00035** | **0.0016** | **0.0015** | **0.012** | **-0.0005** | **0.0008** |
| **Put options** | **-0.00035** | **-0.0016** | **-0.0055** | **-0.0005** | **0.012** | **-0.0008** |
| **Gold** | **0.0004** | **0.0006** | **-0.0007** | **0.0008** | **-0.0008** | **0.005** |

The excel solver is used to calculate the risks involved in investing $10000 and allocation of the investment is calculated to have a minimum baseline expected return of 11%, and at the same time, at a minimum risk.

***Table-6***

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Stocks** | |  | **Allocation** |  | **Investment** |
| **Bonds** | **X1** | 0.16041996 | 16.04199566 |  | $ 1,604.20 |
| **High tech stocks** | **X2** | 0.16791621 | 16.79162087 |  | $ 1,679.16 |
| **Foreign stocks** | **X3** | 0.16641696 | 16.64169583 |  | $ 1,664.17 |
| **Call options** | **X4** | 0.17091471 | 17.09147095 |  | $ 1,709.15 |
| **Put options** | **X5** | 0.17091471 | 17.09147095 |  | $ 1,709.15 |
| **Gold** | **X6** | 0.16341846 | 16.34184574 |  | $ 1,634.18 |
|  | **Total** | 1.000001 | = | 1 | $ 10,000.01 |
|  |  | **Return** |  |  |  |
| **Protfolio** |  | 11% | ≥ | 11.0% |  |

The excel solver has given the amount of money to be invested in different stocks in order to have a minimum baseline expected return of 11%.

The investment in Bonds ($ 1,604.20), High Tech Stocks ($ 1,679.16), Foreign Stocks ($ 1,664.17), Call Options ($ 1,709.15), Put Options ($ 1,709.15), and Gold ($ 1,634.18) sums up to $10,000.

**The Portfolio mean is** **0.112248987**

**Variance is 0.001117236**

**And the Standard Deviation is 0.033425086**

After computing the variance for an 11 percent risk, we'll use the following values to determine the portfolio value: 10 percent, 10.5 percent, 11 percent, 11.5 percent, 12 percent, 12.5 percent, 13 percent, and 13.5 percent.

***Table-7***

|  |  |
| --- | --- |
| Expected Portfolio Returns "r" | Minimized Risk "e" |
| 10 | 0.00051391 |
| 10.5 | 0.00060327 |
| 11 | 0.00073564 |
| 11.5 | 0.00091099 |
| 12 | 0.00112935 |
| 12.5 | 0.00112935 |
| 13 | 0.00112935 |
| 13.5 | 0.00349625 |
| 14 | 0.00567575 |

***Figure-1***

The figure shows the change in minimum risk for different percentages of portfolio returns.

As we can see that from 13% the minimized risk is skyrocketed. It can be observed that there is a positive correlation between the Risk and the Percentage of Portfolio Returns.

**Conclusion**

To conclude the assignment, numerous strategies are to solve the transshipment and risk minimization problems. It has been discovered that the costs of various shipping choices by completing this assignment, and we can claim that the cost of transportation of the barrels to various intermediate nodes, i.e. plants and dumping sites is cheaper than the cost of directly transporting the barrels to the dispose sites. When it comes to investment allocation, we can state that the risk is low when the return is low, and as the return increases, the risk increases as well.