Contents

Course description

Chapter 1

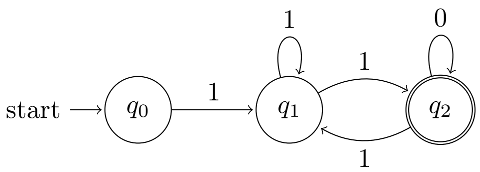
Exercises 2019

Week 1

page 84, question 1.7

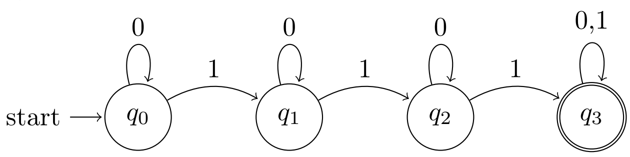
a

w|w begins with a 1 and ends with a 0



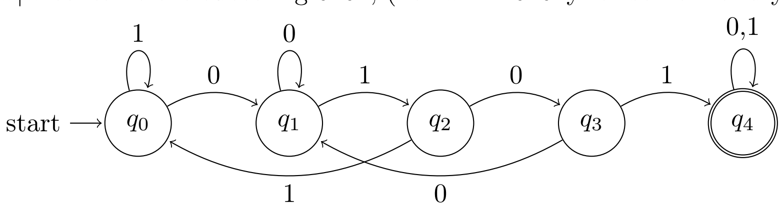
b

w|w contains at least 3 1's



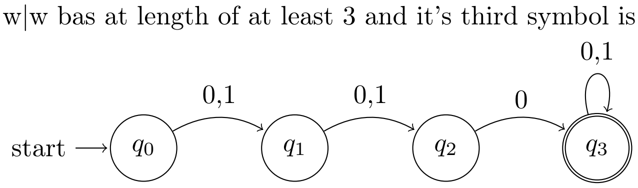
c

w|w contains the substring 0101, (i.e. w = x0101y for some x and y)



d

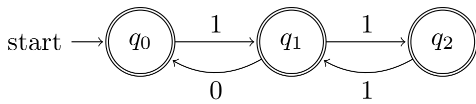
w|w bas at length of at least 3 and it's third symbol is 0



f

w|w doesn't contain the substring 001

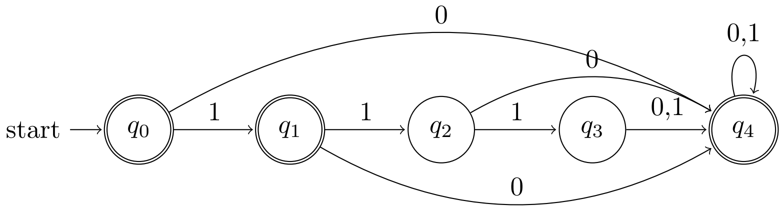
Accepts any string that doesn't contain the substring 001, and loops in rejecting state if this state is



found, was unsure if the looping was needed but included it if it was the case.

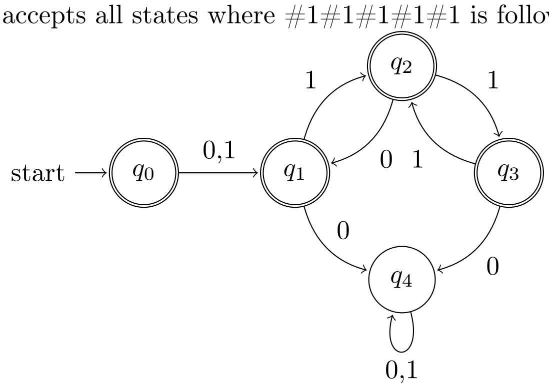
h

w|w is any string except 11 and 111
accepts any string that isn't 11, 111 including the empty string ϵ



