

3T1: Fourier Transform properties (1 of 2)

Xavier Serra

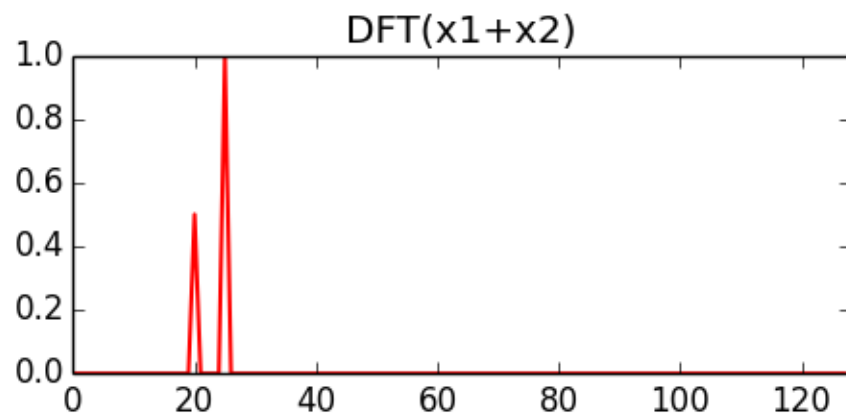
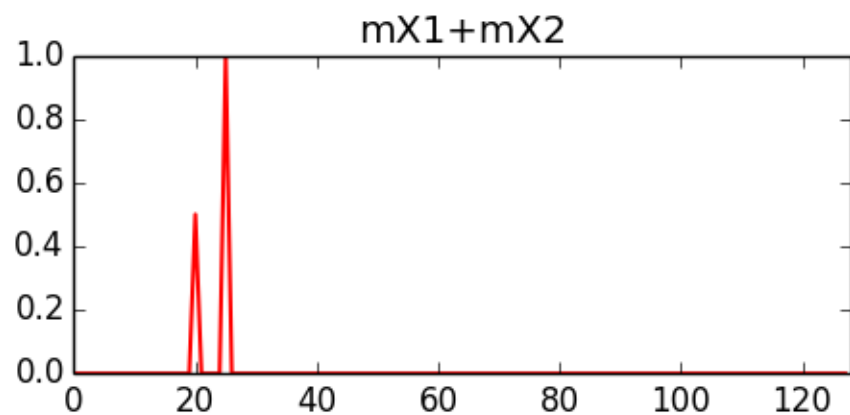
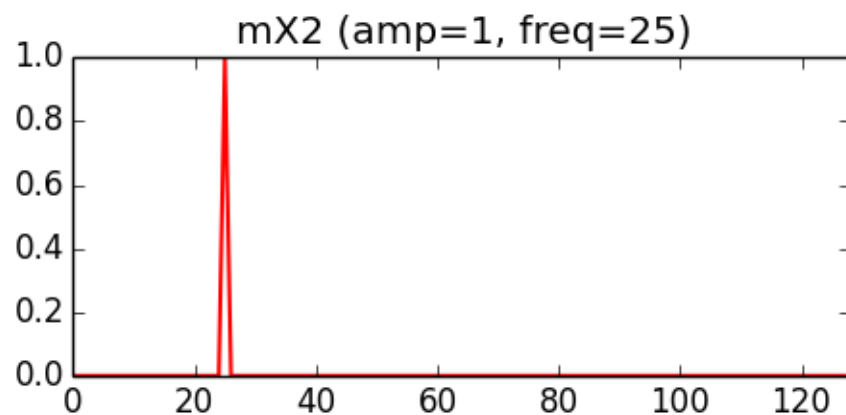
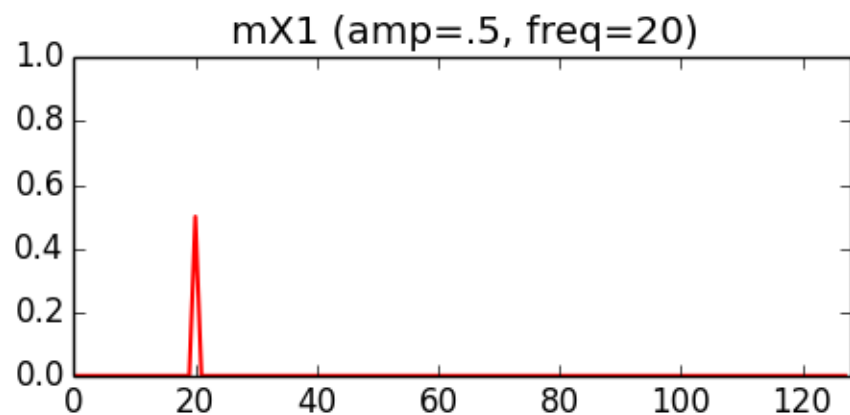
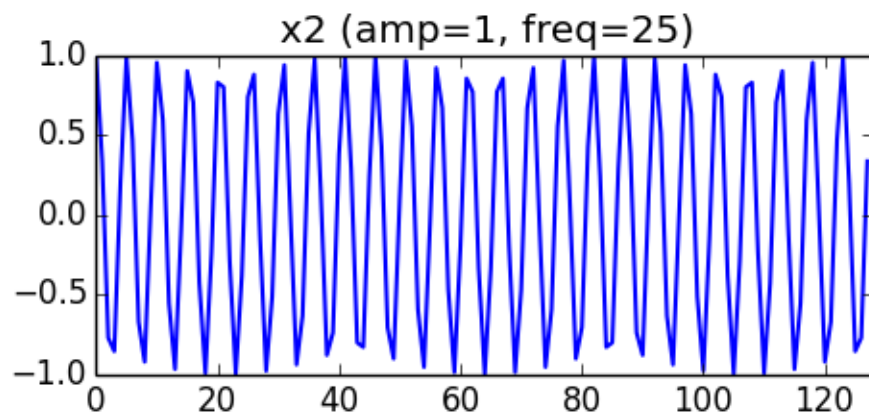
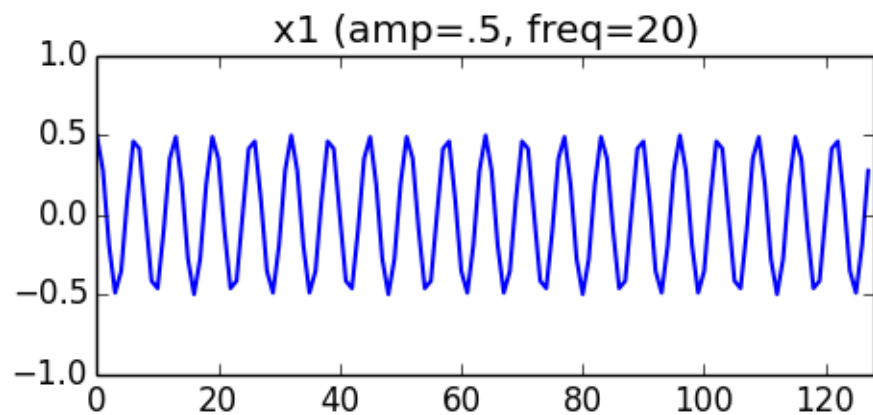
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Linearity: $a x_1[n] + b x_2[n] \Leftrightarrow a X_1[k] + b X_2[k]$

