7T1: Stochastic Model

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Stochastic signals

- Described by the laws of probability; mean, variance, probability distributions

- Autocorrelation

$$Z_{xx}[k] = \sum_{n=0}^{n=N-1} x[n]x[n+k] \qquad k = -N+1, \ldots, N-1$$

- Power spectral density

$$Xp[k] = \lim_{N \to \infty} |X[k]|^2$$

where $$X[k] = \frac{1}{N} \sum_{n=0}^{N-1} x[n] e^{-j 2\pi kn/N} \qquad k = 0, \ldots, N-1$$

Stochastic model

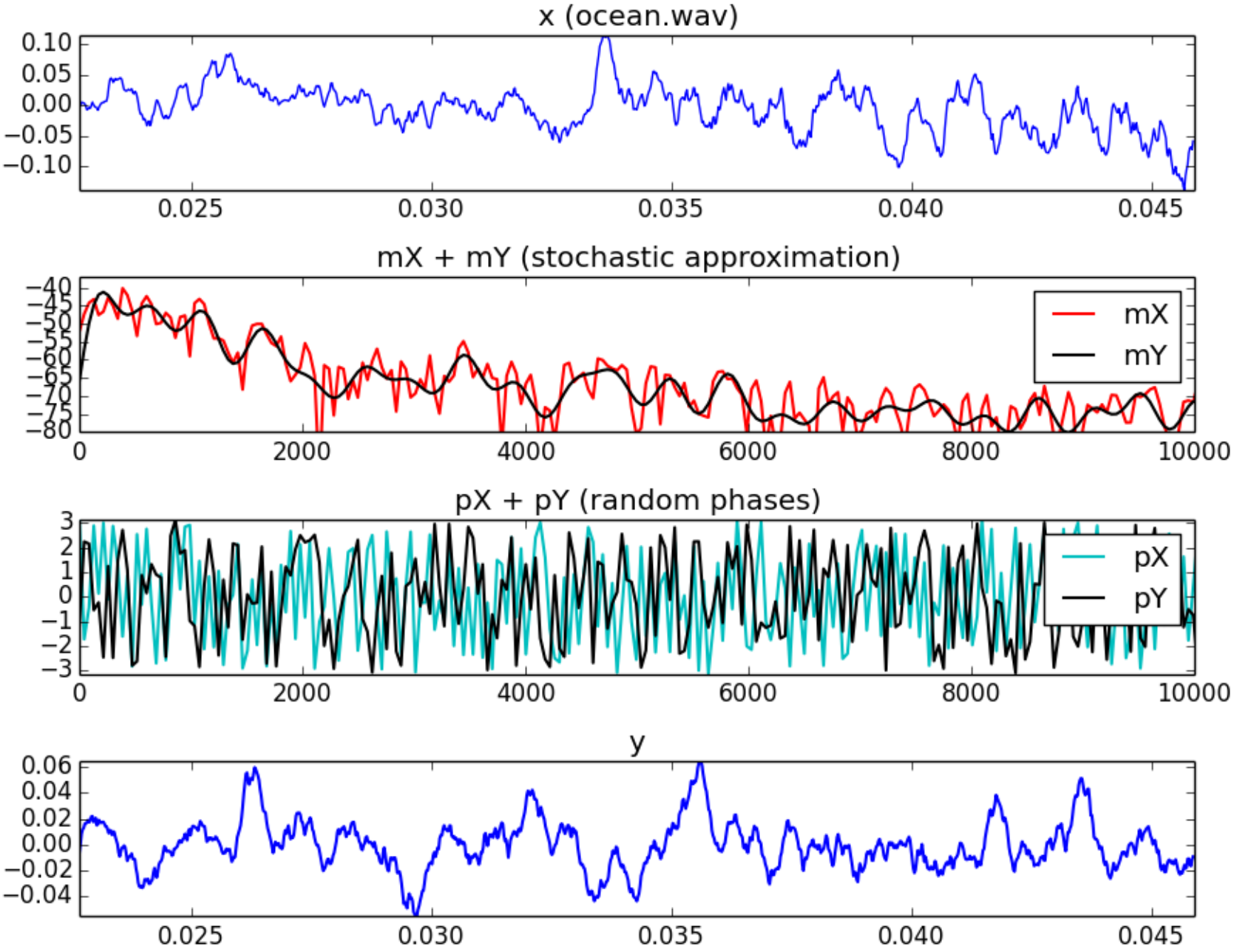
$$yst[n] = \sum_{k=0}^{N-1} u[k]h[n-k]$$

