Dynamique des Systèmes Mécaniques

Project 1

One degree of freedom damped system

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December 11, 2018

Setup

The system consists in a mass m supported by a spring of stiffness k and a viscous damper of unknown damping c.

We assume that the system has only one degree of freedom x and we know that

$$m = 4.484 \text{ kg}$$
$$k = 10\,500 \text{ N/m}$$

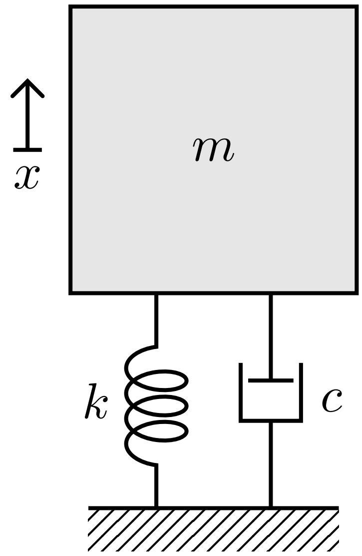

Figure 1 – System

Computations

1 Natural frequency

The natural (or undamped) frequency[1] w_0 of the system, which would have been the frequency of the motion if the system wasn't damped, is given by

$$\omega_0 = \sqrt{\frac{k}{m}}$$
$$= 48.3907 \text{ rad/s}$$

3 Time response

First of all, we observe that our system performs damped harmonic oscillations. It means that ε, the damping ratio of the system, is strictely between 0 and 1.

Further, to compute experimental system parameters, we have to select the 1^{st} and the n^{th} [2] maxima of the dataset[3] and interpolate around both peaks to obtain more accurate values of time and acceleration :

$$t_1 = 0.1621 \text{ s} \qquad a_1 = 8.793 \times 10^{-2} \text{ g}$$
$$t_n = 6.3645 \text{ s} \qquad a_n = 0.299 \times 10^{-2} \text{ g}$$

[1] In this document, by convenience, angular frequency is shortened to frequency, despite their different units of measurement.

[2] In the matlab implementation, we have chosen $n = 50$.

[3] We have shrunken the dataset to the time interval [0.140, 9.850] [s] because extreme values were unreliable.

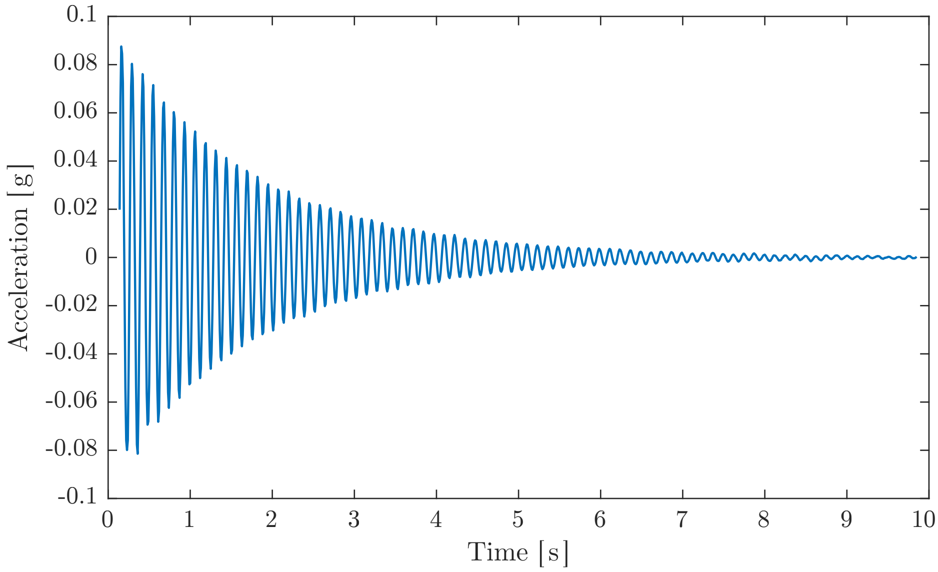

Figure 2 – Acceleration response of the system from time 0.140 s to 9.850 s.