$$\begin{aligned} & DFT(a x_1[n] + b x_2[n]) \\ &= \sum_{n=0}^{N-1} (a x_1[n] + b x_2[n]) e^{-j 2 \pi k n / N} \\ &= \sum_{n=0}^{N-1} a x_1[n] e^{-j 2 \pi k n / N} + \sum_{n=0}^{N-1} b x_2[n] e^{-j 2 \pi k n / N} \\ &= a \sum_{n=0}^{N-1} x_1[n] e^{-j 2 \pi k n / N} + b \sum_{n=0}^{N-1} x_2[n] e^{-j 2 \pi k n / N} \\ &= a X_1[k] + b X_2[k] \end{aligned}$$



Shift: $x[n - n_0] \Leftrightarrow e^{-j2\pi kn_0/N} X[k]$

$$DFT(x[n - n_0])$$

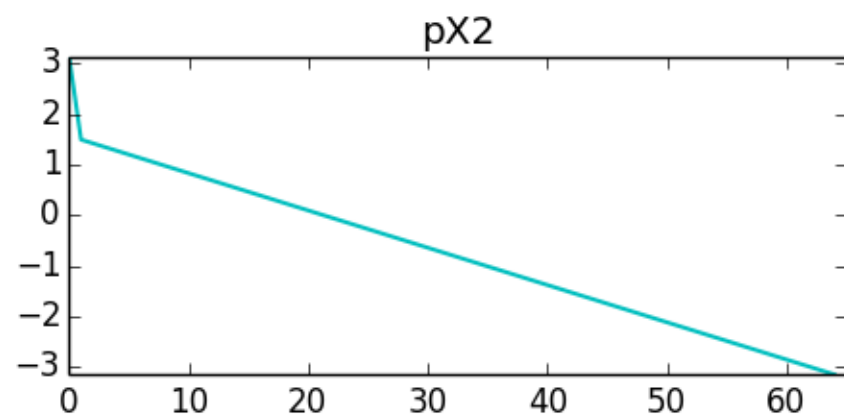
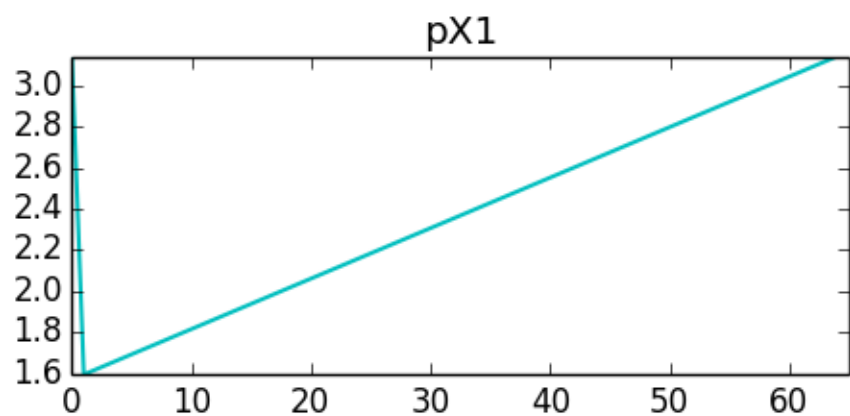
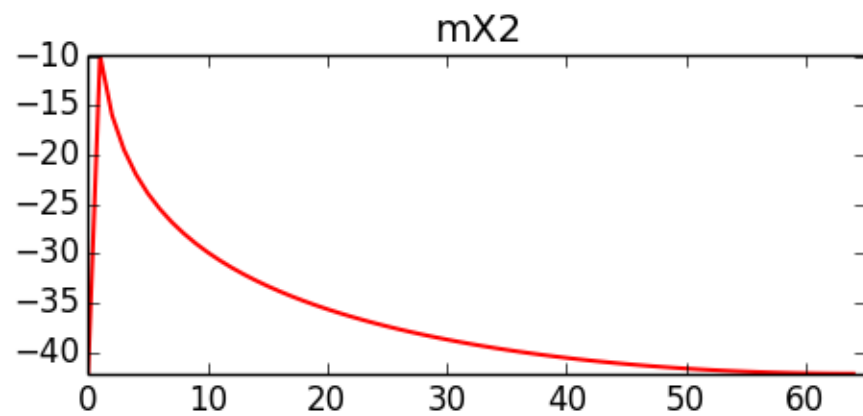
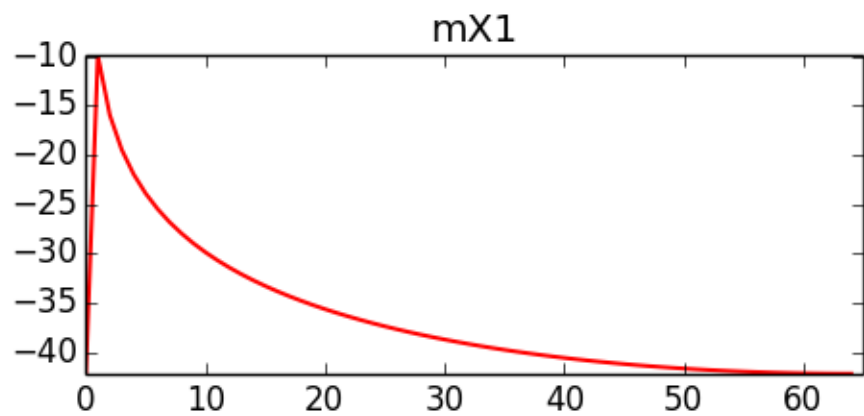
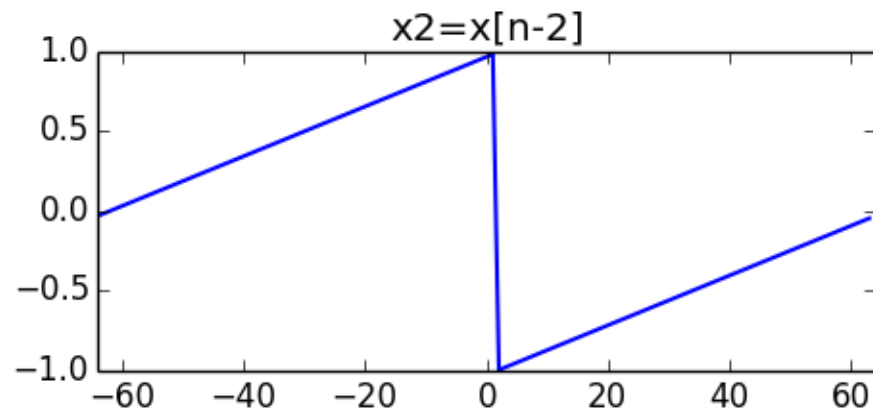
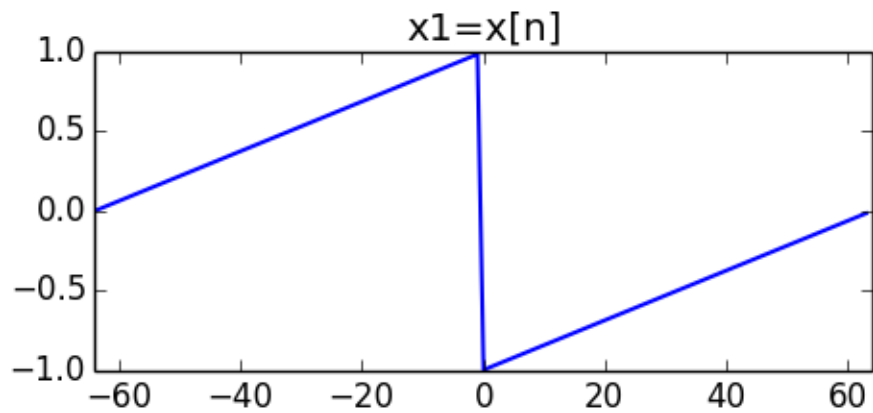
$$= \sum_{n=0}^{N-1} x[n - n_0] e^{-j2\pi kn/N}$$

