

5T1: Sinusoidal model (1 of 3)

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Index

- Sinusoidal model
- Spectrum of sinewave
- Sinewaves as spectral peaks

Sinusoidal model

$$y[n] = \sum_{r=1}^R A_r[n] \cos(2\pi f_r[n]n)$$

R : number of sinewaves

$A_r[n]$: instantaneous amplitude

$f_r[n]$: instantaneous frequency (Hz)

Spectrum of sinewave

$$x[n] = A \cos(2\pi k_0 n/N + \phi)$$

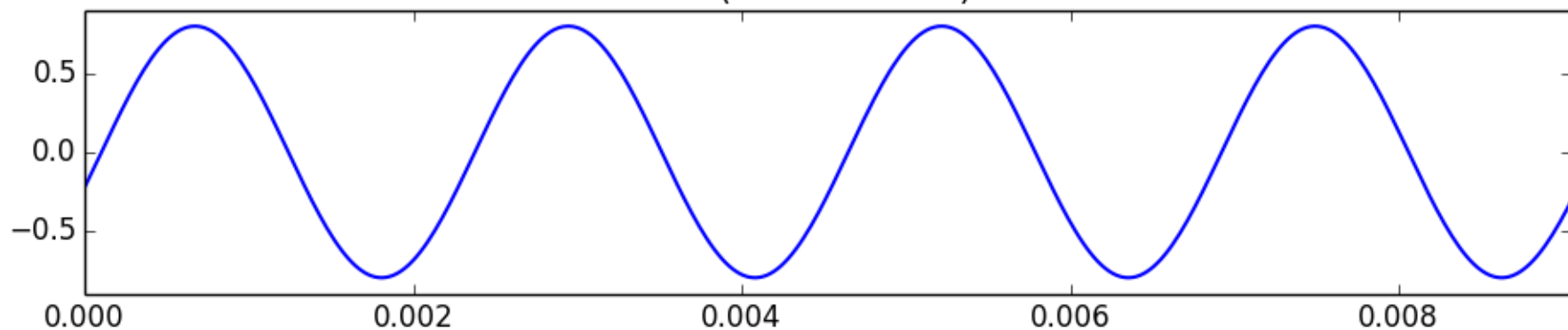
$$X[k] = A \sum_{n=0}^{N-1} w[n] \frac{1}{2} (e^{j2\pi k_0 n/N} + e^{-j2\pi k_0 n/N}) e^{-j2\pi kn/N}$$

$$= \frac{A}{2} \sum_{n=0}^{N-1} w[n] e^{j2\pi k_0 n/N} e^{-j2\pi kn/N} + \frac{A}{2} \sum_{n=0}^{N-1} w[n] e^{-j2\pi k_0 n/N} e^{-j2\pi kn/N}$$

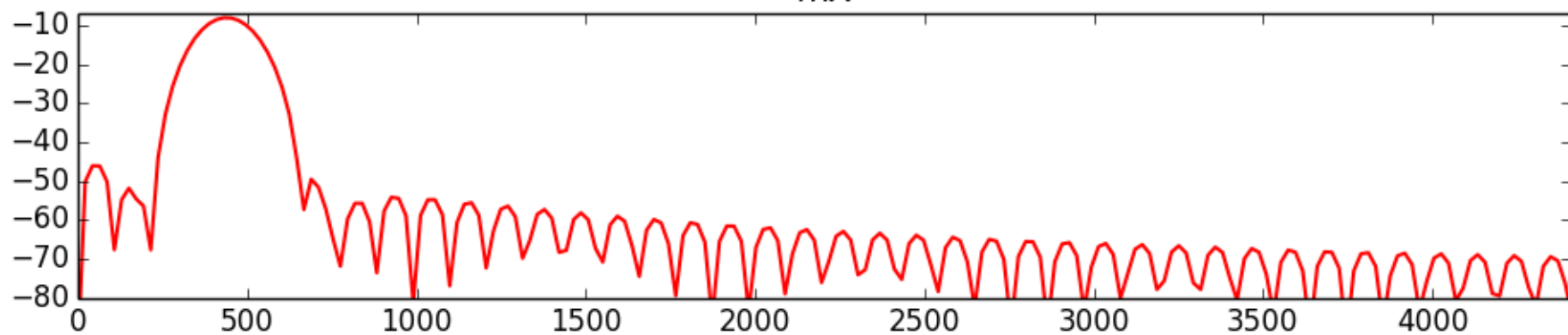
$$= \frac{A}{2} \sum_{n=0}^{N-1} w[n] e^{-j2\pi(-k_0+k)n/N} + \frac{A}{2} \sum_{n=0}^{N-1} w[n] e^{-j2\pi(k_0+k)n/N}$$