3.1 Frequency

The frequency f of an harmonic motion is the ratio between the number of full oscillations performed by the system during a time period and that time period length.

$$\begin{aligned} f_d &= \frac{n}{t_n - t_1} \\ &= 7.9001 \text{ Hz} \\ \Leftrightarrow \quad \omega_d &= 2\pi f_d = 49.6380 \text{ rad/s} \end{aligned}$$

We see that the experimental frequency ω_d is a bit larger than the undamped frequency ω_0, altought it should be smaller or equal given that

$$\omega_d = \omega_0 \sqrt{1 - \varepsilon^2} \leq \omega_0$$

It means that either our measures or our parameters (m or k) are imprecise.

3.2 Logartihmic decrement

The logarithmic decrement Δ is defined by the neperian logarithm of the ratio between the amplitudes of two successive maxima. That ratio, and thus Δ, is theoretically constant all along the harmonic motion. From its definition, it is easy to show that

$$\Delta = \ln\left(\frac{a_k}{a_{k+1}}\right) = \ldots = \frac{\ln\left(\frac{a_1}{a_k}\right)}{k-1} \quad \forall k > 1$$

Replacing with our computed values a_1 and a_n, we obtain

$$\Delta = \frac{\ln\left(\frac{a_1}{a_n}\right)}{n-1} = 6.9002 \times 10^{-2}$$

3.3 Damping ratio

The value of the damping ratio ε is usually computed according to its definition :

$$\varepsilon = \frac{c}{2m\omega_0}$$

In this case, the damping c of the system is unknown. However, the damping ratio can still be estimated by the logarithmic decrement divided by 2π.

$$\varepsilon \simeq \frac{\Delta}{2\pi} = 1.0982 \times 10^{-2}$$

5 Bode diagram

In the amplitude Bode diagram, we can observe a spike in amplitude near the natural frequency of the system. Searching for more accuracy, we can interpolate around the peak and obtain X, the maximum amplitude, and ω_b, its associated frequency.

