

5T3: Sinusoidal Model

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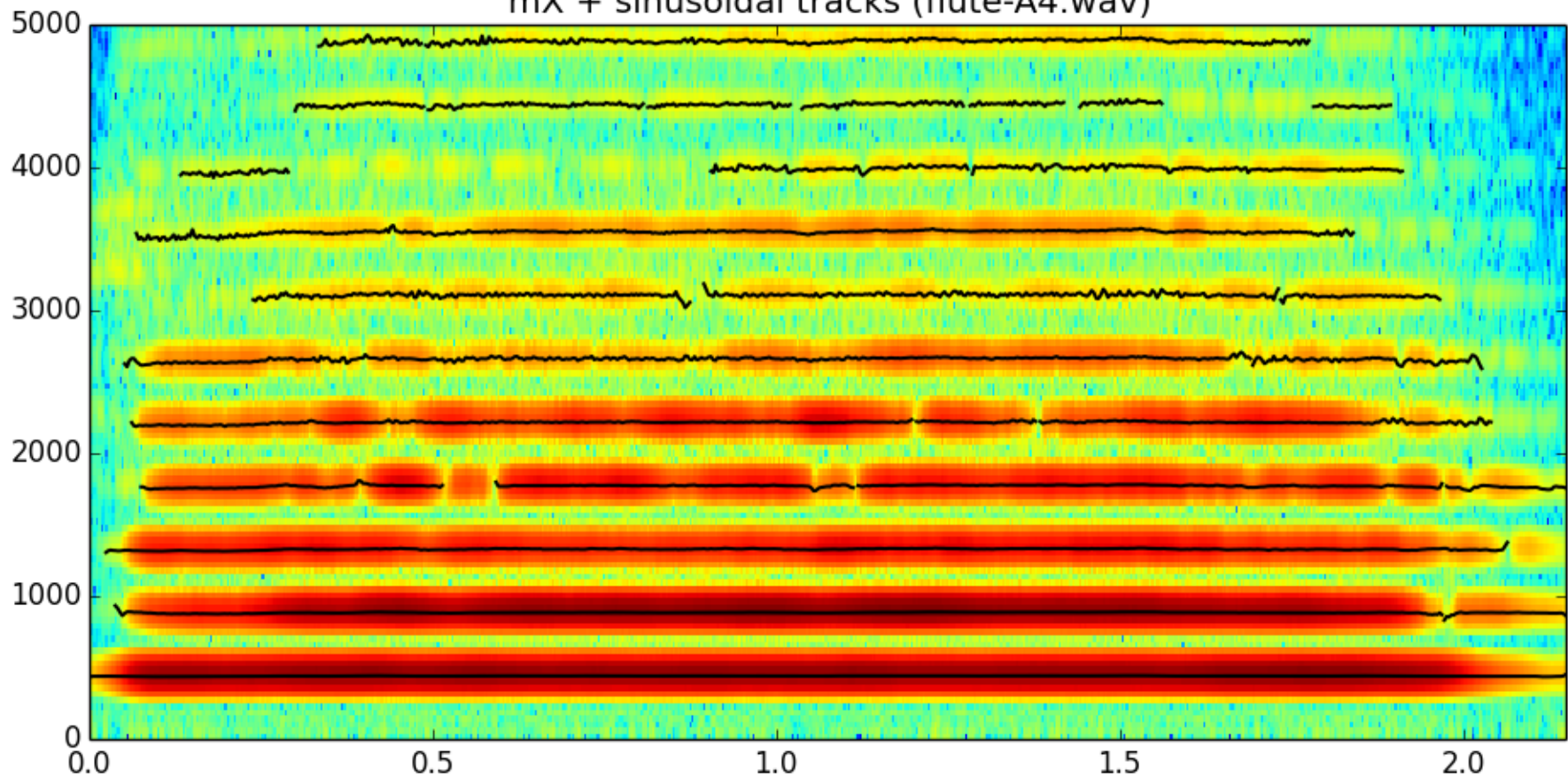
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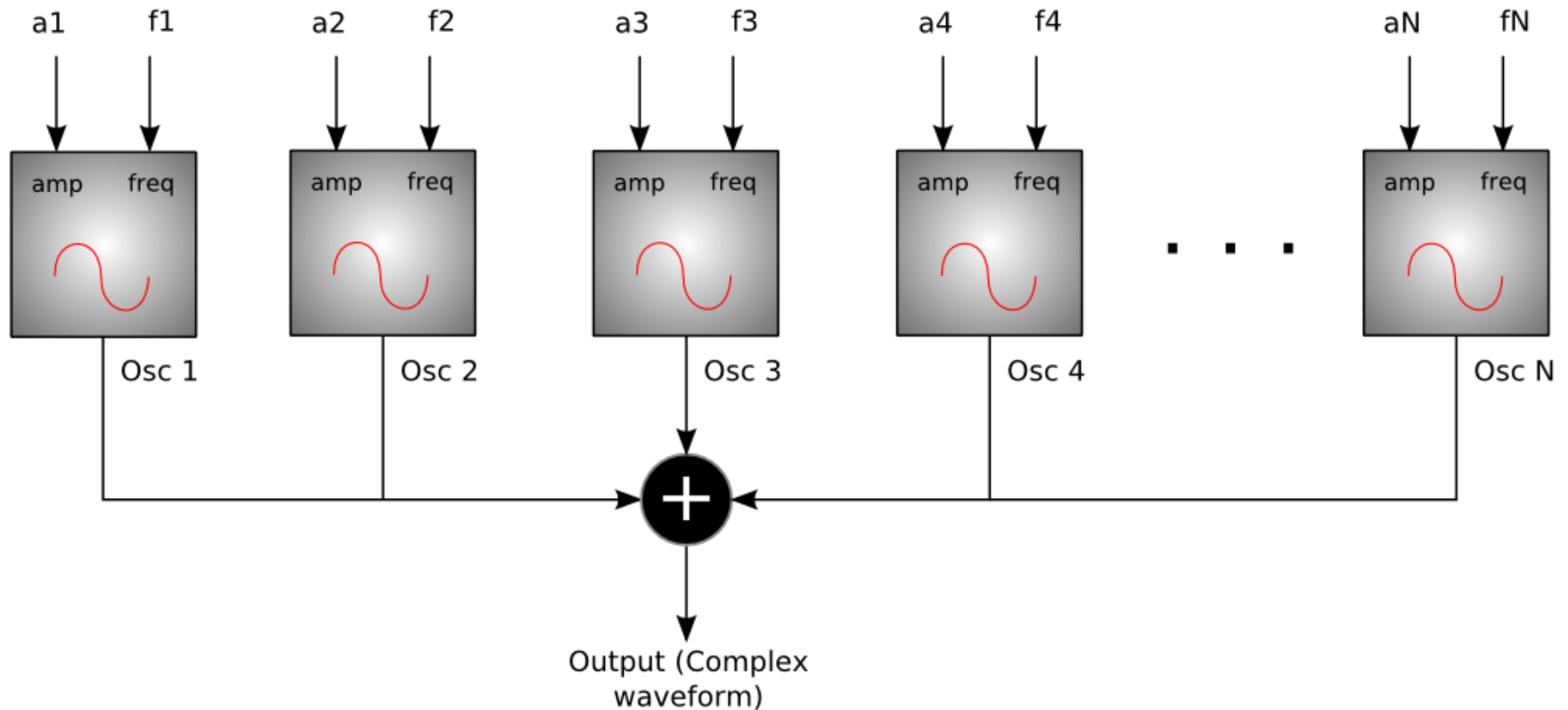
Sinusoidal model

