5T2: Sinusoidal model (2 of 3)
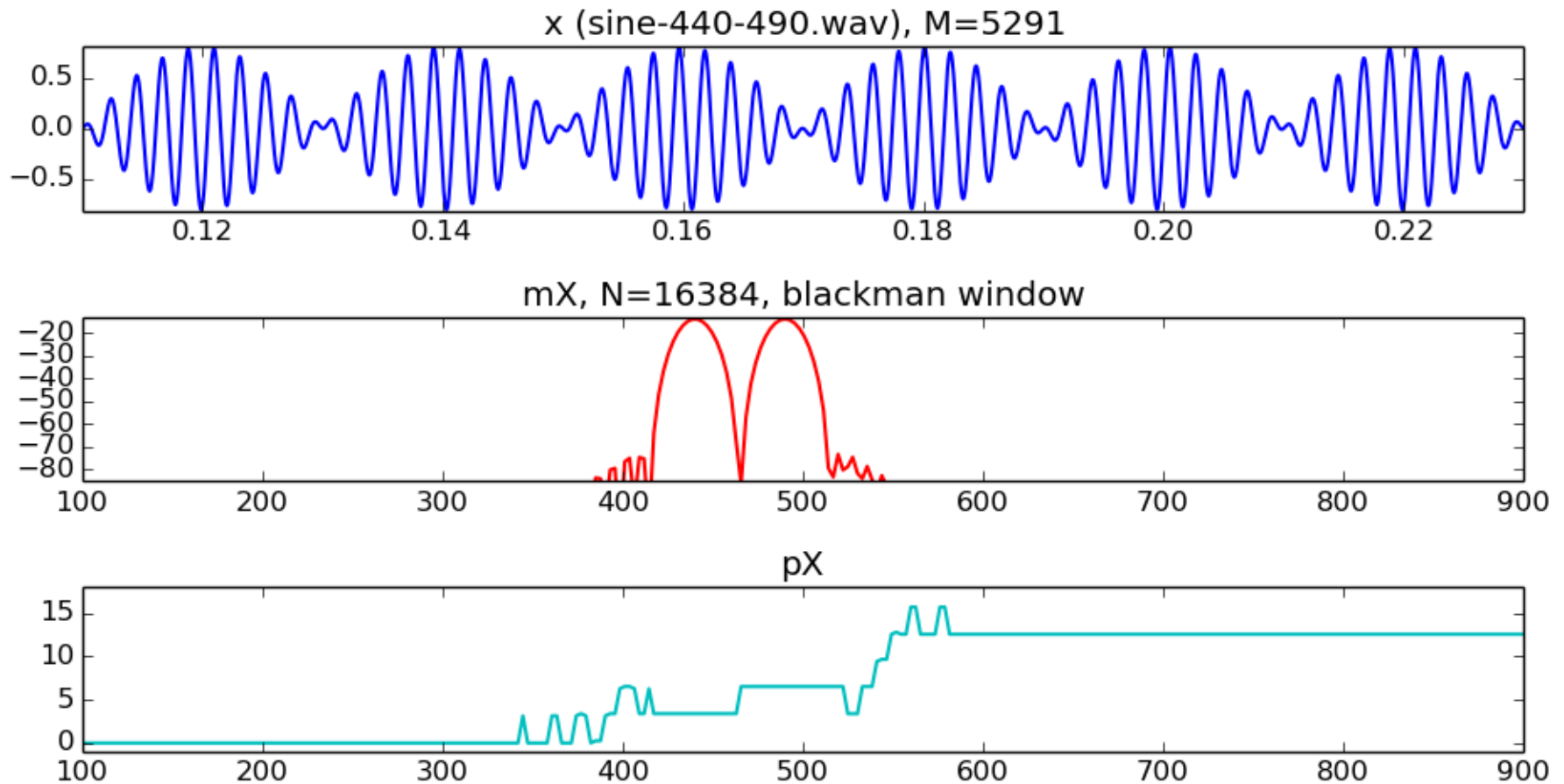
Xavier Serra

