BANGLADESH UNIVERSITY OF ENGINEERING AND TECHNOLOGY

A REPORT ON

MINIMIZING OVERALL COST OF TRANSPORTED GOODS OVER LONG-DISTANCE DISTRIBUTION

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Abstract

Operations research is a discipline that uses mathematical and analytical methods to optimize complex systems and decision-making processes. In operations research, problems are broken down into basic components and then solved in defined steps by mathematical analysis.

The prime objective of this project is to implement an optimization technique to a real-life problem. We opted to implement the concept of Transportation Problems on the data we obtained on PepsiCo, Inc's factories and warehouses. The case study is presented to illustrate how the transportation problem can be applied in different industries and contexts.

In any country, factories need to make a decision whether to accept an order or not. Or at what price they should accept. If the price they were offered is feasible (or optimal) or not. Such decisions cannot be taken on a whim or without foresight. These decisions are needed to be taken in a methodical approach. This is where Transportation Problem Algorithms come in.

The transportation problem is a fundamental issue in operations research that involves determining the optimal way to transport goods from a set of suppliers to a set of destinations while minimizing the overall transportation costs. It is a special class of Linear programming problem (LPP). The report provides a comprehensive overview of the problem, including its mathematical formulation, and solution method.

Overall, this project report provides a comprehensive understanding of the transportation problem, including its formulation, solution methods, and application in supply chain management.

Methodology Used

The general transportation problem is concerned (literally or figuratively) with distributing any commodity from any group of supply centers, called **sources**, to any group of receiving centers, called **destinations,** in such a way as to minimize the total distribution cost.

As indicated by the fourth and fifth rows of the table, each source has a certain **supply** of units to distribute to the destinations, and each destination has a certain **demand** for units to be received from the sources. The model for a transportation problem makes the following assumption about these supplies and demands:

The Requirements Assumption: Each source has a fixed supply of units, where this entire supply must be distributed to the destinations. (We let s_i denote the number of units being supplied by source i, for $i = 1, 2, \ldots, m$.) Similarly, each destination has a fixed demand for units, where this entire demand must be received from the sources. (We let d_j denote the number of units being received by destination j, for $j = 1, 2, \ldots, n$.)

This assumption holds for the Pepsi Co. problem since each Factory (source) has a fixed output and each warehouse (destination) has a fixed allocation.

This assumption that there is no leeway in the amounts to be sent or received means that there needs to be a balance between the total supply from all sources and the total demand at all destinations.

The Feasible Solutions Property: A transportation problem will have feasible solutions if and only if

$$\sum_{i=1}^{m} s_i = \sum_{j=1}^{n} d_j$$

Unfortunately, these sums are not equal for the Pepsi Co.

In some real problems, the supplies actually represent maximum amounts (rather than fixed amounts) to be distributed. Similarly, in other cases, the demands represent maximum amounts (rather than fixed amounts) to be received. Such problems do not quite fit the model for a transportation problem because they violate the requirements assumption. However, it is possible to reformulate the problem so that they then fit this model by introducing a dummy destination or a dummy source to take up the slack between the actual amounts and maximum amounts being distributed. We will illustrate how this is done with two examples at the end of this section.

The Cost Assumption: The cost of distributing units from any particular source to any particular destination is directly proportional to the number of units distributed. Therefore, this cost is just the unit cost of distribution times the number of units distributed. (We let c_{ij} denote this unit cost for source i and destination j.) This assumption holds for the Pepsi Co. problem since the cost of shipping peas from any Factory to any warehouse is directly proportional to the number of truckloads being shipped.

The only data needed for a transportation problem model are the supplies, demands, and unit costs. These are the parameters of the model. All these parameters can be summarized conveniently in a single parameter table in case of our Pepsi Co. Problem.

The Model: Any problem (whether involving transportation or not) fits the model for a transportation problem if it can be described completely in terms of a parameter table like the one stated below and it satisfies both the requirements assumption and the cost assumption. The objective is to minimize the total cost of distributing the units. All the parameters of the model are included in this parameter table.

		Cost per Unit Distributed			
		Destination			
	1	2	⋯	n	Supply
Source 1	c_{11}	c_{12}	⋯	c_{1n}	s_1
2	c_{21}	c_{22}	⋯	c_{2n}	s_2
⋮					⋮
m	c_{m1}	c_{m2}	⋯	c_{mn}	s_m
Demand	d_1	d_2	⋯	d_n	

Therefore, formulating a problem as a transportation problem only requires filling out a parameter table in the format of the parameter table for the PepsiCo. problem. Alternatively, the same information can be provided by using the network representation of the problem. Some problems that have nothing to do with transportation also can be formulated as a transportation problem in either of these two ways.

Since a transportation problem can be formulated simply by either filling out a parameter table or drawing its network representation, it is not necessary to write out a formal mathematical model for the problem. However, to emphasize that it is indeed a special type of linear programming problem, the Model for a transportation Problem is as follows:

Letting Z be the total distribution cost and x_{ij} ($i = 1, 2, \ldots, m$; $j = 1, 2, \ldots, n$) be the number of units to be distributed from source i to destination j, the linear programming formulation of this problem is

Minimize:

$$Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$

subject to

$$\sum_{j=1}^{n} x_{ij} = s_i \qquad \text{for, } i = 1, 2, \ldots, m$$

$$\sum_{j=1}^{n} x_{ij} \leq d_j \qquad \text{for, } j = 1, 2, \ldots, n$$

$$x_{ij} \geq 0 \text{, for all i and j.}$$

As with other linear programming problems, the usual software options (Excel with either the standard Solver or ASPE, LINGO/LINDO, MPL/Solvers) are available to you for setting up and solving transportation problems

Problem Description

Transcom Beverage Limited (TBL) is the exclusive PepsiCo Franchise for Bangladesh. TBL owns and operates modern plants in Dhaka, Chittagong and Konabari to manufacture Pepsi, 7UP, Mirinda, Slice, Mountain Dew, Aquafina and more. Transcom Beverages has three manufacturing sites in Gazipur (Konabari and Bagher Bazar) and Chittagong. These plants have been recognized by PepsiCo globally with the Gold Accreditation and the Bagher Bazar plant is 1 of 12 plants in the world to have the Platinum Accreditation. In order to maintain its reputation, TBL needs to constantly monitor the demands of the crowd and maintain their production and distribute the products according to their demands in respected areas. We talked with Mr. Romel regarding this and came upon these following points that TBL maintains while their distribution of their products.