$$y[n] = \sum_{r=1}^R A_r[n] \cos(2\pi f_r[n]n)$$

mX + sinusoidal tracks (flute-A4.wav)



Sinusoidal (additive) synthesis

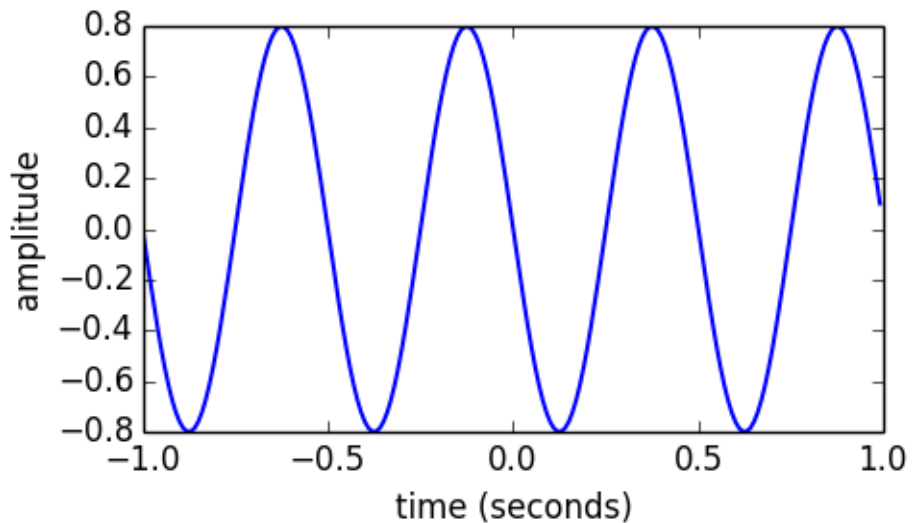


Sinusoidal synthesis

$$y[n] = A_r[n] \cos(2\pi f_r[n]n + \phi_r)$$

$A_r[n]$: instantaneous amplitude ; $f_r[n]$: instantaneous frequency
 ϕ_r : initial phase

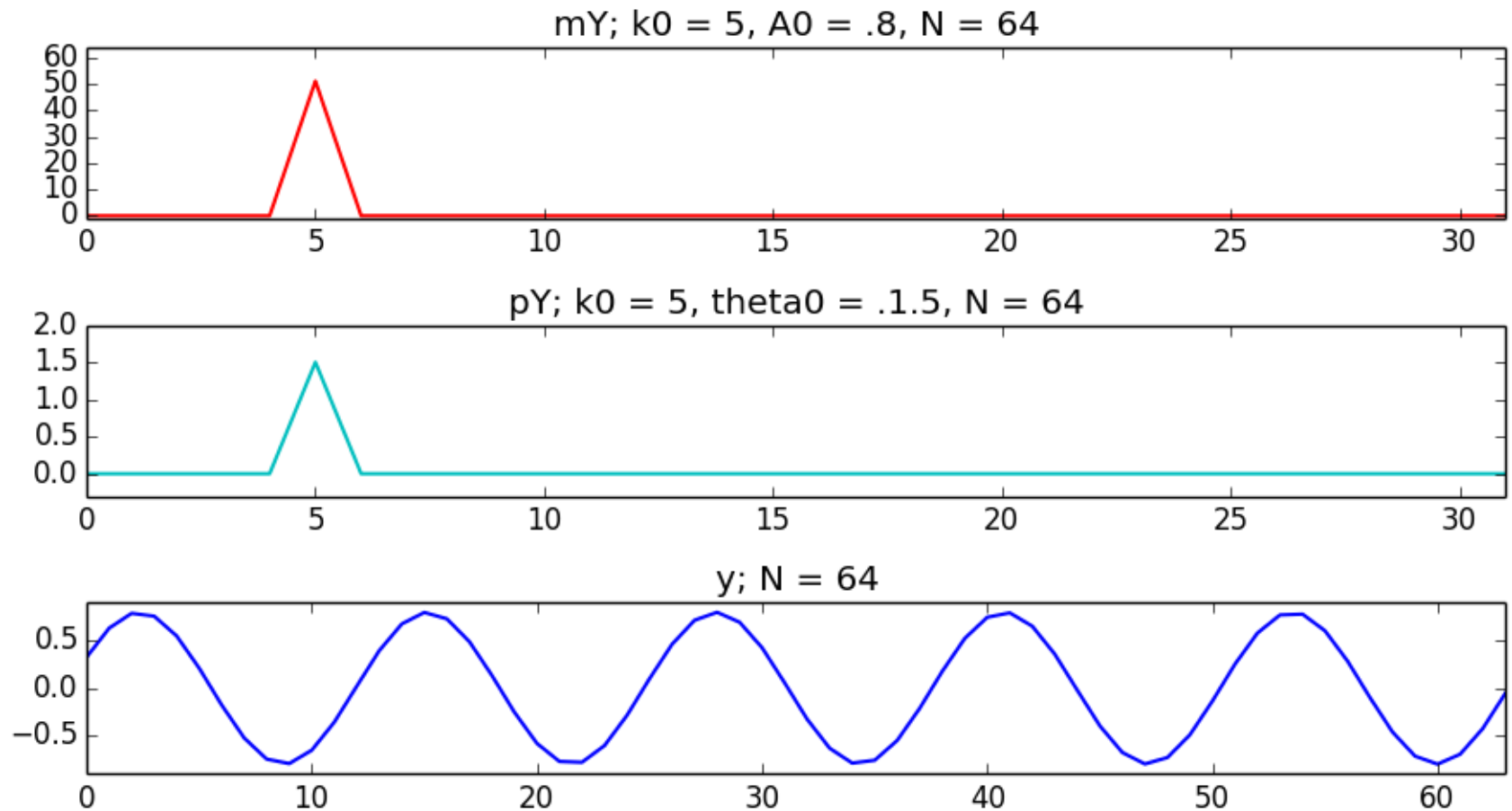
```
Ar = .8  
fr = 2.0  
phi = pi/2  
fs = 100  
t = arange(-1, 1, 1.0/fs)  
x = Ar * cos(2*pi*fr*t+phi)
```