Universitat Pompeu Fabra, Barcelona

Index

- Sinusoidal model
- Peak detection
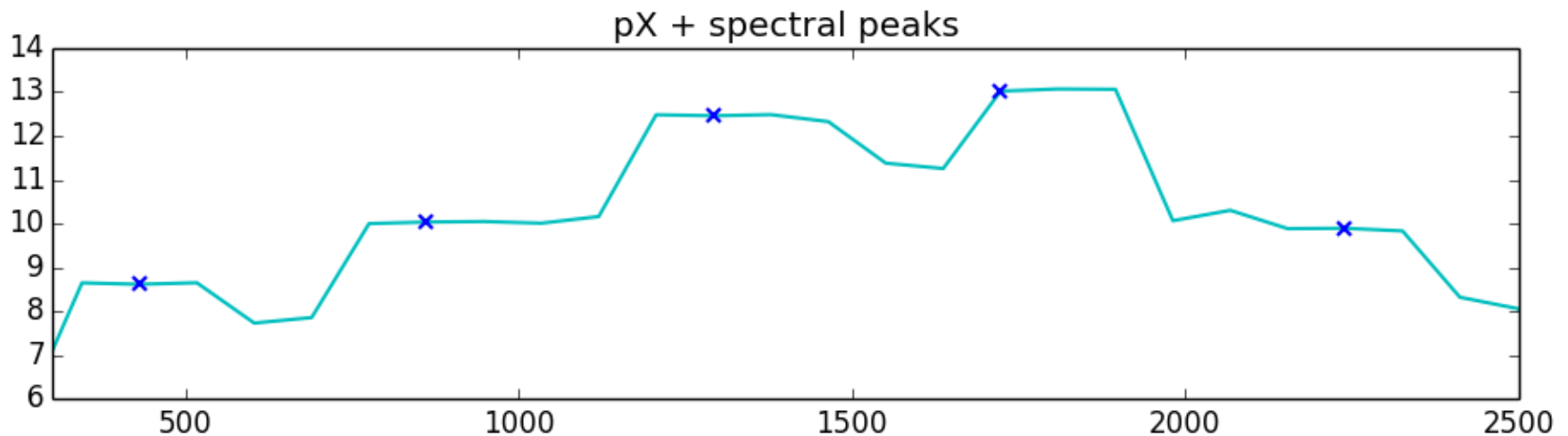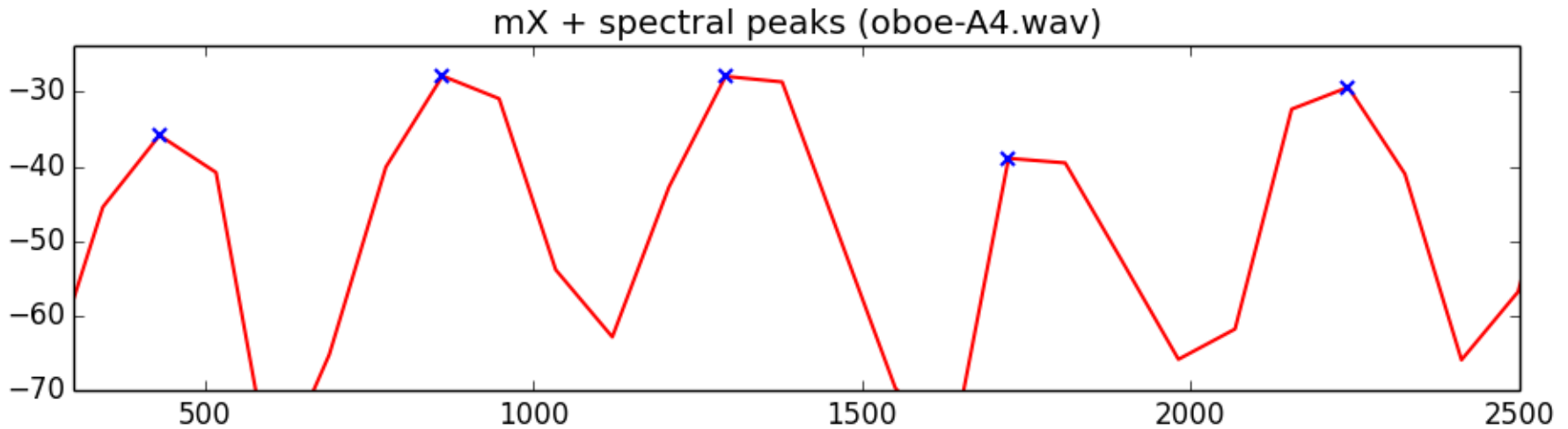- Time-varying sinewaves in spectrogram

