ICBV HW #4

Dor Litvak Gal Elgavish

Question 1 – Line Drawing Interpretation

Section A

There is a consistent labeling assignment for the given shape.

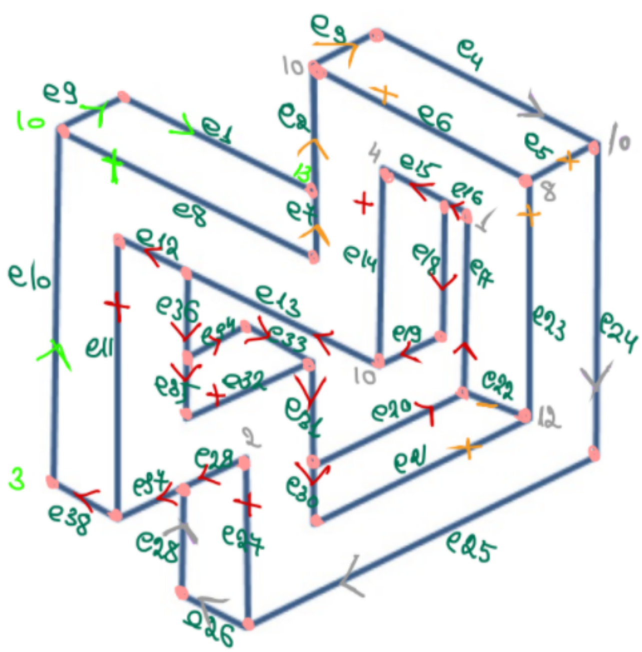

Figure 1: Labeling assignment.

Section B

1. What was the initial guess? How was it chosen?

Our initial guess was chosen by the catalog, each vertex has a unique shapes and possible labels.

As you can see in the figure below, vertices 1,2,3 are arrow type vertices, they can get in 1/3 present the label 10/11/12. The 4th vertex is a Y-shape vertex and can get 1/3 present the labels 7/8/9. This is how

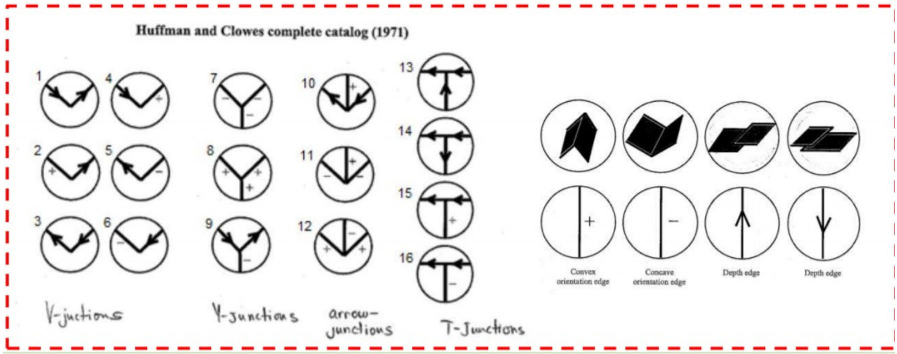

Figure 2: Catalog

we choose our initial guesses.

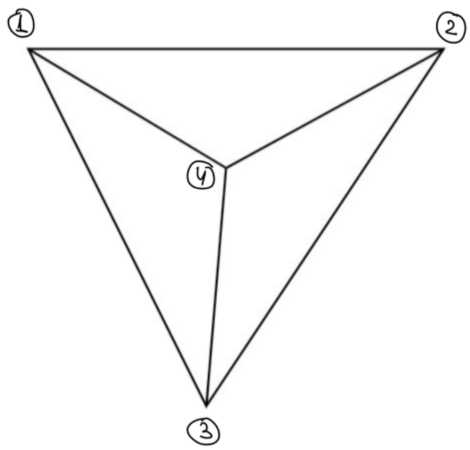

Figure 3: Provided line drawing image

2. What was the compatibility function? How was it chosen?

The compatibility function was chosen by the possible pairs of vertices types that can get non-contradicting assignments. In the provided image, all the vertices are connected to all. For 1,2,3 vertices, the assignments can be (10,10), (11,11), (12,12). For 4 with the other vertices the assignments can be (10, 8), (11, 8), (12, 7), (12, 9).