$$= \frac{A}{2} W[k - k_0] + \frac{A}{2} W[k + k_0]$$

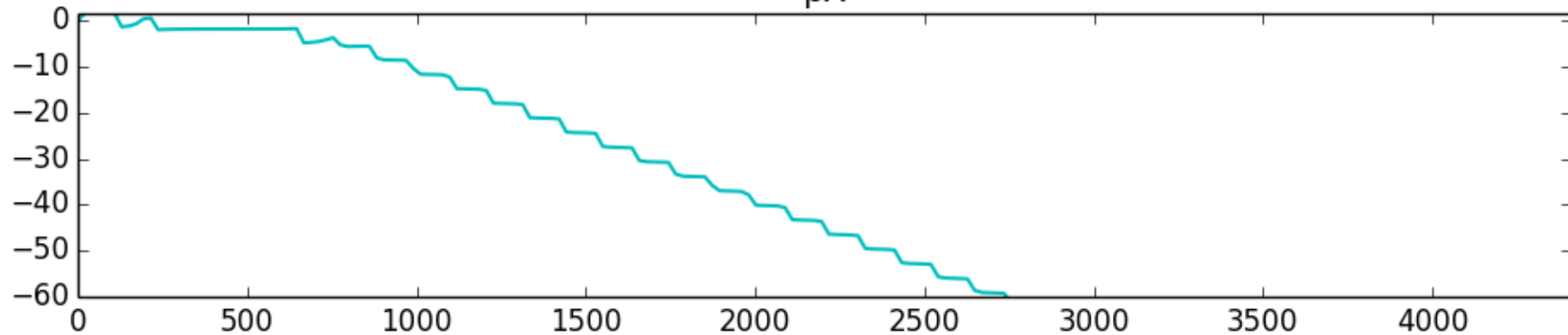
x (sine-440.wav)