Sinusoidal model

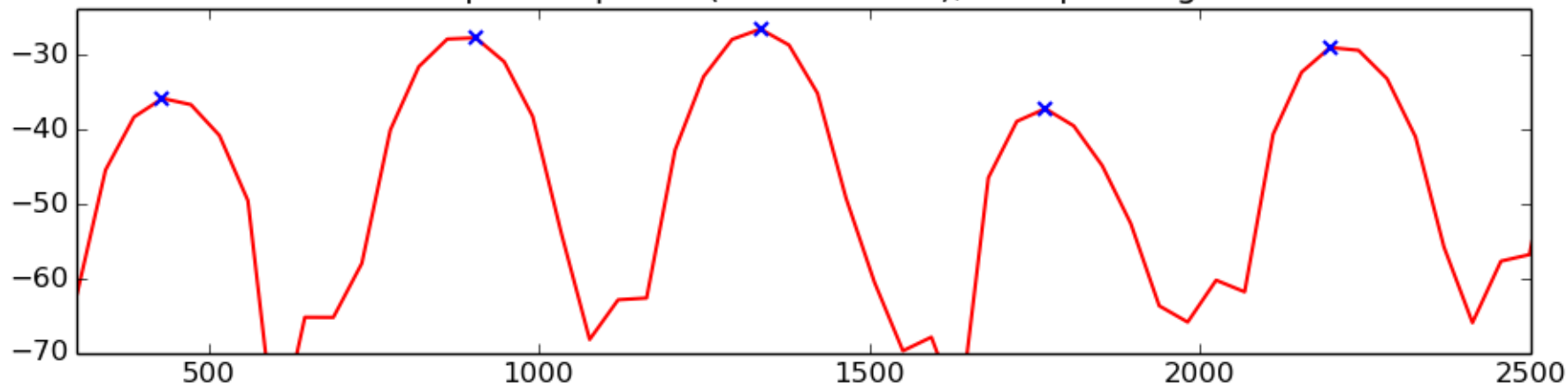$$y[n] = \sum_{r=1}^{R} A_r[n] \cos(2\pi f_r[n] n)$$

x (sine-440-490.wav), M=5291

mX, N=16384, blackman window

pX

Peak detection

$$p_r = mX[k_0] \quad \text{when} \quad mX[k_0 - 1] < mX[k_0] > mX[k_0 + 1]$$

$$mX = \text{magnitude spectrum}; r = \text{peak number}; k_0 = \text{peak location}$$
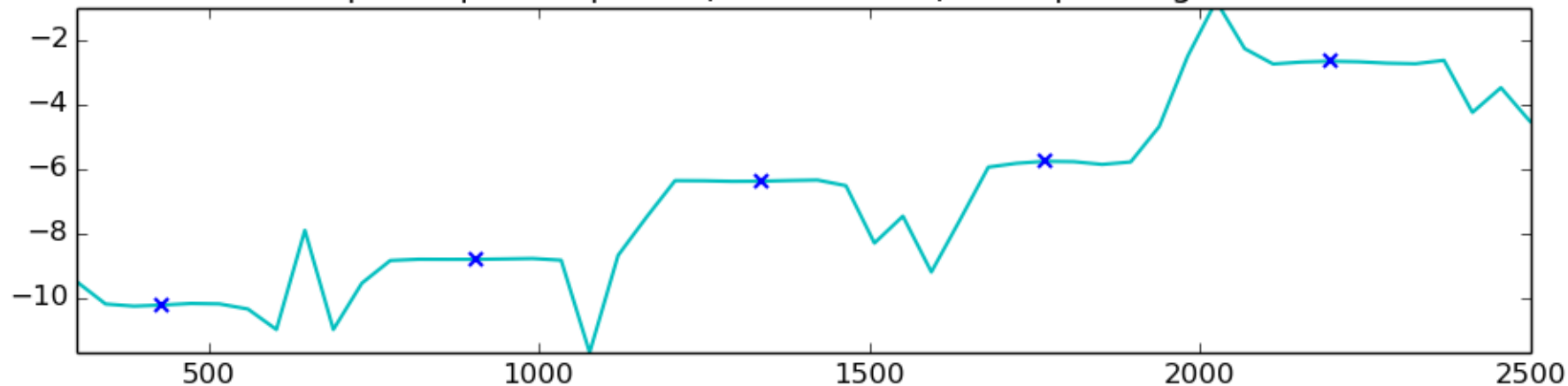
Peak detection with zero-padding

