Brandon Walters

CS 3200

Prof. Berzins

Homework 4 Report

1. Using our program, we can see that the Jacobi method takes 50 iterations to get a residual infinity norm of 1.0e-15, while the Gauss-Seidel method only takes 18, and the Gradient Descent method takes 41. This fits with what we know about these methods, where the Jacobi and Gradient Descent methods are slow but typically easier to implement and more accurate when given more iterations, while the Gauss-Seidel is tougher to implement but runs much faster.
2. Given our discrete model, we can see that while our Gauss-Seidel method converges properly on this problem (doing so in 125 iterations), we can also see that our Jacobi implementation does not converge on this specific matrix. This occurs because we do not have diagonal dominance with this matrix: if we look at row two, we can see that the absolute value sum of all values outside of the diagonal value of 6 adds up to 10. Since 10 is bigger than 6, we know that we do not have diagonal dominance in our matrix, and therefore we know that the Jacobi method cannot converge.
3. Looking at the solutions generated by the Gradient Descent and Conjugate Gradient methods for the same problem, we can see that compared with the results that we generate in our xReal array, which gives the true answers for x using the linsolve() function, we notice that our Gradient Descent method gives us the correct values, while the Conjugate Gradient method does not work. To

explain why the Conjugate Gradient method does not converge properly on this problem, we can examine the eigenvalues of the A matrix using our AEig variable. We can use this to determine our matrix's condition number, which in this case is (15.0747/0.0853) = 176.72. This is high, which means that we have issues with our iterations being maxed out before the algorithm can close in on more accurate values since a higher condition number means that our convergence will take much longer. However, we can also use the raw matrix values to definitely determine that the Conjugate Gradient method will never converge: the Conjugate Gradient method only works on symmetric, positive-definite matrices, however, if we look at the values in matrix positions (7,8) and (8,7), we can see that they are not the same value, and therefore our matrix is not symmetric, therefore meaning that the Conjugate Gradient method will never converge on this specific matrix. If we now go back to look at the matrix from Question 1, we can determine that the Conjugate Gradient method does work for the matrix provided there, as not only is the matrix positive-definite and symmetric, but we can see that our condition number is now (20.6845/5.6673) = 3.65, which is a very low condition number. We can therefore conclude that the Conjugate Gradient will converge for the matrix provided in Question 1.

4. With the provided table below, we can compare the residual norms of both the Conjugate Gradient and the Gradient Descent methods to see how they compare on Hilbert matrices of different dimensions. Looking at the table, we can immediately see that our Conjugate Gradient Method has a higher accuracy on nearly every Hilbert matrix iteration, with the residual norm hanging around the

1.0e-8 area on average. By contrast, we can see that the Gradient Descent method tends to be about two or three powers higher, meaning that our results generated by the Gradient Descent method is slightly less accurate. I attempted to make the tolerance even smaller, but it led to the program taking several minutes and billions of iterations for even the beginning matrices, so I tried to strike a balance between acceptable accuracy and speed with the tolerance chosen. If we look at the iteration numbers, we can also see a large difference: the Conjugate Gradient method hits its max iterations cap of 1000 at just the third matrix, whereas our uncapped Gradient Descent method goes up as far as a whopping 17,496,871 iterations on our final 40x40 Hilbert matrix. Overall, we can conclude that our Gradient Descent method is not able to get results that are as accurate as the Conjugate Gradient method, although the results themselves and the residual norms are not too far apart.

Hilbert Matrix Size	Conjugate Gradient Iterations	Residual Norm for Conjugate Gradient	Gradient Descent Iterations	Residual Norm for Gradient Descent
4	7	7.1054e-15	46648	4.5931e-07
8	44	6.3665e-12	9962813	9.2656e-05
12	1000	1.3201e-07	12242461	1.0697e-04
32	1000	2.0875e-04	13790792	1.2567e-04
40	1000	1.9816e-06	17496871	1.1780e-04