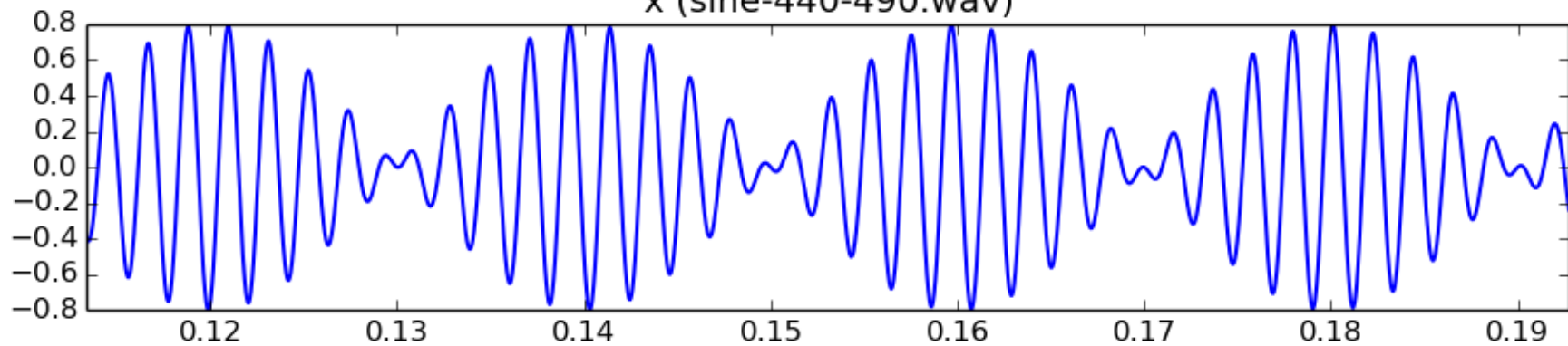
mX



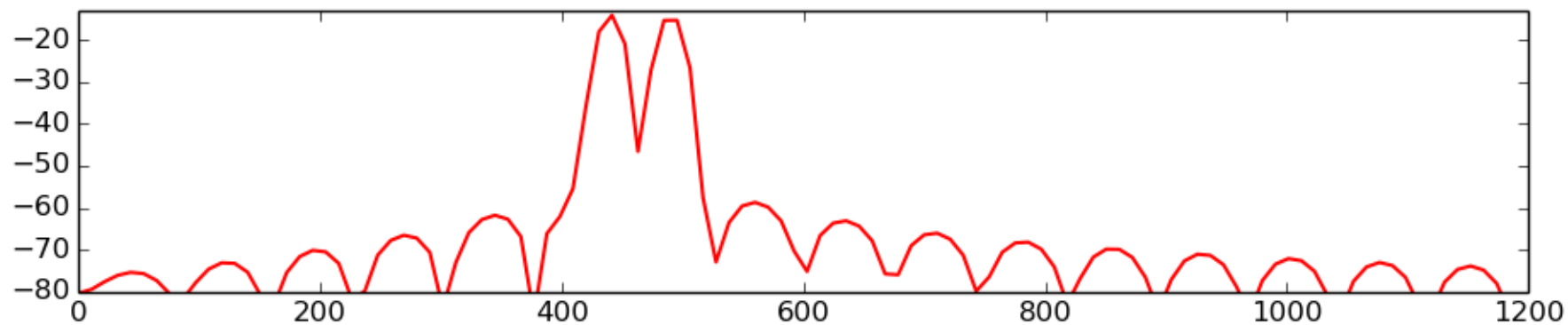
pX



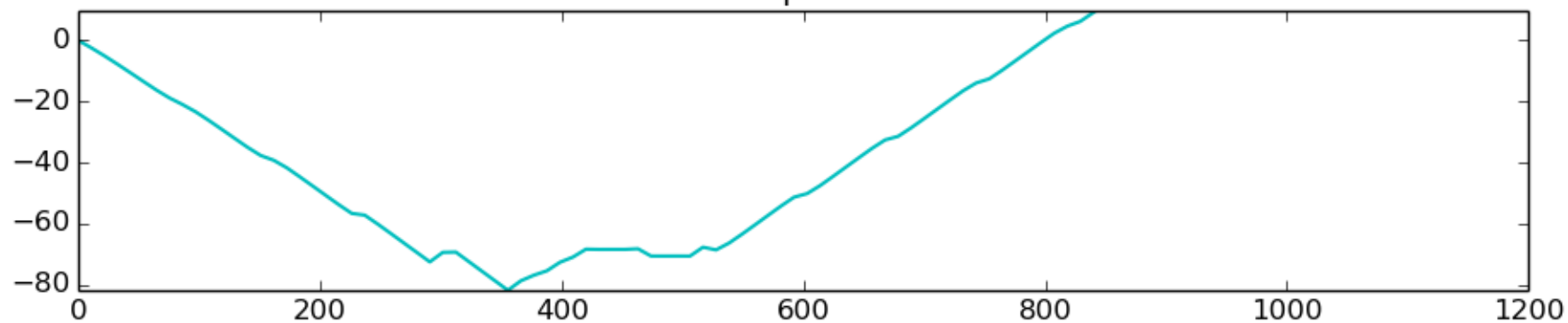
x (sine-440-490.wav)