mX + spectral peaks (oboe-A4.wav), zero padding = 2
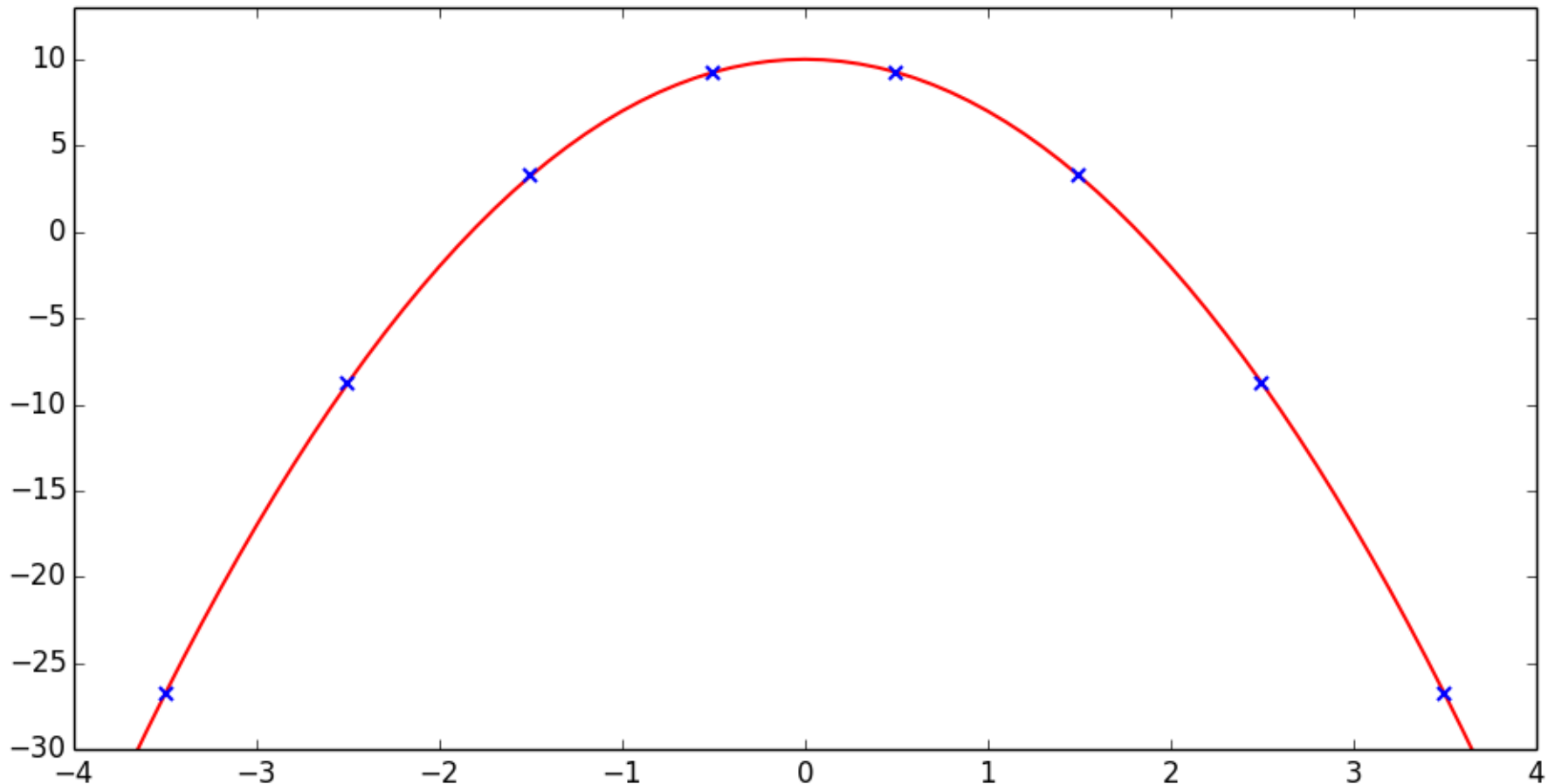
pX + spectral peaks (oboe-A4.wav), zero padding = 2

