ECE 365 **Fall 2020**

September 29, 2020

QUIZ 2: 8:00–8:40 PM (*60 points total*)

Note: This is a **closed-book** exam. Please write your answers on clean sheets of letter paper using a pen or pencil, and upload the scanned pages on Gradescope, indicating the page numbers corresponding to your answers when you upload on Gradescope.

1. **[Handling Data (5 points)]**
   State whether the following statement is true or false, and explain your reasoning:

   In the model selection stage, the model with the smallest training error is chosen for final model assessment.

2. **[Vector Quantization (10 points)]**
   Describe in words how clustering can be used for vector quantization based lossy compression of images.

3. **[Regression (10 points)]**

   (a) Explain how linear regression can be used for nonlinear prediction from the data..

   (b) Explain the purpose of regularization in the linear regression optimization.

4. **[Eigen-decomposition (10 points)]**
   Find the eigenvalues and orthonormal eigenvectors of the matrix

$$A = \begin{bmatrix} 8 & -2 \\ -2 & 8 \end{bmatrix}.$$

   Hint: First show that the eigenvalues are $\lambda_1 = 10$ and $\lambda_2 = 6$.

5. **[SVD (10 points)]**
   Suppose the matrix A can be written as

$$A = \begin{bmatrix} 2 \\ 0 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 2 \end{bmatrix}$$

   (a) What are the left singular vectors of A?

   (b) What are the right singular vectors of A?

   (c) What are the singular values of A?

   Hint: Recall that the SVD of a $m \times n$ matrix can be written as

$$A = \sum_{i=1}^{\min\{m,n\}} \sigma_i \underline{u}_i \underline{v}_i^\top,$$

   where the $\underline{u}_i$'s and $\underline{v}_i$'s are the orthonormal left and right eigenvectors, respectively.

6. **[SVD and Image Compression (10 points)]**
   Consider an image as an $m \times n$ matrix of pixel values. Explain how a rank-k approximation of the SVD can be used to make the number of stored values less than mn.

7. **[Choosing Number of Principal Components in PCA (5 points)]**
   Explain how we may use the cost function

$$J(k) = \frac{1}{N} \sum_{i=1}^{N} \|\underline{x}_i - W_k^\top W_k \underline{x}_i\|^2$$

   to choose the value of k for PCA.