$u[n]$: white noise
$h[n]$: impulse response of filter approximating input signal $x[n]$

Spectral view:

$$Yst_l[k] = |H_l[k]||U[k]|e^{j(\measuredangle H[k] + \measuredangle U[k])} = |\tilde{X}_l[k]|e^{j\measuredangle U[k]}$$

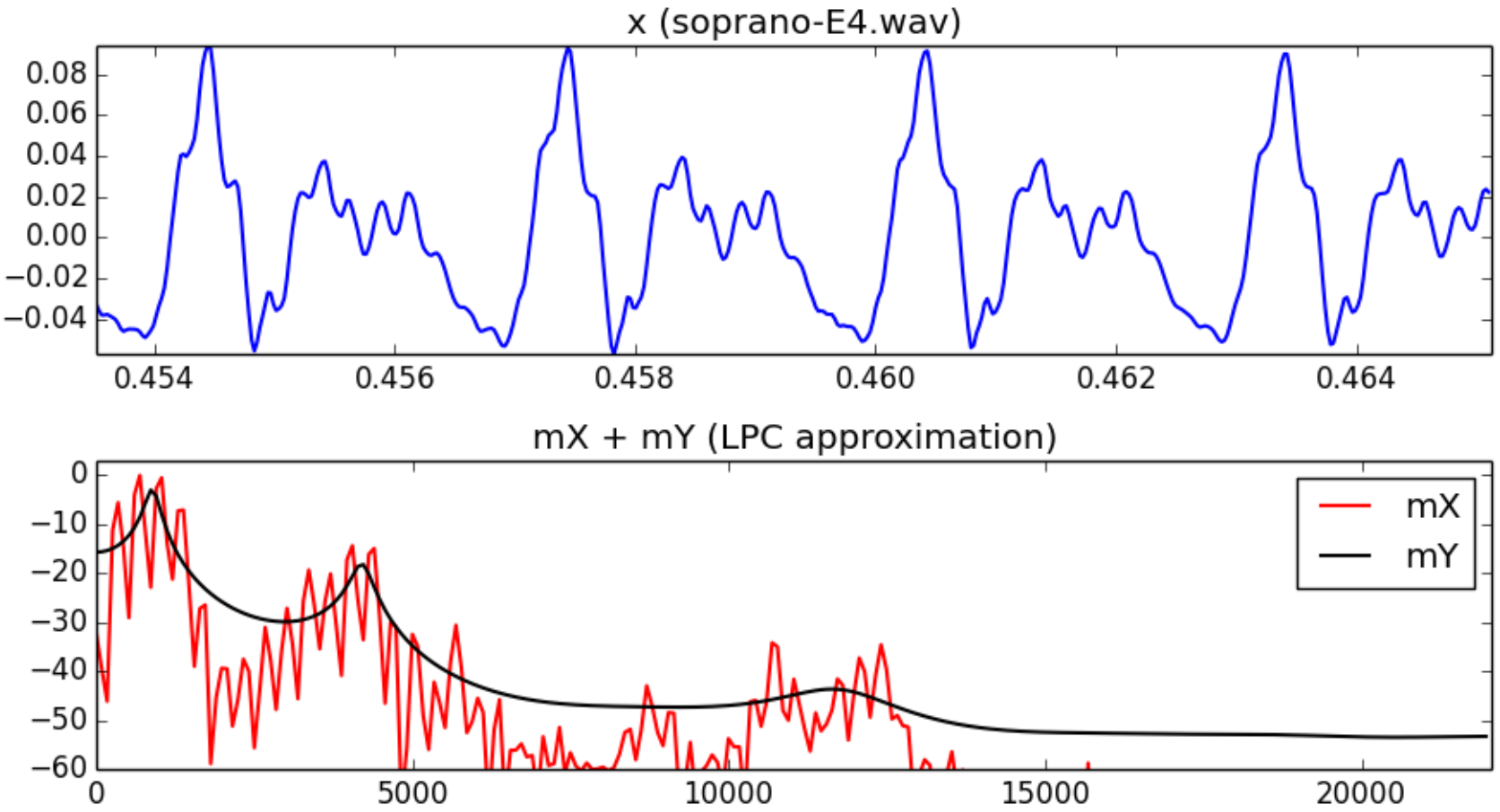
$|\tilde{X}[k]|$: approximation of magnitude spectrum of input signal $x[n]$
$\measuredangle U[k]$: spectral phases of noise signal
l: frame number


x (ocean.wav)
0.10
0.05
0.00
−0.05
−0.10
0.025
0.030
0.035
0.040
0.045
mX + mY (stochastic approximation)
−40
−45
−50
−55
−60
−65
−70
−75
−80
0
2000
4000
6000
8000
10000
mX
mY
pX + pY (random phases)
3
2
1
0
−1
−2
−3
0
2000
4000
6000
8000
10000
pX
pY
y
0.06
0.04
0.02
0.00
−0.02
−0.04
0.025
0.030
0.035
0.040
0.045