Sinusoidal synthesis: discrete frequency

$$y[n] = \text{IDFT} \left(mY[k] * e^{j * pY[k]} \right)$$

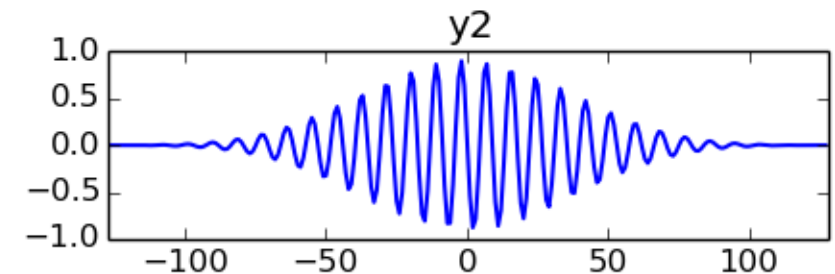
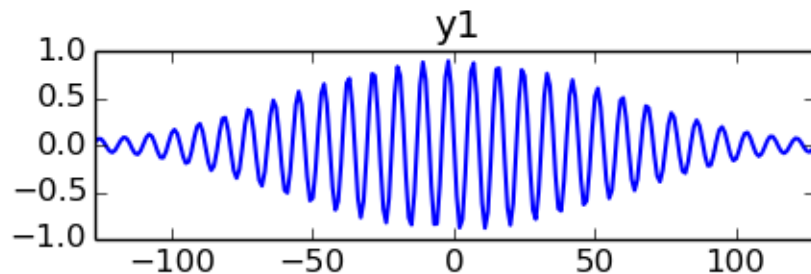
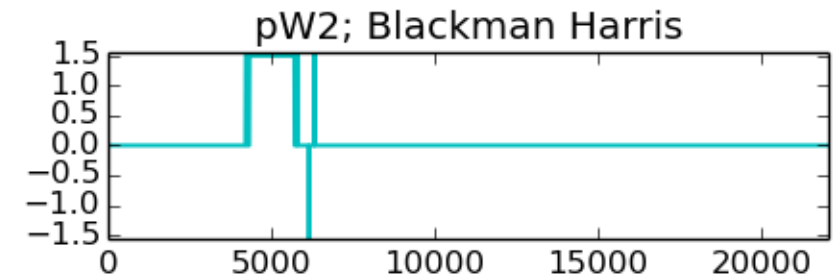
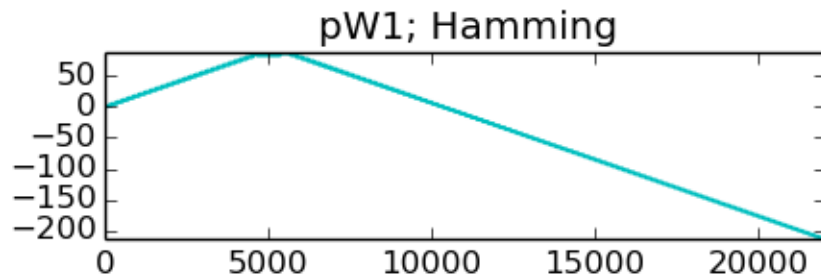
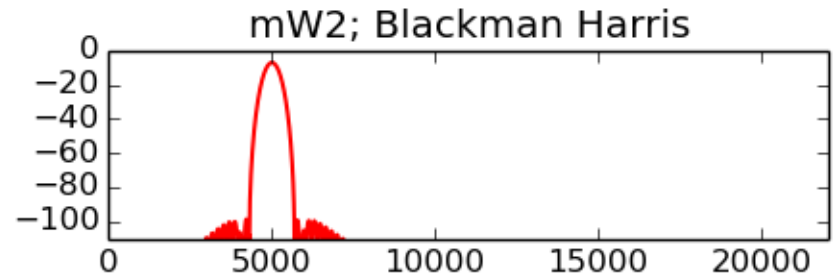
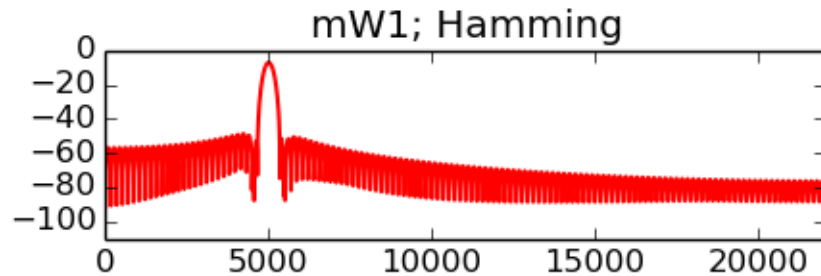
$mY[k] = A_0$ for $k = k_0$ and 0 for $k \neq k_0$; $pY[k] = \phi_0$ for $k = k_0$ and 0 for $k \neq k_0$