- The raw materials are imported from America. Upon arriving, the products are sent to the manufacturing sites or factories.
- Transcom Beverages has three manufacturing sites situated in Gazipur (Konabari and Bagher Bazar) and Chittagong.
- TBL has five warehouses to store its beverages from which they are distributed to the distributors. The warehouses are situated in Sylhet, Chittagong, Bogura, Khulna, Gazipur.
- Then the products are assigned to the distributors. The distributors invest in the company and take a share in the sell by keeping the products in their own warehouses on which TBL has no sight upon.
- The minimum cost for transportation of a 1.5 ton covered van per kilometer is 140tk. For short distance warehouses (10-120km), a 16ft covered vans are used. And for long distance (greater or equal than 120km), 22 feet covered vans are used.
- A 22ft covered van can carry up to 1.5 ton (roughly 2720 500ml Pepsi bottles, around 136 cartoons each consisting of 20 bottles).
- A 16ft covered van can carry up to 1.1 ton (roughly 2000 500ml Pepsi bottles, around 100 cartoons each consisting of 20 bottles).

Data Collection

The average distance from each manufacturing site to each distributor is given in the table below:

	Sylhet	Bogura	Chittagong	Khulna	Gazipur
Konabari	232	157	292	250	21
Bagher Bazar	235	184	311	270	40
Chittagong	378	441	20	450	274

The toll costs for each transportation methods are as below:

Factory	Routes	Toll Roads	Toll Costs	Total Cost
Bagher Bazar	Sylhet	Aushkandi - Sherpur Rd	1200	3000
		Dhaka - Sylhet Highway	1000	
		Jamtola-Charshindur Road	800	
	Bogura	Jamuna Bridge	1400	1400
	Chittagong	Bostail-Madanpur Highway	800	2400
		Dhaka - Chittagong Highway	1600	
	Khulna	Padma Brg Rd	2000	3900
		Dhaka - Mawa Highway	1200	
		Mayor Hanif Flyover	700	
	Gazipur	N/A	0	0
Chittagong	Sylhet	N/A	0	0
	Bogura	Jamuna Bridge	1400	3000
		Kaptan Bazar Flyover	700	
		Kachpur Road	900	
	Chittagong	N/A	0	0
	Khulna	Khulna City Bypass	1100	4300
		Padma Bridge Road	2000	
		Dhaka Mawa Highway	1200	
		Kachpur Road	900	
	Gazipur	Kaptan Bazar Flyover	700	1600
		Kachpur Road	900	
KonaBari	Sylhet	Aushkandi - Sherpur Rd	1200	3000
		Dhaka - Sylhet Highway	1000	
		Jamtola-Charshindur Road	800	
	Bogura	Jamuna Bridge	1400	1400
	Chittagong	Bostail-Madanpur Highway	800	2400
		Dhaka - Chittagong Highway	1600	
	Khulna	Padma Brg Rd	2000	3900
		Dhaka - Mawa Highway	1200	
		Mohammad Hanif Flyover	700	
	Gazipur	N/A	0	0

The daily supply from each factory and daily demand for each warehouse alongside transportation costs for manufacturing sites to warehouses are given below:

	Sylhet	Bogura	Chittagong	Khulna	Gazipur	Supply (Cartoons)
Konabari	35480	23380	43280	38900	2940	45000
Bagher Bazar	35900	27160	45940	41700	5600	90000
Chittagong	52920	62040	2800	67300	39960	80000
Demand (Cartoons)	52000	35000	50000	50000	60000	

Formulation

The objective of this formulation is to assign specific amounts of supply cartoons to each warehouse to minimize the cost for transporting finished goods. The problem can be formulated as follows:

Minimize

$$Z = 35480x_{11} + 23380x_{12} + 43280x_{13} + 38900x_{14} + 2940x_{15} + 35900x_{21} + 27160x_{22} + 45940x_{23} + 41700x_{24} + 5600x_{25} + 52920x_{31} + 62040x_{32} + 2800x_{33} + 67300x_{34} + 39960x_{35}$$

Subjected to:

$$x_{11} + x_{12} + x_{13} + x_{14} + x_{15} = 45000$$

$$x_{21} + x_{22} + x_{23} + x_{24} + x_{25} = 90000$$

$$x_{31} + x_{32} + x_{33} + x_{34} + x_{35} = 80000$$

$$x_{11} + x_{21} + x_{31} \leq 52000$$

$$x_{12} + x_{22} + x_{32} \leq 35000$$

$$x_{13} + x_{23} + x_{33} \leq 50000$$

$$x_{14} + x_{24} + x_{34} \leq 50000$$

$$x_{15} + x_{25} + x_{35} \leq 60000$$

Solution

We can solve the transportation Problem through the transportation simplex table.To get the initial BFS, we used Russel's Approximation method.

	D1	D2	D3	D4	D5	Supply
S1	35480	23380	43280	38900	2940	45000
S2	35900	27160	45940	41700	5600	90000
S3	52920	62040	2800	67300	39960	80000
Demand	52000	35000	50000	50000	60000	

Solution:
TOTAL number of supply constraints : 3
TOTAL number of demand constraints : 5
Problem Table is

	D_1	D_2	D_3	D_4	D_5	Supply
S_1	35480	23380	43280	38900	2940	45000
S_2	35900	27160	45940	41700	5600	90000
S_3	52920	62040	2800	67300	39960	80000
Demand	52000	35000	50000	50000	60000	

Here Total Demand = 247000 is greater than Total Supply = 215000. So We add a dummy supply constraint with 0 unit cost and with allocation 32000.
Now, The modified table is

	D_1	D_2	D_3	D_4	D_5	Supply
S_1	35480	23380	43280	38900	2940	45000
S_2	35900	27160	45940	41700	5600	90000
S_3	52920	62040	2800	67300	39960	80000
S_{dummy}	0	0	0	0	0	32000
Demand	52000	35000	50000	50000	60000	

Table-1: Calculate $\bar{U}_i$ and $\bar{V}_j$ (where $\bar{U}_i$ is the largest cost in row and $\bar{V}_j$ is the largest cost in column)

	D_1	D_2	D_3	D_4	D_5	Supply	$\bar{U}_i$
S_1	35480	23380	43280	38900	2940	45000	43280
S_2	35900	27160	45940	41700	5600	90000	45940
S_3	52920	62040	2800	67300	39960	80000	67300
S_{dummy}	0	0	0	0	0	32000	0
Demand	52000	35000	50000	50000	60000		
$\bar{V}_j$	52920	62040	45940	67300	39960		