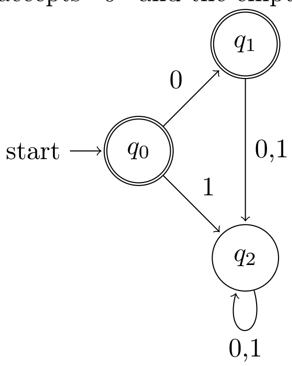
i

w|w every odd position of w is a 1
accepts all states where #1#1#1#1#1 is followed also accepts the empty string ϵ



k

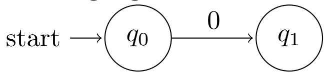
w|w is only the empty string and 0
accepts "0" and the empty string ϵ



page 84, question 1.7

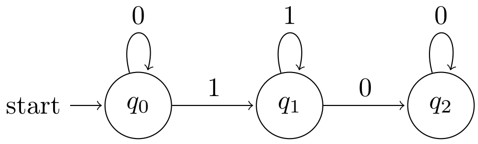
d

The language 0 with two states,



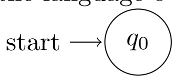
e

the language 0*1*0* with 3 states



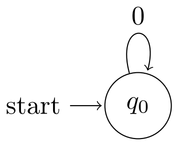
g

the language ϵ with 1 state



h

The language 0* with 1 state



Solve the following problem,

A man is travelling with a wolf (w) and a goat (g). He also brings along a nice big cabbage (c). He encounters a small river which he must cross to continue his travel. Fortunately, there is a small boat at the shore which he can use. However, the boat is so small that the man cannot bring more than himself and exactly one more item along (from w, g, c). The man knows that if left alone with the goat, the wolf will surely eat it and the goat if left alone with the cabbage will also surely eat that. The man's task is hence to device a transportation scheme in which, at any time, at most one item from w, g, c is in the boat and the result is that they all crossed the river and can continue unharmed.

a

Describe a solution to the problem which satisfies the rules of the "game". You may use your answer to (b) to find a solution.

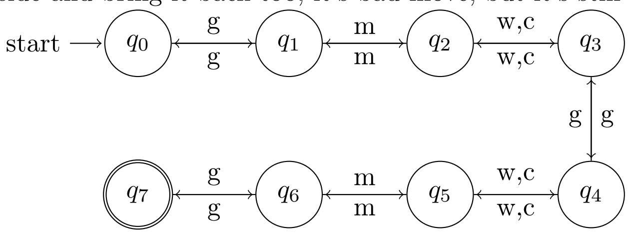
- First you carry the goat to the other side, and go back empty.
- The you ferry the wolf to the other side, and swap with the goat and bring the goat back.
- you then swap the goat with the cabbage and bring it to the other side.
- lastly you head back empty and bring the goat.
- you now have all the items on the other side of the river.

b

The string all of the valid moves are

$$\begin{aligned}(g(m(x(g(y(m(g)*)*)*)*)*)*)*\\ x,y=(w|c)\\ x\neq y\end{aligned}\tag{1.1}$$

This is due to the fact that it's legal moves in the game, like the man can bring the goat to the other side and bring it back too, it's bad move, but it's still a valid move.



