Linear Algebra II (MATH1049) — Coursework Sheet 1 — 2020/21

Submit a single pdf with scans of your work to Blackboard by Monday, 15 February 2021, 17:00.

Exercise 1

Let G and H be groups with binary operations $\boxplus$ and $\odot$, respectively. We define a binary operation $*$ on the cartesian product $G \times H$ by

$$(a, b) * (a', b') := (a \boxplus a', b \odot b') \quad (\text{for } a, a' \in G \text{ and } b, b' \in H).$$

Show that $G \times H$ together with this operation is a group.

Exercise 2

For $a, b \in \mathbb{R}$ we define $a * b := a + b + ab \in \mathbb{R}$. Furthermore let $G := \mathbb{R} \backslash \{-1\}$.

(a) Show that $a * b \in G$ for all $a, b \in G$.

(b) Show that G together with the binary operation $G \times G \to G$, $(a, b) \mapsto a * b$, is a group.

Exercise 3

Let $G = \{s, t, u, v\}$ be a group with $s * u = u$ and $t * t = v$. Determine the group table of G. (*There is only one way of completing the group table for G. Give a reason for each step.*)

Exercise 4

Write down the group tables for the groups C_4 and $C_2 \times C_2$ (cf. Exercise 1). For every element a in C_4 and $C_2 \times C_2$ determine the smallest positive integer m such that ma equals the identity element.

Extra question (not assessed — no need to submit)

Let G be a group whose binary operation is written additively, i.e. $G \times G \to G$, $(a, b) \mapsto a + b$. Show that $m(na) = (mn)a$ for all $a \in G$ and $m, n \in \mathbb{Z}$. (*Hint: You need to distinguish up to 9 cases.*) Write down the other two exponential laws in additive notation as well. (*Formulate these laws as complete mathematical statements including all quantifiers. No proofs are required.*)