$$= \sum_{m=-n_0}^{N-1-n_0} x[m] e^{-j2\pi k(m+n_0)/N} \quad (m = n - n_0)$$

$$= \sum_{m=0}^{N-1} x[m] e^{-j2\pi km/N} e^{-j2\pi kn_0/N}$$

$$= e^{-j2\pi kn_0/N} \sum_{m=0}^{N-1} x[m] e^{-j2\pi km/N}$$

$$= e^{-j2\pi kn_0/N} X[k]$$



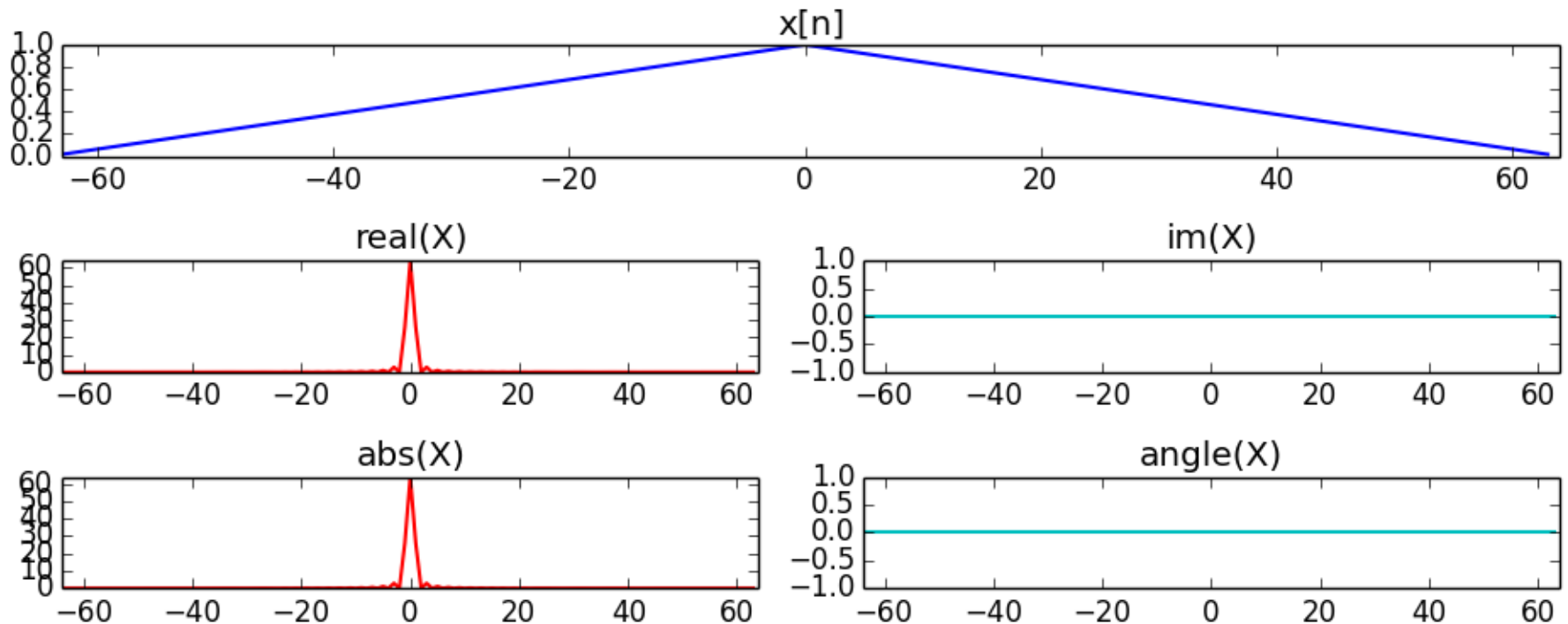
Symmetry:

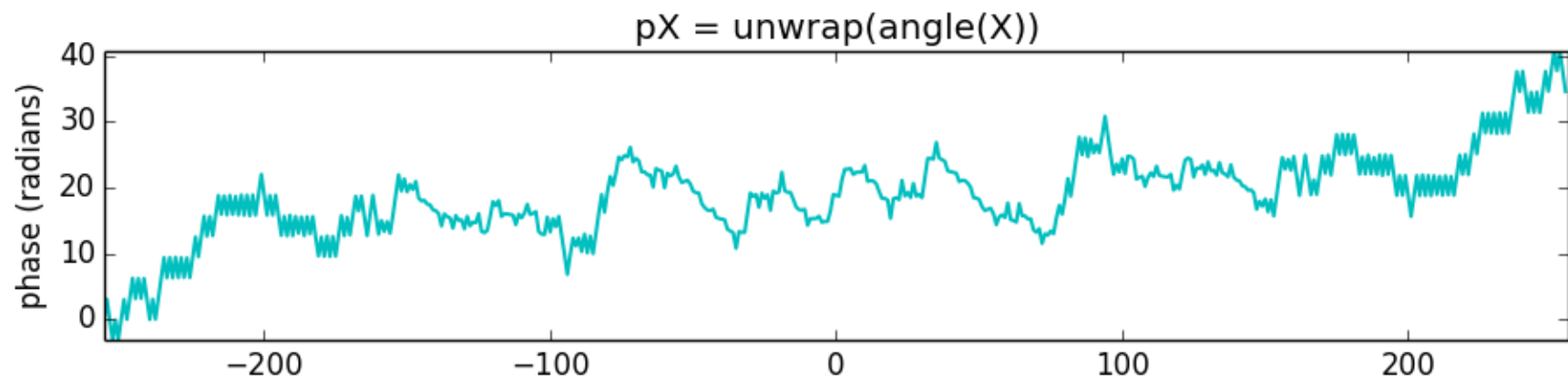
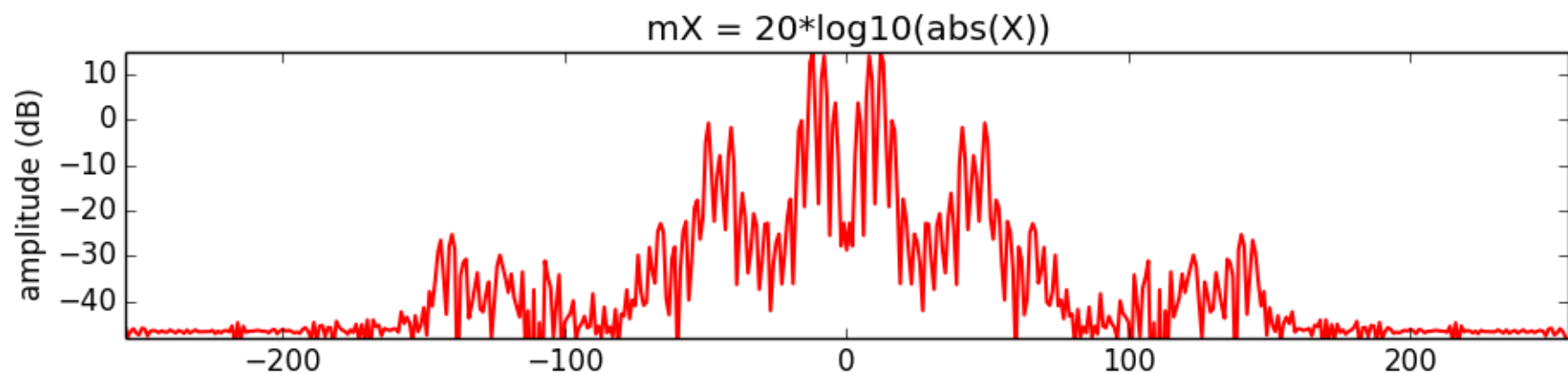
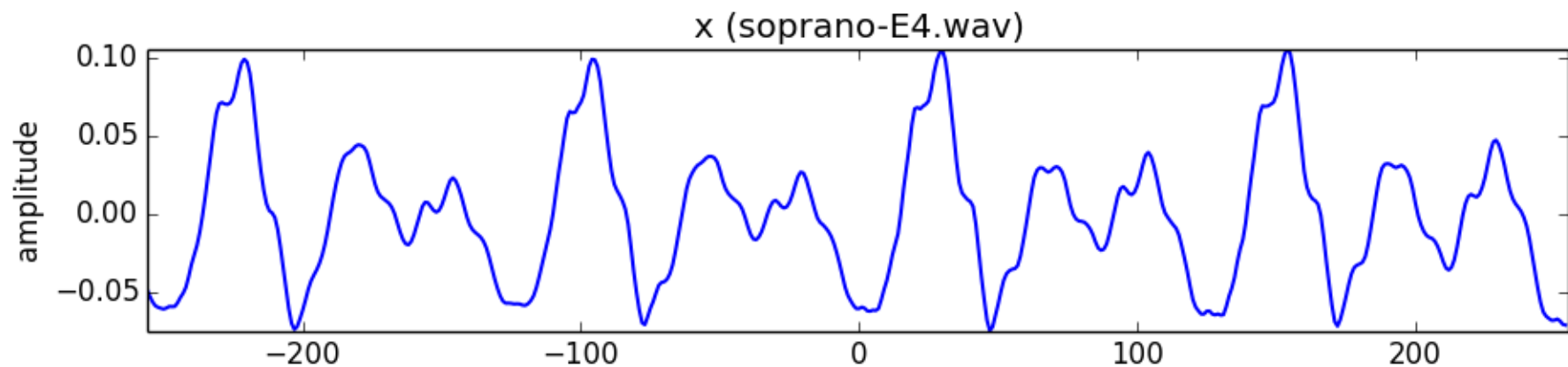
$$x[n] \text{ real} \Leftrightarrow \Re\{X[k]\} \text{ even and } \Im\{X[k]\} \text{ odd}$$

$$\Leftrightarrow |X[k]| \text{ even and } \angle X[k] \text{ odd}$$

$$x[n] \text{ real and even} \Leftrightarrow \Re\{X[k]\} \text{ even and } \Im\{X[k]\} = 0$$