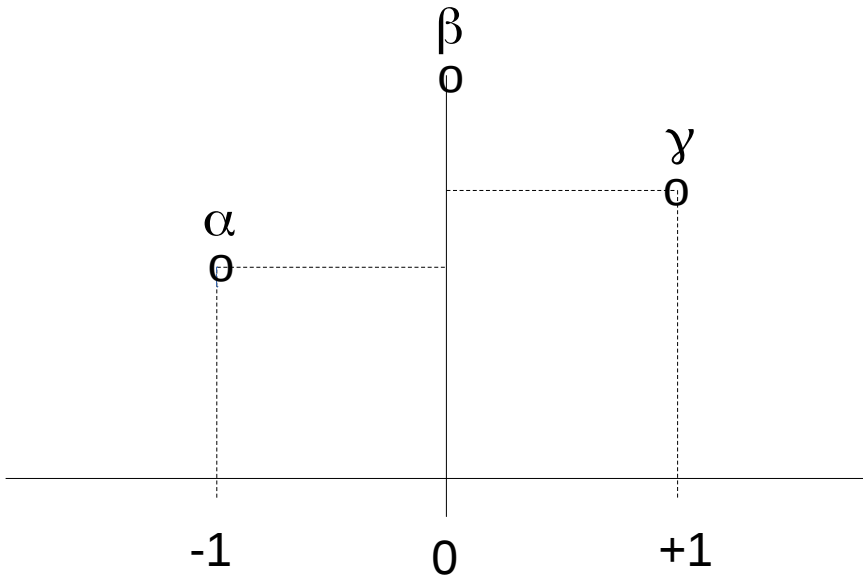# Parabola

$$x[n] = a(n - p)^2 + b$$

p: center of parabola
a: concavity measure
b: offset

Peak interpolation

$$x[-1] \;=\; \alpha \;=\; mX[k_\beta - 1]$$
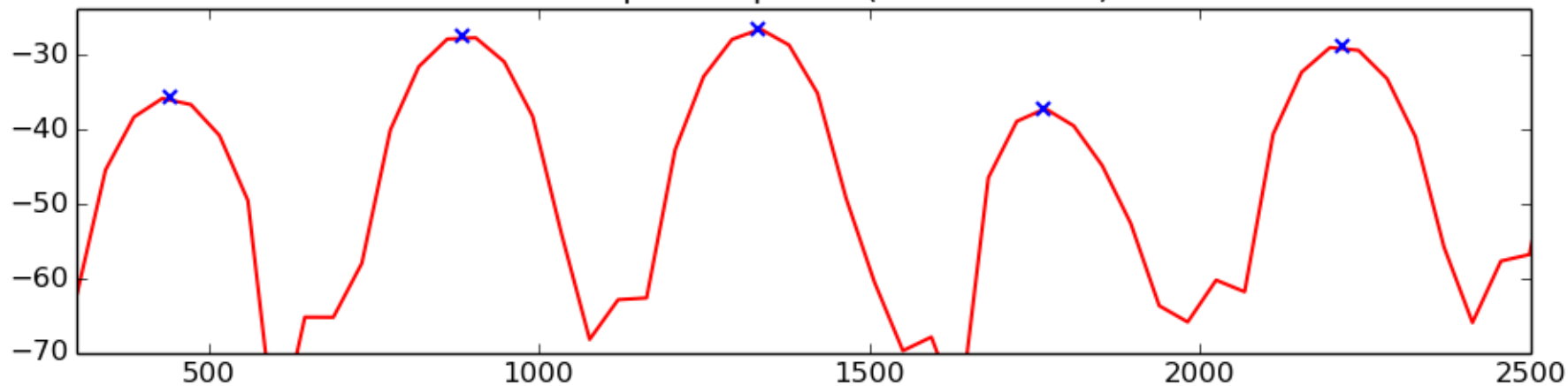$$x[0] \;=\; \beta \;=\; mX[k_\beta]$$
$$x[1] \;=\; \gamma \;=\; mX[k_\beta + 1]$$

