

## 5주차. 만유인력과 천체의 운동 II

$$\vec{F} = -\frac{Gm_1m_2}{|\vec{r}|^2}\hat{r}$$



# 3체 운동

$$\vec{F} = -\frac{Gm_1m_2}{|r|^2}\hat{r}$$







GlowScript 2.7 VPython

#Creating Objects

$r = 385000 \times 10^3 / 2$

earth = sphere(pos = vector(r,0,0), radius = 6400000,  
texture = textures.earth)

moon = sphere(pos = vector(-r,0,0), radius = 6400000,  
make\_trail = True)

sat = sphere(pos = vector(0,0,0), radius = 1737000, color  
= color.yellow, make\_trail = True)

sf = 3 #scaling factor  
earth.radius = sf\*earth.radius  
moon.radius = sf\*moon.radius  
sat.radius = sf\*sat.radius

#Physical Properties

$G = 6.67 \times 10^{-11}$

earth.m =  $5.972 \times 10^{24}$

moon.m =  $5.972 \times 10^{24}$

sat.m =  $1/10 \times \text{earth.m}$

#earth.v = vec(0,0,0)

moon.v = vec(0,1000,0)

sat.v = vec(0,0,0)

#Momentum Conservation

earth.v = -moon.v\*moon.m/earth.m

attach\_trail(earth)

#time

t = 0

dt = 60\*5

#Simulation Loop

while True:

rate(1000)

#Forces

r\_me = moon.pos-earth.pos

f\_me =  $-G \times \text{earth.m} \times \text{moon.m} / \text{mag}(\text{r21})^2 \times \text{norm}(\text{r21})$

r\_sm = sat.pos - moon.pos

f\_sm =  $-G \times \text{sat.m} \times \text{moon.m} / \text{mag}(\text{r32})^2 \times \text{norm}(\text{r32})$

r\_se = sat.pos - earth.pos

f\_se =  $-G \times \text{sat.m} \times \text{earth.m} / \text{mag}(\text{r31})^2 \times \text{norm}(\text{r31})$

#Time Integration

earth.v = earth.v + (-f21-f31)/earth.m\*dt

moon.v = moon.v + (f21-f32)/moon.m\*dt

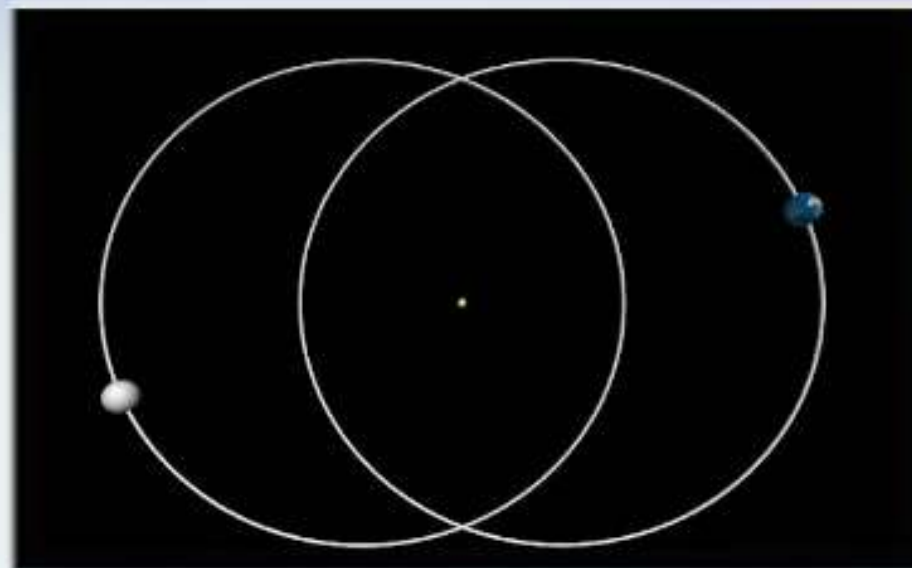
sat.v = sat.v + (f32+f31)/sat.m\*dt

earth.pos = earth.pos + earth.v\*dt

moon.pos = moon.pos + moon.v\*dt

sat.pos = sat.pos + sat.v\*dt

t = t + dt



달과 지구의 질량이 같아 위성을 잡아당기는 힘도 평형



## GlowScript 2.7 VPython

#Creating Objects

$r = 385000e3/2$

earth = sphere(pos = vector(r,0,0), radius = 6400000,  
texture = textures.earth)

moon = sphere(pos = vector(-r,0,0), radius = 6400000,  
make\_trail = True)

