#### 1

# Assignment 4

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### Download latex-tikz codes from

https://github.com/Dontha-Aarthi/AI1103/blob/main/Assignment4/assignment4.tex

#### 1 GATE 2020 (XE-C), Q.14

In an industry, the probability of an accident occurring in a given month is  $\frac{1}{100}$ . Let Pr(n) denote the probability that there will be no accident over a period of 'n' months. Assume that the events of individual months are independent of each other. The smallest integer value of 'n' such that  $Pr(n) \leq \frac{1}{2}$  is ......(round off to the nearest integer).

#### 2 Solution

Let A be the event of an accident occurring in a given month. So,

$$\Pr(A) = \frac{1}{100} \tag{2.0.1}$$

$$Pr(A') = 1 - Pr(A)$$
 (2.0.2)

$$\Pr(A') = \frac{99}{100} \tag{2.0.3}$$

So, Pr(n) can be written as:

$$Pr(n) = Pr(A' \times A' \cdots A')_{A' \ n \ times}$$
 (2.0.4)

Its given that events of individual months are independent of each other, so

$$Pr(n) = Pr(A') \cdot Pr(A') \cdot \cdot \cdot Pr(A')_{A' \ n \ times} \quad (2.0.5)$$

$$= (\Pr(A'))^n \tag{2.0.6}$$

Given:

$$\Pr\left(n\right) \le \frac{1}{2} \tag{2.0.7}$$

So, from (2.0.6),

$$(\Pr(A'))^n \le \frac{1}{2}$$
 (2.0.8)

$$\implies \ln \left( \Pr \left( A' \right) \right)^n \le \ln \frac{1}{2} \tag{2.0.9}$$

$$\implies n \cdot \ln \frac{99}{100} \le \ln \frac{1}{2} \tag{2.0.10}$$

$$\implies n \ge \frac{\ln\frac{1}{2}}{\ln\frac{99}{100}} \tag{2.0.11}$$

$$\implies n \ge 68.9675$$
 (2.0.12)

:. The smallest integer value of n is 69.