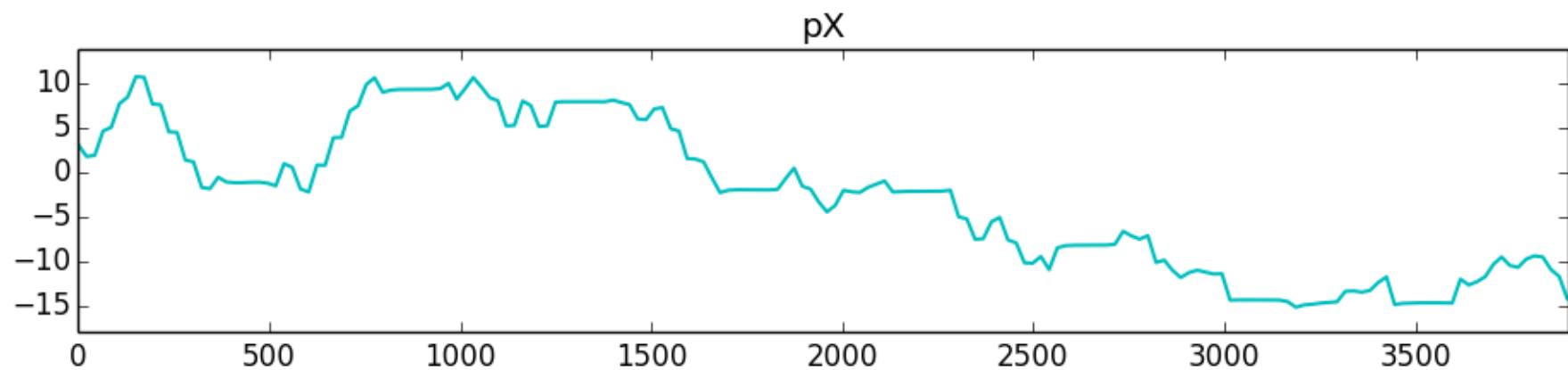
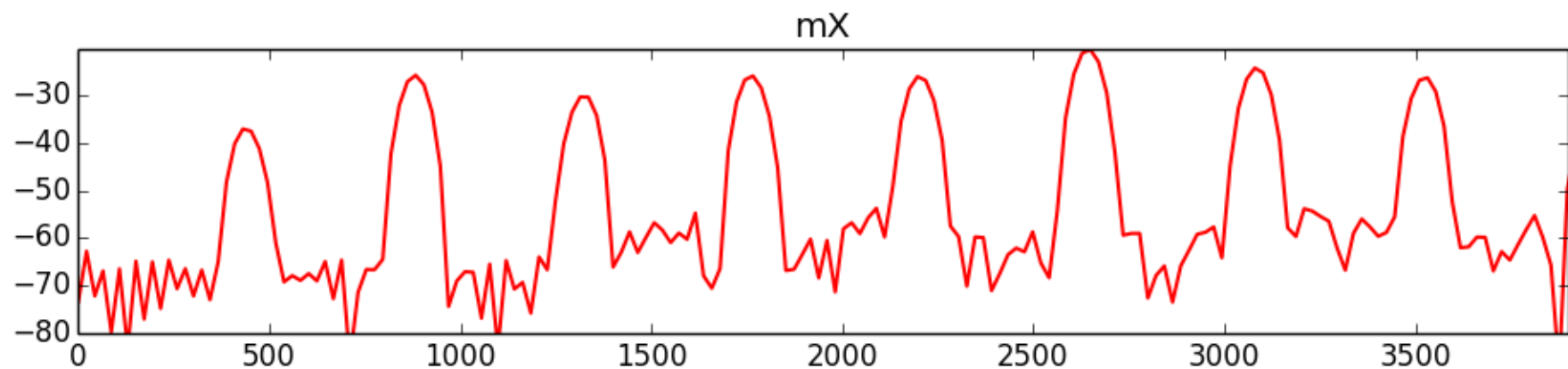
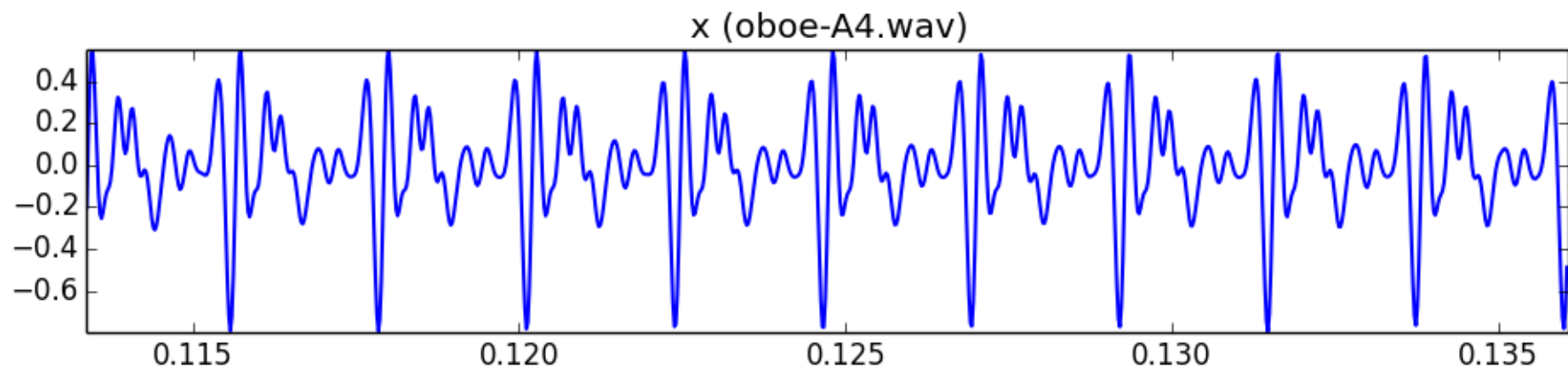


mX



pX





Sinewaves as spectral peaks

- Sinusoid \rightarrow peak in magnitude spectrum
- Frequency resolution: $1/2$ bin
- Improvement of frequency resolution by:
 - zero-padding
 - spectral interpolation