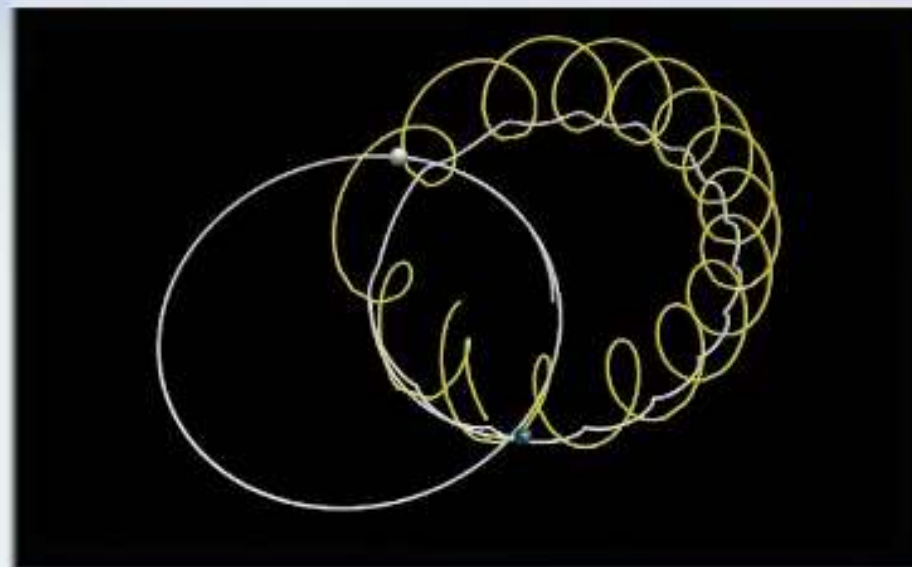
sat = sphere(pos = vector(-0.01\*r,0,0),  
radius = 1737000, color = color.yellow,  
make\_trail = True)

sf = 3 #scaling factor

earth.radius = sf\*earth.radius

moon.radius = sf\*moon.radius

sat.radius = sf\*sat.radius





## GlowScript 2.7 VPython

### #Creating Objects

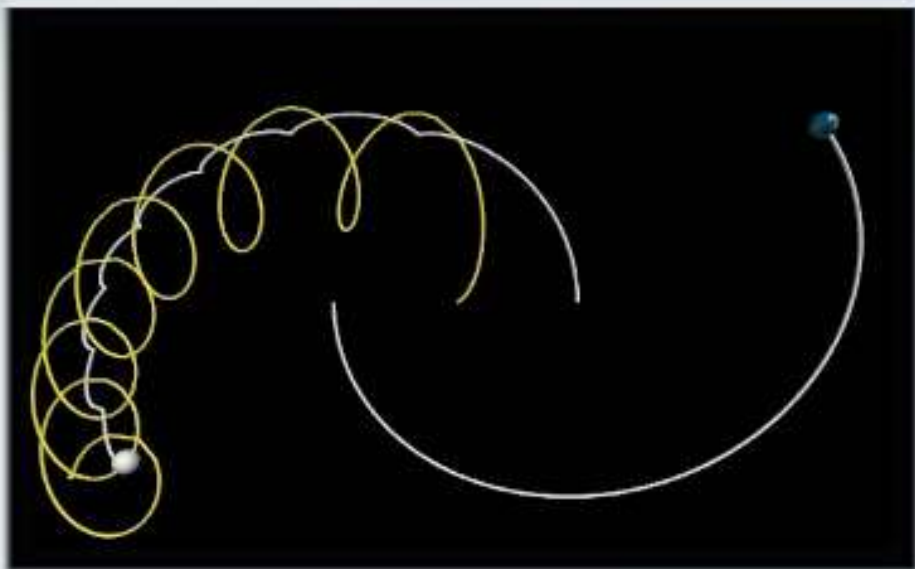
```
r = 385000e3/2
earth = sphere(pos = vector(r,0,0), radius = 6400000,
texture = textures.earth)
moon = sphere(pos = vector(-r,0,0), radius = 6400000,
make_trail = True)
```

```
sat = sphere(pos = vector(0.01*r,0,0),
radius = 1737000, color = color.yellow, make_trail
= True)
```

```
sf = 3 #scaling factor
earth.radius = sf*earth.radius
moon.radius = sf*moon.radius
sat.radius = sf*sat.radius
```







