Project Report

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Stochastic optimization

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Introduction

Overview of Emergency Department Overcrowding

Emergency Department (ED) overcrowding is a critical issue in healthcare systems worldwide, particularly pronounced in the U.S. This problem occurs when the number of patients surpasses the ED's operational capacity, leading to prolonged waiting times and potential delays in necessary medical treatments. The key reasons for this issue include an increase in non-emergency patient visits, a decline in available hospital beds causing extended ED stays, and a decrease in the number of operational emergency facilities, despite rising healthcare demands. The ramifications of overcrowding are extensive, impacting patient health outcomes due to delayed care, increasing stress levels among healthcare staff, and leading to inefficiencies within the healthcare system.

Significance of Effective Resource Allocation in Healthcare

In healthcare, particularly within emergency departments, effective resource allocation is essential for maximizing patient care and enhancing operational efficiency. Properly managing resources such as personnel, medical supplies, and infrastructure in response to dynamic patient needs is vital. For instance, judiciously transferring patients among hospitals in a network can reduce the burden on overwhelmed EDs, thereby optimizing resource usage and improving patient care by decreasing waiting periods. This approach is bolstered by using advanced modeling and optimization methods, such as capacitated network design and integer programming, which support strategic decision-making in resource distribution. These methodologies help ensure that resources are utilized where they are most needed, promoting better patient outcomes and more efficient hospital operations.

Article and Model Description

This study presents a mathematical model designed to optimize resource allocation and patient flow in multi-emergency department (ED) settings through strategic patient transfers. With the increasing challenge of ED overcrowding, which leads to prolonged wait times and potentially compromises patient care, the model aims to alleviate pressure by enabling the transfer of non-emergency patients to hospitals within the network that have available capacity.

The model employs a capacitated network design approach, incorporating various costs associated with increasing hospital capacity, patient treatment, inter-hospital transfers, patient waiting times, unmet demand, and staff overtime. Through a series of constraints, the model ensures operational feasibility, including adherence to hospital capacity limits, budget constraints, and transfer policies. The objective is to minimize the total operational cost while improving patient flow and reducing wait times.

Experimental results demonstrate the model's effectiveness in decreasing the number of **waiting patients by up to 35% and** reducing **resource requirements by about 10%** without compromising patient care. This study contributes to healthcare operations research by offering a viable strategy to address the critical issue of ED overcrowding, emphasizing the importance of integrated healthcare networks and efficient resource utilization.

Deterministic Model Formulation based on article

This model meticulously defines the structure and cost components to manage and optimize emergency department operations. Each equation is a cog in the overall machinery of resource allocation, patient throughput, and cost minimization, central to addressing the challenges posed by ED overcrowding.

Parameters

- i, j = Hospital and demand area indices
- t = Time indices
- k = Patient type indices (1: emergency patients, 2: non-emergency patients)

- H = Total number of hospitals
- T = Optimization period
- T' = Period in/during which accepting patients will require over-time
- α_i = Cost of increasing one unit capacity in hospital i
- β_i = Cost or income of accepted patients in hospital i
- ϑ_i = Cost of overtime in hospital i
- $\gamma_{i,j}$ = Cost of transferring patients from hospital i to hospital j
- $\delta_{i,k}$ = Penalty cost of waiting patient type k in hospital i for one time interval
- $\eta_{i,k}$ = Penalty cost for unmet demand of patient type k in hospital i
- $\lambda_{i,k,t}$ = Arrival rate of patient type k in hospital i at time t
- μ_i = Expected service time in hospital i
- $d_{i,j}$ = Time required to transfer a patient(s) from hospital i to hospital j l_i Number of allowed patients to be transferred each time from hospital i
- B_i = Available budget for resource allocation in hospital i.

The decision variables in this optimization model are as follows:

- x_i = Capacity considered for hospital i
- $Y_{i,k,t}$ = Number of patient type k admitted to hospital i at time t
- $Z_{i,k,t}$ = Number of patient type k discharged from hospital i at time t
- $c_{i,t}$ = Available capacity in hospital i at time t
- $u_{i,k,t}$ = Number of unmet non-emergency demand of patient type k in hospital i at time t
- $w_{i,k,t}$ = Number of waiting non-emergency patient type k in hospital i at time t
- $f_{i,j,t}$ = Number of non-emergency patients (k = 2) sent from hospital i to hospital j at time t
- o_i = Overtime for hospital i.

Objective Function: Minimize the total operational costs including capacity increase, patient treatment, transfers, waiting times, unmet demands, and overtime.

(1)

$$\text{Minimize} \sum_{i\in H} \alpha_i x_i + \sum_{k\in K} \sum_{t\in T} \sum_{i\in H} \beta_i y_{i,k,t} + \sum_{t\in T} \sum_{i\in H} \sum_{j\in H\neq i} \gamma_{i,j} f_{i,j,t} + \sum_{k\in K} \sum_{t\in T} \sum_{i\in H} \delta_{i,k} w_{i,k,t} + \sum_{k\in K} \sum_{t\in T} \sum_{i\in H} \eta_{i,k} u_{i,k,t} + \sum_{i\in H} \psi_i o_i$$

Subject To:

Balance Constraints for Emergency and Non-Emergency Patients: For both emergency and non-emergency patients.