LPC approximation

$$\hat{x}[n] = \sum_{k=1}^{K} a_k \, x[n-k]$$

$$Error = \sum_{n=-\infty}^{\infty} \left(x[n] - \sum_{k=1}^{K} a_k \, x[n-k] \right)^2$$



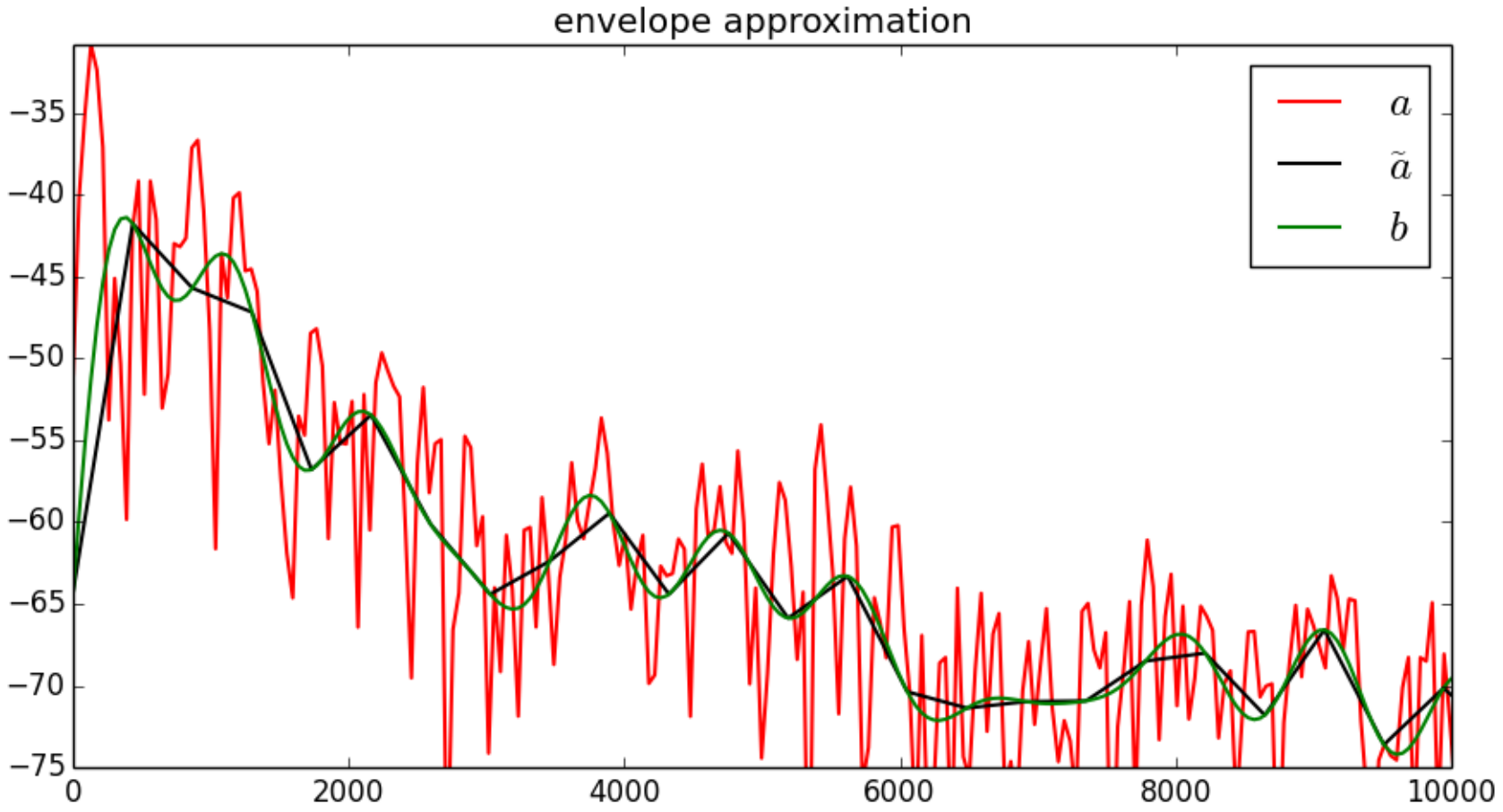
Envelope approximation

$\tilde{a}[k] = IDFT(LP(DFT(a[k])))$

