4.4.2(c):
Prove that for any positive integer n,
$\Sigma_{j=1}^{n} j^3 = (\frac{n(n+1)}{2})^2$
case 1: n = 1
$\Sigma_{j=1}^{n} j^3 = 1^3 = 1$ and $(\frac{1(1+1)}{2})^2 = (\frac{1 \cdot 2}{2})^2 = 1^2 = 1$
case 2: n = k+1
$\Sigma_{j=1}^{n} j^3 = (\frac{n(n+1)}{2})^2$
$\Sigma_{j=1}^{k+1} j^3 = (\frac{(k+1)(k+2)}{2})^2$
$\Sigma_{j=1}^{k+1} j^3 = \Sigma_{j=1}^{k} j^3 + (k+1)^3$
$\Sigma_{j=1}^{k+1} j^3 = (\frac{k(k+1)}{2})^2 + (k+1)^3$
$(\frac{k(k+1)}{2})^2 + (k+1)^3 = \frac{k^2(k+1)^2}{4} + (k+1)^3$
$= (k+1)^2(\frac{k^2}{4})(\frac{4k}{4})(\frac{4}{4}) = (k+1)^2(\frac{k^2+4k+4}{4})$
$= (k+1)^2(\frac{(k+2)^2}{4}) = \frac{(k+1)^2(k+2)^2}{4}$
$\Sigma_{j=1}^{k+1} j^3 = (\frac{(k+1)(k+2)}{2})^2$

4.5.1(e):
Prove that for any positive integer n, 2 evenly divides n^2 - 5n + 2
Case1: n is even
n ≡ 0 (mod 2)
1. $n^2 \equiv 0^2 \equiv 0\ (mod\ 2)$
2. 5n ≡ 5 · 0 ≡ 0 (mod 2)
3. $n^2 - 5n + 2 \equiv 0 - 0 + 2 \equiv 2 \equiv 0\ (mod\ 2)$
making $n^2 - 5n + 2 =$ even when n is even

Case2: n is odd
n ≡ 1 (mod 2)
1. $n^2 \equiv 1^2 \equiv 1 (mod 2)$
2. 5n ≡ 5 · 1 ≡ 5 ≡ 1 (mod 2)
3. $n^2 - 5n + 2 \equiv 1 - 1 + 2 \equiv 2 \equiv 0\ (mod\ 2)$
making $n^2 - 5n + 2$ odd when n is odd

Therefore, for any positive integer n, n^2 - 5n + 2 is even, proving that 2 evenly divides n^2 - 5n + 2 is divisible by 2 for any positive integer n.

4.4.3(c):

Prove that for n $\geq$ 1,

$\Sigma_{j=1}^{n} \frac{1}{j^2} \leq 2$ - $\frac{1}{n}$

Consider f(x) = $\frac{1}{x^2}$

$\int_1^{n+1} \frac{1}{x^2} dx < \Sigma_{j=1}^{n} \frac{1}{j^2} < 1 + \int_1^{n} \frac{1}{x^2} dx$

$\int \frac{1}{x^2} dx =$ -$\frac{1}{x}$ + C

$\int_1^{n+1} \frac{1}{x^2}\, dx = \left[-\frac{1}{x}\right]_1^{n+1} = -\frac{1}{n+1} + 1 = 1 - \frac{1}{n+1}$

$\int_1^{n} \frac{1}{x^2}\, dx = \left[-\frac{1}{x}\right]_1^{n} = -\frac{1}{n} + 1 = 1 - \frac{1}{n}$

From the integrals, we now get

$1 - \frac{1}{n+1} < \sum_{j=1}^{n} \frac{1}{j^2} < 1 + \left(1 - \frac{1}{n}\right) = 2 - \frac{1}{n}$

thus for all n $\geq$ 1:

$\sum_{j=1}^{n} \frac{1}{j^2} < 2 - \frac{1}{n}$