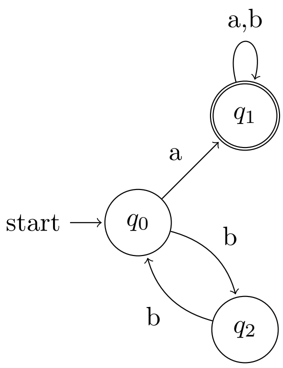
Week 2

page 86, question 1.16

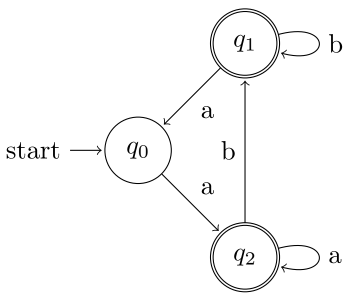
* is any number
! is one or more

a

it accepts (bb)*a!(a|b)*
the language 0*1*0* with 3 states



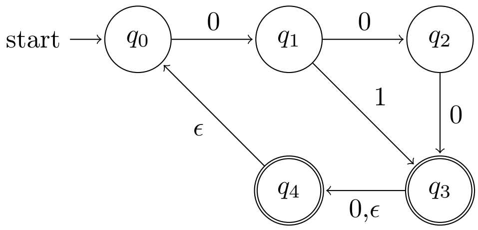
b



page 86, question 1.17

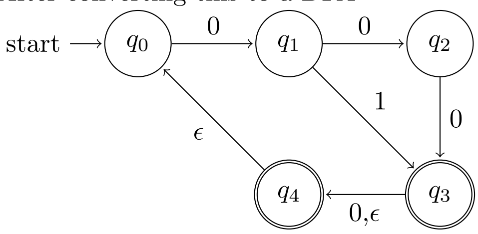
a

The first task is to make a NFA recognizing (01u001u010)*



b

After converting this to a DFA



page 86, question 1.18

Predefined terms

$$x = (0, 1)^* \tag{1.2}$$

$$y = (0|1) \tag{1.3}$$

a

$1x0$

b

$x1x1x1x$

c

$x0101x$

d

$yy0x$

e

$(0(yy)*)|1y(yy)^*)$

f

$0^*(1 + 10)^*$

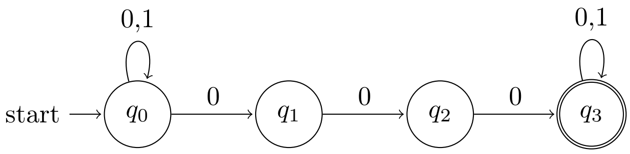
g

$yyyyy$

page 86, question 1.19 a

Convert the following regex to a NFA via lemma 1.55

$$(0 \cup 1)^{*}000(0 \cup 1)^{*} \tag{1.4}$$



page 86, question 1.20

For each of the following expressions give two strings that are and two that are not in the languages. assume that the alphabet is {a,b}

a

$$a^{*}b^{*} \tag{1.5}$$

member
aa, ab, aabb, aab, abbbb
not member
ba, abab,

b

$$a(ba)^{*}b \tag{1.6}$$

member
abab, ababababab,
not member
aaba baba

c

$$a^{*} \cup B^{*} \tag{1.7}$$

member
ab, aabb
not member
ba, aabba

d

$$(aaa)^{*} \tag{1.8}$$

member
aaa, aaaaaa
not member
a, aaaa

e

$$\sum^{*} a \sum^{*} b \sum^{*} a \sum^{*} \tag{1.9}$$

member
todo
not member
todo

f

$$aba \cup bab \tag{1.10}$$

member
todo
not member
todo

g

$$(\epsilon \cup a)b \tag{1.11}$$

member
ab, b
not member
abb, aab

h

$$(a \cup ba \cup bb) \sum^{*} \tag{1.12}$$

member
todo
not member
todo

page 86, question 1.21 b

The solution is

$$((a|b)(a, bb) * b(a)?)* \tag{1.13}$$

(a or b, followed by any number of (a,bb)* followed by a single b (as it matches uneven numbers of b and followed by 0 or 1 a*

page 86, question 1.29

a

For the task a we have the sting

$$0^n1^n2^n \tag{1.14}$$

the first contradiction is that when we have a string eg. 000111222 and we split it so we have the floowing, x = 000, y = 111, z = 222 and we pump y so we have the string xyyz this violates the first condition of the pumping lemma, as we'll have more 1s then 0s and 2s.
the string y only contains 1s which also causes a contradiction.
and the 3rd case if we have the string x = 00, y = 011 and z = 1222 where we'll get out of order letters so we'll again reach a contradiction.

b

For assignment b i find it a bit odd, i may misunderstand the exercise but w/e
We want to pump the language

$$a_2 = \{www|w \in \{a,b\}^*\} \tag{1.15}$$

But from my understanding is the * a kleen star? eg then one w and www are equivalent as {a,b} is all possibly strings in the library w. ? or is it understand such that w is equal to either a or b but any number of them eq $\{a|b\}^*$,
How i choose to intercept it for now is that w is equal to either the string a* or b* and if this is the case then some of the same argumentation as for task a is valid.
www can give us the string 111000111 and we can split it as follows. 111|1000|11111, This means that zyyx gives of a out of order 1 and we therefor get a contradiction wrt. to the pumping lemma.

page 88, question 1.30

The error There's a few things here, the first case from 1:73 fail right away as the string 0001111 is in the lang, the case of out of order fails as if we can get the string 00100111 ten we'll reach a contradiction but the case of

page 89, question 1.36

For the language w there exist a DFA that accepts if, for the revere language w^r the DFA which a reversed edges and the start state is now our accept state.

week 3

page 88, question 1.29(a),(b) and 1.30

Already done, see above.

page 89. question 1.35

ILONA As B is in bijection of A there for it's a regular.

page 91. question 1.51

a

This can be proved via the pumping lemma. where you can get out of order 1 and/or 0s and that causes a contradiction.

b

Same as a wrt. our of order.

c

This is this is either the unbounded string 0 or 1.

