Advanced Topics in Concurrent Systems
DM869

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IMADA

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Contents

notes

w.r.t With regards to
s.t. such that

Introduction to inference systems

Rules

Theorem 1. $\frac{\text{ﬄ}}{num(Z)}[Zero]$

Theorem 2. $\frac{num(Z)}{num(Sx)}[Succ]$

num(Z)

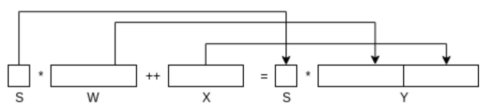
Num(Z) is derivable iff x encodes a nautral number, if any derivation for number x has exatly height n, then x encodes n proff by induction, on the structure pf the given deviration for num(x)
Care Zero The derivation starts with rule[Zero] hence X must be >, the height must be 1
case one Num(Z) is derivable iff x encodes a nautral number, if any derivation for number x has exatly height n+1, then x encodes n proff by induction, on the structure pf the given deviration for num(x)
Care Zero The derivation starts with rule[Zero] hence X must be n+1, the height must be n+1

succ

Case Zero
The derivation rulestarts with rule[succ] hence X=Sy for somey, we have a derivation for num(Y), Its height, Say m.
By induction HP, y encodes m-1 thus Sy, encodes m-1

Add

Theorem 3. $\frac{add(w,x,y)}{add(Sw,X,Sy)}[Succ]$



add(W,X,Y) is derivable iff W+X = Y
Prove by induction on the derication of add(W,X,Y)
Case $+\mathbb{Z}$ There is no inductive step
W = Z, X = x = Y, O+X=X˜
Case +S W= Sw, X = x, Y = Sy,
We have a serivation for add(w,x,y) We can apply the inductive hypothesis(ind. HP) w+x=y, W=1+w, Y=1+y, we conclude that 1+w+x=1+y, W+X =Y

sub

Theorem 4. $\frac{num(Z)}{num(Sx)}[Succ]$

sub(w,x,y) def, rules s.t. sub(q,x,y) is derivable iff w-x=y.
It can be proved that there is no proof for this.

if then

Theorem 5. $\frac{if x+y=z}{then w-x=y}\frac{add(x,y,w)}{sub(w,x,y)}$

Conculus of the cumunication system

C is a channel
C = new channel ("IP eg 10.130.10.42")
C.open(); connect C.send(42);
x: c.recv() P:
Def. a labelled transition sstem is (S, L, $\rightarrow$)

- S is a set of steates (processes)
- K us a set of lables (Actions)
- $\rightarrow \subseteq S \times L \times S$ is trasition relation

Natation S $\xrightarrow{\text{e}}$ S' means (S,e, S')$\in\rightarrow$

$P ::= \emptyset$ //Termination program
$\overline{C}.P$ //send on channel c and continue as P
$\overline{C}.P$ // Recieve on channel c and continue as P
$\frac{}{C.P \xrightarrow{\text{c}} P}$ [Send]
$\frac{}{\overline{C}.P \xrightarrow{\overline{\text{c}}} P}$ [Recieve]
$\frac{P \xrightarrow{\text{c}} P' \quad Q \xrightarrow{\text{a}} Q'}{P|Q \xrightarrow{\text{t}} P'|Q'}$ [Com]
How can i see that two programs are running at the same time.
P | Q // P and Q urn concurrently
$c.P|\overline{C}.Q \rightarrow P|Q$ //If we have two nodes, c, and $\overline{c}$ and c wants to send to $\overline{c}$ and $\overline{c}$ want's to recieve from c, this is syncronys transmition. eg, a communication can't fail.
Spec R spec 1 .... Secn Rimpl
R should be an equvalence relation between two processes -transitive: reason about chain refinemnt
Reflexivity PRP Symmytry Spec R impl. -> Impl R spec
Def A cotntect is any term generated $C := [-]|\alpha.C|C + P|(P|C)$
$P ::= \alpha.P|O|P + P'|(P|P')$
Def A trace is a sequence$\alpha_1.......\alpha_n \in ACT$
(ACT)* is the finite word containing all possible programs
The set of traces of a process P is the set of traces(P) $\{\alpha_1.....\alpha_n\}$

$$Traces(User) = \{\in; \overline{P}; \overline{P}.enter; \overline{P}.enter.exit; .....\} = \{\in\} \cup \{\overline{P}t| \in traces(U_1)\} \tag{1}$$