2. Compute reduced cost of each cell Δ_{ij}, where $\Delta_{ij} = c_{ij} - \left(\bar{U}_i + \bar{V}_j\right)$

1. $\Delta_{11} = c_{11} - \left(\bar{U}_1 + \bar{V}_1\right) = 35480 - (43280 + 52920) = -60720$

2. $\Delta_{12} = c_{12} - \left(\bar{U}_1 + \bar{V}_2\right) = 23380 - (43280 + 62040) = -81940$

3. $\Delta_{13} = c_{13} - \left(\bar{U}_1 + \bar{V}_3\right) = 43280 - (43280 + 45940) = -45940$

4. $\Delta_{14} = c_{14} - \left(\bar{U}_1 + \bar{V}_4\right) = 38900 - (43280 + 67300) = -71680$

5. $\Delta_{15} = c_{15} - \left(\bar{U}_1 + \bar{V}_5\right)$ = 2940 - (43280 + 39960) = -80300

6. $\Delta_{21} = c_{21} - \left(\bar{U}_2 + \bar{V}_1\right)$ = 35900 - (45940 + 52920) = -62960

7. $\Delta_{22} = c_{22} - \left(\bar{U}_2 + \bar{V}_2\right)$ = 27160 - (45940 + 62040) = -80820

8. $\Delta_{23} = c_{23} - \left(\bar{U}_2 + \bar{V}_3\right)$ = 45940 - (45940 + 45940) = -45940

9. $\Delta_{24} = c_{24} - \left(\bar{U}_2 + \bar{V}_4\right)$ = 41700 - (45940 + 67300) = -71540

10. $\Delta_{25} = c_{25} - \left(\bar{U}_2 + \bar{V}_5\right)$ = 5600 - (45940 + 39960) = -80300

11. $\Delta_{31} = c_{31} - \left(\bar{U}_3 + \bar{V}_1\right)$ = 52920 - (67300 + 52920) = -67300

12. $\Delta_{32} = c_{32} - \left(\bar{U}_3 + \bar{V}_2\right)$ = 62040 - (67300 + 62040) = -67300

13. $\Delta_{33} = c_{33} - \left(\bar{U}_3 + \bar{V}_3\right)$ = 2800 - (67300 + 45940) = -110440

14. $\Delta_{34} = c_{34} - \left(\bar{U}_3 + \bar{V}_4\right)$ = 67300 - (67300 + 67300) = -67300

15. $\Delta_{35} = c_{35} - \left(\bar{U}_3 + \bar{V}_5\right)$ = 39960 - (67300 + 39960) = -67300

16. $\Delta_{41} = c_{41} - \left(\bar{U}_4 + \bar{V}_1\right)$ = 0 - (0 + 52920) = -52920

17. $\Delta_{42} = c_{42} - \left(\bar{U}_4 + \bar{V}_2\right)$ = 0 - (0 + 62040) = -62040

18. $\Delta_{43} = c_{43} - \left(\bar{U}_4 + \bar{V}_3\right)$ = 0 - (0 + 45940) = -45940

19. $\Delta_{44} = c_{44} - \left(\bar{U}_4 + \bar{V}_4\right)$ = 0 - (0 + 67300) = -67300

20. $\Delta_{45} = c_{45} - \left(\bar{U}_4 + \bar{V}_5\right)$ = 0 - (0 + 39960) = -39960

	D_1	D_2	D_3	D_4	D_5	Supply	$\bar{U}_i$
S_1	35480 [-56340]	23380 [-77560]	43280	38900 [-67300]	2940 [-75920]	45000	38900
S_2	35900 [-58720]	27160 [-76580]	45940	41700 [-67300]	5600 [-76060]	90000	41700
S_3	52920 [-67300]	62040 [-67300]	2800(**50000**)	67300 [-67300]	39960 [-67300]	30000	67300
S_{dummy}	0 [-52920]	0 [-62040]	0	0 [-67300]	0 [-39960]	32000	0
Demand	52000	35000	0	50000	60000		
$\bar{V}_j$	52920	62040	--	67300	39960		

The most negative Δ_{ij} is -77560 in cell S_1D_2

The allocation to this cell is min(45000,35000) = **35000**.
This satisfies the entire demand of D_2 and leaves 45000 - 35000=10000 units with S_1

Table-2: This leads to the following table

	D_1	D_2	D_3	D_4	D_5	Supply
S_1	35480	23380 (**35000**)	43280	38900	2940	10000
S_2	35900	27160	45940	41700	5600	90000
S_3	52920	62040	2800 (**50000**)	67300	39960	30000
S_{dummy}	0	0	0	0	0	32000
Demand	52000	0	0	50000	60000	

Table-3: Calculate $\bar{U}_i$ and $\bar{V}_j$ (where $\bar{U}_i$ is the largest cost in row and $\bar{V}_j$ is the largest cost in column)

	D_1	D_2	D_3	D_4	D_5	Supply	$\bar{U}_i$
S_1	35480	23380(**35000**)	43280	38900	2940	10000	38900
S_2	35900	27160	45940	41700	5600	90000	41700
S_3	52920	62040	2800(**50000**)	67300	39960	30000	67300
S_{dummy}	0	0	0	0	0	32000	0
Demand	52000	0	0	50000	60000		
$\bar{V}_j$	52920	--	--	67300	39960		

2. Compute reduced cost of each cell Δ_{ij}, where $\Delta_{ij} = c_{ij} - \left(\bar{U}_i + \bar{V}_j\right)$

1. $\Delta_{11} = c_{11} - \left(\bar{U}_1 + \bar{V}_1\right)$ = 35480 - (38900 + 52920) = -56340

2. $\Delta_{14} = c_{14} - \left(\bar{U}_1 + \bar{V}_4\right)$ = 38900 - (38900 + 67300) = -67300

3. $\Delta_{15} = c_{15} - \left(\bar{U}_1 + \bar{V}_5\right)$ = 2940 - (38900 + 39960) = -75920

4. $\Delta_{21} = c_{21} - \left(\bar{U}_2 + \bar{V}_1\right)$ = 35900 - (41700 + 52920) = -58720

5. $\Delta_{24} = c_{24} - \left(\bar{U}_2 + \bar{V}_4\right)$ = 41700 - (41700 + 67300) = -67300

6. $\Delta_{25} = c_{25} - \left(\bar{U}_2 + \bar{V}_5\right)$ = 5600 - (41700 + 39960) = -76060

7. $\Delta_{31} = c_{31} - \left(\bar{U}_3 + \bar{V}_1\right)$ = 52920 - (67300 + 52920) = -67300