## GlowScript 2.7 VPython

### #Creating Objects

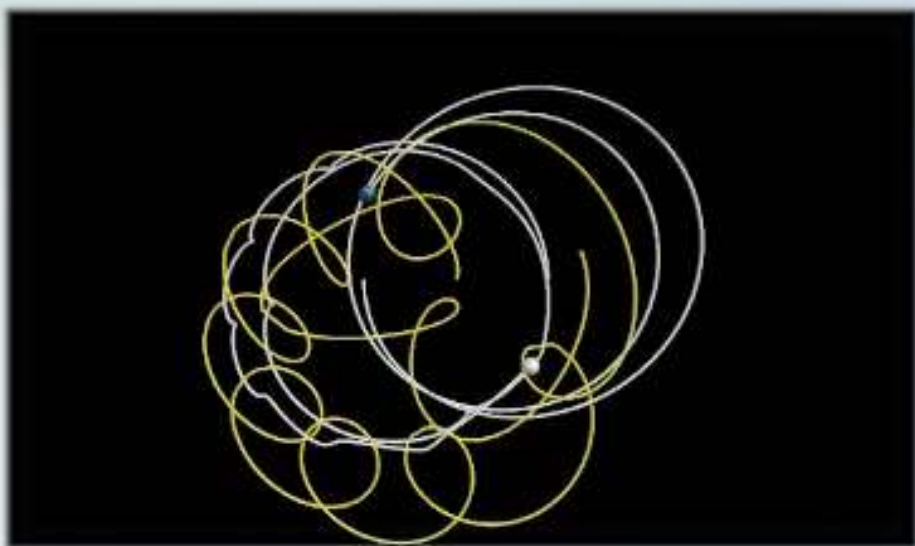
```
r = 385000e3/2
earth = sphere(pos = vector(r,0,0), radius = 6400000,
texture = textures.earth)
moon = sphere(pos = vector(-r,0,0), radius = 6400000,
make_trail = True)
```

```
sat = sphere(pos = vector(0.001*r,0,0),
radius = 1737000, color = color.yellow, make_trail
= True)
```

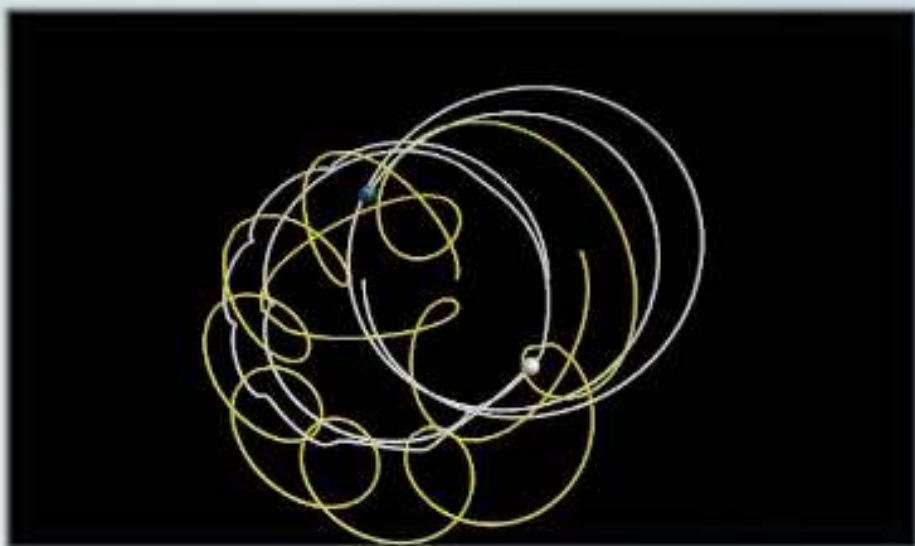
```
sf = 3 #scaling factor
earth.radius = sf*earth.radius
moon.radius = sf*moon.radius
sat.radius = sf*sat.radius
```