(2)

$$w_{i,k,t} + y_{i,k,t} + u_{i,k,t} = w_{i,k,t-1} + \lambda_{i,k,t} \quad \forall i \in H, \forall t > 1, k = 1$$

(3)

$$\sum_{j\in H\neq i} f_{i,j,t} + y_{i,k,t} + w_{i,k,t} + u_{i,k,t} = w_{i,k,t-1} + \lambda_{i,k,t} + \sum_{j\in H|t>d_{i,j}} f_{j,i,t-d_{i,j}} \quad \forall i \in H, \forall t > 1, k = 2$$

Initial Condition Constraints for Emergency and Non-Emergency Patients: Set the starting operational conditions for EDs.

(4)

$$w_{i,k,t} + y_{i,k,t} + u_{i,k,t} = \lambda_{i,k,t} \quad \forall i \in H, t = 1, k = 1$$

(5)

$$\sum_{j\in H\neq i} f_{i,j,t} + y_{i,k,t} + w_{i,k,t} + u_{i,k,t} = \lambda_{i,k,t} \quad \forall i \in H, t = 1, k = 2$$

Capacity Constraints: Ensure hospital capacities are not exceeded.

(6)

$$y_{i,k,t} \leq c_{i,t} \quad \forall i \in H, \forall t \in T$$

Hospital Capacity Limits Over Time: Monitor and adjust capacities over the optimization period.

(7)
$$c_{i,t} \leq x_i \quad \forall i \in H, t > 1$$
(8)
$$c_{i,t} = x_i \quad \forall i \in H, t = 1$$
(9)
$$c_{i,t} = c_{i,t-1} - \sum_{k \in K} y_{i,k,t-1} + \sum_{k \in K} z_{i,k,t-1} \quad \forall i \in H, \forall t > 1$$

Service and Discharge Constraints: Manage the flow of patients being admitted and discharged.

(10)
$$z_{i,k,t} \leq y_{i,k,t-\mu_i} \quad \forall i \in H, \forall t > \mu_i, \forall k$$
(11)
$$z_{i,k,t} = 0 \quad \forall i \in H, \forall t \leq \mu_i$$

Budget limitation: Adhere to the financial constraints set for each hospital.

(12)
$$\alpha_i x_i \leq B_i \quad \forall i \in H$$
(13)
$$\sum_{i \in H} \alpha_i x_i \leq B$$

Transfer Restrictions: Regulate the transfer of patients between hospitals.

(14)
$$f_{i,j,t} \leq \min(c_{j,t+d_{i,j}}, l_i) \quad \forall i, j \in H, \forall t \in T$$
(15)
$$f_{i,j,t} \leq l_i \quad \forall i, j \in H, \forall t \in T$$
(16)
$$f_{i,j,t} \leq c_{j,t+d_{i,j}} \quad \forall i, j \in H, \forall t \in T$$

Overtime Calculation: Calculate the necessary overtime based on demand and capacity.

(17)
$$o_i = \sum_{t \in T'} \sum_{k \in K} y_{i,k,t}(t + \mu_i - T) \quad T' : \{t + \mu_i > T\}$$

Integrality Constraints: Maintain integer values for decision variables where required.

(18)

$$x_i, y_{i,k,t}, z_{i,k,t}, u_{i,k,t}, f_{i,j,t}, w_{i,k,t}, o_i \in \mathbb{Z}^+$$

Methodology

For this project we employed a multi-stage stochastic model tailored to manage the dynamic and uncertain environment of emergency department (ED) operations. The decision to use this model stemmed from an extensive review of patient data, operational dynamics, and previous research outcomes, highlighting the need for an approach that could adeptly handle the complexities of ED management.

The multi-stage stochastic model is particularly suited for EDs due to its ability to explicitly incorporate and manage the uncertainties associated with patient arrival rates, which can vary due to daily fluctuations, seasonal effects, and sporadic health emergencies. Unlike traditional models that rely on static assumptions and average conditions, this model allows for iterative decision-making across various stages of the planning horizon. Such flexibility is crucial in EDs where decisions need to adapt in real-time to new information and changing operational conditions.

Additionally, the model facilitates future scenario planning by integrating multiple potential future scenarios at each decision point. This foresight enhances decision-making, enabling a proactive approach to resource management and patient care, rather than a reactive one. It prepares the ED to maintain service levels across varying patient demand scenarios, thereby bolstering the resilience of healthcare operations against potential disruptions.

Moreover, the multi-stage stochastic model enables optimal resource allocation over both short and long terms. It assesses the impacts of immediate decisions on future resource needs and patient outcomes, ensuring a strategic balance that maximizes healthcare service effectiveness across time. This feature is vital in managing the finite resources typical of ED settings, where policy compliance and budget constraints are continually evolving.