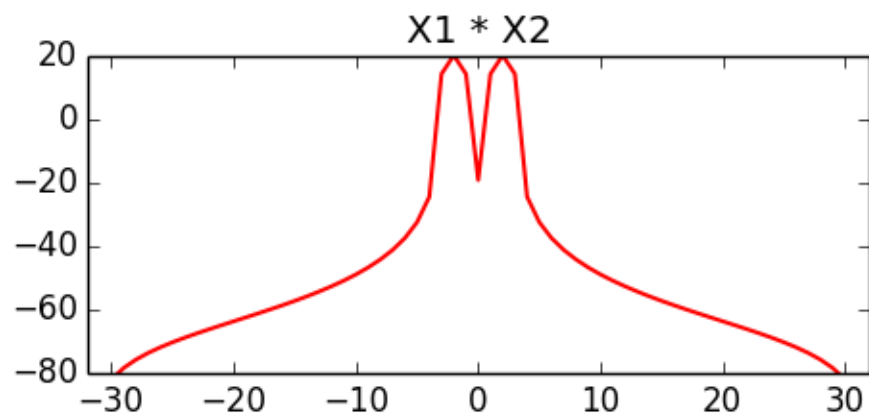
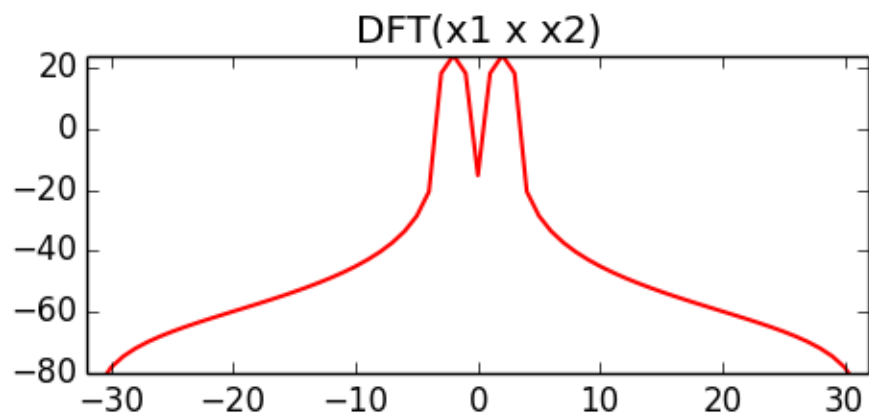
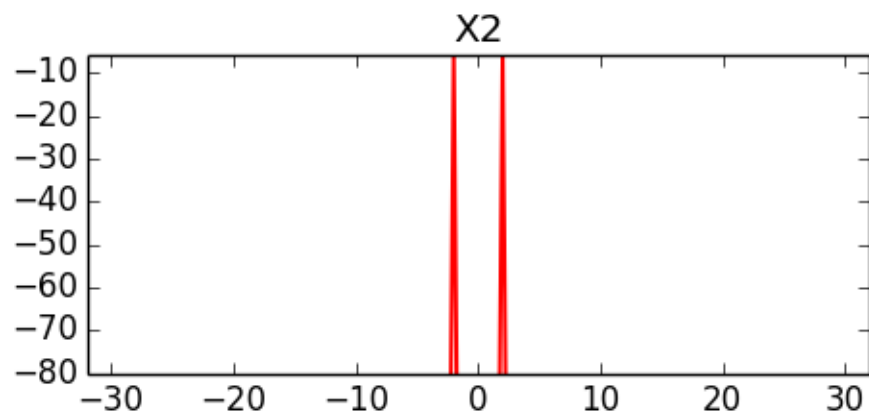
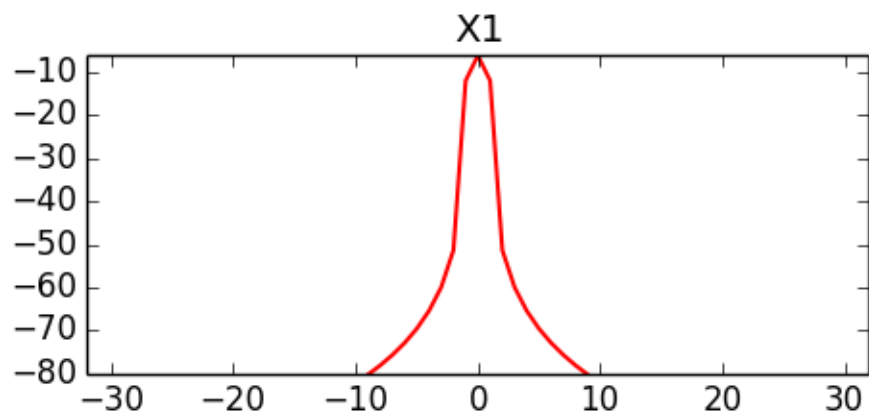
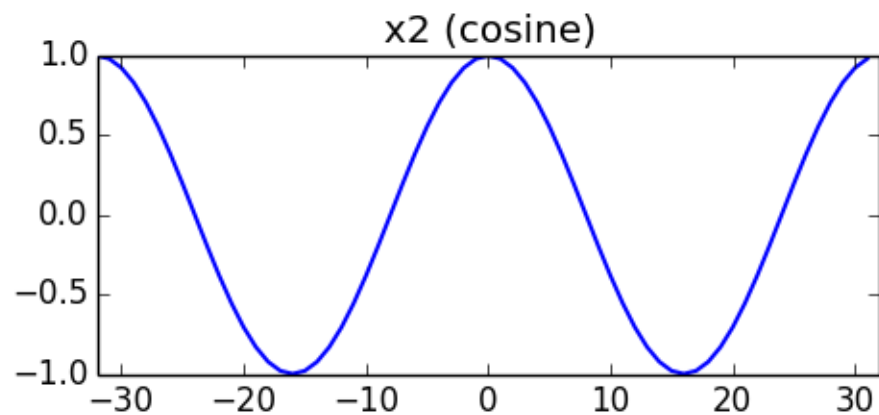
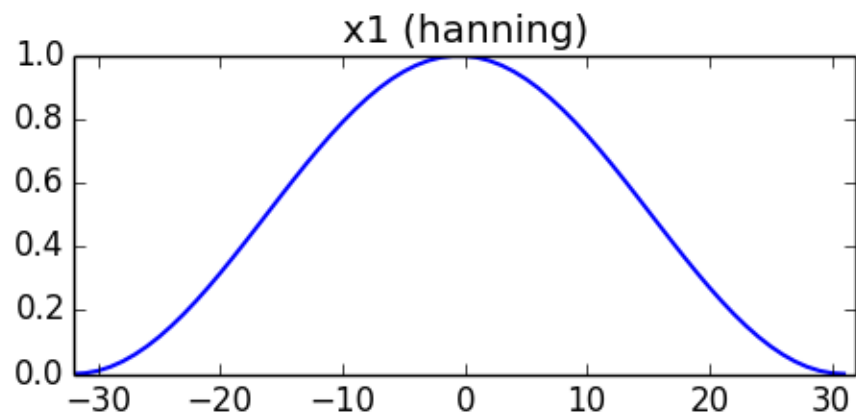
$$\Leftrightarrow |X[k]| \text{ even and } \angle X[k] = n\pi$$