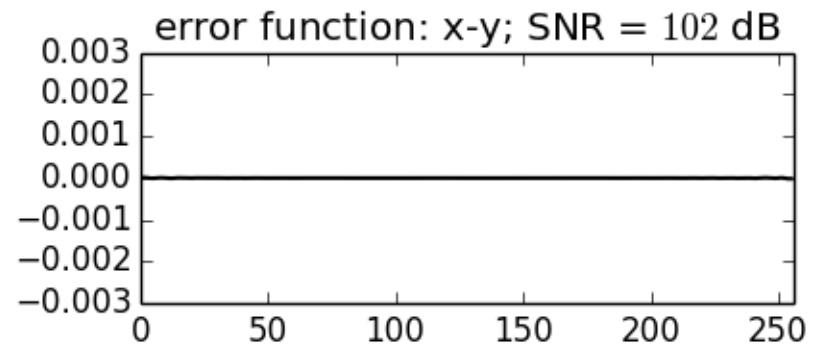
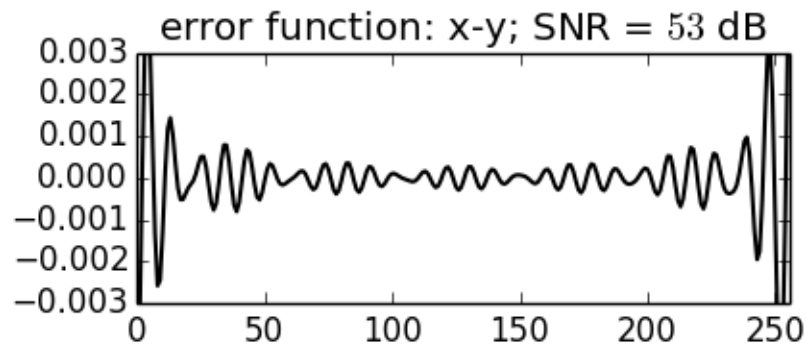
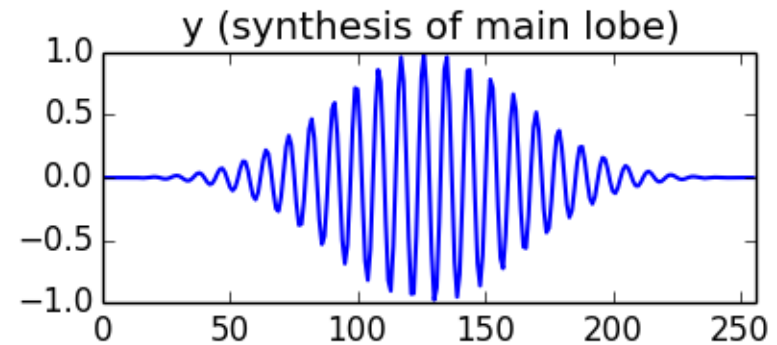
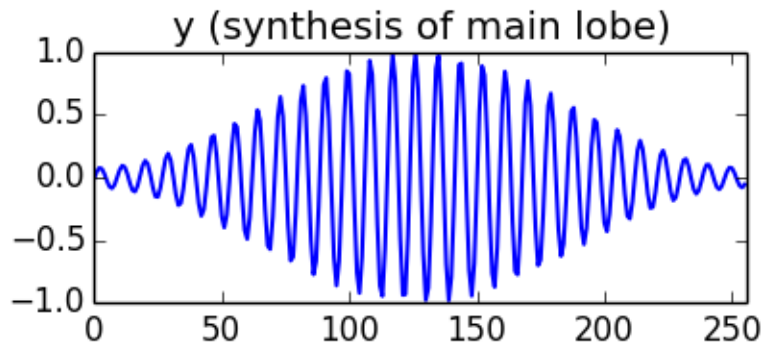
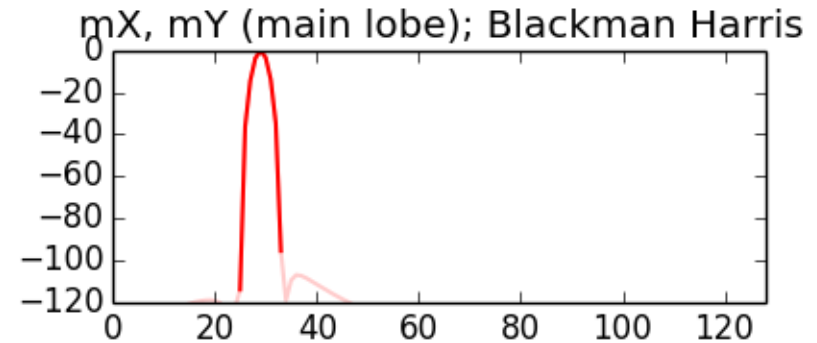
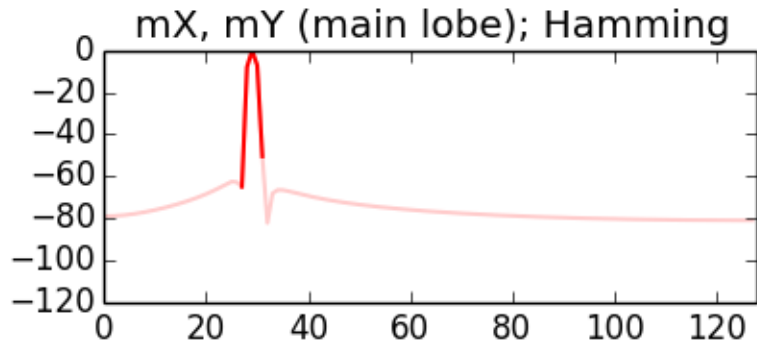
Sinusoidal synthesis: any frequency

$$y[n] = \text{IDFT} \left(A_0 * mW[k - k_0] * e^{j * (pW[k - k_0] + \phi_0)} \right)$$