By updating the available capacity in each time slot based on observed uncertainties from previous periods, the multi-stage model enhances decision-making accuracy. This dynamic adjustment to capacity and resources ensures that the ED's operations are optimized continuously, aligning closely with the actual needs and constraints of the healthcare environment. This methodology not only supports the operational goals of reducing wait times and managing patient flows but also provides a robust framework for the strategic and efficient management of emergency healthcare services.

Implementing Multi-Stage Stochastic Models in Emergency Departments

Patient Arrival Rates

The foundation of the multi-stage stochastic model is built upon the understanding of patient arrival rates within the Emergency Department (ED). Table 3 presents a detailed breakdown of these rates across different time intervals throughout a typical day. The intervals capture peak and off-peak hours, reflecting the real-world variability in ED visits. To facilitate the modeling process, we consider every two time slots as one stage. The arrival rates during these stages are subject to stochastic variability with a set minimum of 20 and a maximum of 40 patients, establishing the boundaries for our scenario planning. You can observe the assumptions we considered for stages and stationary duration between each stage in the transformed table below.

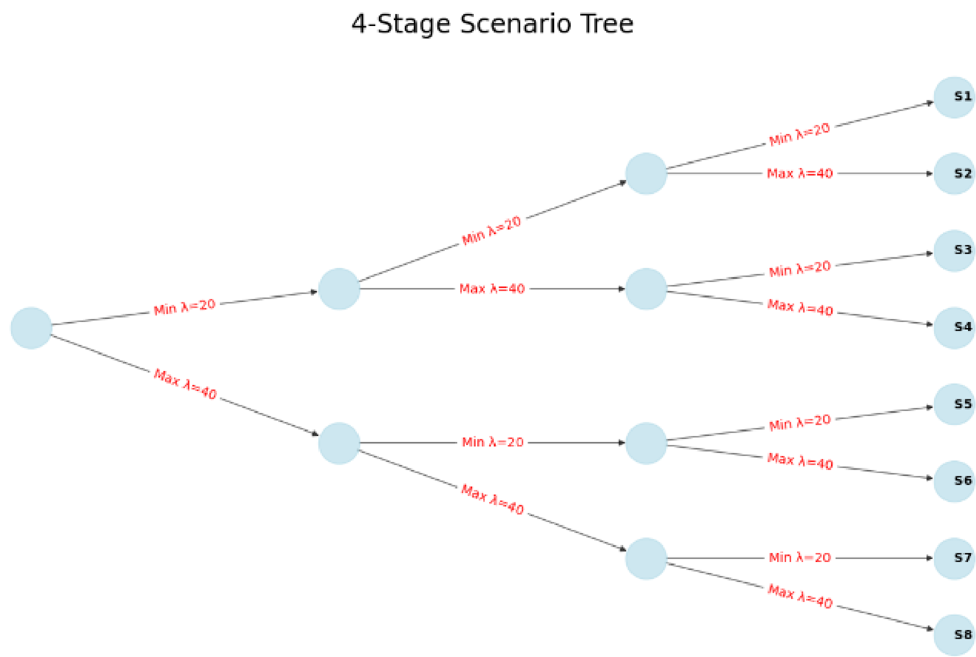
Table 3
Patient arrival rates.

Time	8:00–9:00	9:00–10:00	10:00–11:00	11:00–12:00	12:00–1:00	1:00–2:00	2:00–3:00	3:00–4:00
Arrival rate	10	15	19	20	19	17	15	15

Stages	Stage 1 = 8:00-10:00	Stage 2 = 10:00-12:00	Stage 3 = 12:00-14:00	Stage 4 = 14:00-16:00
Minimum Arrival Rate	20	20	20	20
Maximum Arrival Rate	40	40	40	40

Scenario Tree for Patient Arrivals

The scenario tree visually represents the potential variations in patient arrival rates and forms the backbone of the multi-stage stochastic model. Each node within the tree depicts a stage in the model, with the branches emanating from each node representing different scenarios based on the minimum and maximum arrival rates. The tree structure embodies the decision-making process at each stage, allowing the model to account for a range of possible future events and adapt the resource allocation accordingly.



stage 1 stage 2 stage 3 stage 4

Multistage Stochastic Model – Split Variable Formulation

By incorporating these components, the stochastic model takes a more refined approach to ED management. It considers the inherent variability and unpredictability of healthcare settings, allowing for more flexible and robust decision-making. The model's design reflects actual ED operations, aiming to improve patient outcomes while maintaining operational efficiency and financial viability.

Assumptions:

The probability of all scenarios are equal.

Patient arrival rate for period 1 is known and is equal to 20.

Budget of all hospitals are equal.