Def a Relation R is a visimulation if Whenever two processes PRQ it holds that : We have two conditions if P can proform some transition

$$\text{if } p \xrightarrow{\alpha} P' \text{ then there is } Q \xrightarrow{\alpha} Q' \wedge R'RQ' \tag{2}$$

$$\text{if } Q \xrightarrow{\alpha} Q' \text{ then there is } P \xrightarrow{\alpha} P \wedge' R'RQ' \tag{3}$$

Def P,Q are called Bisimilar (P Q) iff there is a bisimulation r st. PRQ
Desiderata for behavioval eq.
-Equvilance relation
-Congruence (w.r.t to the syntax for out programming language)
def. A relation R over CCS processes is a bisimulation if whenever $(P, Q) \in R$ it hold that for any lable α :

Theorem 6. *1) If p $\xrightarrow{\alpha}$ p' then there is q' s.t. Q $\xrightarrow{\alpha}$ Q' and* $(P', Q') \in R$

Theorem 7. *2) symmetric of 1)*

P and Q are called bisimilar (P Q) if there a bisimulation R. S.t. $(P, Q) \in R$
Lemma: Bisimularirt is an equivilance relation
Proof -Reflexivity $\forall P$, P P $p \xrightarrow{\alpha} p'$, $p \xrightarrow{\alpha} p'$ - $I \subseteq$ -Prove that I is a bisimulartion: $(P, Q) \in I =>$ P=Q $=> p \xrightarrow{\alpha} p'$ then $Q \xrightarrow{\alpha} p'$ $(P', P') \in I$

-Symmetry: P Q => Q P. - By def. of bisimilairty there is a bisimulation R such that $(P, Q) \in R$ Claim $R^{-}1$ (The relation where you flip every pair) $= \{(x, y)|(y, z) \in R\}$ is a bisimulatin $(Q, P) \in R^{-}1$ is bisimular, You can conclude Q P
Given that R is a bisimulation then R^{-1} is a bisimulation for any R. let $(s, t) \in R^{-1}$. 1) For any $S \xrightarrow{\alpha} S'$,let T' be any s.t. $(S', T') \in R$(There is at least one since R is a Bisiulation). (Since S is in relation with T by our definition, by this point by construction.) It follows that $T \xrightarrow{\alpha} T'$, $(S', T') \in R^{-1}$ by The above formula. this proves the first conditioning of the def. of bisumulation.
2.) Likewise: given $T \xrightarrow{\alpha} T'$, there is a transition $S \xrightarrow{\alpha} S'$ s.t. $(T', S') \in R$ hence $(T', S') \in R^{-1}$
-Transitivity: P Q, Q S => P S, **The strategy here is to show that it is similar to what we did before.** -From P Q it follows that $\exists R_1$ s.t. R_1 is a bisumulation and $(P, Q) \in R_1$ -From Q S it follows that $\exists R_2$ s.t.R_1 is a bisumulation and $(Q, S) \in R_2$
Define $R = R_1 \dot{R}_2 = \{(x, <)|\exists y.(x, y) \in R_1 \wedge (y, z) \in R_2\}$ If R is a bisimulation by constrction $(P, S) \in R$ hence P S
$(T_1, T_2) \in R$ **If t1 can preform an action, then t2 can do the same with the same lables.** 1) let $T_1 \xrightarrow{\alpha} T_2$, by construction of R. T_3 s.t. $(T_1, T_3) \in R_1$, therefor T_3' s.t. $T_3 \xrightarrow{\alpha} T_3' \wedge (T_1', T_3') \in R_1$ $(T_3, T_2) \in R_2$, since R_2 is a bimiulation, $T_2 \xrightarrow{\alpha} T_2'$ s.t. $(T_3', T_2') \in R$ **Inset picture here**
$T_1' R_1 T_3' R_2 T_2' => T_1' R T_2'$
2) $T_2 \xrightarrow{\alpha} T_2'$,
<u>Lemma:</u> Is a bisimulartion See notes
<u>Lemma:</u> is a congruence $\forall C, P\ Q\ =>\ C[P]\ C[Q]$ The result is a result for a weaker/eqivilant one.
Proof: by structual induction Let P Q, Proceed by strucurtal induction on C, <u>Base:</u> C C, (No holes) It follow by reflexivity of if C= [_] then C[P] = P Q = C[Q] <u>induction case:</u>