Over all our compatibility function get 1 only if a legal assignment as follows occurs:

```
{(1, 1): [(10, 10), (11, 11), (12, 12)],
 (1, 2): [(10, 10), (11, 11), (12, 12)],
 (1, 3): [(10, 10), (11, 11), (12, 12)],
 (1, 4): [(10, 8), (11, 8), (12, 7), (12, 9)],
 (2, 1): [(10, 10), (11, 11), (12, 12)],
 (2, 2): [(10, 10), (11, 11), (12, 12)],
 (2, 3): [(10, 10), (11, 11), (12, 12)],
 (2, 4): [(10, 8), (11, 8), (12, 7), (12, 9)],
 (3, 1): [(10, 10), (11, 11), (12, 12)],
 (3, 2): [(10, 10), (11, 11), (12, 12)],
 (3, 3): [(10, 10), (11, 11), (12, 12)],
 (3, 4): [(10, 8), (11, 8), (12, 7), (12, 9)],
 (4, 1): [(8, 10), (8, 11), (7, 12), (9, 12)],
 (4, 2): [(8, 10), (8, 11), (7, 12), (9, 12)],
 (4, 3): [(8, 10), (8, 11), (7, 12), (9, 12)],
 (4, 4): [(7, 7), (8, 8), (9, 9)]}
```

Figure 4

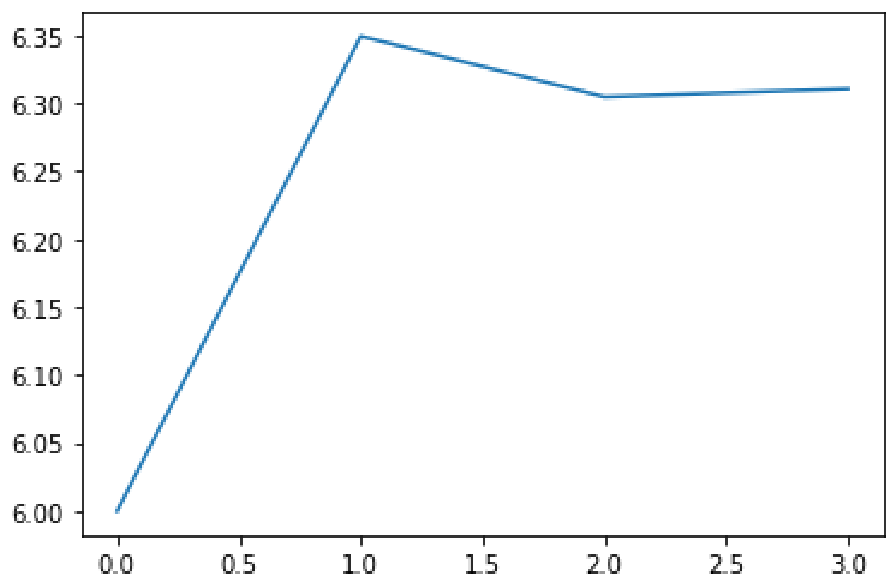

Figure 5: The average local consistency as a function of the iteration number

	b=1	b=2	b=3	b=4
λ=1	0.000000	0.000000	0.000000	0.000000
λ=2	0.000000	0.000000	0.000000	0.000000
λ=3	0.000000	0.000000	0.000000	0.000000
λ=4	0.000000	0.000000	0.000000	0.000000
λ=5	0.000000	0.000000	0.000000	0.000000
λ=6	0.000000	0.000000	0.000000	0.000000
λ=7	0.000000	0.000000	0.000000	0.274604
λ=8	0.000000	0.000000	0.000000	0.450792
λ=9	0.000000	0.000000	0.000000	0.274604
λ=10	0.310745	0.310745	0.310745	0.000000
λ=11	0.310745	0.310745	0.310745	0.000000
λ=12	0.378510	0.378510	0.378510	0.000000
λ=13	0.000000	0.000000	0.000000	0.000000
λ=14	0.000000	0.000000	0.000000	0.000000
λ=15	0.000000	0.000000	0.000000	0.000000
λ=16	0.000000	0.000000	0.000000	0.000000

Figure 6: The final labeling distribution

Final labels are: b1=12, b2=12, b3=12, b4=8 which are not consistent.

3. Is your labeling consistent? If not, explain shortly why you did not get good results. Otherwise, describe the obtained 3D shape. In our compatibility function, we have the pair (12, 9) for vertex 4 and vertices 1/2/3, Although that if we are looking at the image, this assignment is not possible and is not consistent with the other vertices. This is because the compatibility function is a pairwise element consistent function. If we will use a different consistent function or remove the pair (12, 9), we would get a correct assignment. b1=10, b2=10, b3=10, b4=8. which is a pyramid.