Parameters

- s= Scenario Number
- i, j = Hospital and demand area indices
- t = Time indices
- k = Patient type indices (1: emergency patients, 2: non-emergency patients)
- H = Total number of hospitals
- T = Optimization period
- T' = Period in/during which accepting patients will require over-time
- $\lambda_{i,k}(s)$ =rrival rate of patient type k in hospital i for scenario s
- $\gamma_{i,j}$ = cost from hospital i to j
- $\delta_{i,k}$ = Penalty cost of waiting patient type k in hospital i for each time interval
- $\eta_{i,k}$ = Penalty cost of unmet demand of patient type k in hospital i
- μ_i = expected service time in hospital i
- $d_{i,j}$ = time required to transfer from hospital i to hospital j
- l_i = number of allowed patients to be transferred each time from hospital i
- B_i = available budget for resource allocation in hospital i

Decisions Variables

- x_i = Capacity considered for hospital i

Main decision Variable:

- $c_{i,t}(s)$ = available capacity in hospital i at time t for scenario s

Recourse Actions:

- $y_{i,k,t}(s)$ = number of patient type k admitted to hospital i at time t for scenario s
- $z_{i,k,t}(s)$ = number of patient type k discharged from hospital i at time t for scenario s
- $u_{i,k,t}(s)$ = number of unmet non-emergency demand of patient type k in hospital i at time t for scenario s

- wi,k,t(s) = number of waiting non-emergency patient type k in hospital i at time t for scenario s
- fi,j,t(s) = number of non-emergency patients (k=2) sent from hospital i to hospital j at time t for scenario s

Recourse Costs

The recourse costs used in this model are given in the below table:

Cost of Patient Transfer	50
Waiting Cost	[800, 300]
Unmet Demand Cost	[2000, 1000]

Objective Function: The goal is to minimize costs, considering the probabilities Ps of various scenarios s.

$$\text{Min} \sum_i \alpha_i x_i + \sum_s p^s \left(\sum_k \sum_t \sum_i \beta_i y^s_{i,k,t} + \sum_k \sum_t \delta_k w^s_{i,k,t} + \sum_i \sum_j \gamma_{i,j} f^s_{i,j,t} + \sum_k \sum_t \eta_k u^s_{i,k,t} \right)$$

Subject To:

Balance Constraints for Emergency and Non-Emergency Patients: These constraints ensure the flow of emergency and non-emergency patients through the system is consistent with the rate of arrivals and departures.

$$w^s_{i,1,t} + y^s_{i,1,t} + u^s_{i,1,t} = w^s_{i,1,t-1} + \lambda^s_{i,1,t}, \quad \forall i \in H, \forall t > 1, k = 1$$

$$w^s_{i,2,1} + y^s_{i,2,1} + u^s_{i,2,1} + \sum_{j \neq i} f^s_{j,i,1} = \lambda^s_{i,2,1}, \quad \forall i \in H, t = 1, k = 2$$

Initial Condition Constraints for Emergency and Non-Emergency Patients: These conditions define the state of the ED at the start of the optimization period.

$$w_{i,1,1}^s + y_{i,1,1}^s + u_{i,1,1}^s = \lambda_{i,1,1}^s, \quad \forall i \in H, t = 1, k = 1$$

$$w_{i,2,1}^s + y_{i,2,1}^s + u_{i,2,1}^s + \sum_{j \neq i} f_{j,i,1}^s = \lambda_{i,2,1}^s, \quad \forall i \in H, t = 1, k = 2$$

Capacity Constraints: These constraints ensure that the number of patients admitted for treatment at any given time does not exceed the ED's capacity.

$$y_{i,k,t}^s \leq C_{i,t}^s, \quad \forall i \in H, \forall k \in \{1,2\}, \forall t \in T, \forall s \in S$$

Hospital Capacity Limits Over Time: These limits control the availability of hospital resources throughout the optimization period. By dynamically updating capacity based on patient admissions and discharges, the model maintains a balance between demand and resource availability, ensuring efficient operation over time.

$$\begin{array}{l} C_{i,k}^s = x_i, \quad \text{for } t = 1, \quad \forall i \in H, \forall k \in \{1,2\} \\ C_{i,t}^s = C_{i,t-1}^s - \sum_{k \in \{1,2\}} y_{i,k,t-1}^s + \sum_{k \in \{1,2\}} z_{i,k,t-1}^s, \quad \forall i \in H, \forall t > 1, \forall s \in S \end{array}$$

Service and Discharge Constraints: These are critical for managing the flow of patients into and out of the hospital.

$$z_{i,k,t}^s \leq y_{i,k,t-1}^s, \quad \forall i \in H, \forall k \in \{1,2\}, \forall t > 1, \forall s \in S$$

Budget Limitation: A financial constraint is imposed to ensure that the cost of capacity expansion stays within the predetermined budget for each hospital.

$$\sum_{i \in H} \alpha_i x_i \leq B$$

Transfer Restrictions: These restrictions govern the movement of patients between hospitals, ensuring that transfers do not exceed the predefined limits set by hospital policy or the available capacity at the destination hospital.

$$f^s_{i,j,t} \leq \min(C^s_{j,t+d_{i,j}}, l_i), \quad \forall i \in H, \forall j \in H \setminus \{i\}, \forall t > 1, \forall s \in S$$
$$f^s_{i,j,t} \leq C^s_{j,t+d_{i,j}}, \quad \forall i \in H, \forall j \in H \setminus \{i\}, \forall t > 1, \forall s \in S$$
$$f^s_{i,j,t} \leq l_i, \quad \forall i \in H, \forall j \in H \setminus \{i\}, \forall t > 1, \forall s \in S$$