8. $\Delta_{34} = c_{34} - \left(\bar{U}_3 + \bar{V}_4\right)$ = 67300 - (67300 + 67300) = -67300

9. $\Delta_{35} = c_{35} - \left(\bar{U}_3 + \bar{V}_5\right)$ = 39960 - (67300 + 39960) = -67300

10. $\Delta_{41} = c_{41} - \left(\bar{U}_4 + \bar{V}_1\right)$ = 0 - (0 + 52920) = -52920

11. $\Delta_{44} = c_{44} - \left(\bar{U}_4 + \bar{V}_4\right)$ = 0 - (0 + 67300) = -67300

12. $\Delta_{45} = c_{45} - \left(\bar{U}_4 + \bar{V}_5\right)$ = 0 - (0 + 39960) = -39960

	D_1	D_2	D_3	D_4	D_5	Supply	$\bar{U}_i$
S_1	35480 [-56340]	23380(**35000**)	43280	38900 [-67300]	2940 [-75920]	10000	38900
S_2	35900 [-58720]	27160	45940	41700 [-67300]	5600 [-76060]	90000	41700
S_3	52920 [-67300]	62040	2800(**50000**)	67300 [-67300]	39960 [-67300]	30000	67300
S_{dummy}	0 [-52920]	0	0	0 [-67300]	0 [-39960]	32000	0
Demand	52000	0	0	50000	60000		
$\bar{V}_j$	52920	--	--	67300	39960		

The most negative Δ_{ij} is -76060 in cell S_2D_5

The allocation to this cell is min(90000,60000) = **60000**.
This satisfies the entire demand of D_5 and leaves 90000 - 60000=30000 units with S_2

Table-3: This leads to the following table

	D_1	D_2	D_3	D_4	D_5	Supply
S_1	35480	23380 (**35000**)	43280	38900	2940	10000
S_2	35900	27160	45940	41700	5600 (**60000**)	30000
S_3	52920	62040	2800 (**50000**)	67300	39960	30000
S_{dummy}	0	0	0	0	0	32000
Demand	52000	0	0	50000	0	

Table-4: Calculate $\bar{U}_i$ and $\bar{V}_j$ (where $\bar{U}_i$ is the largest cost in row and $\bar{V}_j$ is the largest cost in column)

	D_1	D_2	D_3	D_4	D_5	Supply	$\bar{U}_i$
S_1	35480	23380(**35000**)	43280	38900	2940	10000	38900
S_2	35900	27160	45940	41700	5600(**60000**)	30000	41700
S_3	52920	62040	2800(**50000**)	67300	39960	30000	67300
S_{dummy}	0	0	0	0	0	32000	0
Demand	52000	0	0	50000	0		
$\bar{V}_j$	52920	--	--	67300	--		

2. Compute reduced cost of each cell Δ_{ij}, where $\Delta_{ij} = c_{ij} - \left(\bar{U}_i + \bar{V}_j\right)$

1. $\Delta_{11} = c_{11} - \left(\bar{U}_1 + \bar{V}_1\right) = 35480 - (38900 + 52920) = -56340$

2. $\Delta_{14} = c_{14} - \left(\bar{U}_1 + \bar{V}_4\right) = 38900 - (38900 + 67300) = -67300$

3. $\Delta_{21} = c_{21} - \left(\bar{U}_2 + \bar{V}_1\right) = 35900 - (41700 + 52920) = -58720$

4. $\Delta_{24} = c_{24} - \left(\bar{U}_2 + \bar{V}_4\right) = 41700 - (41700 + 67300) = -67300$

5. $\Delta_{31} = c_{31} - \left(\bar{U}_3 + \bar{V}_1\right) = 52920 - (67300 + 52920) = -67300$

6. $\Delta_{34} = c_{34} - \left(\bar{U}_3 + \bar{V}_4\right) = 67300 - (67300 + 67300) = -67300$

7. $\Delta_{41} = c_{41} - \left(\bar{U}_4 + \bar{V}_1\right) = 0 - (0 + 52920) = -52920$

8. $\Delta_{44} = c_{44} - \left(\bar{U}_4 + \bar{V}_4\right) = 0 - (0 + 67300) = -67300$

	D_1	D_2	D_3	D_4	D_5	Supply	$\bar{U}_i$
S_1	35480 [-56340]	23380(**35000**)	43280	38900 [-67300]	2940	10000	38900
S_2	35900 [-58720]	27160	45940	41700 [-67300]	5600(**60000**)	30000	41700
S_3	52920 [-67300]	62040	2800(**50000**)	67300 [-67300]	39960	30000	67300
S_{dummy}	0 [-52920]	0	0	0 [-67300]	0	32000	0
Demand	52000	0	0	50000	0		
$\bar{V}_j$	52920	--	--	67300	--		

The most negative Δ_{ij} is -67300 in cell $S_{dummy}D_4$

The allocation to this cell is min(32000,50000) = **32000**.
This exhausts the capacity of S_{dummy} and leaves 50000 - 32000=18000 units with D_4

Table-4: This leads to the following table

	D_1	D_2	D_3	D_4	D_5	Supply
S_1	35480	23380 (**35000**)	43280	38900	2940	10000
S_2	35900	27160	45940	41700	5600 (**60000**)	30000
S_3	52920	62040	2800 (**50000**)	67300	39960	30000
S_{dummy}	0	0	0	0 (**32000**)	0	0
Demand	52000	0	0	18000	0	

	D_1	D_2	D_3	D_4	D_5	Supply	$\bar{U}_i$
S_1	35480 [-56340]	23380(**35000**)	43280	38900 [-67300]	2940	10000	38900
S_2	35900 [-58720]	27160	45940	41700 [-67300]	5600(**60000**)	30000	41700
S_3	52920 [-67300]	62040	2800(**50000**)	67300 [-67300]	39960	30000	67300
S_{dummy}	0 [-52920]	0	0	0 [-67300]	0	32000	0
Demand	52000	0	0	50000	0		
$\bar{V}_j$	52920	--	--	67300	--		

The most negative Δ_{ij} is -67300 in cell $S_{dummy}D_4$

The allocation to this cell is min(32000,50000) = **32000**.
This exhausts the capacity of S_{dummy} and leaves 50000 - 32000=18000 units with D_4

Table-4: This leads to the following table

	D_1	D_2	D_3	D_4	D_5	Supply
S_1	35480	23380 **(35000)**	43280	38900	2940	10000
S_2	35900	27160	45940	41700	5600 **(60000)**	30000
S_3	52920	62040	2800 **(50000)**	67300	39960	30000
S_{dummy}	0	0	0	0 **(32000)**	0	0
Demand	52000	0	0	18000	0	