- $C = \alpha.[_]\alpha.P\ \alpha.Q$
- $C = S|[_]S|P\ S|Q$
- $C = [_]|SS|P\ S|Q$
- $C = S + [_]$
- $C = [_]+S$
- $C = [_]\backslash L$

Lemma: If P Q then: 1) a.P a.Q 2) $\forall S S|P\ S|Q$ 3) $\forall S + P\ S + Q$
1) P Q => a.P A.Q let R be a Bisimulation s.t. $(P, Q) \in R$. Def. $R' = R \cup \{(a.P, a, Q)\}$ The two conditions

- $a.P \xrightarrow{\alpha} p$ the other process can only reply with $a.Q \wedge a.Q \xrightarrow{\alpha} Q$. $(P, Q) \in R'$,

2)$P\ Q\ =>\ \forall S, S|P|S|Q$.Let R be a bisimulation $(P, Q) \in R$. -Def. R as the following relation$R = \{(S|P, S|Q)|P\ Q$, P,Q,S are CCS Processes } Assume that $S|P \xrightarrow{\alpha} S'|P'$. Derivations for $S|P \xrightarrow{\alpha} S'|P'$ have thererfo: the first rule applied is 1) Lpar, 2) Rpar, 3, com
1) Lpar

$$\frac{S \xrightarrow{\alpha} S'}{S|P \xrightarrow{\alpha} S'|P}(P = P')\frac{S \xrightarrow{\alpha} S'}{S|q \xrightarrow{\alpha} S'|Q} \tag{4}$$

$(S'|P, S'|Q) \in R$ by def of R
2) Rpar

$$\frac{P \xrightarrow{\alpha} P'}{S|P \xrightarrow{\alpha} S|P'}(S = S') \text{ Since } P\ Q \exists Q \xrightarrow[(*)]{\alpha} Q' P' \dagger Q' \tag{5}$$

$$\text{From}(\star)\frac{Q \xrightarrow{\alpha} Q'}{S|Q \xrightarrow{\alpha} S|Q'}\text{And from } (\dagger)(S|PTEST', S|Q') \in R \tag{6}$$

3) com

$$\frac{S \xrightarrow{\alpha} S' \ \ P \xrightarrow{\overline{\alpha}} P'}{S|P \xrightarrow{\tau} S'|P'} \text{ Since } P \ \ Q, Q \xrightarrow[(\star)]{\overline{\alpha}} Q' \ \ P' {}_{(\dagger)} Q' \tag{7}$$

$$\text{From } (\star) \frac{S \xrightarrow{\alpha} S' \ \ Q \xrightarrow{\overline{\alpha}} Q'}{S|Q \xrightarrow{\tau} S|Q'} \text{ And from } (\dagger)(S'|P', S'|Q') \in R. \tag{8}$$

Likewise $S|Q \xrightarrow{\alpha} S'|Q'$....

PFor S+P S+Q use the relation **Picture from phone**

P|Q Q|P prove that R = { (P|Q, Q|P) } is a bisimular

For an LTS (S, ACT, ->) It's saturation is the LTS S, ACT, =>) where => is the least relation s.t.

We can ignore internail computation

$$\frac{\text{ffl}}{S \rightarrow []\tau S} \frac{S_1 \rightarrow []\tau S_1' \ \ S_1' \xrightarrow{[]\alpha} S_2' \ \ S_2' \xrightarrow{[]\alpha S_2}}{S_1 \rightarrow []\alpha S_2} \tag{9}$$