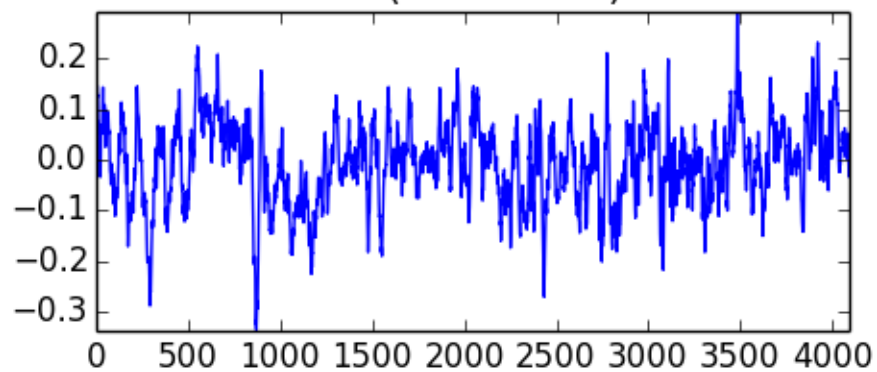


Convolution: $x_1[n] * x_2[n] \Leftrightarrow X_1[k] \times X_2[k]$

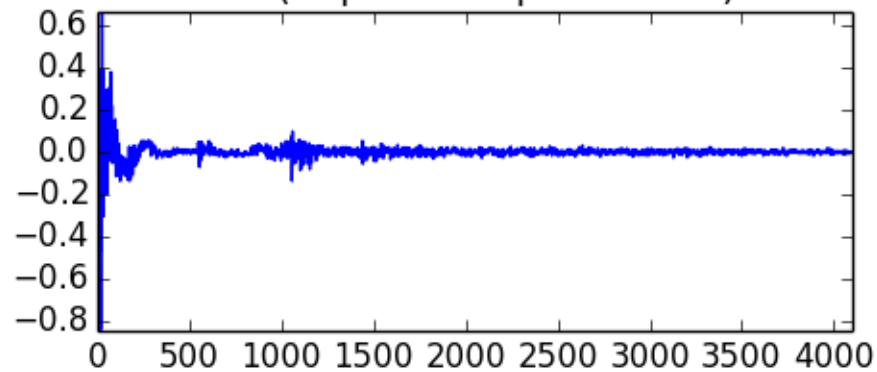
$$\begin{aligned} & DFT(x_1[n] * x_2[n]) \\ &= \sum_{n=0}^{N-1} (x_1[n] * x_2[n]) e^{-j2\pi kn/N} \\ &= \sum_{n=0}^{N-1} \sum_{m=0}^{N-1} x_1[m] x_2[n-m] e^{-j2\pi kn/N} \\ &= \sum_{m=0}^{N-1} x_1[m] \sum_{n=0}^{N-1} x_2[n-m] e^{-j2\pi kn/N} \\ &= \left(\sum_{m=0}^{N-1} x_1[m] e^{-j2\pi km/N} \right) X_2[k] \\ &= X_1[k] \times X_2[k] \end{aligned}$$