$mW[k]$, $pW[k]$ magnitude and phase spectrum of window



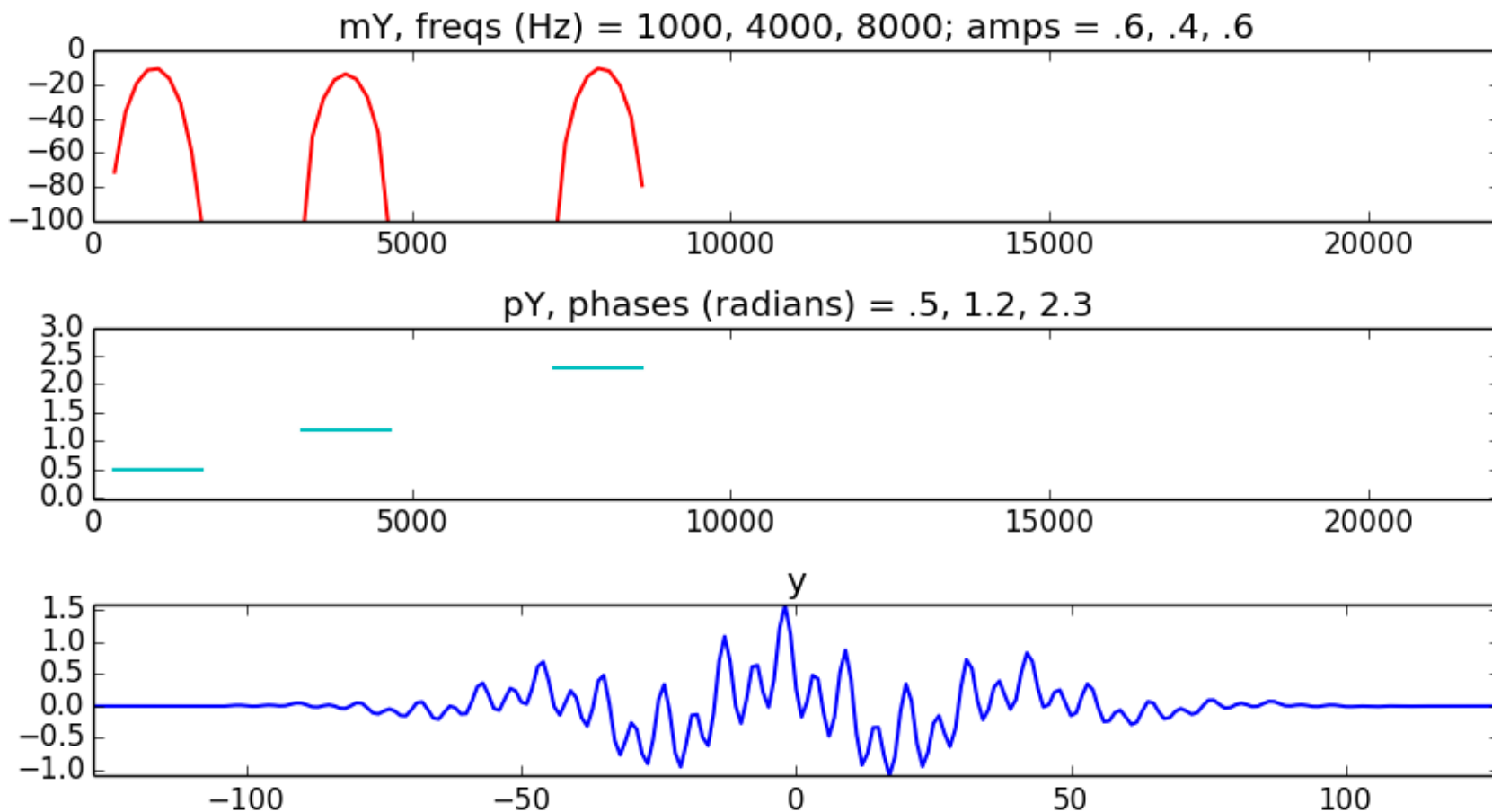
Sinusoidal synthesis: only main lobe



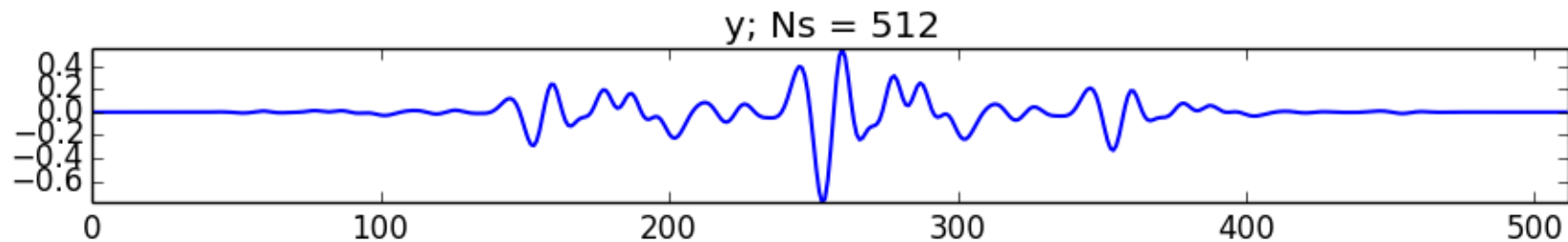
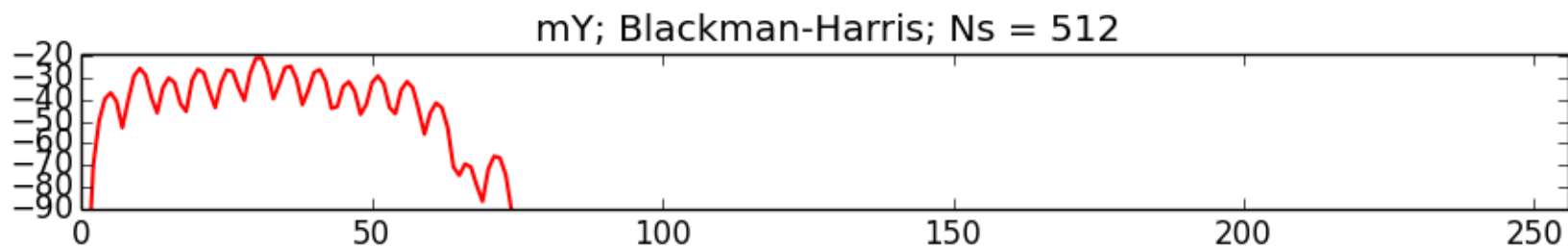
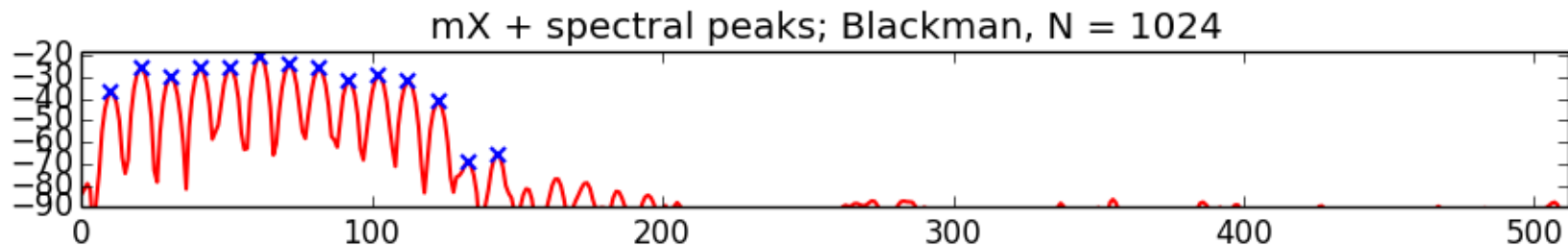
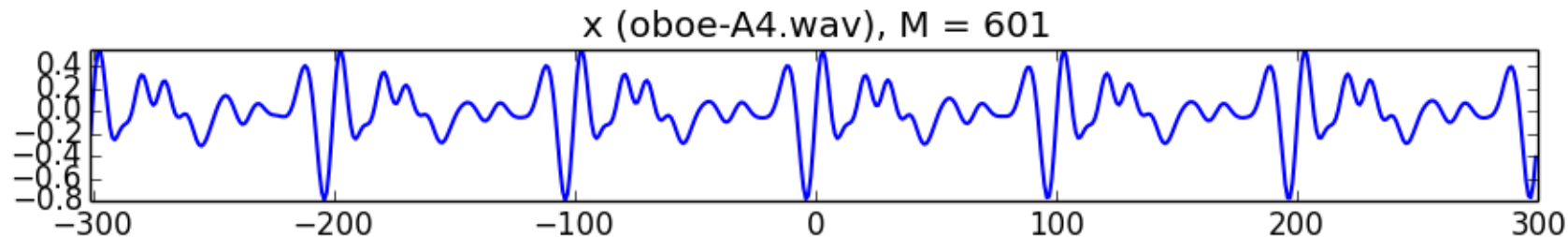
Additive synthesis