Spectral peaks and window-size

If $B_f = B_s f_s / M$ and $\Delta = |f_{k+1} - f_k|$

B_s = main-lobe bandwidth of window

f_s = sampling rate in Hz

M = window size

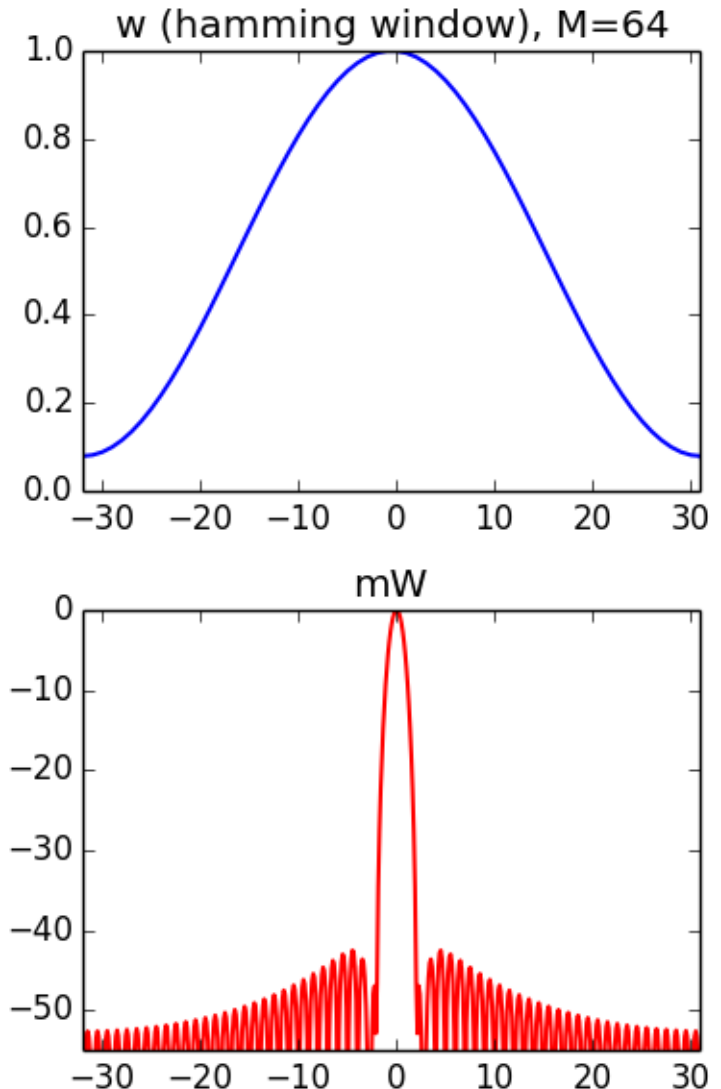
f_k and f_{k+1} = frequency of sinusoids in Hz