Question 2 – Shape from Shading

Section A

Since the object is described as a height function above a reference plane XY, we can say that the equation of the normal is defined:

$$\hat{N} = \frac{(-p, -q, 1)}{\sqrt{p^2 + q^2 + 1}}$$

We are given that:

$$I_1(x_0, y_0) = 2 \quad I_2(x_0, y_0) = \frac{1}{\sqrt{3}}$$

$$R_1(p, q) = p + q \quad R_2(p, q) = \frac{1}{\sqrt{p^2 + q^2 + 1}}$$

So, the equations are:

$$I_1(x_0, y_0) = R_1(p_0, q_0) \longrightarrow p_0 + q_0 = 2$$

$$I_2(x_0, y_0) = R_2(p_0, q_0) \longrightarrow \frac{1}{\sqrt{p_0^2 + q_0^2 + 1}} = \frac{1}{\sqrt{3}}$$

And after solving the equations we get: $p_0 = q_0 = 1$.
So the normal is:

$$\hat{N}(x_0, y_0) = \frac{(-1, -1, 1)}{\sqrt{1^2 + 1^2 + 1}} = \frac{(-1, -1, 1)}{\sqrt{3}}$$

Section B

We are given:

$$\begin{aligned} I(x, y) &= \cos x \cdot \sin y \\ R(p, q) &= p + 2q + pq \\ (x_0, y_0, H(x_0, y_0)) &= (3, 3, 5) \\ n(x_0, y_0) &= (-0.5345, -0.8017, 0.2672) \\ \delta_s &= 1 \end{aligned}$$

Since the object is described as a height function above the plane R^2, we can say that the equation of the normal is defined:

$$\hat{N} = \frac{(-p, -q, 1)}{\sqrt{p^2 + q^2 + 1}}$$

And we get the following system:

$$n(x_0, y_0) = \frac{(-p_0, -q_0, 1)}{\sqrt{p_0^2 + q_0^2 + 1}} = (-0.5345, -0.8017, 0.2672)$$

After solving we are left with:

$$p_0 = \frac{0.5345}{0.2672} \quad q_0 = \frac{0.8017}{0.2672}$$

We will also use the following equations:

$$\begin{aligned}
R_p &= 1+q \\
R_q &= 2+p \\
I_x &= -\sin y \cdot \sin x \\
I_y &= \cos y \cdot \cos x \\
\delta_x &= R_p\delta_s = 1+q \\
\delta_y &= R_q\delta_s = 2+p \\
\delta H &= (pR_p+qR_q)\delta_s = p+2q+2pq \\
\delta_p &= I_x\delta_s = -\sin y\cdot\sin x \\
\delta_q &= I_y\delta_s = \cos y\cdot\cos x
\end{aligned}$$

<u>Iteration #1:</u>

$$\begin{aligned}
\delta_x &= 1+q_0 = 1+\frac{0.8017}{0.2672} = \frac{1.0689}{0.2672} = 4.00037 \\
\delta_y &= 2+p_0 = 2+\frac{0.5345}{0.2672} = \frac{1.0689}{0.2672} = 4.00037 \\
\delta H &= p_0+2q_0+2p_0q_0 = \frac{0.5345}{0.2672}+2\cdot\frac{0.8017}{0.2672}+2\cdot\frac{0.5345}{0.2672}\cdot\frac{0.8017}{0.2672} = 20.00486 \\
\delta_p &= -\sin y_0\cdot\sin x_0 = -\sin 3\cdot\sin 3 = -0.01991 \\
\delta_q &= \cos y_0\cdot\cos x_0 = \cos 3\cdot\cos 3 = 0.98008 \\
\hline
x_1 &= x_0+\delta_x = 3+4.00037 = 7.00037 \\
y_1 &= y_0+\delta_y = 3+4.00037 = 7.00037 \\
H(x_1,y_1) &= H(x_0,y_0)+\delta H = 5+20.00486 = 25.00486 \\
p(x_1,y_1) &= p_0+\delta_p = \frac{0.5345}{0.2672}-0.01991 = 1.98046 \\
q(x_1,y_1) &= q_0+\delta_q = \frac{0.8017}{0.2672}+0.98008 = 3.98045
\end{aligned}$$