LP : low-pass filter

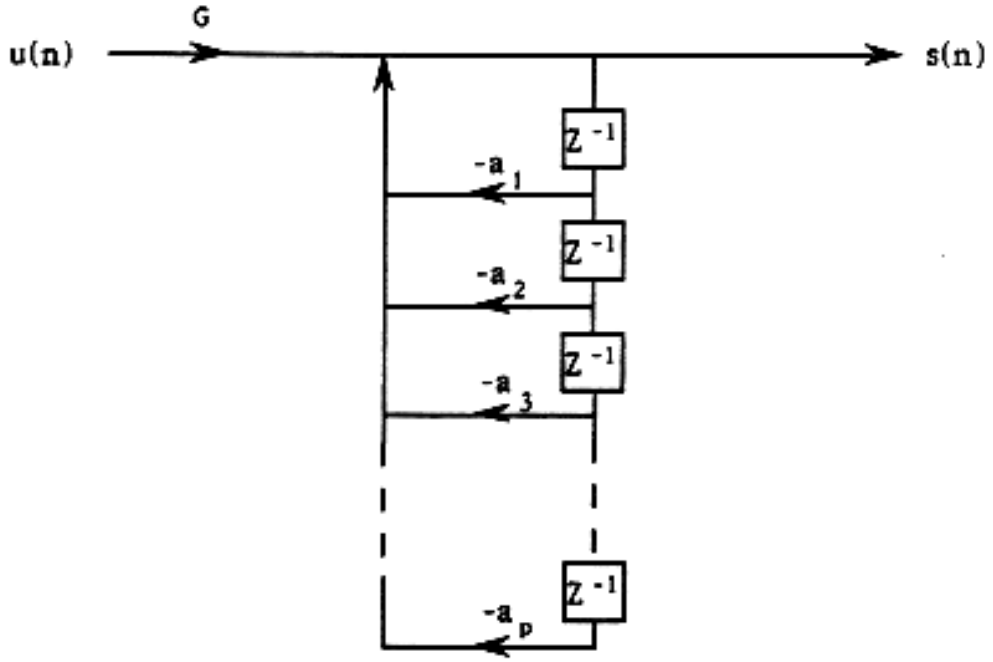
$b[k] = IDFT(ZP(DFT(\tilde{a}[k])))$

ZP : zero-padding



Stochastic synthesis using LPC

$$yst[n] = \sum_{k=1}^{K} a_k \, u[n-k], \quad a_k : \text{filter coefficients}; \; u[n] : \text{white noise}$$