Table-5: Calculate $\bar{U}_i$ and $\bar{V}_j$ (where $\bar{U}_i$ is the largest cost in row and $\bar{V}_j$ is the largest cost in column)

	D_1	D_2	D_3	D_4	D_5	Supply	$\bar{U}_i$
S_1	35480	23380**(35000)**	43280	38900	2940	10000	38900
S_2	35900	27160	45940	41700	5600**(60000)**	30000	41700
S_3	52920	62040	2800**(50000)**	67300	39960	30000	67300
~~S_{dummy}~~	~~0~~	~~0~~	~~0~~	~~0**(32000)**~~	~~0~~	0	--
Demand	52000	0	0	18000	0		
$\bar{V}_j$	52920	--	--	67300	--		

2. Compute reduced cost of each cell Δ_{ij}, where $\Delta_{ij} = c_{ij} - \left(\bar{U}_i + \bar{V}_j\right)$

1. $\Delta_{11} = c_{11} - \left(\bar{U}_1 + \bar{V}_1\right) = 35480 - (38900 + 52920) = -56340$

2. $\Delta_{14} = c_{14} - \left(\bar{U}_1 + \bar{V}_4\right) = 38900 - (38900 + 67300) = -67300$

3. $\Delta_{21} = c_{21} - \left(\bar{U}_2 + \bar{V}_1\right) = 35900 - (41700 + 52920) = -58720$

4. $\Delta_{24} = c_{24} - \left(\bar{U}_2 + \bar{V}_4\right) = 41700 - (41700 + 67300) = -67300$

5. $\Delta_{31} = c_{31} - \left(\bar{U}_3 + \bar{V}_1\right) = 52920 - (67300 + 52920) = -67300$

6. $\Delta_{34} = c_{34} - \left(\bar{U}_3 + \bar{V}_4\right) = 67300 - (67300 + 67300) = -67300$

	D_1	D_2	D_3	D_4	D_5	Supply	$\bar{U}_i$
S_1	35480 [-56340]	23380**(35000)**	43280	38900 [-67300]	2940	10000	38900
S_2	35900 [-58720]	27160	45940	41700 [-67300]	5600**(60000)**	30000	41700
S_3	52920 [-67300]	62040	2800**(50000)**	67300 [-67300]	39960	30000	67300
~~S_{dummy}~~	~~0~~	~~0~~	~~0~~	~~0**(32000)**~~	~~0~~	0	--
Demand	52000	0	0	18000	0		
$\bar{V}_j$	52920	--	--	67300	--		

The most negative Δ_{ij} is -67300 in cell S_3D_1

The allocation to this cell is min(30000,52000) = **30000**.
This exhausts the capacity of S_3 and leaves 52000 - 30000=22000 units with D_1

	D_1	D_2	D_3	D_4	D_5	Supply	$\bar{U}_i$
S_1	35480 [-56340]	23380**(35000)**	43280	38900 [-67300]	2940	10000	38900
S_2	35900 [-58720]	27160	45940	41700 [-67300]	5600**(60000)**	30000	41700
S_3	52920 [-67300]	62040	2800**(50000)**	67300 [-67300]	39960	30000	67300
~~S_{dummy}~~	~~0~~	~~0~~	~~0~~	~~0**(32000)**~~	~~0~~	0	--
Demand	52000	0	0	18000	0		
$\bar{V}_j$	52920	--	--	67300	--		

The most negative Δ_{ij} is -67300 in cell S_3D_1

The allocation to this cell is min(30000,52000) = **30000**.
This exhausts the capacity of S_3 and leaves 52000 - 30000=22000 units with D_1

Table-5: This leads to the following table

	D_1	D_2	D_3	D_4	D_5	Supply
S_1	35480	23380 **(35000)**	43280	38900	2940	10000
S_2	35900	27160	45940	41700	5600 **(60000)**	30000
S_3	52920 **(30000)**	62040	2800 **(50000)**	67300	39960	0
S_{dummy}	0	0	0	0 **(32000)**	0	0
Demand	22000	0	0	18000	0	

Table-6: Calculate $\bar{U}_i$ and $\bar{V}_j$ (where $\bar{U}_i$ is the largest cost in row and $\bar{V}_j$ is the largest cost in column)

	D_1	D_2	D_3	D_4	D_5	Supply	$\bar{U}_i$
S_1	35480	23380**(35000)**	43280	38900	2940	10000	38900
S_2	35900	27160	45940	41700	5600**(60000)**	30000	41700
~~S_3~~	~~52920**(30000)**~~	~~62040~~	~~2800**(50000)**~~	~~67300~~	~~39960~~	0	--
~~S_{dummy}~~	~~0~~	~~0~~	~~0~~	~~0**(32000)**~~	~~0~~	0	--
Demand	22000	0	0	18000	0		
$\bar{V}_j$	35900	--	--	41700	--		

2. Compute reduced cost of each cell Δ_{ij}, where $\Delta_{ij} = c_{ij} - \left(\bar{U}_i + \bar{V}_j\right)$

1. $\Delta_{11} = c_{11} - \left(\bar{U}_1 + \bar{V}_1\right) = 35480 - (38900 + 35900) = -39320$

2. $\Delta_{14} = c_{14} - \left(\bar{U}_1 + \bar{V}_4\right) = 38900 - (38900 + 41700) = -41700$

3. $\Delta_{21} = c_{21} - \left(\bar{U}_2 + \bar{V}_1\right) = 35900 - (41700 + 35900) = -41700$

4. $\Delta_{24} = c_{24} - \left(\bar{U}_2 + \bar{V}_4\right) = 41700 - (41700 + 41700) = -41700$

	D_1	D_2	D_3	D_4	D_5	Supply	$\bar{U}_i$
S_1	35480 [-39320]	23380**(35000)**	43280	38900 [-41700]	2940	10000	38900
S_2	35900 [-41700]	27160	45940	41700 [-41700]	5600**(60000)**	30000	41700
~~S_3~~	~~52920**(30000)**~~	~~62040~~	~~2800**(50000)**~~	~~67300~~	~~39960~~	0	--
~~S_{dummy}~~	~~0~~	~~0~~	~~0~~	~~0**(32000)**~~	~~0~~	0	--
Demand	22000	0	0	18000	0		
$\bar{V}_j$	35900	--	--	41700	--		