물체의 개수가 많아지면 움직임을 수학적인 일반으로 표현 불가



컴퓨터 계산으로 근사적인 형태로만 알 수 있음





15,000kg



지구 반지름에서 10배 떨어진 곳에서 출발  
연료 없이 지구와 달의 **인력** 으로만 이동

GlowScript 2.7 VPython

```
#drawing obj.
Earth = sphere(pos=vec(0,0,0), radius=6.4e6,
color=color.blue)
Moon = sphere(pos=vec(4e8, 0,0), radius=1.75e6)
#constants
G = 6.7e-11
Earth.m = 6e24
Moon.m = 7e22

#craft
craft = sphere(pos=vec(-10*Earth.radius, 0,0),
radius=1e6, color=color.yellow, make_trail=True)
craft.m = 15e3

Earth.radius *=sf
Moon.radius *=sf

#initial vel.
craft.v = vec(0,2e3,0) # initial vel without moon
```

```
#time
t = 0
dt = 60

#time integration
while t < 10*365*24*60*60:
    rate(500)
    ##Force
    r = craft.pos - Earth.pos
    rmag = mag(r)
    rhat = r/rmag
    Earth.f = -G*Earth.m*craft.m/rmag**2*rhat
    rmoon = craft.pos - Moon.pos
    rmoon_mag = mag(rmoon)
    rmoon_hat = rmoon/rmoon_mag
    Moon.f = -G*Moon.m*craft.m/rmoon_mag**2
        *rmoon_hat
    craft.f = Earth.f + Moon.f
    craft.v = craft.v + craft.f/craft.m*dt
    craft.pos = craft.pos + craft.v*dt
    t = t+dt
```





GlowScript 2.7 VPython

```
#drawing obj.
Earth = sphere(pos=vec(0,0,0), radius=6.4e6,
color=color.blue)
Moon = sphere(pos=vec(4e8, 0,0), radius=1.75e6)
#constants
G = 6.7e-11
Earth.m = 6e24
Moon.m = 7e22

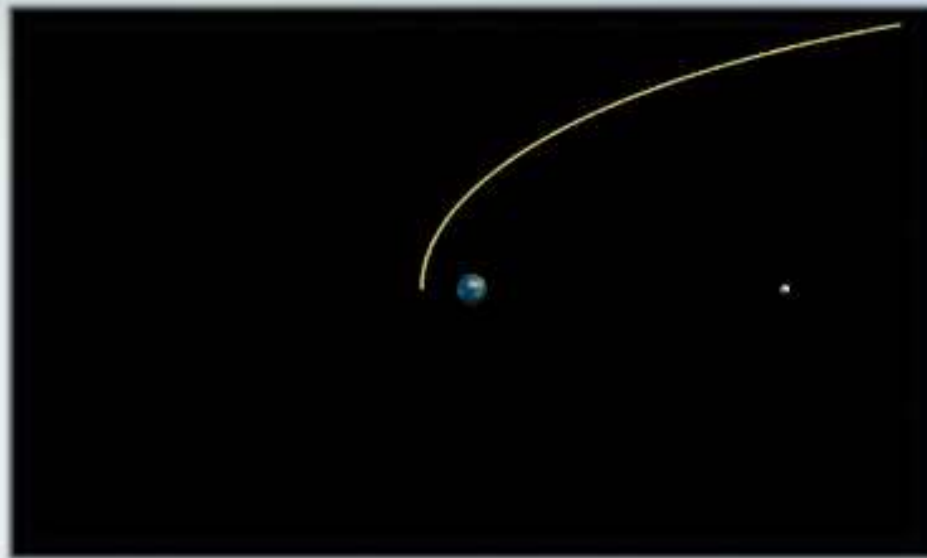
#craft
craft = sphere(pos=vec(-10*Earth.radius, 0,0),
radius=1e6, color=color.yellow, make_trail=True)
craft.m = 15e3

Earth.radius *=sf
Moon.radius *=sf

#initial vel.
craft.v = vec(0,2e3,0) # initial vel without moon
craft.v = vec(0,3.5e3,0) # hyperbolic
```