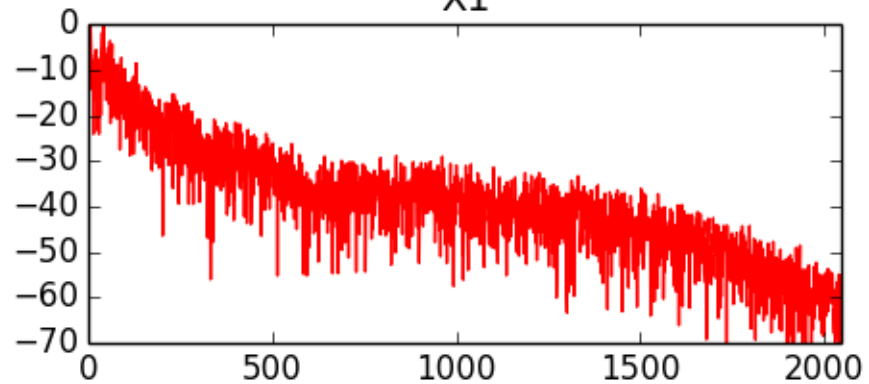
x1 (ocean.wav)



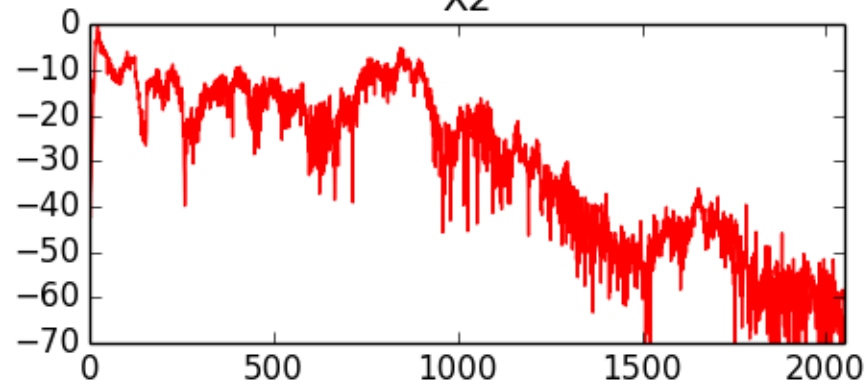
x2 (impulse-response.wav)



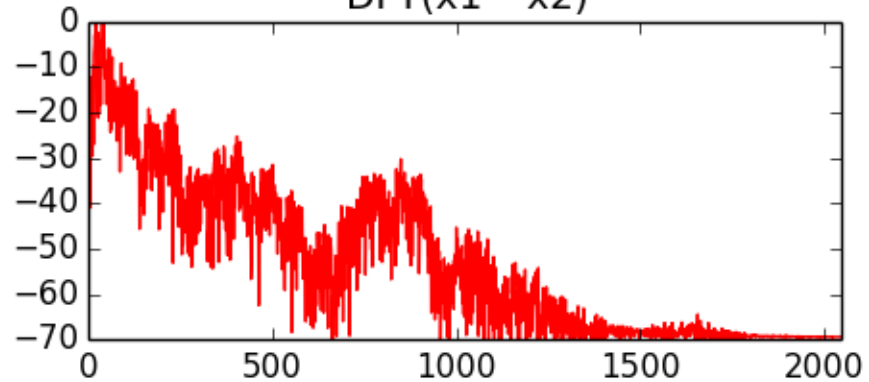
X1



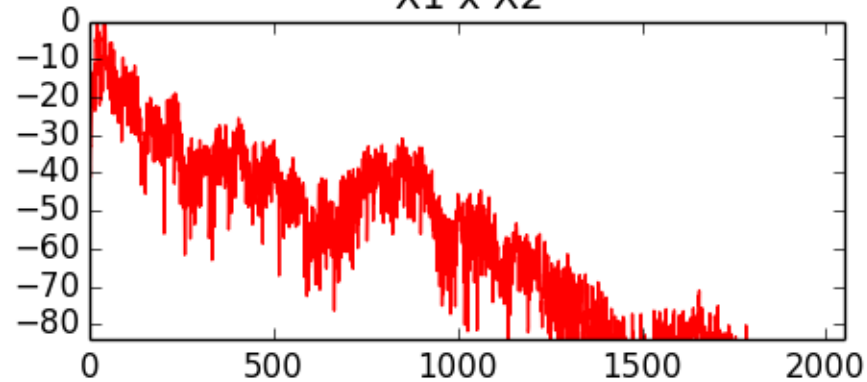
X2



DFT(x1 * x2)



X1 x X2



References and credits

- More information in:
 - https://en.wikipedia.org/wiki/Discrete_Fourier_transform
- Reference on the DFT by Julius O. Smith:
<https://ccrma.stanford.edu/~jos/mdft/>
- Sounds from:
<http://www.freesound.org/people/xserra/packs/13038/>
- Slides released under CC Attribution-Non Commercial-Share Alike license and code under Affero GPL license; available from <https://github.com/MTG/sms-tools>

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