Challenging Problem 3

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Download latex-tikz codes from

https://github.com/Dontha−Aarthi/AI1103/blob/main/Challenging%20Problem3/Challenge%20Problem3.tex

1 UGC/MATH 2019, Q.50

Let X_1, X_2, X_3, X_4, X_5 be *i.i.d.* random variables having a continuous distribution function. Then

$$\Pr\left(X_1 > X_2 > X_3 > X_4 > X_5 | X_1 = max(X_1, X_2, X_3, X_4, X_5)\right)$$

equals _____.

(A) $\frac{1}{4}$

(B) $\frac{1}{5}$

(C) $\frac{1}{4!}$

(D) $\frac{1}{5!}$

2 Solution

Required probability

$$= \Pr\left(X_1 > X_2 > X_3 > X_4 > X_5 | X_1 = max\left(X_1, X_2, X_3, X_4, X_5\right)\right) \tag{2.0.1}$$

$$= \Pr\left(X_2 > X_3 > X_4 > X_5\right) \tag{2.0.2}$$

CDF in terms of PDF is

$$F(x) = \int_{-\infty}^{x} f(t)dt \tag{2.0.3}$$

And, PDF in terms of CDF is

$$f(x) = \frac{d}{dx}F(x) \tag{2.0.4}$$

The PDF of X is

$$F_X(x) = \int_{-\infty}^{\infty} f(x)dx \tag{2.0.5}$$

(2.0.2) can be written in terms of PDF as

$$= \int_{-\infty}^{\infty} f_X(x)\left(\int_{-\infty}^{x} f_X(t)\left(\int_{-\infty}^{t} f_X(z)\left(\int_{-\infty}^{x} f_X(k)dk\right)dz\right)dt\right)dx \tag{2.0.6}$$

$$= \int_{-\infty}^{\infty} f_X(x)\left(\int_{-\infty}^{x} f_X(t)\left(\int_{-\infty}^{t} f_X(z)\left(F_X(z)\right)dz\right)dt\right)dx \tag{2.0.7}$$

$$= \int_{-\infty}^{\infty} f_X(x)\left(\int_{-\infty}^{x} f_X(t)\frac{(F_X(t))^2}{2}dt\right)dx \tag{2.0.8}$$

$$= \int_{-\infty}^{\infty} f_X(x)\frac{(F_X(x))^3}{2\times 3}dx \tag{2.0.9}$$

$$= \frac{F_X^4(x)}{4\times 3\times 2}\Big|_{-\infty}^{\infty} \tag{2.0.10}$$

$$= \frac{1}{4!} \tag{2.0.11}$$