Bayesian Framework of Learning

Probability

Bayesian learning is based on probability.

If events A and B are independent, then knowledge about event A does not change the probability of event B.

Instead if A and B are not independent from each other, then it is possible to quantify the conditional probability of A subject to our knowledge about the probability of B.

Bayes' Theorem:

$$\Pr(h|\mathcal{D}) = \frac{\Pr(\mathcal{D}|h) \cdot \Pr(h)}{\Pr(\mathcal{D})}$$

Example - Election

There has been an election between Republicans and Democrats. You know nothing about who won, but months later a law has been passed saying taxes has been lowered. Based on knowledge about the two parties, it is most probable that Republicans won the election.

Now turn it around. Now you have to vote. Now we can calculate the probability that the taxes will be lowered.

For simplicity, numbers are very simple.

Both candidates have a 0.5 ratio of winning. This means that P(Democrat)=0.5 and P(Republican)=0.5.

Now the probability that the Republican will lower taxes is 0.85 and the probability that he will NOT lower taxes is 0.15. Therefore P(a|R)=0.85 and P(a'|R)=0.15.

In the same way, the Democrat has a 0.25 probability that the taxes will be lowered and a probability of 0.75 that taxes will not be lowered. Therefore P(a|D)=0.25 and P(a'|D)=0.75.

Based of that information, the probability that taxes will be lowered can be calculated:

P(a)=P(R) * P(a|R) + P(D) * P(a|D)

P(a)=P(0.5 * 0.85) + P(0.5 * 0.25)

P(a)=0.475 * 0.125

P(a)=0.55

There is therefore 55% chance that taxes will be lowered.

Now what is the probability that the Democrat or Republican was elected, if the taxes were lowered?

P(D|a)=(0.85 * 0.5)/(0.55) = 0.772

P(R|a)=(0.25 * 0.5)/(0.55) = 0.227

There is a 77.2% probability that the winner of the election was a Republican and 22.7% probability that the winner of the election was a Democrat.