Assignment 1

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Download all python codes from

```
https://github.com/Dontha-Aarthi/AI1103-Assignment-1/blob/main/Assignment1/code-assignment1.py
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and latex-tikz codes from

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https://github.com/Dontha-Aarthi/AI1103-Assignment-1/blob/main/Assignment1/main.tex
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1 Problem

Question 2.1:
Bag I contains 3 red and 4 black balls and Bag II contains 4 red and 5 black balls. One ball is transferred from Bag I to Bag II and then a ball is drawn from Bag II. The ball so drawn is found to be red in colour. Find the probability that the transferred ball is black.

2 Solution

Let X $\in \{0, 1\}$ represent the bags and Y $\in \{0, 1\}$ where 0 denotes black and 1 denotes red.

Bags(X)	n(Y=0)	n(Y=1)
0	4	3
1	5	4

$$Pr(Y = 0, X = 0) = \frac{4}{7} \tag{2.0.1}$$

$$Pr(Y = 1, X = 0) = \frac{3}{7} \tag{2.0.2}$$

There are 2 cases,

1) **Transferring a black ball**.

   Probability of transferring a black ball from bag 1 to bag 2 is:

   $$Pr(Y = 0, X = 0) = \frac{4}{7} \tag{2.0.3}$$

   Now, after transferring black ball to bag 2, the probability of picking a red ball from bag 2 is:
   Let X $\in \{0, 1\}$ represent the bags and $Y_1 \in \{0, 1\}$ where 0 denotes black and 1 denotes red.

Bags(X)	n(Y_1 = 0)	n(Y_1 = 1)
0	3	3
1	6	4

   $$Pr(Y_1 = 1, X = 1) = \frac{4}{10} = \frac{2}{5} \tag{2.0.4}$$

2) **Transferring a red ball.**

   Probability of transferring a red ball from bag 1 to bag 2 is

   $$Pr(Y = 1, X = 0) = \frac{3}{7} \tag{2.0.5}$$

   Now, after transferring red ball to bag 2, the probability of picking a red ball from bag 2 is:
   Let X $\in \{0, 1\}$ represent the bags and $Y_2 \in \{0, 1\}$ where 0 denotes black and 1 denotes red.

Bags(X)	n(Y_2 = 0)	n(Y_2 = 1)
0	4	2
1	5	5

   $$Pr(Y_2 = 1, X = 1) = \frac{5}{10} = \frac{1}{2} \tag{2.0.6}$$

   Now, we have to find probability of black ball being transferred from bag 1 to bag 2 if a red ball is being drawn from bag 2.
   Let T be the event of drawing a red ball from bag 2 after transferring a random ball from bag 1 to bag 2.
   Using Baye's theorem,

   $$Pr((Y = 0, X = 0)|T) =$$

   $$\frac{Pr((Y = 0, X = 0), T)}{Pr((Y = 0, X = 0), (Y_1 = 1, X = 1)) + Pr((Y = 1, X = 0), (Y_2 = 1, X = 1))}$$

   $$= \frac{Pr((Y = 0, X = 0) * Pr(Y_1 = 1, X = 1))}{Pr((Y = 0, X = 0) * Pr(Y_1 = 1, X = 1)) + Pr((Y = 1, X = 0) * Pr(Y_2 = 1, X = 1))} \tag{2.0.7}$$

   On substituting the values in equation (2.0.7), we get

$$Pr((Y=0,X=0)|T)=\frac{\frac{4}{7}\cdot\frac{2}{5}}{\frac{4}{7}\cdot\frac{2}{5}+\frac{3}{7}\cdot\frac{1}{2}}$$

$$\implies Pr((Y=0,X=0)|T)=\frac{\frac{8}{35}}{\frac{8}{35}+\frac{3}{14}}$$

$$\implies Pr((Y=0,X=0)|T)=\frac{16}{31}$$