$$X = 0.8116 \text{ g/N} \qquad \omega_b = 49.7242 \text{ rad/sec}$$

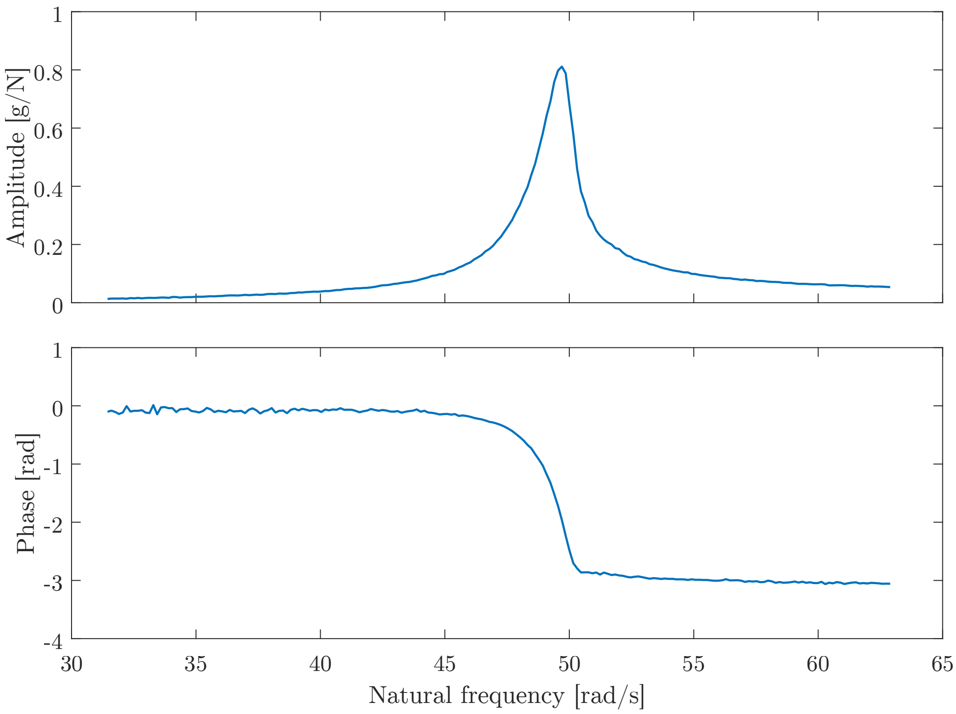

Figure 3 – Bode diagrams.

5.1 Quality factor

The half-power method's first step is to find the frequencies ω_- and ω_+ for which the amplitude is equal to $\frac{X}{\sqrt{2}}$. These can be found by interpolating separately each side of the amplitude Bode diagram's peak. In doing so, we obtain

$$\omega_- = 48.9125 \text{ rad/ sec} \qquad \omega_+ = 50.1696 \text{ rad/ sec}$$

We can now compute the quality factor Q of the system

$$\begin{aligned} Q &= \frac{\omega_b}{\Delta\omega} = \frac{\omega_b}{\omega_+ - \omega_-} \\ &= 38.494 \end{aligned}$$

5.2 Damping ratio

The damping ratio can be directly estimated from the quality factor.

$$\varepsilon \simeq \frac{1}{2Q} = 1.299 \times 10^{-2}$$

6 Nyquist diagram

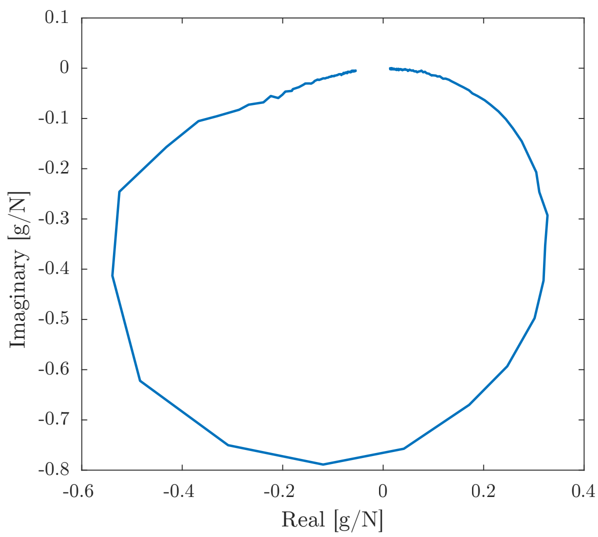

Figure 4 – Nyquist diagram.

6.1 Quality factor

The quality factor can also be estimated by the amplitude[4] at the phase resonance, described by a null real part. Firstly, we have to find the phase resonance frequency ω_n by

[4]The amplitude has to be normed. In our case it means to multiply the amplitude by m.

interpolating around the real part root, which gives us

$$\omega_n = 49.4366 \text{ rad/ sec}$$

It is now possible to determine the amplitude X_n associated to ω_n.

$$\begin{aligned} X_n &= \sqrt{\Re(\boldsymbol{X})^2 + \Im(\boldsymbol{X})^2} = |\Im(\boldsymbol{X})| \\ &= 0.7723 \text{ g/N} = 7.5759 \text{ m/s}^2 \cdot \text{N} \end{aligned}$$

And, therefore, the quality factor.

$$Q \simeq mX_n = 33.9704$$

6.2 Damping ratio

Finally, with the same estimation than before

$$\varepsilon \simeq \frac{1}{2Q} = 1.472 \times 10^{-2}$$

7 Conclusion

We can observe that either the time response, the Bode diagram or the Nyquist diagram gave us approximatively the same results :

$$\varepsilon \approx 1\%$$

This shows that, as expected, all these methods are effective. Therefore, the choice between one or another should be lead by its convenience which varies from case to case.