Linear Algebra II (MATH1049) — Coursework Sheet 5 — 2020/21

Submit a single pdf with scans of your work to Blackboard by Monday, 12 April 2021, 17:00.

Exercise 1

Which of the following are spanning sets for the vector space $\mathbb{P}_2$ of polynomial functions of degree at most 2? (*Give reasons for your answers.*)

(a) $\frac{1}{2}$, t^2+t, t^2-1

(b) 1, $2t$, t^2, $3t^2+5$

(c) $t+1$, t^2+t

Exercise 2

Determine whether the following are linearly independent sets of vectors in the vector space $\mathbb{R}^{\mathbb{R}}$ of all functions from $\mathbb{R}$ to $\mathbb{R}$. (*Give reasons for your answers.*)

(a) $1+t$, $1+t+t^2$, $1+t+t^2+t^3$, $1+t+t^2+t^4$

(b) $\sin, \cos^2, \sin^3$

(c) $1, \sin^2, \cos^2$

(*Here for example* $\sin^2$ *denotes the function* $\mathbb{R} \to \mathbb{R}, s \mapsto (\sin(s))^2$.)

Exercise 3

Find a basis of the null space $N(A) \subset \mathbb{R}^5$ of the matrix

$$A = \begin{pmatrix} 1 & -3 & 3 & -1 & -1 \\ -2 & 6 & -1 & -3 & -8 \\ 3 & -9 & 10 & -4 & -5 \end{pmatrix} \in M_{3\times 5}(\mathbb{R})$$

and hence determine its dimension.

Exercise 4

(a) Determine whether the following (2×2)-matrices form a basis of the vector space $M_{2\times 2}(\mathbb{R})$ of all (2×2)-matrices over $\mathbb{R}$:

$$A_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad A_2 = \begin{pmatrix} 2 & 1 \\ 0 & 0 \end{pmatrix}, \quad A_3 = \begin{pmatrix} 3 & 2 \\ 1 & 0 \end{pmatrix}, \quad A_4 = \begin{pmatrix} 4 & 3 \\ 2 & 1 \end{pmatrix}.$$

(b) Find a basis of the subspace $W := \{A \in M_{2\times 2}(\mathbb{R}) \mid \operatorname{trace}(A) = 0\}$ of the vector space $M_{2\times 2}(\mathbb{R})$ and hence determine the dimension of W (see Exercise 1 on Coursework Sheet 4 for the definition of trace).

Extra exercise (not marked, do not submit)

We view $\mathbb{C}^2 = \left\{ \begin{pmatrix} w \\ z \end{pmatrix} : w, z \in \mathbb{C} \right\}$ as a vector space over $\mathbb{C}$, $\mathbb{R}$ and $\mathbb{Q}$ (cf. Example 3.16 (b)). Let $\mathbf{x}_1 := \begin{pmatrix} i \\ 0 \end{pmatrix}$, $\mathbf{x}_2 := \begin{pmatrix} \sqrt{2} \\ \sqrt{5} \end{pmatrix}$, $\mathbf{x}_3 := \begin{pmatrix} 0 \\ 1 \end{pmatrix}$, $\mathbf{x}_4 := \begin{pmatrix} i\sqrt{3} \\ \sqrt{3} \end{pmatrix}$, $\mathbf{x}_5 := \begin{pmatrix} 1 \\ 3 \end{pmatrix} \in \mathbb{C}^2$. Determine $\dim_F(\operatorname{Span}_F(\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \mathbf{x}_4, \mathbf{x}_5))$ for $F = \mathbb{C}$, $\mathbb{R}$ and $\mathbb{Q}$.