The most negative Δ_{ij} is -41700 in cell S_2D_1

The allocation to this cell is min(30000,22000) = **22000**.
This satisfies the entire demand of D_1 and leaves 30000 - 22000=8000 units with S_2

Table-6: This leads to the following table

	D_1	D_2	D_3	D_4	D_5	Supply
S_1	35480	23380 **(35000)**	43280	38900	2940	10000
S_2	35900 **(22000)**	27160	45940	41700	5600 **(60000)**	8000
S_3	52920 **(30000)**	62040	2800 **(50000)**	67300	39960	0
S_{dummy}	0	0	0	0 **(32000)**	0	0
Demand	0	0	0	18000	0	

2. Compute reduced cost of each cell Δ_{ij}, where $\Delta_{ij} = c_{ij} - \left(\bar{U}_i + \bar{V}_j\right)$

1. $\Delta_{11} = c_{11} - \left(\bar{U}_1 + \bar{V}_1\right) = 35480 - (38900 + 35900) = -39320$

2. $\Delta_{14} = c_{14} - \left(\bar{U}_1 + \bar{V}_4\right) = 38900 - (38900 + 41700) = -41700$

3. $\Delta_{21} = c_{21} - \left(\bar{U}_2 + \bar{V}_1\right) = 35900 - (41700 + 35900) = -41700$

4. $\Delta_{24} = c_{24} - \left(\bar{U}_2 + \bar{V}_4\right) = 41700 - (41700 + 41700) = -41700$

	D_1	D_2	D_3	D_4	D_5	Supply	$\bar{U}_i$
S_1	35480 [-39320]	23380(**35000**)	43280	38900 [-41700]	2940	10000	38900
S_2	35900 [-41700]	27160	45940	41700 [-41700]	5600(**60000**)	30000	41700
S_3	52920(**30000**)	62040	2800(**50000**)	67300	39960	0	--
S_{dummy}	0	0	0	0(**32000**)	0	0	--
Demand	22000	0	0	18000	0		
$\bar{V}_j$	35900	--	--	41700	--		

The most negative Δ_{ij} is -41700 in cell S_2D_1

The allocation to this cell is min(30000,22000) = **22000**.
This satisfies the entire demand of D_1 and leaves 30000 - 22000=8000 units with S_2

Table-6: This leads to the following table

	D_1	D_2	D_3	D_4	D_5	Supply
S_1	35480	23380 **(35000)**	43280	38900	2940	10000
S_2	35900 **(22000)**	27160	45940	41700	5600 **(60000)**	8000
S_3	52920 **(30000)**	62040	2800 **(50000)**	67300	39960	0
S_{dummy}	0	0	0	0 **(32000)**	0	0
Demand	0	0	0	18000	0	

Table-7: Calculate $\bar{U}_i$ and $\bar{V}_j$ (where $\bar{U}_i$ is the largest cost in row and $\bar{V}_j$ is the largest cost in column)

	D_1	D_2	D_3	D_4	D_5	Supply	$\bar{U}_i$
S_1	35480	23380(**35000**)	43280	38900	2940	10000	38900
S_2	35900(**22000**)	27160	45940	41700	5600(**60000**)	8000	41700
S_3	52920(**30000**)	62040	2800(**50000**)	67300	39960	0	--
S_{dummy}	0	0	0	0(**32000**)	0	0	--
Demand	0	0	0	18000	0		
$\bar{V}_j$	--	--	--	41700	--		

2. Compute reduced cost of each cell Δ_{ij}, where $\Delta_{ij} = c_{ij} - \left(\bar{U}_i + \bar{V}_j\right)$

1. $\Delta_{14} = c_{14} - \left(\bar{U}_1 + \bar{V}_4\right) = 38900 - (38900 + 41700) = -41700$

2. $\Delta_{24} = c_{24} - \left(\bar{U}_2 + \bar{V}_4\right) = 41700 - (41700 + 41700) = -41700$

	D_1	D_2	D_3	D_4	D_5	Supply	$\bar{U}_i$
S_1	35480	23380(**35000**)	43280	38900 [-41700]	2940	10000	38900
S_2	35900(**22000**)	27160	45940	41700 [-41700]	5600(**60000**)	8000	41700
S_3	52920(**30000**)	62040	2800(**50000**)	67300	39960	0	--
S_{dummy}	0	0	0	0(**32000**)	0	0	--
Demand	0	0	0	18000	0		
$\bar{V}_j$	--	--	--	41700	--		

The most negative Δ_{ij} is -41700 in cell S_1D_4

The allocation to this cell is min(10000,18000) = **10000**.
This exhausts the capacity of S_1 and leaves 18000 - 10000=8000 units with D_4

Table-7: This leads to the following table

	D_1	D_2	D_3	D_4	D_5	Supply
S_1	35480	23380 **(35000)**	43280	38900 **(10000)**	2940	0
S_2	35900 **(22000)**	27160	45940	41700	5600 **(60000)**	8000
S_3	52920 **(30000)**	62040	2800 **(50000)**	67300	39960	0
S_{dummy}	0	0	0	0 **(32000)**	0	0
Demand	0	0	0	8000	0	

Table-8: Calculate $\bar{U}_i$ and $\bar{V}_j$ (where $\bar{U}_i$ is the largest cost in row and $\bar{V}_j$ is the largest cost in column)

	D_1	D_2	D_3	D_4	D_5	Supply	$\bar{U}_i$
S_1	35480	23380(**35000**)	43280	38900(**10000**)	2940	0	--
S_2	35900(**22000**)	27160	45940	41700	5600(**60000**)	8000	41700
S_3	52920(**30000**)	62040	2800(**50000**)	67300	39960	0	--
S_{dummy}	0	0	0	0(**32000**)	0	0	--
Demand	0	0	0	8000	0		
$\bar{V}_j$	--	--	--	41700	--		

2. Compute reduced cost of each cell Δ_{ij}, where $\Delta_{ij} = c_{ij} - \left(\bar{U}_i + \bar{V}_j\right)$

1. $\Delta_{24} = c_{24} - \left(\bar{U}_2 + \bar{V}_4\right) = 41700 - (41700 + 41700) = -41700$

	D_1	D_2	D_3	D_4	D_5	Supply	$\bar{U}_i$
S_1	35480	23380(**35000**)	43280	38900(**10000**)	2940	0	--
S_2	35900(**22000**)	27160	45940	41700 [-41700]	5600(**60000**)	8000	41700
S_3	52920(**30000**)	62040	2800(**50000**)	67300	39960	0	--
S_{dummy}	0	0	0	0(**32000**)	0	0	--
Demand	0	0	0	8000	0		
$\bar{V}_j$	--	--	--	41700	--		

