Discrete Math HW #2

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10. a) $(p \wedge q) \implies (q \vee r)$ is false. b) $(q \wedge r) \implies (p \wedge q)$ is false.

    a) The value of p could not be determined for the implication to evaluate as false. For an implication to be false, the hypothesis needs to be true and the conclusion needs to be false. For the hypothesis to be true, both p and q must be true since they are expressed using a conjunction. However, given that both p and q are true, the conclusion's disjunction would evaluate to true as well, as $(q \vee r)$ when q is true evaluates to true. Hence $(p \wedge q) \implies (q \vee r)$ is a tautology which is true for all values of p, q, and r and cannot evaluate to false - thus the question is unanswerable.

    b) p must be false for $(q \wedge r) \implies (p \wedge q)$ to evaluate to false. This is because for an implication to evaluate to false, the hypothesis must be true and the conclusion must be false. To make the hypothesis true, both q and r which are operated on by a conjunction must be true. So for the conclusions conjunction to be false (and for the implication as a whole to be false), p must be false to get the expression into a $T \implies F$ format.

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11. Answer each of the following tasks:

    (a) Find the contrapositive of the following implication:
    If x^2 is an even integer, then x is an even integer.

    (b) Write the biconditional as two implications:
    A right triangle with legs a and b and hypotenuse c exists if and only if $a^2 + b^2 = c^2$.

a) The contrapositive statement would be: If x is not an even integer, then x^2 is not an even integer.

b) If $a^2 + b^2 = c^2$ then a right triangle with legs a, b, and hypotenuse c exists.

If a right triangle with legs a, b, and hypotenuse c exists, then $a^2 + b^2 = c^2$

12. Determine whether formulas u and v are logically equivalent (you may use an argument, a truth table or properties of logical equivalences). Determine whether formulas u and v are logically equivalent (you may use an argument, a truth table or properties of logical equivalences).

$$
\begin{aligned}
(a)\;\; u : (p \implies q) \wedge (p \implies \overline{q}) &\qquad v : \overline{p} \\
(b)\;\; u : p \implies q &\qquad v : q \implies p \\
(c)\;\; u : (p \implies q) \implies r &\qquad v : p \implies (q \implies r) \\
(d)\;\; u : p \implies (q \vee r) &\qquad v : (p \implies q) \vee (p \implies r) \\
(e)\;\; u : \overline{p \iff q} &\qquad v : \overline{p} \iff \overline{q}
\end{aligned}
$$

a)

To prove that the left expression is logically equivalent to the right expression, we algebraically manipulate the left expression to look identical to the right expression.

$$
\begin{aligned}
(p \implies q) \wedge (p \implies \overline{q}) &\equiv (\overline{p} \;\vee\; q) \;\wedge\; (\overline{p} \;\wedge\; q) && \textit{implication as disjunction} \\
&\equiv \overline{p} \vee (q \wedge \overline{q}) && \textit{distributivity} \\
&\equiv \overline{p} && \textit{inverse laws}
\end{aligned}
$$

As can be seen both sides can be manipulated to equal $\overline{p}$ and are thus equivalent in all aspects.

b)

A truth table will be usde to show that u is equvivalent to v.

p	$\overline{p}$	q	$\overline{q}$	p $\implies q$	q $\implies p$
T	T	F	F	T	F
T	F	F	T	F	T
F	T	T	F	T	T
F	F	T	T	T	T

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c)

p	q	r	p $\implies q$	q $\implies r$	(p $\implies q) \implies r$	p $\implies (q \implies r)$
T	T	T	T	T	T	T
T	T	F	T	F	F	F
T	F	T	F	T	T	T
T	F	F	F	T	T	T
F	T	T	T	T	T	T
F	T	F	T	F	F	T
F	F	T	T	T	T	T
F	F	F	T	T	F	T

d)

To prove that the left expression is logically equivalent to the right expression, we algebraically manipulate both expressions starting with the left one followed by the right one.

$$p \implies (q \vee r) \equiv \overline{p} \vee q \vee r \qquad \textit{implication as disjunction}$$

$$\begin{aligned}(p \implies q) \vee (p \implies r) &\equiv \overline{p} \vee q \vee \overline{p} \vee r && \textit{implication as disjunction}\\ &\equiv \overline{p} \vee q \vee r && \textit{Idempotent Laws}\end{aligned}$$

As can be seen both sides can be manipulated to equal $\overline{p}$ and are thus equivalent in all aspects.

e)

A truth table will be usde to show that u is equvivalent to v.

p	$\overline{p}$	q	$\overline{q}$	$\overline{p \leftrightarrow q}$	$\overline{q} \leftrightarrow \overline{p}$
T	F	T	F	F	T
T	F	F	T	T	F
F	T	T	F	T	F
F	T	F	T	F	T