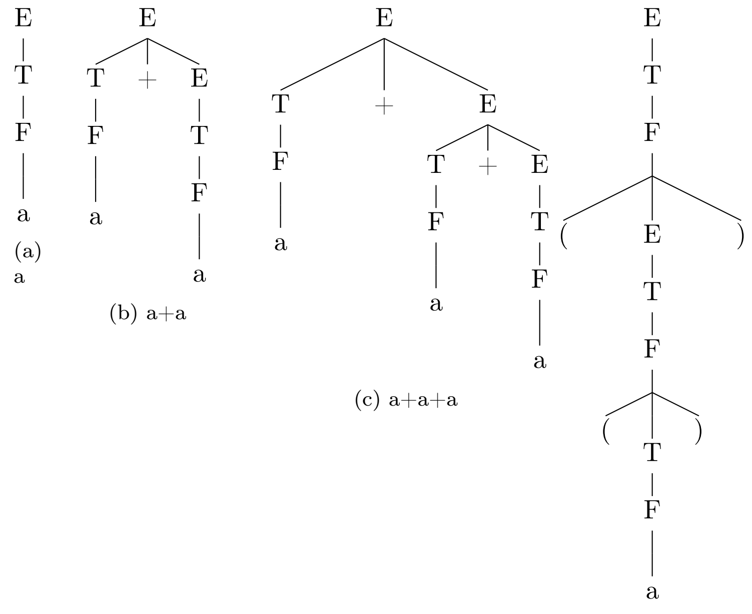
d

this is again wrt. out of order 0.1 eg. if we can get the string 01010 by umping as this is not in the lang.

page 154. question 2.2

Give parse trees and derivations for each string using the following Context free gramma (CFG)

$$E \to E + T | T$$
$$T \to E \times F | F$$
$$F \to (E) | a$$



(a) a

(b) a+a

(c) a+a+a

(d) ((a))

Figure 1.1: 2 Figures side by side

page 154. question 2.4

a

$S \to TTT$
$T \to E1E$
$E \to FTF$
$F \to 0|\epsilon$

b

$S \to 0F0|1F1$
$F \to F0F|F1F|\epsilon$

c

$S \to EFE$
$E \to FEF|EFF|FFE|\epsilon$
$F \to 0|1$

d

$S \to E0E$
$E \to FEF|\epsilon$
$F \to 0|1$

e

$S \to E$
$E \to FEF|GEG|\epsilon$
$F \to 0$ $G \to 1$

f

$S \to \epsilon$

2.6

Give the CFG that generates the following languages

b

The complment of the langauges $\{a^n b^n | n \geq 0\}$

$S \rightarrow FG$
$G \rightarrow GbG$
$F \rightarrow FaF$

d

For the third one it's a unbounded string with up to k alternations of 0*1*0*1*0*
This can simply be defined using the following grammar.
$S \rightarrow FG$
$E \rightarrow GE|FE|\epsilon$
$G \rightarrow 1G|\epsilon$
$F \rightarrow 0F|\epsilon$

2.14

We start in the initial state
$A \rightarrow BAB|B|\epsilon$
$B \rightarrow 00|\epsilon$

From here we put a new start state S

$S \rightarrow A$
$A \rightarrow BAB|B|\epsilon$
$B \rightarrow 00|\epsilon$

From here we can eliminate the first ϵ

$S \rightarrow A$
$A \rightarrow BAB|B|\epsilon|BA|AB$
$B \rightarrow 00 \cancel{|\epsilon}$
From here we can eliminate the ϵ in our 2nd rule,

$S \rightarrow \cancel{|A}|BAB|\cancel{B}|BA|AB|\epsilon|CC$
$A \rightarrow BAB|\cancel{B}|\cancel{\epsilon}|BA|AB|CC$
$B \rightarrow \cancel{00}|\cancel{\epsilon}|CC$
$C- > 0$

page 156, question 2.16

Show that the class of context free languages are closed under the regular operations Concatenation, union and Star
Using the following grammar