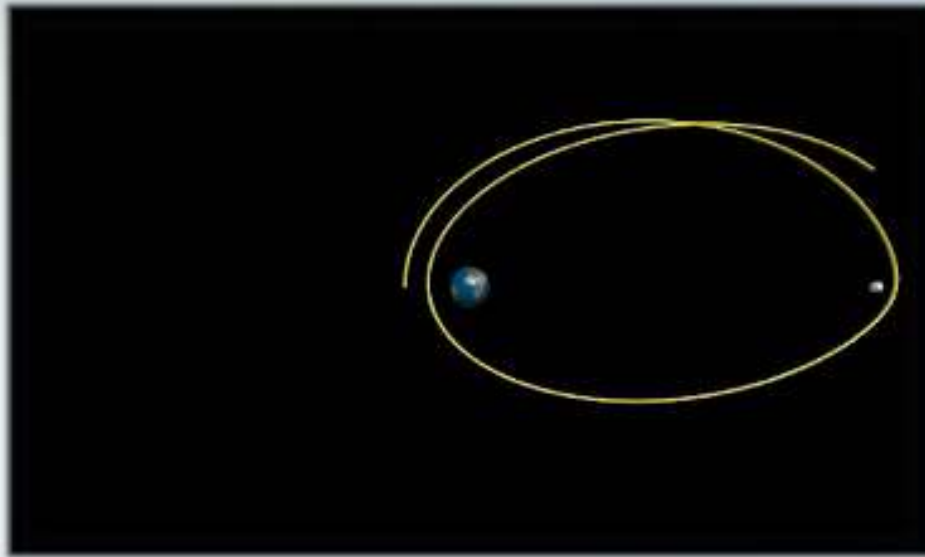






```
#initial vel.  
craft.v = vec(0,3.3e3,0)
```





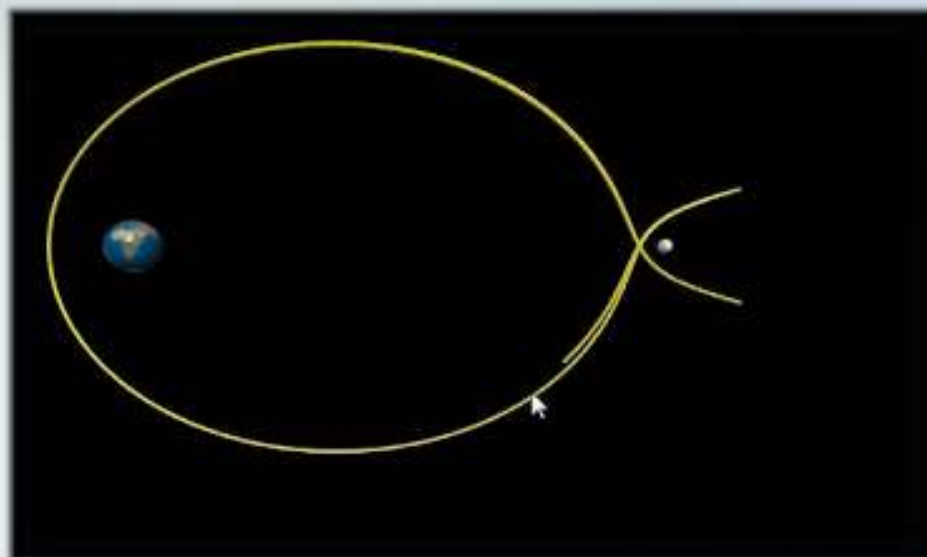


```
craft.v = vec(0,3.27e3,0) #critical pt.
```





만유인력으로 만들어지는 궤도는 원, 타원, 포물선 형태



3체에서는 다양한 형태의 궤도 생성







수학적 일반 해를 가지지 못하는 **3체 운동**의  
움직임을 확신 할 수 있는 이유는?



시간 간격을 줄여 시뮬레이션을 진행해도  
**궤도**가 크게 달라지지 않음을 확인할 수 있다.



시뮬레이션을 통해 **근사적인 방법**으로 진행



시간 간격을 줄여 보며 시뮬레이션

궤도로의 움직임을 확인후 확신





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