Overtime Calculation: Overtime is calculated to meet the demand that exceeds regular operational capacity, especially during peak periods.

$$o^s_i = \sum_{t \in T} y^s_{i,k,t}(t + \mu_i - T), \quad \forall i \in H, \forall s \in S, \text{ where } t + \mu_i > T$$

Integrality Constraints:

$$x_i, y^s_{i,k,t}, z^s_{i,k,t}, u^s_{i,k,t}, w^s_{i,k,t}, o^s_i \in \mathbb{Z}^+, \quad \forall i \in H, \forall k \in \{1,2\}, \forall t \in T, \forall s \in S$$

Non-anticipativity constraints (NAC):

Stage 1 NAC Constraints: At the initial stage, for every element i in the index set I, all eight decision variables $Ci11$, $Ci12$,...,$Ci18$ are set to be equal. This enforces the non-anticipativity constraint that at the first stage, before any new information has been revealed, all decisions across all scenarios indexed by i are the same.

$$C^1_{i1} = C^2_{i1} = C^3_{i1} = C^4_{i1} = C^5_{i1} = C^6_{i1} = C^7_{i1} = C^8_{i1}, \quad \forall i \in I$$

Stage 2 NAC Constraints: As the second stage begins, the decision variables are grouped into two sets of four: $Ci21$, $Ci22$, $Ci23$, $Ci24$ are equal for each i, and $Ci25$, $Ci26$, $Ci27$, $Ci28$ are equal for each i. This represents the scenario where some new information has possibly been revealed, leading to a bifurcation of decision paths. Nonetheless, decisions within each branch remain non-anticipative of future information.

$$C_{i2}^1 = C_{i2}^2 = C_{i2}^3 = C_{i2}^4, \quad \forall i \in I$$
$$C_{i2}^5 = C_{i2}^6 = C_{i2}^7 = C_{i2}^8, \quad \forall i \in I$$

Stage 3 NAC Constraints: At this stage, the non-anticipativity constraints are further refined. Each pair of decision variables within the same subset are set equal: $Ci31$=$Ci32$, $Ci33$=$Ci34$, $Ci35$=$Ci36$, and $Ci37$=$Ci38$ for all i. This delineation reflects an additional layer of scenario splitting due to new information being available, requiring decisions to remain consistent within each newly defined scenario.

$$C_{i3}^1 = C_{i3}^2, \quad \forall i \in I$$
$$C_{i3}^3 = C_{i3}^4, \quad \forall i \in I$$
$$C_{i3}^5 = C_{i3}^6, \quad \forall i \in I$$
$$C_{i3}^7 = C_{i3}^8, \quad \forall i \in I$$

Using CPLEX solver to solve multi-stage stochastic problem with recourse:

```
/*********************************************
 * OPL 22.1.1.0 Model
 * Author: Reyhane Alimohammadi and Donya Razinejad
 * Creation Date: Apr 16, 2024 at 3:16:09 PM
 *********************************************/
range I = 1..3;  // Set I is {1, 2, 3, 4}
range K = 1..2;  // Set K is {1, 2}
range T = 1..4;  // Set T is {1, 2, 3, 4}
range S = 1..8;
int waiting_cost[K] = ...;
int unmet_demand_cost[K] = ...;
int B = ...;
int l = ...;
int d[S][T] = ...;

// Decision variables
dvar int+ x[I];
dvar int+ w[I][K][T][S];
dvar int+ y[I][K][T][S];
dvar int+ f[I][I][T][S];
dvar int+ u[I][K][T][S];
dvar int+ z[I][K][T][S];
dvar int+ c[I][T][S];

/// Objective function
minimize 1/8 *
    sum(s in S) 0.125 * (
        sum(i in I) (200 * x[i]) +
        sum(i in I, k in K, t in T) (1 * y[i][k][t][s]) +
        sum(i in I, j in I, t in T : i != j) (50 * f[i][j][t][s]) +
        sum(i in I, k in K, t in T) (waiting_cost[k] * w[i][k][t][s]) +
        sum(i in I, k in K, t in T) (unmet_demand_cost[k] * u[i][k][t][s])
    );

// Constraints
subject to
{
    forall (s in S) {
        forall (i in I, t in 2..4) {
```

```
            w[i][1][t][s] + y[i][1][t][s] + u[i][1][t][s] == w[i][1][t-1][s] + 0.3 * d[s][t];
            sum(j in I : j != i) (f[i][j][t][s]) + y[i][2][t][s] + u[i][2][t][s] == w[i][2][t-1][s] +
             0.7 * d[s][t] + sum(j in I : j != i) (f[j][i][t-1][s]);
        }
        forall (i in I) {
            w[i][1][1][s] + y[i][1][1][s] + u[i][1][1][s] == 0.3 * d[s][1];
            sum(j in I : j != i) (f[i][j][1][s]) + w[i][2][1][s] + y[i][2][1][s] + u[i][2][1][s] ==
             0.7 * d[s][1];
        }
        forall (i in I, t in T) {
            sum(k in K) (y[i][k][t][s]) <= c[i][t][s];
        }
        forall (i in I, t in 2..4) {
            c[i][t][s] <= x[i];
        }
        forall (i in I) {
            c[i][1][s] == x[i];
        }
        forall (i in I, t in 2..4) {
            c[i][t][s] == c[i][t-1][s] - sum(k in K) (y[i][k][t-1][s]) + sum(k in K) (z[i][k][t-1][s]);
        }
        forall (i in I, t in 2..4) {
            forall (k in K) {
                z[i][k][t][s] <= y[i][k][t-1][s];
            }
        }
        forall (i in I, k in K) {
            z[i][k][1][s] == 0;
        }
        sum(i in I) (200 * x[i]) <= B;
        forall (i in I, j in I, t in T, s in S) {
            f[i][j][t][s] <= 1;
        }
```

```

        //NAC

        forall (i in I) {
          c[i][2][1] == c[i][2][2] == c[i][2][3] == c[i][2][4];
          c[i][2][5] == c[i][2][6] == c[i][2][7] == c[i][2][8];
          c[i][3][1] == c[i][3][2];
          c[i][3][3] == c[i][3][4];
          c[i][3][5] == c[i][3][6];
          c[i][3][7] == c[i][3][8];
        }
}}
```