center of the parabola: $\hat{k}_p = k_p + \dfrac{1}{2}\dfrac{\alpha - \gamma}{(\alpha - 2\beta + \gamma)}$
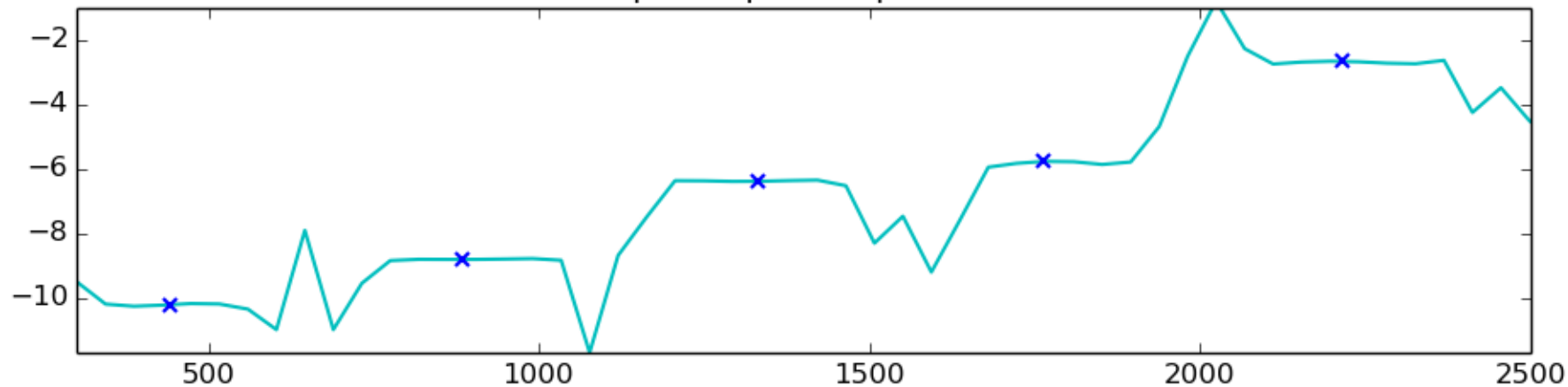
amplitude: $\hat{a} = \beta - \dfrac{1}{4}(\hat{k}_p - k_p)(\alpha - \gamma)$

Peak detection with interpolation

mX + spectral peaks (oboe-A4.wav)

pX + spectral peaks

Sinusoidal parameters from peaks

$$\hat{k}_p = k_p + \frac{0.5 * (|X[k_p-1]| - |X[k_p+1]|)}{|X[k_p-1]| - 2*|X[k_p]| + |X[k_p+1]|}$$