<u>Iteration #2:</u>

$$\begin{aligned}
\delta_x &= 1+q_1 = 1+3.98045 = 4.98045 \\
\delta_y &= 2+p_1 = 2+1.98046 = 3.98046 \\
\delta H &= p_1+2q_1+2p_1q_1 = 1.98046+2\cdot 3.98045+2\cdot 1.98046\cdot 3.98045 = 25.70760 \\
\delta_p &= -\sin y_1\cdot\sin x_1 = -\sin 7.00037\cdot\sin 7.00037 = -0.431998 \\
\delta_q &= \cos y_1\cdot\cos x_1 = \cos 7.00037\cdot\cos 7.00037 = 0.56800 \\
\hline
x_2 &= x_1+\delta_x = 7.00037+4.98045 = 11.98082 \\
y_2 &= y_1+\delta_y = 7.00037+3.98046 = 10.98083 \\
H(x_2,y_2) &= H(x_1,y_1)+\delta H = 25.00486+25.70760 = 50.71246 \\
p(x_2,y_2) &= p_1+\delta_p = 1.98046-0.431998 = 1.548462 \\
q(x_2,y_2) &= q_1+\delta_q = 3.98045+0.56800 = 4.54845
\end{aligned}$$

So, after two iterations:

$$\begin{aligned}
(x_2,y_2,H(x_2,y_2)) &= (11.98082, 10.98083, 50.71246) \\
n(x_2,y_2) &= \frac{(-1.548462,-4.54845,1)}{\sqrt{(1.548462)^2+(4.54845)^2+1}}
\end{aligned}$$

Section C

Given to us:

- light sources: $E_1, \quad E_2, \quad L_1, \quad L_2, \quad \|L_1\| = 1, \quad \|L_2\| = 1$
- Lambertian material with albedo $= \rho$
- normal vectors: $\hat{n_1}, \quad \hat{n_2}, \quad \|\hat{n_1}\| = 1, \quad \|\hat{n_2}\| = 1$
- image pixels: $I_1, \quad I_2$

1

$$
\begin{aligned}
I_1 = R_1 = \rho\frac{1}{\pi}E_1(\hat{n_1} \cdot L_1) + \rho\frac{1}{\pi}E_2(\hat{n_1} \cdot L_2) \\
I_2 = R_2 = \rho\frac{1}{\pi}E_1(\hat{n_2} \cdot L_1) + \rho\frac{1}{\pi}E_2(\hat{n_2} \cdot L_2)
\end{aligned}
$$

2

The new single light source (*) will create the following brightness levels:

$$
\begin{aligned}
I_1^* = R_1^* = \rho\frac{1}{\pi}E^*(\hat{n_1} \cdot L^*) \\
I_2^* = R_2^* = \rho\frac{1}{\pi}E^*(\hat{n_2} \cdot L^*)
\end{aligned}
$$

Now we will create the following system of equations:

$$
\begin{aligned}
\text{I)} \quad I_1^* = I_1 \longrightarrow \rho\frac{1}{\pi}E^*(\hat{n_1} \cdot L^*) = \rho\frac{1}{\pi}E_1(\hat{n_1} \cdot L_1) + \rho\frac{1}{\pi}E_2(\hat{n_1} \cdot L_2) \\
\text{II)} \quad I_2^* = I_2 \longrightarrow \rho\frac{1}{\pi}E^*(\hat{n_2} \cdot L^*) = \rho\frac{1}{\pi}E_1(\hat{n_2} \cdot L_1) + \rho\frac{1}{\pi}E_2(\hat{n_2} \cdot L_2)
\end{aligned}
$$

Simplification of Eq.I

$$E^* \cdot \hat{n_1}^T L^* = E_1 \cdot \hat{n_1}^T L_1 + E_2 \cdot \hat{n_1}^T L_2$$

After multiplying by $\hat{n_1}$ from the left and dividing by E^*:

$$L^* = \frac{E_1}{E^*} \cdot L_1 + \frac{E_2}{E^*} \cdot L_2$$

Simplification of Eq.II after multiplying by $\hat{n_2}$ from the left and by L^{*T} from the right:

$$E^* = E_1 \cdot L_1 L^{*T} + E_2 \cdot L_2 L^{*T}$$

Plugging in the simplification of Eq.I into the simplification of Eq.II:

$$E^* = E_1 \cdot L_1(\frac{E_1}{E^*} \cdot L_1^T + \frac{E_2}{E^*} \cdot L_2^T) + E_2 \cdot L_2(\frac{E_1}{E^*} \cdot L_1^T + \frac{E_2}{E^*} \cdot L_2^T)$$

After some more algebraic simplifications we are ended up with:

$$
\begin{aligned}
E^* = \sqrt{{E_1}^2 + 2 \cdot E_1 \cdot E_2 \cdot (L_1 \cdot L_2) + {E_2}^2} \\
L^* = \frac{E_1}{E^*} \cdot L_1 + \frac{E_2}{E^*} \cdot L_2
\end{aligned}
$$