Cplex .dat file

```
 * OPL 22.1.1.0 Data
 * Author: Reyhane Alimohammadi and Donya Razinejad
 * Creation Date: Apr 16, 2024 at 3:16:09 PM
 *********************************************/

waiting_cost = [800, 300];
unmet_demand_cost = [2000, 1000];
l = 7;
B = 13000;
d = [
    [20, 20, 20, 20],
    [20, 20, 20, 40],
    [20, 20, 40, 20],
    [20, 20, 40, 40],
    [20, 40, 20, 20],
    [20, 40, 20, 40],
    [20, 40, 40, 20],
    [20, 40, 40, 40]
];
```

Cplex Code Solution Multi-Stage Model:

```
// solution (optimal) with objective 181888.5
// Quality Incumbent solution:
// MILP objective                                1.8188850000e+05
// MILP solution norm |x| (Total, Max)           4.59800e+03  2.80000e+01
// MILP solution error (Ax=b) (Total, Max)       0.00000e+00  0.00000e+00
// MILP x bound error (Total, Max)               0.00000e+00  0.00000e+00
// MILP x integrality error (Total, Max)         0.00000e+00  0.00000e+00
// MILP slack bound error (Total, Max)           0.00000e+00  0.00000e+00
// MILP indicator slack bound error (Total, Max) 0.00000e+00  0.00000e+00
//

x = [21
         21 21];
y = [[[[6 6 6 6 6 6 6 6]
                    [6 6 1 1 12 12 1 1]
                    [6 6 17 17 6 6 20 20]
                    [0 0 0 0 0 0 0 0]]
                [[9 9 14 14 3 3 14 14]
                    [0 0 0 0 0 0 0 0]
                    [9 9 3 0 3 1 0 0]
                    [6 6 1 4 12 14 1 1]]]
            [[[6 6 6 6 6 6 6 6]
                    [6 6 1 1 12 12 1 1]
                    [6 6 17 17 6 6 20 20]
                    [0 0 0 0 0 0 0 0]]
                [[9 9 14 14 3 3 14 14]
                    [0 0 0 0 0 0 0 0]
                    [9 9 3 0 3 1 0 0]
                    [6 6 1 4 12 14 1 1]]]
            [[[6 6 6 6 6 6 6 6]
                    [6 6 1 1 12 12 1 1]
                    [6 6 17 17 6 6 20 20]
                    [0 0 0 0 0 0 0 0]]
                [[9 9 14 14 3 3 14 14]
                    [0 0 0 0 0 0 0 0]
                    [9 1 3 0 3 1 0 0]
                    [6 14 1 4 12 14 1 1]]]];
```

```
f = [[[[0 0 0 0 0 0 0 0]
       [0 0 0 0 0 0 0 0]
       [0 0 0 0 0 0 0 0]
       [0 0 0 0 0 0 0 0]]
      [[0 0 0 0 0 0 0 0]
       [0 0 0 0 3 0 0 0]
       [6 0 1 0 6 0 0 0]
       [7 7 7 7 7 7 7 7]]
      [[0 0 0 0 0 0 0 0]
       [0 0 0 0 0 0 0 0]
       [0 0 0 0 5 0 1 0]
       [7 7 7 7 7 7 7 7]]]
     [[[0 0 0 0 0 0 0 0]
       [0 0 0 0 0 0 0 0]
       [0 0 0 0 7 0 1 0]
       [7 7 7 7 7 7 7 7]]
      [[0 0 0 0 0 0 0 0]
       [0 0 0 0 0 0 0 0]
       [0 0 0 0 0 0 0 0]
       [0 0 0 0 0 0 0 0]]
      [[0 0 0 0 0 0 0 0]
       [1 0 0 0 0 0 0 0]
       [6 0 1 0 7 0 0 0]
       [7 7 7 7 7 7 7 7]]]
     [[[0 0 0 0 0 0 0 0]
       [1 0 0 0 0 0 0 0]
       [6 0 1 0 5 0 0 0]
       [7 7 7 7 7 7 7 7]]
      [[0 0 0 0 0 0 0 0]