$$M \geq B_s \frac{f_s}{\Delta} = B_s \frac{f_s}{|f_{k+1} - f_k|}$$

If $f_0 = \Delta$, then $B_f \leq f_0$

and $M \geq B_s f_s / f_0$, or $M \geq B_s P$,

where P = period in samples

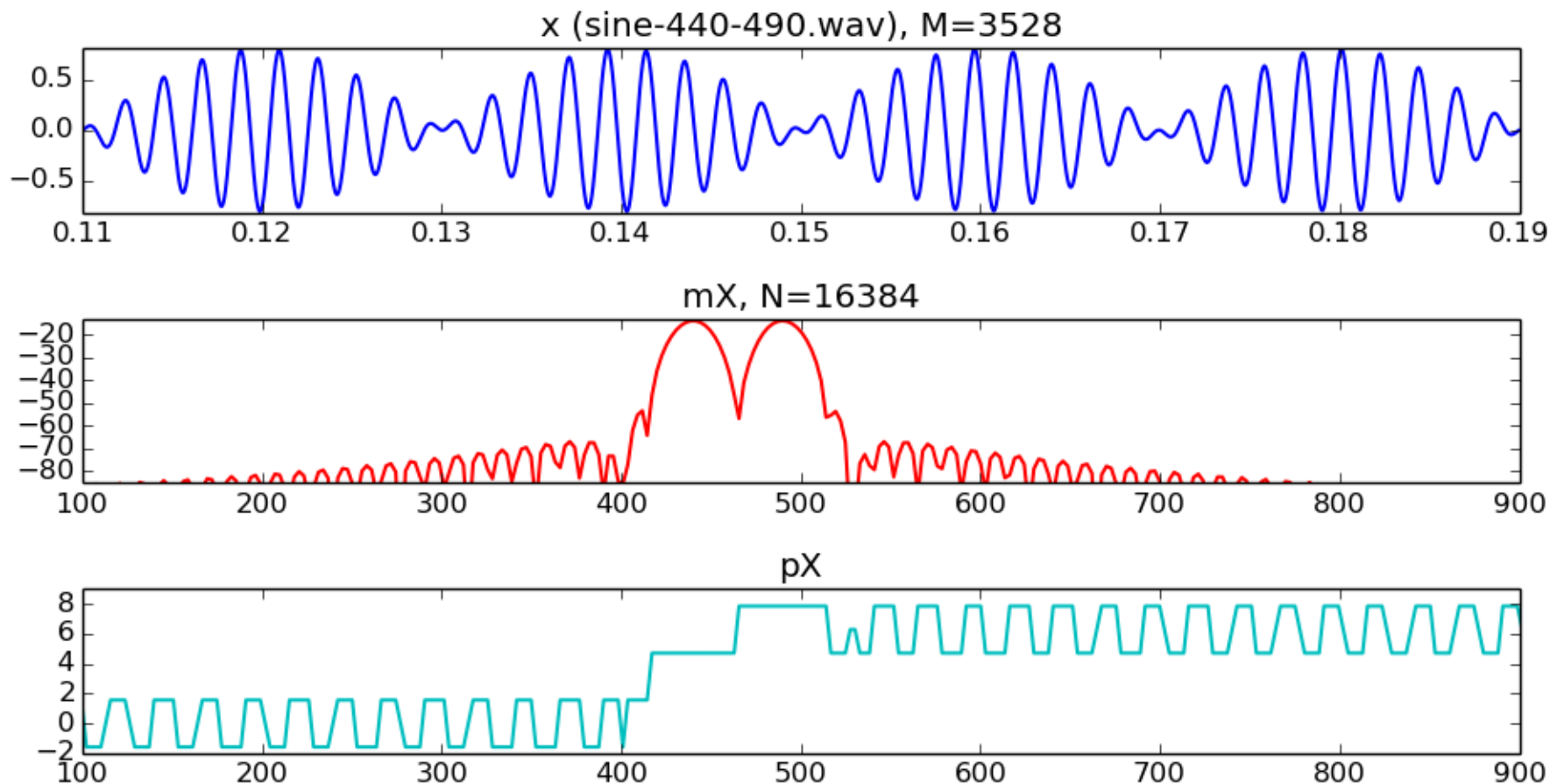


