Assignment 4

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Download latex-tikz codes from

https://github.com/Dontha−Aarthi/AI1103/blob/main/Assignment4/assignment4.tex

1 GATE 2020 (XE-C), Q.14

In an industry, the probability of an accident occurring in a given month is $\frac{1}{100}$. Let $\Pr(n)$ denote the probability that there will be no accident over a period of 'n' months. Assume that the events of individual months are independent of each other. The smallest integer value of 'n' such that $\Pr(n) \leq \frac{1}{2}$ is ....(*round off to the nearest integer*).

2 Solution

Let A be the event of an accident occurring in a given month. So,

$$\Pr(A) = \frac{1}{100} \tag{2.0.1}$$

$$\Pr(A') = 1 - \Pr(A) \tag{2.0.2}$$

$$\Pr(A') = \frac{99}{100} \tag{2.0.3}$$

So, $\Pr(n)$ can be written as:

$$\Pr(n) = \Pr(A' \times A' \cdots A')_{A' \; n \; times} \tag{2.0.4}$$

Its given that events of individual months are independent of each other, so

$$\Pr(n) = \Pr(A') \cdot \Pr(A') \cdots \Pr(A')_{A' \; n \; times} \tag{2.0.5}$$

$$= (\Pr(A'))^n \tag{2.0.6}$$

Given:

$$\Pr(n) \leq \frac{1}{2} \tag{2.0.7}$$

So, from (2.0.6),

$$(\Pr(A'))^n \leq \frac{1}{2} \tag{2.0.8}$$

Taking **Natural Logarithm** on both sides of (2.0.8)

$$\implies log_e(\Pr(A'))^n \leq log_e\frac{1}{2} \tag{2.0.9}$$

$$\implies n \cdot log_e\frac{99}{100} \leq log_e\frac{1}{2} \tag{2.0.10}$$

$$\implies n \geq \frac{log_e\frac{1}{2}}{log_e\frac{99}{100}} \tag{2.0.11}$$

$$\implies n \geq 68.9675 \tag{2.0.12}$$

$\therefore$ The smallest integer value of n is **69**.