$$y[n] = IDFT \left(\sum_{r=0}^R A_r * mWl[k - k_r] * e^{j * (pWl[k - k_r] + \phi_r)} \right)$$

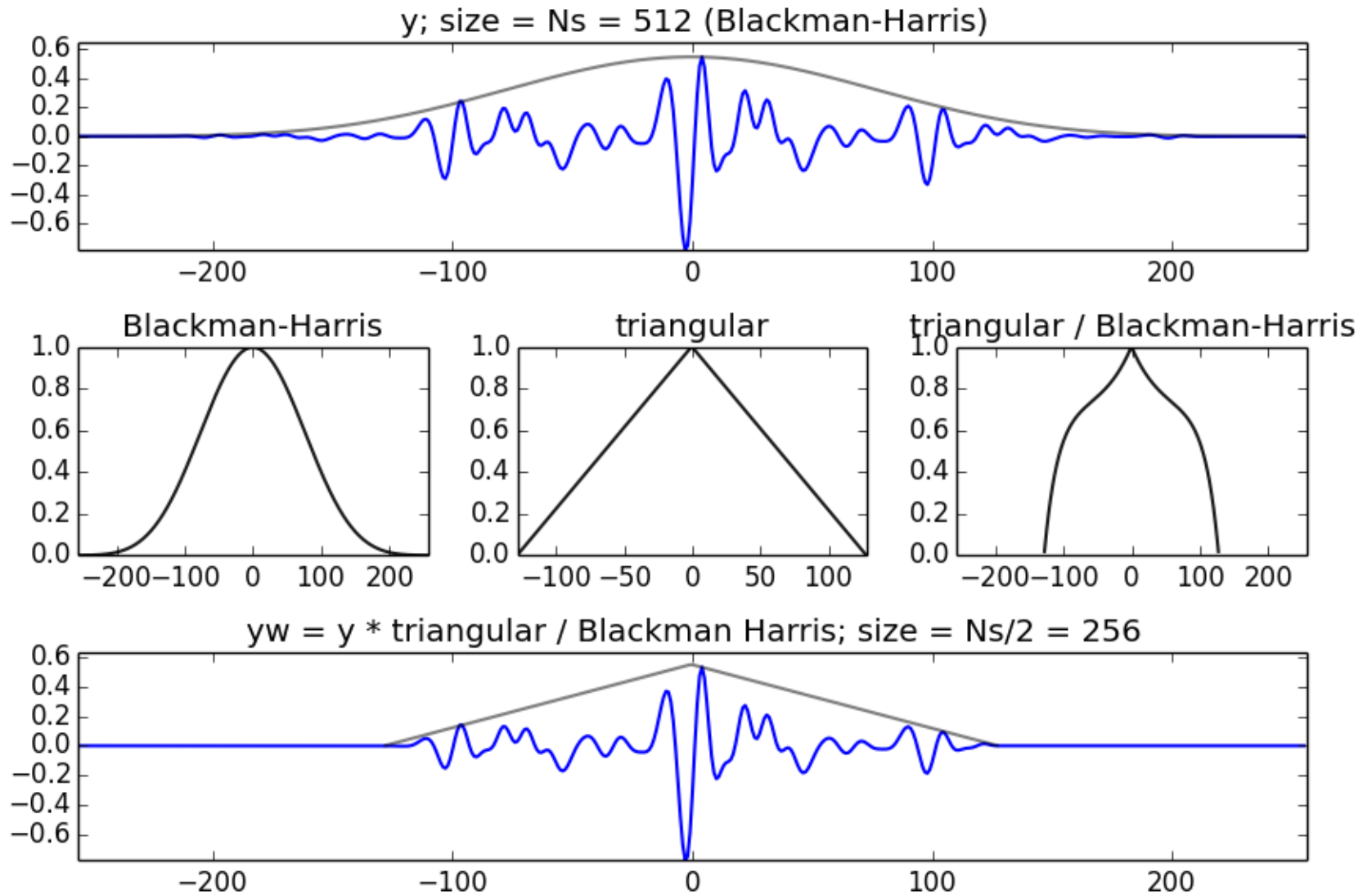
$mWl[k]$, $pWl[k]$ magnitude and phase spectrum of window main lobe