Hamming window : $B_s = 4$

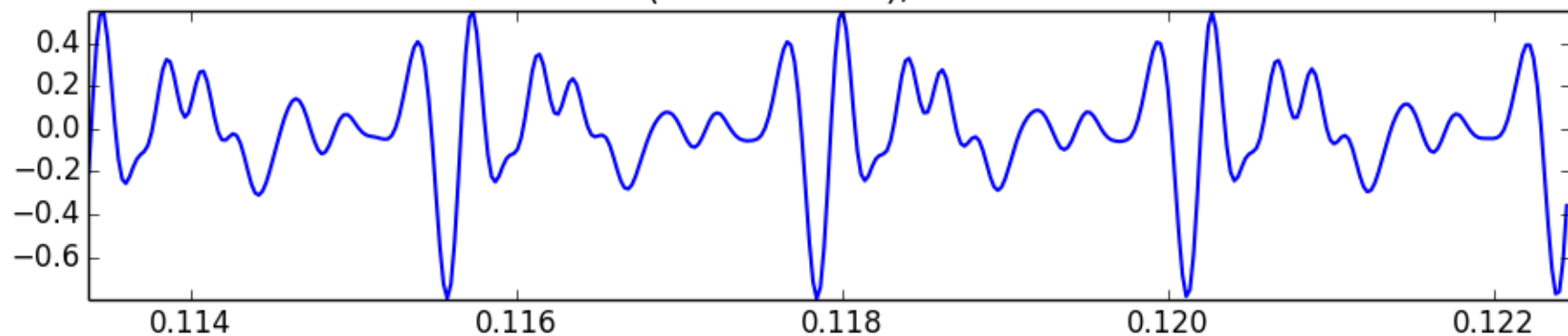
$f_s = 44100$ Hz

$f_k = 440$ Hz ; $f_{k+1} = 490$ Hz

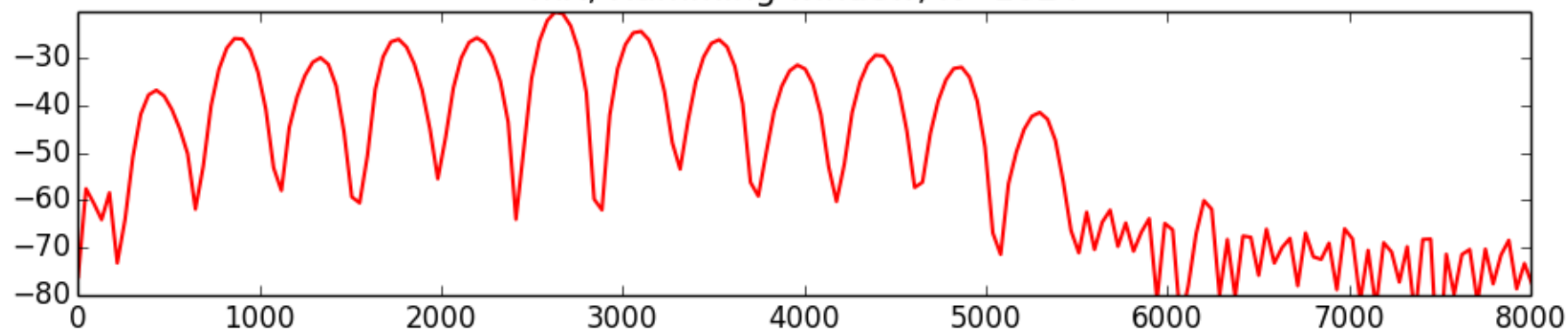
$$M \geq B_s \frac{f_s}{|f_{k+1} - f_k|} = 4 \frac{44100}{|490 - 440|} = 3528$$



