Introduction to Numerical Optimization – Project

Gas Flow Routing and Network Control

A. Gas Trading in Liberalised Markets

In the context of liberalised markets, the trading of gas may take various forms, ranging from long-term, forward contracts to spot market transactions, taking place either through an exchange or over-the-counter (bi-laterally). Transactions involve at least one supplier or trader, usually called *shipper*, and one consumer, also known as end-user. Once transactions have been processed, the modalities of gas delivery are detailed in so-called *nominations*. In their simplest form, nominations essentially indicate the origin of the gas (an entry point in the network), its destination (an exit point), the volume of gas which should be delivered, as well as the timing of the delivery.

B. The Gas System Operator

The gas system operator, which usually owns and operates the gas transmission infrastructure as well as gas storage assets, charges shippers for the timely delivery of the nominated gas volumes to end-users. In other words, the goal of the gas system operator is typically to route the gas so as to honour all nominations whilst minimising operating costs and satisfying a set of operational, security and contractual constraints. Operating costs mostly stem from the use of compressors, which generate pressure differentials across the network, thereby creating the desired flow patterns. Depending on the contract type, the system operator may also incur an economic penalty if the gas is not delivered in the nominated amount and time. On the other hand, typical operational constraints include compressor ramp rates and stability limits, storage injection rates and the physics of gas flow. By contrast, security constraints usually define upper bounds on pressures across the network, whereas contractual constraints require that minimum pressure levels are maintained at various entry/exit points, e.g. at international borders, or that gas injection levels at various points of the network remain within a pre-defined range.

C. Problem Statement & Data

The gas routing problem faced by the system operator in a given hour will be studied in this project. The problem thus reduces to that of finding the network flows and pressures enabling to serve as much of the gas demand as possible while minimising operating costs.

A simplified representation of the Belgian high (calorific value) gas network, which is depicted in Figure 1 along with the low gas network, will be considered. Tables 1 and 2 list the nodes and arcs of the gas network, along with their types. In particular, nodes labelled as "entry" and "exit" designate international connection points from which gas enters/exits the Belgian network. Moreover, nodes labelled as "consumption" represent points where gas should be delivered. "Storage" nodes represent network points where storage assets are available, and from which gas can be injected into the network by the system operator. Finally, "transit" nodes represent points through which gas simply flows. In the present study, only two types of arcs are considered, namely pipes and compressors.

An instance of the Belgian gas network with additional properties of nodes and arcs is provided in the form of a file *test_network.instance*. This instance can be loaded into Python via the *read_gpickle* function of the NetworkX package, which can be installed via *conda*. Before loading the instance, make sure to import the *GasNetwork* class defined in the *gas_network.py* file. The latter should be placed in the working directory. The attributes of the resulting object can be accessed in the usual way, i.e., by typing the name of the object followed by the "." operator and the names of the attributes of interest. The relevant attributes of the gas network instance include *minimum_pressure_bounds*, *maximum_pressure_bounds*, *minimum_nodal_injections*, *maximum_nodal_injections*, *minimum_pressure_ratio*, *maximum_pressure_ratio*, *nodal_demands*, *value_unserved_demand*, *compression_costs*, *friction_coefficients* and *reference_flows*.

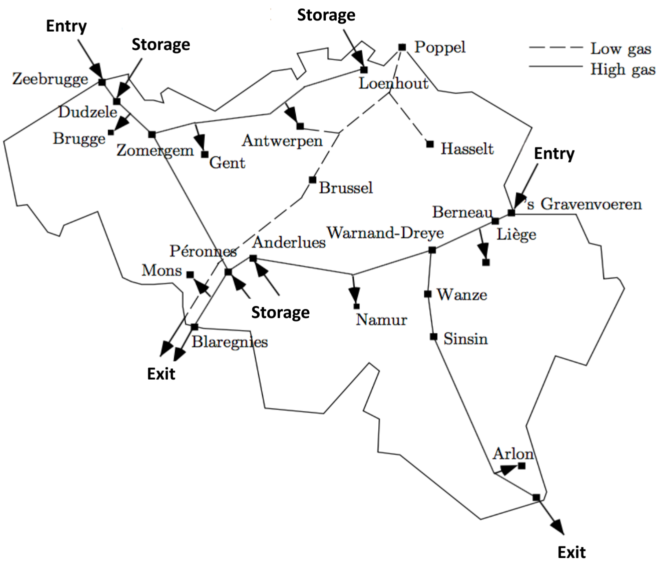

Figure 1: A schematic of the simplified Belgian gas network.