$$f_p = \frac{f_s * \hat{k}_p}{N}$$

$$A_p = |X[k_p]| - 0.25 * (|X[k_p-1]| - |X[k_p+1]|) * (\hat{k}_p - k_p)$$

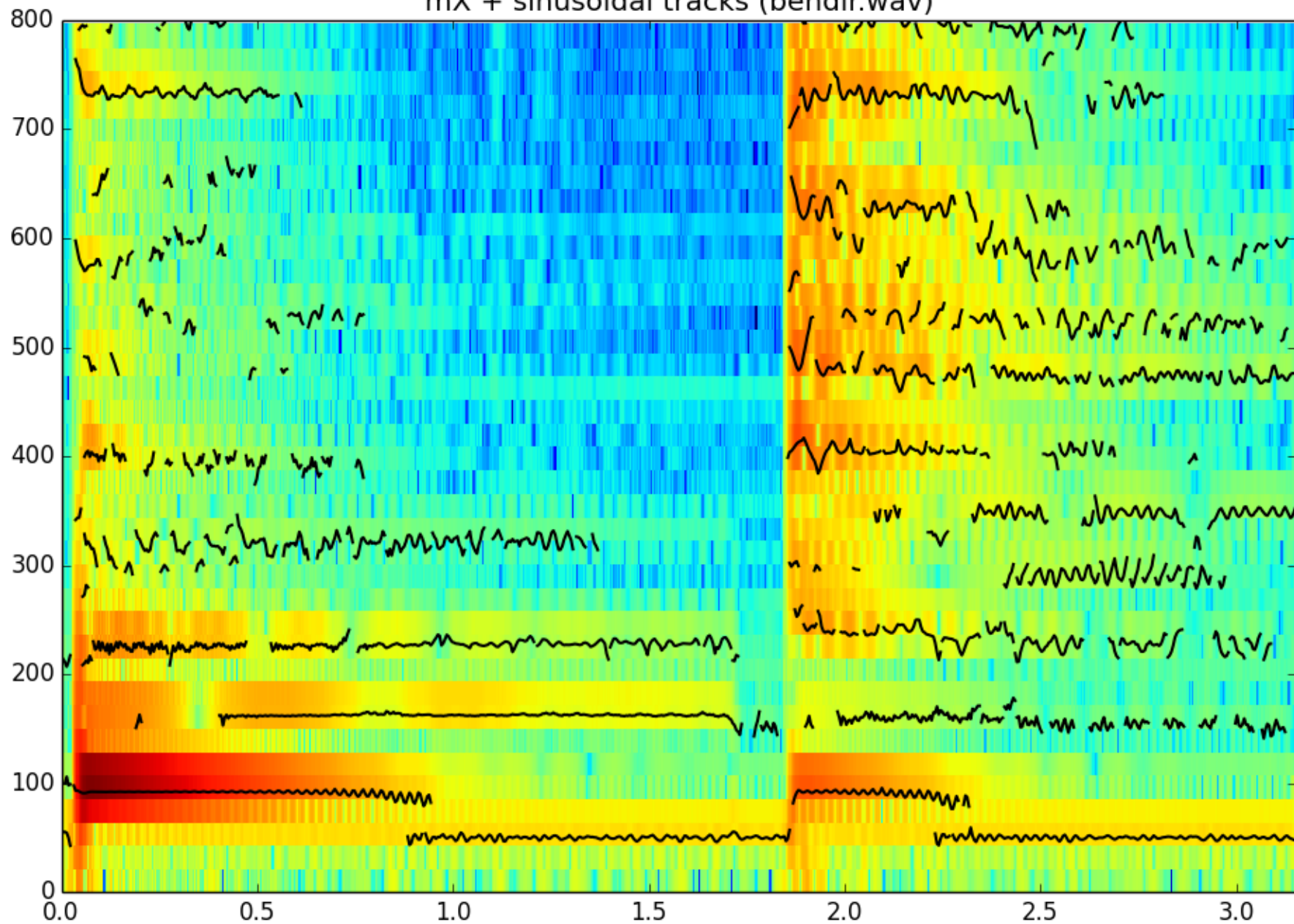$$ph_p = \angle X[\hat{k}_p]$$

Sinewaves in spectrogram

- Sinusoid → **stable** peak track in spectrogram

- Stability identified by
    - frequency and amplitude in succesive frames
    - phase derivative in time/freq

