Discrete Math HW #9

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16. Given any arbitrary positive integers a, b, and c, show that if $a|c$, $b|c$ and $\gcd(a,b) = 1$, then $ab|c$.

Proof: Assume that given any arbitrary $a, b, c \in \mathbb{N}$, $a|c$, $b|c$ and $\gcd(a,b) = 1$. If the $\gcd(a,b) = 1$ $\exists s, t \in \mathbb{Z}$ such that $as + bt = 1$. Multipliying c by both sides yields: $c(as + bt) = c(1)$. Additionally, going by the definition of divisibility, under the assumption that $a|c$, $b|c$, indicates that $c = aj$ and $c = bk$ for some $j, k \in \mathbb{Z}$. Substituting the different values of c with the equation derived above yields:

$$\begin{aligned} c &= c(as + bt) \\ &= cas + cbt \\ &= bkas + ajbt \\ &= abks + abjt \\ &= ab(ks + jt) \end{aligned}$$

Since in the above equation $ab|c$, that shows that if $a|c$, $b|c$ and $\gcd(a,b) = 1$ then $ab|c$. $\square$

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17. Let $a, b, m, n \in \mathbb{Z}$ with $m, n > 0$. Prove that if $a \equiv b \pmod{n}$ and $m|n$, then $a \equiv b \pmod{m}$.

    Proof: Assume $a \equiv b \pmod{n}$ and $m|n$. If $a \equiv b \pmod{n}$ that means $n|(a-b)$; which by the definition of divisibility means that $a - b = nk$ for some $k \in \mathbb{Z}$. If $m|n$ then $n = mj$ for some $j \in \mathbb{Z}$ for the same reason. Substituting mj for n in $a - b = nk$ yields $a - b = mjk$. Therefore, m clearly divides $a - b$ and for that reason $a \equiv b \pmod{m}$. In conclusion, if $a \equiv b \pmod{n}$ and $m|n$ for some $a, b, m, n \in \mathbb{Z}$ with $m, n > 0$ then $a \equiv b \pmod{m}$. $\square$