       [1 0 0 0 0 0 0 0]
       [0 0 0 0 6 0 1 0]
       [7 7 7 7 7 7 7 7]]
      [[0 0 0 0 0 0 0 0]
       [0 0 0 0 0 0 0 0]
       [0 0 0 0 0 0 0 0]
       [0 0 0 0 0 0 0 0]]]];
```

```
w = [[[[0 0 0 0 0 0 0 0]
          [0 0 5 5 0 0 8 8]
          [0 0 0 0 0 0 0 0]
          [6 12 6 12 6 12 6 12]]
        [[0 0 0 0 0 0 0 0]
          [0 0 0 0 0 0 0 0]
          [0 0 0 0 0 0 0 0]
          [0 0 0 0 0 0 0 0]]]
      [[[0 0 0 0 0 0 0 0]
          [0 0 5 5 0 0 8 8]
          [0 0 0 0 0 0 0 0]
          [6 12 6 12 6 12 6 12]]
        [[0 0 0 0 0 0 0 0]
          [0 0 0 0 0 0 0 0]
          [0 0 0 0 0 0 0 0]
          [0 0 0 0 0 0 0 0]]]
      [[[0 0 0 0 0 0 0 0]
          [0 0 5 5 0 0 8 8]
          [0 0 0 0 0 0 0 0]
          [6 12 6 12 6 12 6 12]]
        [[0 0 0 0 0 0 0 0]
          [0 0 0 0 0 0 0 0]
          [0 0 0 0 0 0 0 0]
          [0 0 0 0 0 0 0 0]]]];
```

```
u = [[[[0 0 0 0 0 0 0 0]
        [0 0 0 0 0 0 3 3]
        [0 0 0 0 0 0 0 0]
        [0 0 0 0 0 0 0 0]]
      [[5 5 0 0 11 11 0 0]
        [14 14 14 14 25 28 28 28]
        [0 5 24 28 0 13 27 28]
        [0 8 0 10 0 0 0 13]]]
    [[[0 0 0 0 0 0 0 0]
        [0 0 0 0 0 0 3 3]
        [0 0 0 0 0 0 0 0]
        [0 0 0 0 0 0 0 0]]
      [[5 5 0 0 11 11 0 0]
        [13 14 14 14 28 28 28 28]
        [0 5 24 28 0 13 27 28]
        [0 8 0 10 0 0 0 13]]]
    [[[0 0 0 0 0 0 0 0]
        [0 0 0 0 0 0 3 3]
        [0 0 0 0 0 0 0 0]
        [0 0 0 0 0 0 0 0]]
      [[5 5 0 0 11 11 0 0]
        [12 14 14 14 28 28 28 28]
        [0 13 24 28 0 13 27 28]
        [0 0 0 10 0 0 0 13]]]];
c = [[[21 21 21 21 21 21 21 21]
      [6 6 1 1 12 12 1 1]
      [15 15 20 20 9 9 20 20]
      [6 6 1 4 12 14 1 1]]
    [[21 21 21 21 21 21 21 21]
      [6 6 1 1 12 12 1 1]
      [15 15 20 20 9 9 20 20]
      [6 6 1 4 12 14 1 1]]
    [[21 21 21 21 21 21 21 21]
      [6 6 1 1 12 12 1 1]
      [15 15 20 20 9 9 20 20]
      [6 14 1 4 12 14 1 1]]];
```

```
z = [[[[0 0 0 0 0 0 0 0]
                  [6 6 6 6 6 6 6 6]
                  [6 6 1 1 12 12 1 1]
                  [0 0 0 0 0 0 0 0]]
              [[0 0 0 0 0 0 0 0]
                  [9 9 14 14 3 3 14 14]
                  [0 0 0 0 0 0 0 0]
                  [0 0 0 0 0 0 0 0]]]
          [[[0 0 0 0 0 0 0 0]
                  [6 6 6 6 6 6 6 6]
                  [6 6 1 1 12 12 1 1]
                  [0 0 0 0 0 0 0 0]]
              [[0 0 0 0 0 0 0 0]
                  [9 9 14 14 3 3 14 14]
                  [0 0 0 0 0 0 0 0]
                  [0 0 0 0 0 0 0 0]]]
          [[[0 0 0 0 0 0 0 0]
                  [6 6 6 6 6 6 6 6]
                  [6 6 1 1 12 12 1 1]
                  [0 0 0 0 0 0 0 0]]
              [[0 0 0 0 0 0 0 0]
                  [9 9 14 14 3 3 14 14]
                  [0 0 0 0 0 0 0 0]
                  [0 0 0 0 0 0 0 0]]]];
```