The most negative Δ_{ij} is -41700 in cell S_2D_4

The allocation to this cell is min(8000,8000) = **8000**.
Table-8: This leads to the following table

	D_1	D_2	D_3	D_4	D_5	Supply
S_1	35480	23380 (**35000**)	43280	38900 (**10000**)	2940	0
S_2	35900 (**22000**)	27160	45940	41700 (**8000**)	5600 (**60000**)	0
S_3	52920 (**30000**)	62040	2800 (**50000**)	67300	39960	0
S_{dummy}	0	0	0	0 (**32000**)	0	0
Demand	0	0	0	0	0	

Initial feasible solution is

	D_1	D_2	D_3	D_4	D_5	Supply
S_1	35480	23380 (**35000**)	43280	38900 (**10000**)	2940	45000
S_2	35900 (**22000**)	27160	45940	41700 (**8000**)	5600 (**60000**)	90000
S_3	52920 (**30000**)	62040	2800 (**50000**)	67300	39960	80000
S_{dummy}	0	0	0	0 (**32000**)	0	32000
Demand	52000	35000	50000	50000	60000	

The minimum total transportation cost $= 23380 \times 35000 + 38900 \times 10000 + 35900 \times 22000 + 41700 \times 8000 + 5600 \times 60000 + 52920 \times 30000 + 2800 \times 50000 + 0 \times 32000 = 4394300000$

Here, the number of allocated cells = 8 is equal to m + n - 1 = 4 + 5 - 1 = 8
$\therefore$ This solution is non-degenerate

Optimality test using stepping stone method...
Allocation Table is

	D_1	D_2	D_3	D_4	D_5	Supply
S_1	35480	23380 (**35000**)	43280	38900 (**10000**)	2940	45000
S_2	35900 (**22000**)	27160	45940	41700 (**8000**)	5600 (**60000**)	90000
S_3	52920 (**30000**)	62040	2800 (**50000**)	67300	39960	80000
S_{dummy}	0	0	0	0 (**32000**)	0	32000
Demand	52000	35000	50000	50000	60000	

Iteration-1 of optimality test

1. Create closed loop for unoccupied cells, we get

Unoccupied cell	Closed path	Net cost change
S_1D_1	$S_1D_1 \to S_1D_4 \to S_2D_4 \to S_2D_1$	35480 - 38900 + 41700 - 35900=2380
S_1D_3	$S_1D_3 \to S_1D_4 \to S_2D_4 \to S_2D_1 \to S_3D_1 \to S_3D_3$	43280 - 38900 + 41700 - 35900 + 52920 - 2800=60300
S_1D_5	$S_1D_5 \to S_1D_4 \to S_2D_4 \to S_2D_5$	2940 - 38900 + 41700 - 5600=140
S_2D_2	$S_2D_2 \to S_2D_4 \to S_1D_4 \to S_1D_2$	27160 - 41700 + 38900 - 23380=980
S_2D_3	$S_2D_3 \to S_2D_1 \to S_3D_1 \to S_3D_3$	45940 - 35900 + 52920 - 2800=60160
S_3D_2	$S_3D_2 \to S_3D_1 \to S_2D_1 \to S_2D_4 \to S_1D_4 \to S_1D_2$	62040 - 52920 + 35900 - 41700 + 38900 - 23380=18840
S_3D_4	$S_3D_4 \to S_3D_1 \to S_2D_1 \to S_2D_4$	67300 - 52920 + 35900 - 41700=8580
S_3D_5	$S_3D_5 \to S_3D_1 \to S_2D_1 \to S_2D_5$	39960 - 52920 + 35900 - 5600=17340
$S_{dummy}D_1$	$S_{dummy}D_1 \to S_{dummy}D_4 \to S_2D_4 \to S_2D_1$	0 - 0 + 41700 - 35900=5800
$S_{dummy}D_2$	$S_{dummy}D_2 \to S_{dummy}D_4 \to S_1D_4 \to S_1D_2$	0 - 0 + 38900 - 23380=15520
$S_{dummy}D_3$	$S_{dummy}D_3 \to S_{dummy}D_4 \to S_2D_4 \to S_2D_1 \to S_3D_1 \to S_3D_3$	0 - 0 + 41700 - 35900 + 52920 - 2800=55920
$S_{dummy}D_5$	$S_{dummy}D_5 \to S_{dummy}D_4 \to S_2D_4 \to S_2D_5$	0 - 0 + 41700 - 5600=36100

Since all net cost change ≥ 0

So final optimal solution is arrived.

	D_1	D_2	D_3	D_4	D_5	Supply
S_1	35480	23380 (**35000**)	43280	38900 (**10000**)	2940	45000
S_2	35900 (**22000**)	27160	45940	41700 (**8000**)	5600 (**60000**)	90000
S_3	52920 (**30000**)	62040	2800 (**50000**)	67300	39960	80000
S_{dummy}	0	0	0	0 (**32000**)	0	32000
Demand	52000	35000	50000	50000	60000	

The minimum total transportation cost $= 23380 \times 35000 + 38900 \times 10000 + 35900 \times 22000 + 41700 \times 8000 + 5600 \times 60000 + 52920 \times 30000 + 2800 \times 50000 + 0 \times 32000 = 4394300000$

The Excel Solver, however, deals with the Transportation problem as a Linear programming problem.So, the Optimum Solution is found at once.

	Sylhet	Bogura	Chittagong	Khulna	Gazipur			
Konabari	35480	23380	43280	38900	2940	45000		
Bagher Bazar	35900	27160	45940	41700	5600	90000	total supply=	215000
Chittagong	52920	62040	2800	67300	39960	80000	total demand=	247000
Demand(Cartoons)	52000	35000	50000	50000	60000			