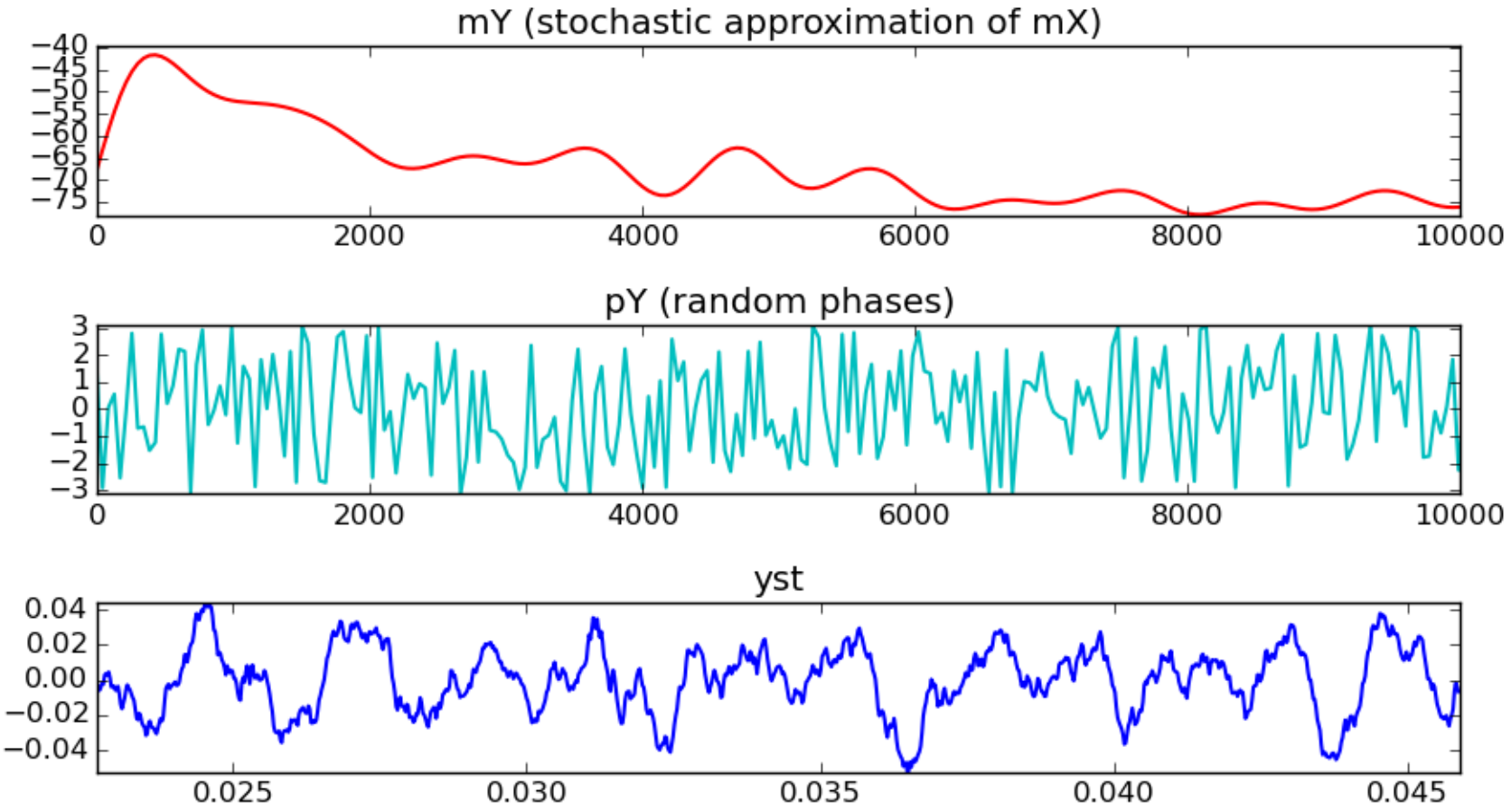


direct form structure

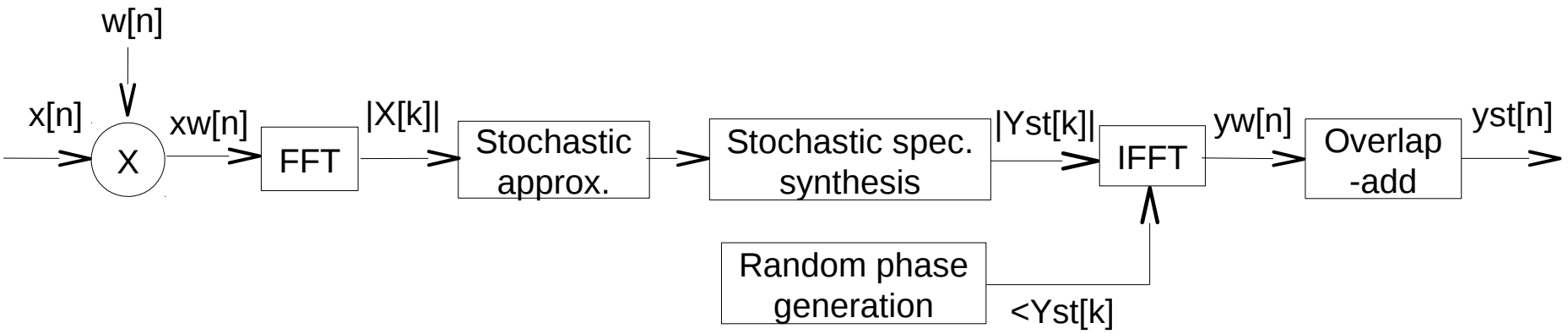
lattice structure

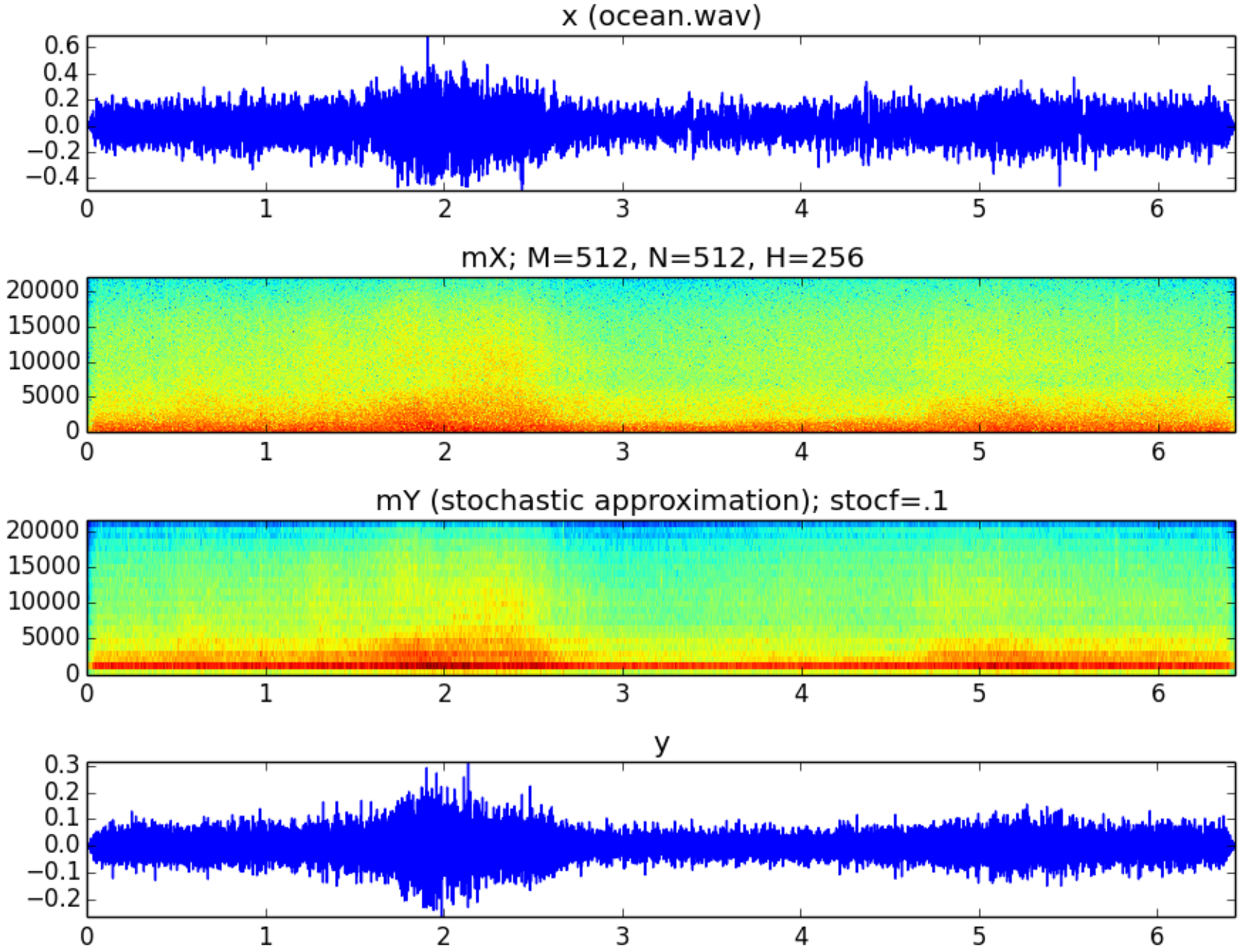
Stochastic synthesis using envelopes

$$yst[n] = IDFT\left(|\widetilde{X}[k]|e^{j \measuredangle U[k]}\right)$$



Stochastic model system




x (ocean.wav)
0.6
0.4
0.2
0.0
−0.2
−0.4
0
1
2
3
4
5
6
mX; M=512, N=512, H=256
20000
15000
10000
5000
0
mY (stochastic approximation); stocf=.1
y
0.3
0.2
0.1
0.0
−0.1
−0.2

References and credits

- More information in:
  - http://en.wikipedia.org/wiki/Statistical_signal_processing
  - http://en.wikipedia.org/wiki/Stochastic_process
  - http://en.wikipedia.org/wiki/Linear_predictive_coding
- Sounds: http://www.freesound.org/people/xserra/packs/13038/
- Slides released under CC Attribution-Noncommercial-Share Alike license and code under Affero GPL license; available from https://github.com/MTG/sms-tools

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