Results

To solve the stochastic model, we used CPLEX software with 35 seconds run time. The results of the stochastic model for all scenarios are given below:

Multi-Stage Stochastic model scenarios	Total required capacity	Total accepted Patient	Total waiting	Total unmet Demand	Total Patient transfer
Scenario 1	144	126	18	54	63
Scenario 2	152	126	36	96	42
Scenario 3	129	126	33	114	45
Scenario 4	138	126	51	156	42
Scenario 5	162	126	18	114	81
Scenario 6	168	126	36	156	42
Scenario 7	129	126	42	174	45
Scenario 8	129	126	60	216	42

Expected Cost of the stochastic solution = 181,888.5

We also solved the Scenario sub-model problems, as well as an Expected Value problem (EV) in order to calculate values of Expected Value of Perfect Information (EVPI) and Value of Stochastic Solution (VSS).

Based on course literature, we have:

$$WS = E_{\omega}\left[\min_{x} z(x,\omega)\right] = E_{\omega}\left[z(\bar{x}(\omega),\omega)\right]$$

$$EV = \min_{x} z(x,\bar{\omega})$$

$$RP = \min_x E_\omega\left[z(x,\omega)\right]$$

$$EVPI=RP-WS$$

$$EEV = E_\omega\left[z(\overline{x}(\overline{\omega}),\omega)\right]$$

$$VSS=EEV-RP.$$

According to these notations, we have:

RP	181,888.5
WS	168,505
EEV	181,888.5
EVPI	13,383.6
VSS	0

Recourse Problem Solution (RP)

The Recourse Problem Solution (RP), which stands for the expected total cost of the solution to the stochastic optimization problem, is calculated at 181,888.5. This figure represents the cost when our model accounts for uncertainty and incorporates reactive strategies, allowing us to adjust our decisions as new information becomes available. The RP embodies the real-world scenario where decisions must be made in the face of uncertainty, with subsequent actions taken as outcomes unfold.

Wait-and-See (WS)

Conversely, the Wait-and-See (WS) value, which is computed at 168,505, represents the hypothetical scenario in which we possess perfect information upfront for each uncertain event. This optimal cost is invariably lower than the RP because it reflects the ideal condition where decisions are made with complete foresight, thus eliminating the cost implications of uncertainty.

Expected Value of Perfect Information (EVPI)

The Expected Value of Perfect Information (EVPI) is a critical metric that quantifies the monetary benefit of obtaining perfect information about uncertain events prior to decision-making. With an EVPI of 13,383.6, it denotes the potential average savings should we have the ability to foresee the uncertain outcomes. Essentially, this value highlights the maximum amount we would theoretically expend to gain such perfect insights.

Value of Stochastic Solution (VSS)

Of particular interest in our analysis is the Value of Stochastic Solution (VSS), which, in this assessment, stands at 0. This result indicates that the adaptive decisions enabled by our stochastic model do not offer a financial advantage over a deterministic approach that uses average values to represent uncertainty. In other words, the stochastic solution does not outperform the expected outcome derived from a model that does not account for uncertainty. This outcome could arise from a variety of factors, including a potential non-sensitivity of the model to the uncertain parameters or the possibility that the costs associated with implementing adaptive strategies negate any expected benefits.

Implications

The analysis of EVPI and VSS has profound implications for our decision-making process under uncertainty. The EVPI underscores the inherent value of information and the potential cost savings that could be achieved with improved forecasting or information-gathering mechanisms. However, the VSS suggests that our current model may require refinement to effectively leverage the information and adaptability that a stochastic framework offers. It prompts us to further investigate whether alternative strategies or model enhancements could yield better performance and result in a positive VSS, thereby truly capitalizing on the benefits of stochastic optimization.

Conclusion

Our study has developed a multi-stage stochastic model tailored for managing patient flow in emergency departments. By mapping patient arrival rates to resource allocation, the model is designed to adapt to the fluctuating nature of ED demand, as depicted in the scenario trees for patient arrivals. These trees form the structural foundation of our model, where each node signifies a decision point, and the diverging branches represent the range of possible patient inflows.

However, we identified an oversight in the definition of unmet demand variable, where emergency patients should not have been considered, and only non-emergency shortage in demand should be allowed. We tried to address this issue and in the same time stay loyal to the original formulation of the problem in the article by increasing the cost of unmet demand for emergency patients in comparison to that of non-emergency patients.

In this report, the values of EVPI and VSS are measured as key metrics to the performance of stochastic model and as a decision-making criterion for investments in data collection, forecasting models, etc. With the Value of Stochastic Solution (VSS) currently stands at zero, it indicates the stochastic model is not better than traditional methods with current assumptions and limitations. The Expected Value of Perfect Information (EVPI) suggests there is room for improvement by refining how we handle information and uncertainty. As we move forward, the recalibrated model will better reflect the operational realities of EDs, ultimately aiming to reduce wait times and improve the overall quality of patient care.

Reference

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- Birge, J. R., & Louveaux, F. (1997). Introduction to Stochastic Programming. Springer-Verlag.