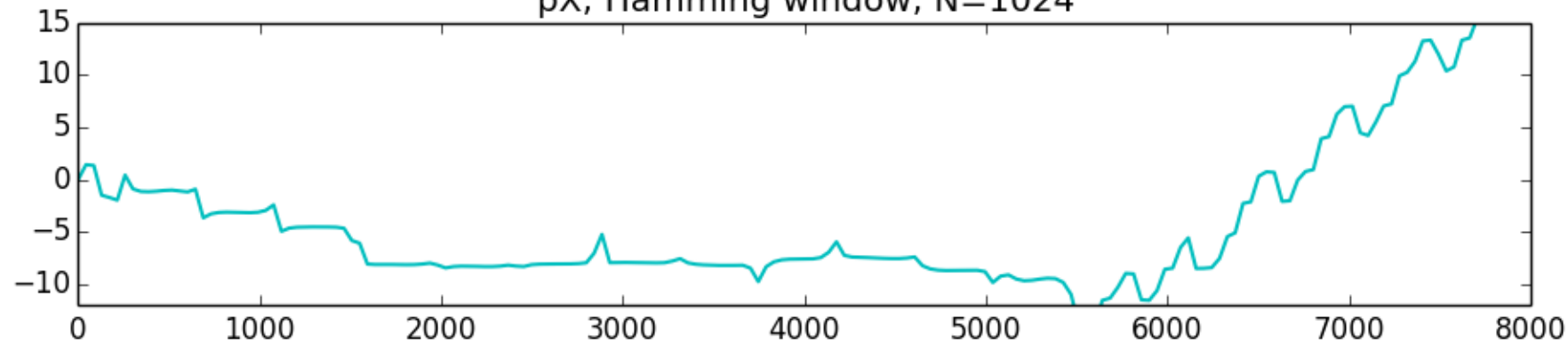
x (oboe-A4.wav), M=401



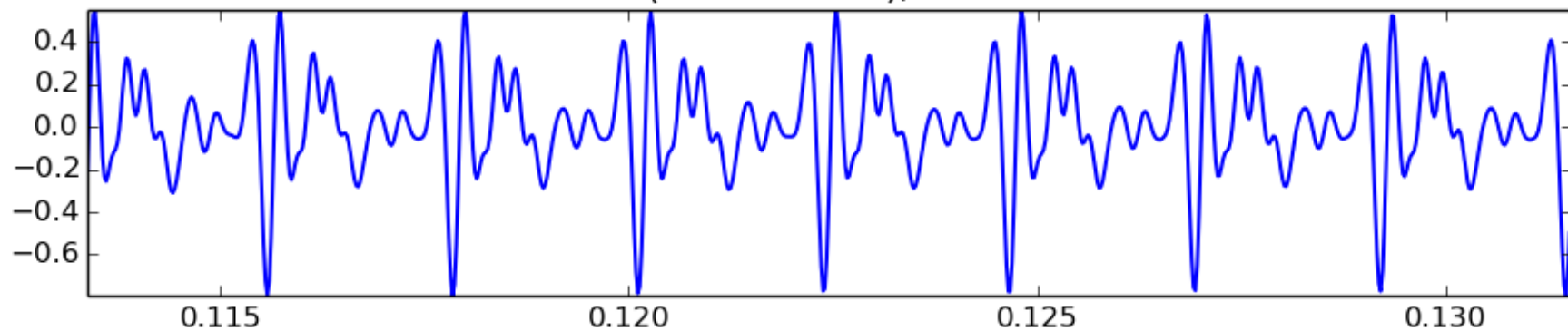
mX; Hamming window, N=1024



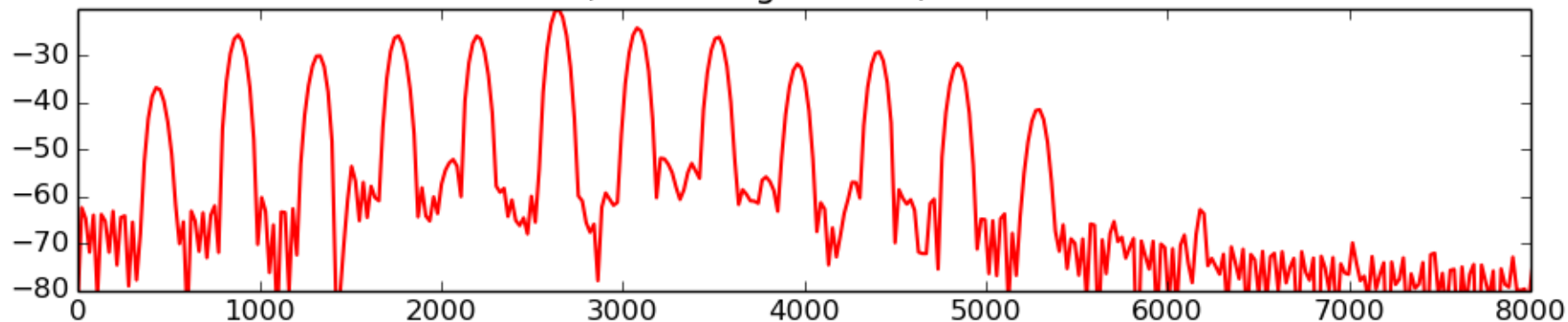
pX; Hamming window, N=1024



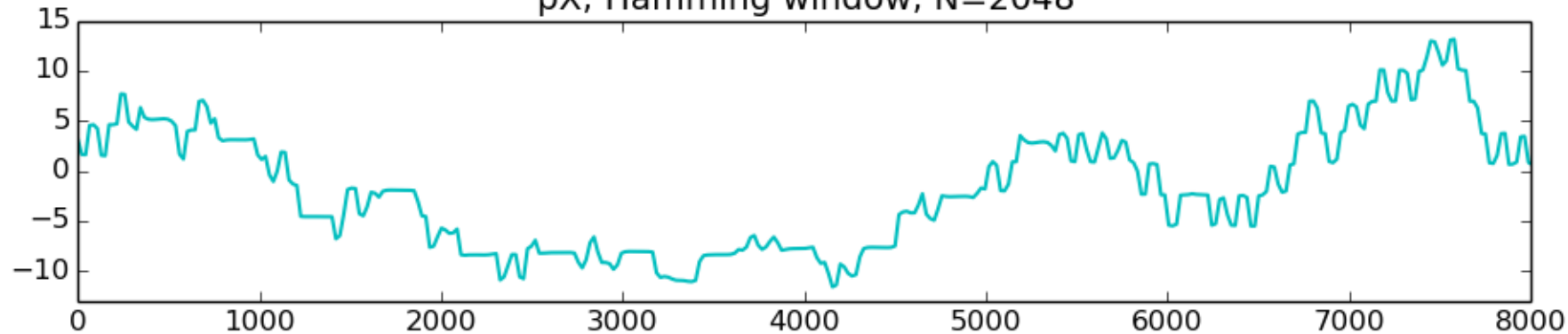
x (oboe-A4.wav), M=801



mX; Hamming window, N=2048



pX; Hamming window, N=2048



References and credits

- More information in:
http://en.wikipedia.org/wiki/Sinusoidal_model
- Reference on sinusoidal modeling by Julius O. Smith:
https://ccrma.stanford.edu/~jos/sasp/Spectrum_Analysis_Sinusoids.html
- Sounds from:
<http://www.freesound.org/people/xserra/packs/13038/>
- Slides released under CC Attribution-Noncommercial-Share Alike license and code under Affero GPL license.
All available from <https://github.com/MTG/sms-tools>

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