- $S_1 \rightarrow aS_1b$
- $S_1 \rightarrow \epsilon$
- $S_2 \rightarrow cS_2d$
- $S_2 \rightarrow \epsilon$

Concatenation

This is simply shown via
Making s start symbol S, and have the two languages follow each other S_1 S_2
$S \rightarrow S_1S_2$

Union

This is simply shown via
Making s start symbol S,
$S \to S_1|S_2$

Star

This is simply shown via
Making s start symbol S,
$S \to S_1 S$

page 158, question 2.32

Let $A/B = \{w|wx \in A$ for some $x \in B\}$, Show that if A is context free and B is regular, then A/B is context free.
"Proof. We can augment the memory of the PDA recognizing the CFL with the DFA recognizing the regular langauge and run both machines in parallel. We accept iff both machines accept.**??**
note the intersection of two CFL's and not necessarily a CFL.!
• 2.32. • 2.38. • 2.42 Hint for (d): first intersect the language with a suitably chosen regular language and then prove that the language you obtain is not context-free.

Chapter 2

Questions 2018

1. Finite automata and regular languages

Introduction

I'm going to talk about Finite automata and regular languages.

Finite automata

Finite automata is the simplest computational model that works via states and transitions, and therefor uses extremely limited memory. Using this model we can recognize and formulate regular languages, This can be a simple tasks a finding a substring, or used as a tool for designing more complex systems. A Finite atomata is defined as a tuple containing the set of states Q, The known alphabet σ, The transition function $\delta : Q \times \sigma \longrightarrow Q$the start state q_1 and the set of accept states F.

Regular languages

A regular languages is a sequence of letters in some alphabet defined by $\sum$ the empty alphabet is defined by the empty set $\varnothing$, and contains letters and the empty string ϵ. they are also closed under the union $\cup$, concatination $\cap$, and kleene star *, and the proceeding order is $^*, \cap, \cup$, A language is Regular is a Finite automata recognizes it.

Pumping lemma for regular languages

The pumping lemma is a way for us to proof if a language is regular. The theorem for the pumping lemma states that the 3 conditions for the lemma is

- for each $i \geq 0, xy^iz \in A$
- $|y| > 0$
- $|xy| \leq p$ where p is the pumpking length.

$0^n 1^n | n \geq 0$
The way you do this is to assume it's regular and make a counter argument. with the pumping lemma we have 3 conditions that our counter argument most meet. the first case we can pump the language to have more 1's than 0s or(2) the other way around.. the third(3) is that we can get out of order letters if we pump a string containing both 0s and 1s,

Deterministic And Non-deterministic

The difference between deterministic and non-deterministic is the way they operate. they recognize the same class of languages as a NFA can be convert into a DFA and the same goes the other way around, it is not a efficiency operation as the algorithm is exponential. there is however some benefits to both. where the non-deterministic preforms branching and could potentially benefit the execution wrt. size, runtime. or recourse consumption.

2. Pushdown automata and context-free languages

What is a push down
What is a context free grammar a -> 0a1 A -> B B -> # ambiguity grammar variables
What can it be used to programming languages Compiler
Pushdown atomata in dept You use a stack.
Pumping lemma Proof
Application

3. Turing machines

Introduction What is a turning machine
Multitape
Nondetermanistic turning machine Faster but less powerfull?

4. Decidability

Introduction If a language defined by a DFA is decidable.
Examples of Decidability and undecidability

5. Reducibility

What is it and how to use it.

6. NP-completeness proofs – examples.

what is np_completeness
Why we use it to reduce,
Proff of np complete. qlique, subset sum, Hamiltonian circuit,

7. Proof that SATISFIABILITY is NP-complete (do not assume that there is a known NP-Complete problem — use the proof in Sipser's book).

Cook-levin theorem - insipset, præsentation use slides on homepage.

8. Information-theoretic lower bounds (lower bounds proven by counting leaves in decision trees), especially the average case bounds for sorting by comparisons.

As is average case.

9. Adversary arguments – technique, examples.

10. Median problem – algorithm and lower bound.

11. Approximation algorithms