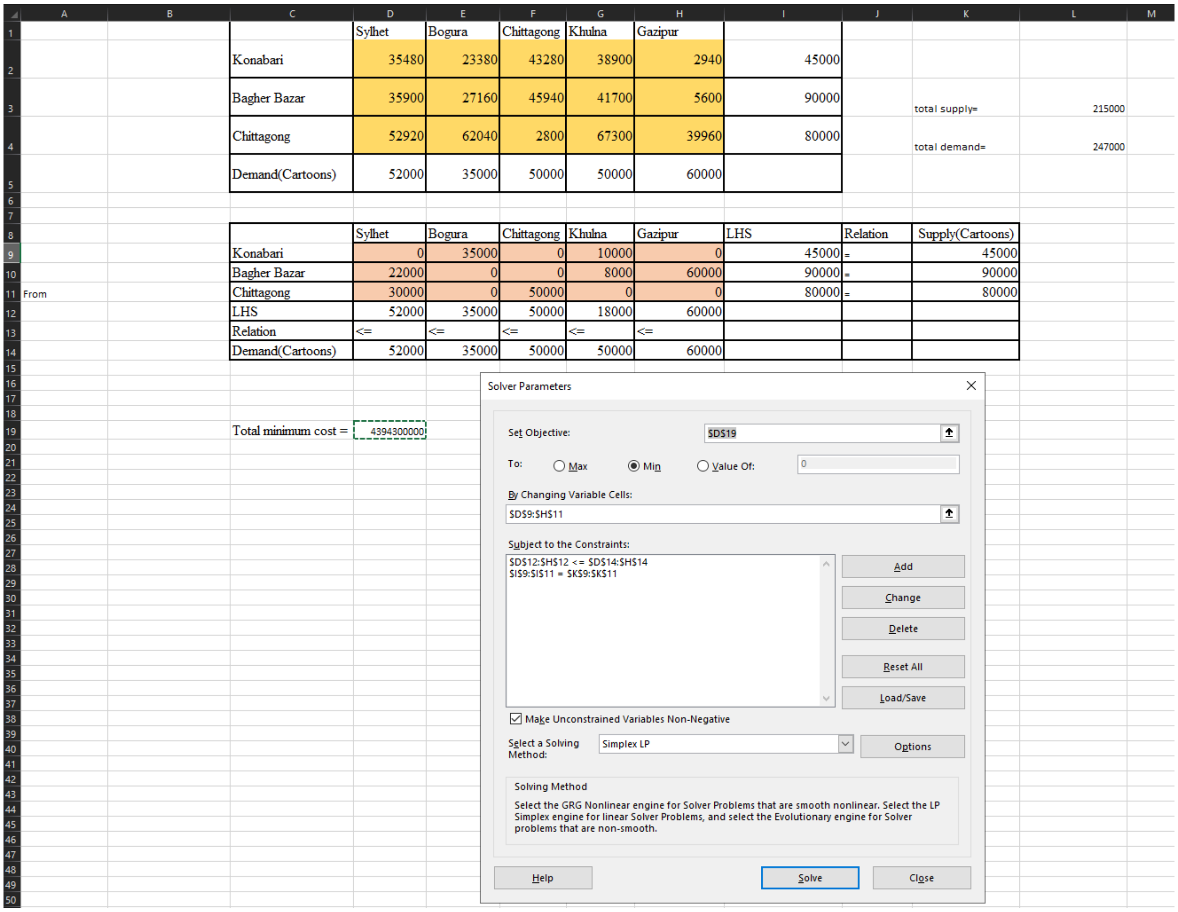
	Sylhet	Bogura	Chittagong	Khulna	Gazipur	LHS	Relation	Supply(Cartoons)
Konabari	0	35000	0	10000	0	45000	=	45000
Bagher Bazar	22000	0	0	8000	60000	90000	=	90000
Chittagong	30000	0	50000	0	0	80000	=	80000
LHS	52000	35000	50000	18000	60000			
Relation	<=	<=	<=	<=	<=			
Demand(Cartoons)	52000	35000	50000	50000	60000			

Optimal Solution(Allocation)

Total minimum cost = 4394300000 → Optimum value of Objective Function

- The Excel file effectively acts as a template for solving transportation problem.
- The table above contatins the unit cost for transportation.
- The table below is The Variable cells that are changed(contains the optimum allocations).
- The LHS column is the Summation of all Allocation from a factory which is related to maximum supply capacity by equality constraint
- The LHS row is the Summation of all Allocation to a warehouse which is related to maximum supply capacity by less than/equality constraint
- The cell containing minimum(Optimum) Cost is the sumproduct of the variable cells form the bottom table and unit cost cells from the table on the top as

$$Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$



	Sylhet	Bogura	Chittagong	Khulna	Gazipur	
Konabari	35480	23380	43280	38900	2940	45000
Bagher Bazar	35900	27160	45940	41700	5600	90000
Chittagong	52920	62040	2800	67300	39960	80000
Demand(Cartoons)	52000	35000	50000	50000	60000	

total supply= 215000

total demand= 247000

	Sylhet	Bogura	Chittagong	Khulna	Gazipur	LHS	Relation	Supply(Cartoons)
Konabari	0	35000	0	10000	0	45000	=	45000
Bagher Bazar	22000	0	0	8000	60000	90000	=	90000
Chittagong	30000	0	50000	0	0	80000	=	80000
LHS	52000	35000	50000	18000	60000			
Relation	<=	<=	<=	<=	<=			
Demand(Cartoons)	52000	35000	50000	50000	60000			

From

Total minimum cost = 4394300000

The Optimum solution is as stated below:

- 22000 units is transported from Bagher Bazar to Sylhet
- 8000 units is transported from Bagher Bazar to Khulna
- 60000 units is transported from Bagher Bazar to Gazipur
- 35000 units is transported from Konabari to Bogura
- 10000 units is transported from Konabari to Khulna
- 30000 units is transported from Chittagong to Sylhet
- 50000 units is transported from Chittagong to Chittagong
- The allocations between other factories and warehouses are null for the optimal solution

Conclusion

While conducting our project, we went through the usual phases of an operations research study. Firstly, we defined the problem, and then gathered relevant data. To represent the problem, we formulated a mathematical model. Finally, we developed a computer-based procedure, i.e., the Excel Solver, to solve the problem from the developed model.

The most formidable phase of this OR study was the mathematical formulation of the problem in a form that's convenient for analysis. After identifying the problem, we located the most influential parameters and the variables which are involved in this problem. Then we stated verbal relationship among these variables based on gathered data. The formulated problem had total demand higher than total supply, giving us an unbalanced transportation problem.

We used Excel Solver as a computer-based Algorithm to find the optimum solution to our formulated Transportation Problem. The Solution was feasible and optimal to our given constraint, giving an assurance about the relevance of the gathered data.

While some paths had lower cost than others, they had zero allocation for the optimal solution, that is because the transportation problem solution looks at the problem as a whole and determines the correct allocation.

After completing this project work, we have gathered a great deal of knowledge which, we believe, we will be able to implement very efficiently in the future. Since OR has its applications in defense, industry and in all public system, the importance of having a clear knowledge about Operations Research is beyond description.