Condition for a peak f_p of frame l to be part of a track t :
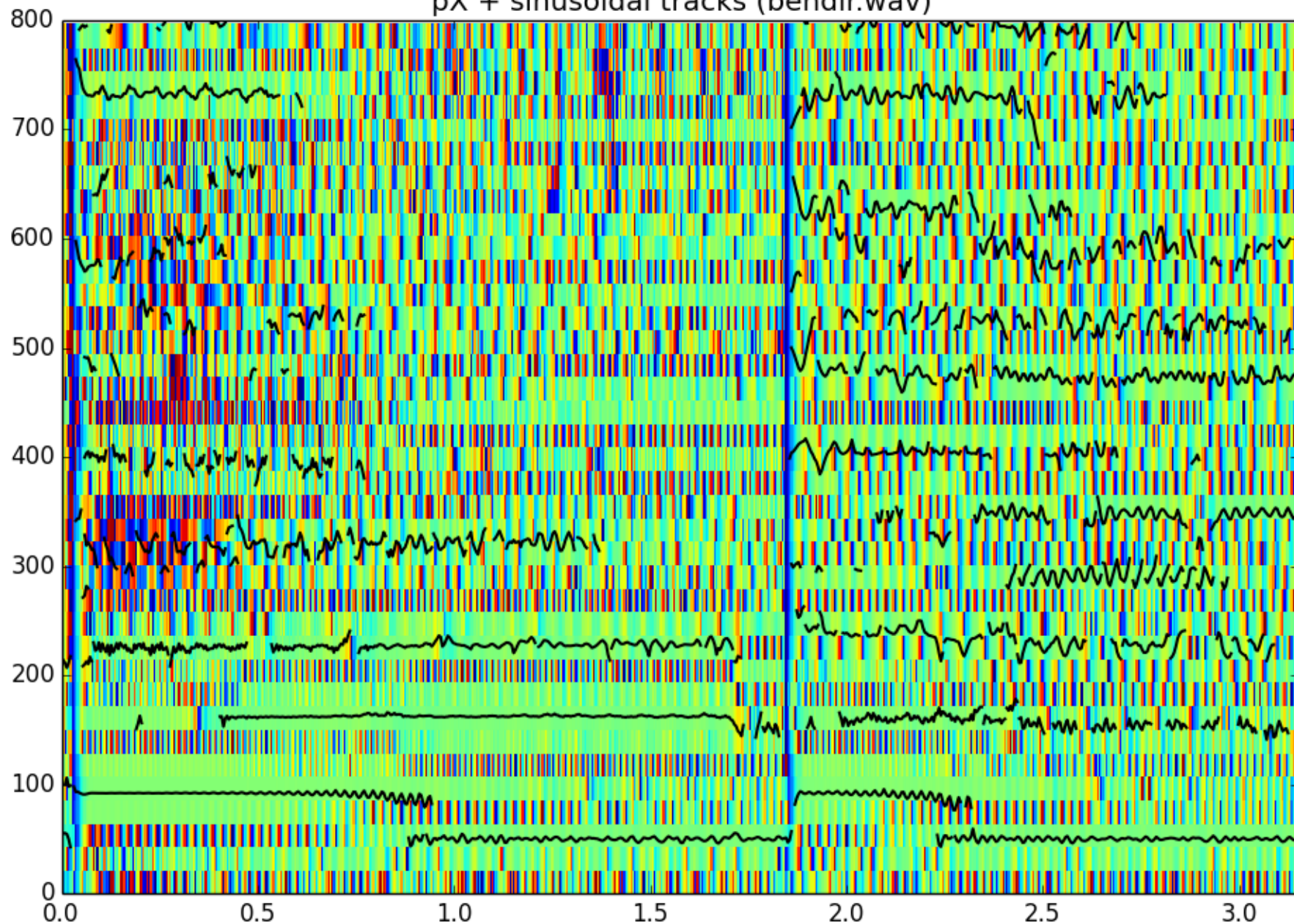
$$f_p[l] = f_t[l] \ \text{if} \ \left(| f_p[l] - f_t[l-1] \| \ < \ threshold \right)$$
$$\text{and if exists} \ \ f_t[l-2], f_t[l-3], ..., f_t[l-L]$$

mX + sinusoidal tracks (bendir.wav)

pX + sinusoidal tracks (bendir.wav)

References and credits

- More information in:
  http://en.wikipedia.org/wiki/Sinusoidal_model

- Reference on sinusoidal modeling by Julius O. Smith:
  https://ccrma.stanford.edu/~jos/sasp/Spectrum_Analysis_
  Sinusoids.html

- Sounds from:
  http://www.freesound.org/people/xserra/packs/13038/

- Slides released under CC Attribution-Noncommercial-
  Share Alike license and code under the Affero GPL
  license; available from https://github.com/MTG/sms-tools

5T2: Sinusoidal model (2 of 3)

Xavier Serra

Universitat Pompeu Fabra, Barcelona