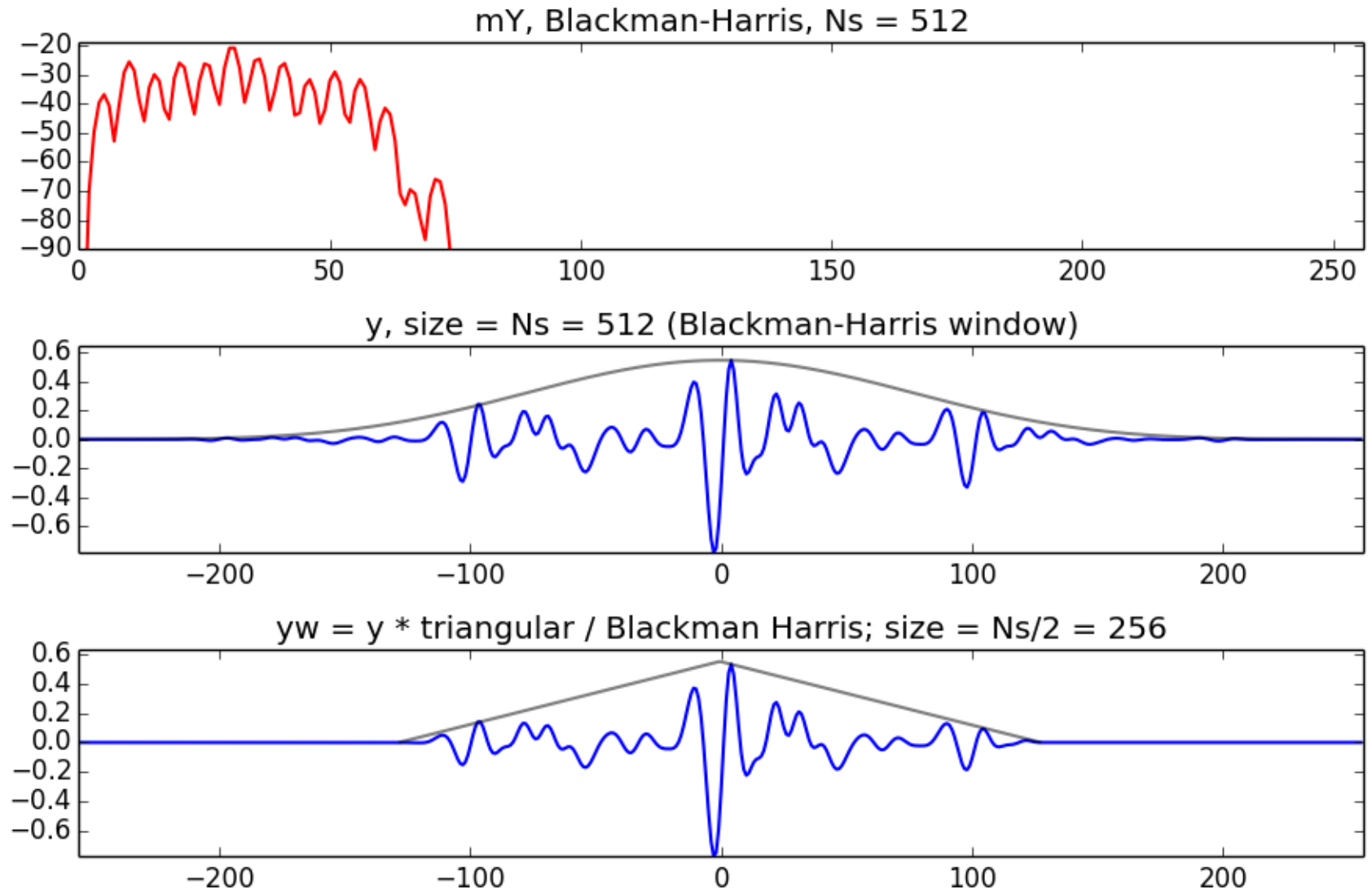
Analysis / Synthesis



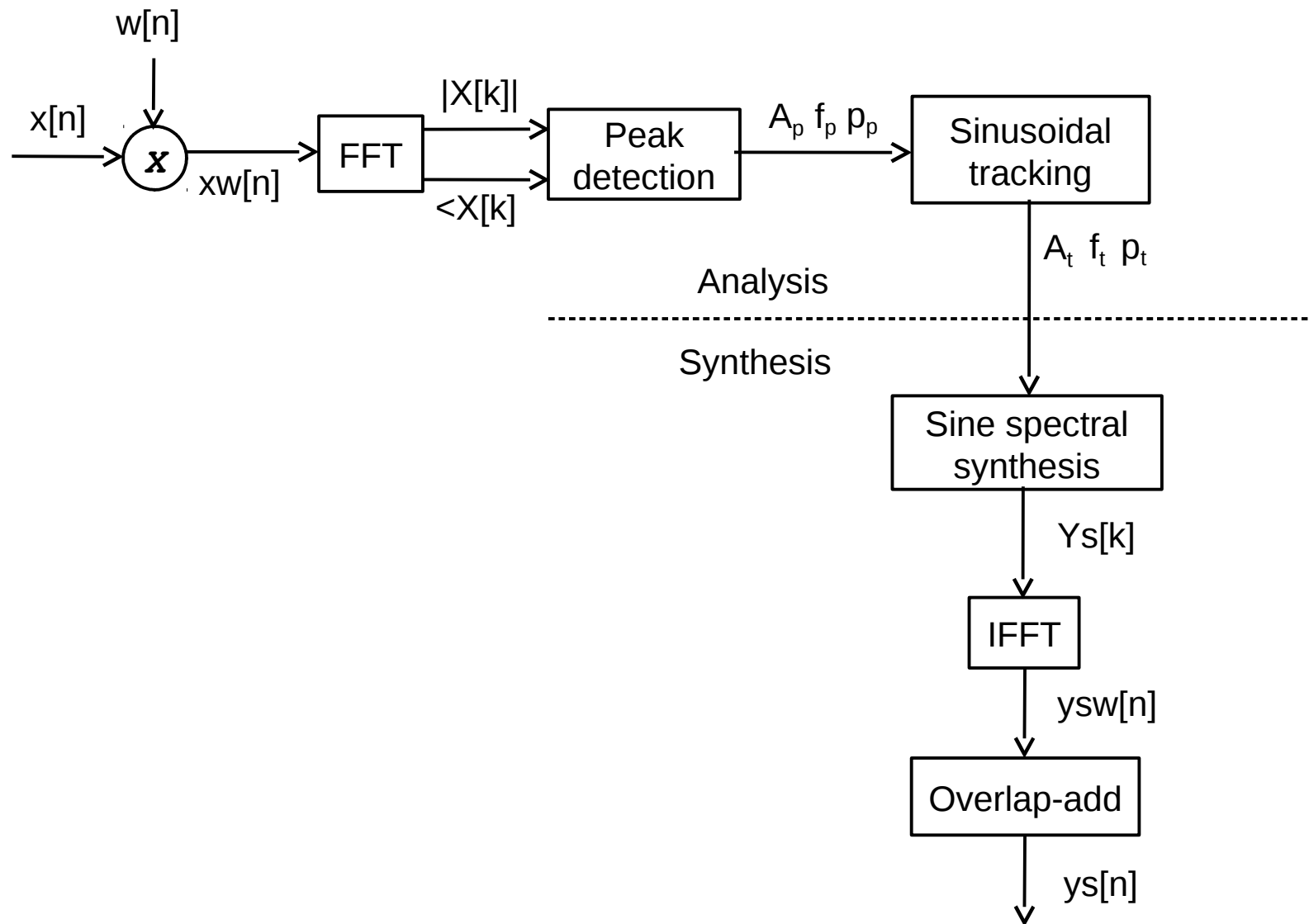
Synthesis window



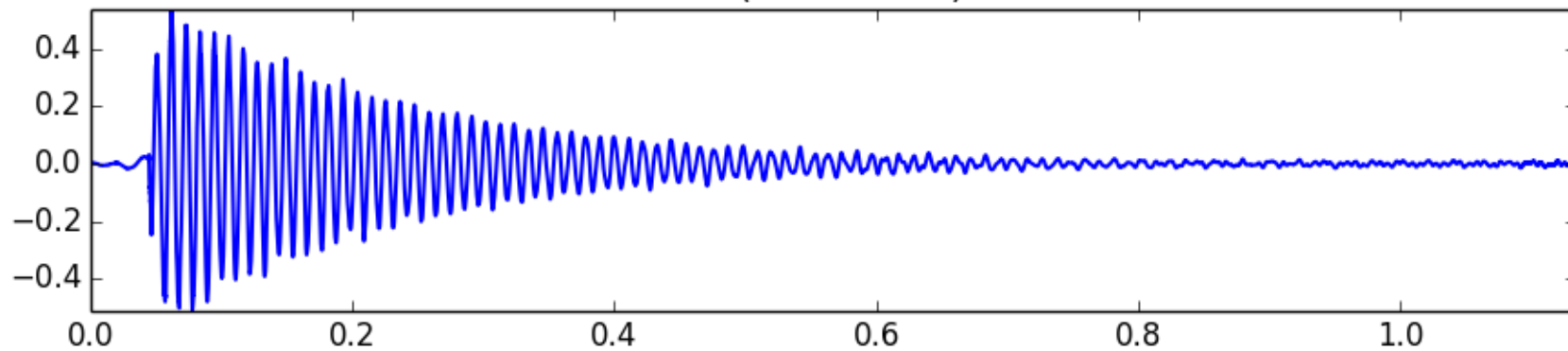
Synthesis for overlap of 25%



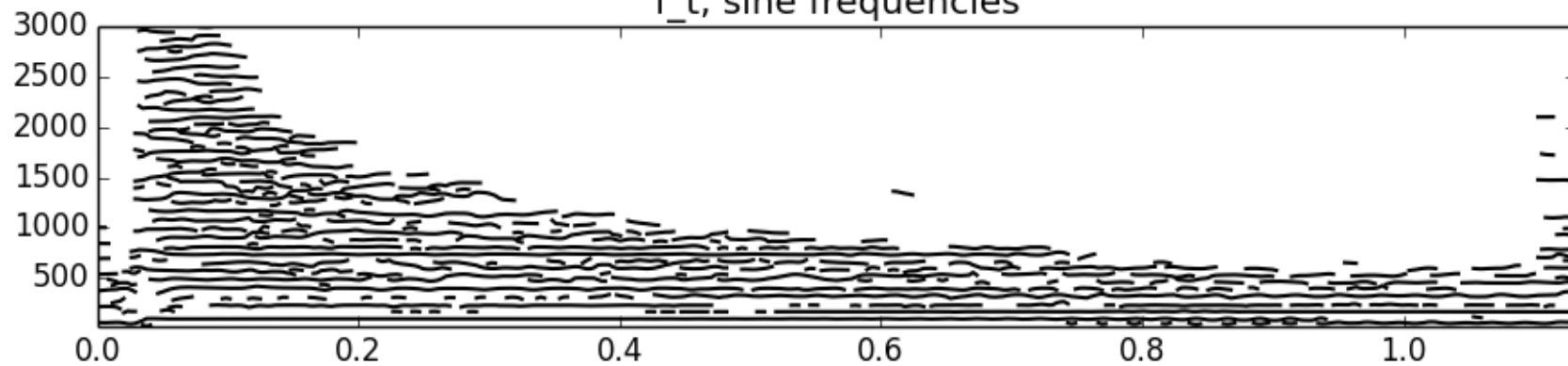
Sinusoidal model system



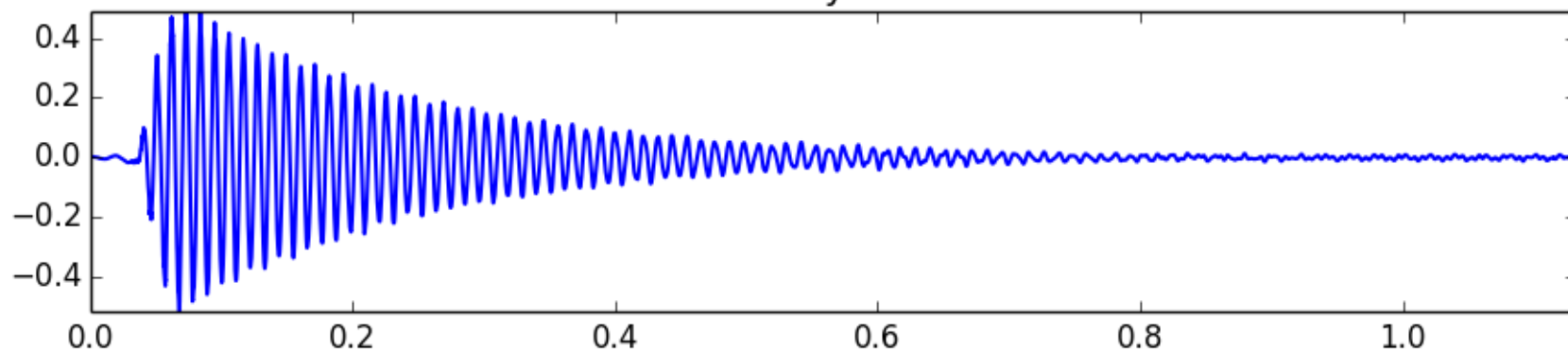
x (bendir.wav)



f_t, sine frequencies



y



References and credits

- More information in:
http://en.wikipedia.org/wiki/Additive_synthesis
http://en.wikipedia.org/wiki/Sinusoidal_model
- Reference on sine modeling by Julius O. Smith:
https://ccrma.stanford.edu/~jos/sasp/Spectrum_Analysis_Sinusoids.html
- Sounds from:
<http://www.freesound.org/people/xserra/packs/13038/>
- Slides released under CC Attribution-Noncommercial-Share Alike license and code under the Affero GPL license; available from <https://github.com/MTG/sms-tools>

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