Node	Name	Type
1	Zeebrugge	Entry
2	Dudzele	Storage
3	Brugge	Consumption
4	Zomergem	Transit
5	Loenhout	Storage
6	Antwerpen	Consumption
7	Gent	Consumption
8	's Gravenvoeren	Entry
9	Berneau	Transit
10	Liege	Consumption
11	Warnand	Transit
12	Namur	Consumption
13	Anderlues	Storage
14	Peronnes	Storage
15	Mons	Consumption
16	Blaregnies	Exit
17	Wanze	Transit
18	Sinsin	Transit
19	Arlon	Consumption
20	Petange	Exit

Table 1: List of nodes in the gas network.

D. Tasks

The project will be performed in pairs or individually. Once teams have been formed, please email your names to mathias.berger@uliege.be.

1. Write a mathematical program expressing the routing problem faced by the gas system operator. In particular:

   - The objective function should include both compression costs and a penalty for failing to deliver the right amount of gas. Compression costs will be assumed proportional to the difference between the squared input and output pressure Δp^2 of the compressor. The penalty incurred for failing to

Arc	From	To	Type
1	Zeebrugge	Dudzele	pipe
2	Zeebrugge	Dudzele	pipe
3	Dudzele	Brugge	pipe
4	Dudzele	Brugge	pipe
5	Brugge	Zomergem	pipe
6	Loenhout	Antwerpen	pipe
7	Antwerpen	Gent	pipe
8	Gent	Zomergem	pipe
9	Zomergem	Peronnes	pipe
10	's Gravenvoeren	Berneau	compressor
11	's Gravenvoeren	Berneau	compressor
12	Berneau	Liege	pipe
13	Berneau	Liege	pipe
14	Liege	Warnand	pipe
15	Liege	Warnand	pipe
16	Warnand	Namur	pipe
17	Namur	Anderlues	pipe
18	Anderlues	Peronnes	pipe
19	Peronnes	Mons	pipe
20	Mons	Blaregnies	pipe
21	Warnand	Wanze	pipe
22	Wanze	Sinsin	compressor
23	Sinsin	Arlon	pipe
24	Arlon	Petange	pipe

Table 2: List of pipes/compressors in the gas network.

deliver the right amount of gas will be assumed proportional to the volume of gas not served.

- The conservation of mass should be enforced at every network node.
- The flow of gas in pipes should be modelled using steady state equations linking pressures and flows, namely $c\Delta p^2 = \phi|\phi|$, with c a coefficient expressing friction losses in pipes.
- A simplified compressor model based on pressure ratios should be adopted. Only unidirectional flows are allowed in compressors, and the output pressure should always be i) greater than the product of the minimum pressure ratio and the input pressure ii) smaller than the product of the maximum pressure ratio and the input pressure.
- Security and contractual constraints should be included.

2. Comment on the structure of the resulting optimisation problem. Is it linear? Is it convex? Motivate the answers to these questions.
3. Produce a linearisation of the flow equations around a set of reference flows. Solve the resulting gas flow model for the data provided.
4. Retrieve the dual variables of all balance equations. Provide a physical interpretation of these variables and their values.
5. Identify the minimum pressure bound value that would allow to satisfy all nominations at node 16. *Hint*: generate a LP file and use the command-line tool of Gurobi.
6. Assuming that gas flow directions remain the same as those found via the linearised formulation, produce a conic relaxation of the steady state flow equations. Solve the resulting model. Compare the solution obtained with that of the linearised problem, and extract the slack variables of the relaxed constraints.
7. Solve the problem formulated in 1. with IPOPT. Compare the solution with those obtained via the linear approximation and convex relaxation. Comment on the results.