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Add the regression line to the scatter plot using geom_abline()

Transforming

line of best fit?

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February 3, 2020

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Excercise and the Brain

PHYS ED

Which Type of Exercise Is Best for the Brain?

BY GRETCHEN REYNOLDS FEBRUARY 17, 2016 5:45 AM 9 509



Getty Images

Figure 1

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Excercise and the Brain

▶ from *The New York Times*, February 2016:

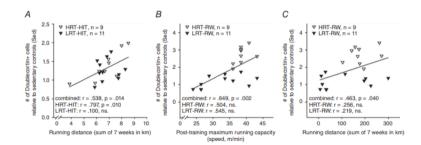
"Some forms of exercise may be much more effective than others at bulking up the brain, according to a remarkable new study in rats. For the first time, scientists compared head-to-head the neurological impacts of different types of exercise: running, weight training and high-intensity interval training. The surprising results suggest that going hard may not be the best option for long-term brain health"

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Excercise and the Brain

► from The Journal of Physiology



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learning objectives

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Final - - II----

Add the regression line to the scatter plot using

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How do outliers affect line of best fit?

- ► Introduce linear regresssion
 - How do we find the line of best fit?
 - What is the slope?
 - What is the intercept?
 - What is the R squred?
- ▶ Using R to run a linear regression and add a regression line to a scatter plot
- ▶ How do we transform data that do not look linear to make a line?
- ▶ How do outliers influence our line of best fit?
- Some Important cautions
 - Association is not causation
 - Do not extrapolate beyond your data
 - ▶ Always consider potential confounders in your interpretation
 - Confirm the shape of your data visually

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Regression

What is a regression line?

- A straight line that is fitted to data to minimize the distance between the data and the fitted line.
- ▶ It is often called the line of best fit.
- ▶ It is also called the least-squares regression line (sometimes refered to as ordinary least squares or ols) this is because mathmatically, the criteria for choosing this line is based on the sum of squares of the vertical distances from the line. We choose the line that minimizes this sum.

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How do outliers affect the line of best fit?

What is a regression line?

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How do outlie

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Once we have calculated this line, the line of best fit can be used to describe the relationship between the explanatory and response variables.

- Can you fit a line of best fit for non-linear relationships?
- Very important to visualize the relationship first. Why?

Equation of the line of best fit

The line of best fit can be represented by the equation for a line:

$$y = a + bx$$

where a is the intercept and b is the slope.

This equation encodes a lot of useful information

In earlier math classes you may have seen this expressed as:

$$y = mx + b$$

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Equation of the line of best fit: the intercept

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$$y = a + bx$$

If x = 0, the equation says that y = a, which is why a is known as the intercept.

Note: Is the value of the intercept always meaningful?

Equation of the line of best fit: the slope

$$y = a + bx$$

b is known as the slope because an increase from x to x+1 is associated with an increase in y by the amount b.

The slope is closely related to the correlation coefficient:

$$b=r\frac{S_y}{S_x}$$

If the correlation coefficient is negative what will be the sign of the *b*?

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The r^2 value or R squared, is the fraction of the variation in the values of y that is explained by the regression of y on x

In a regression where every observation fell exactly on the regression line, the value of r^2 would be 1.

In a linear regression with only one x the r^2 is the square of the correlation coefficient.

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Why the package broom?



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do outliers affect the of best fit?

broom has functions that make the output from the linear model look clean6/64

Add the regression line to the scatter plot using geom_abline()

How do outliers affect

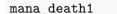
Counfounding

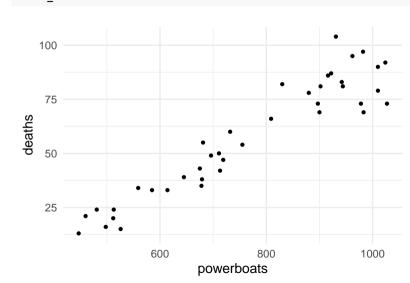
Let's apply the lm() function. Recall the manatee example from Ch.3 that examined the relationship between the number of registered powerboats and the number of manatee deaths in Florida between 1977 and 2016.

Recall that the relationship appeared linear when we examined the scatter plot:

```
library(ggplot2)
mana_death1<-ggplot(mana_data, aes(x = powerboats, y = deaths)) +
   geom_point() +
   theme_minimal(base_size = 15)</pre>
```

Manatee deaths and powerboat purchases





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Regression

Fitting a linear model in R

Add the regression line to the scatter plot using geom_abline()

How do outliers affect the

line of best fit?

```
lm() of manatee deaths and powerboat purchases
```

Calculate the line of best fit:

```
mana_lm <- lm(deaths ~ powerboats, mana_data)
library(broom)</pre>
```

Warning: package 'broom' was built under R version 3.5.3

```
tidy(mana_lm)
```

```
## # A tibble: 2 \times 5
##
    term estimate std.error statistic
                                           p.value
##
    <chr>
                           <dbl>
                                             <dbl>
              <dbl>
                                    <dbl>
  1 (Intercept) -46.8
                         6.03
                                    -7.75 2.43e- 9
  2 powerboats
                  0.136
                         0.00764
                                    17.8
                                          5.21e-20
```

Only pay attention to the term and estimate columns for now.

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Fitting a linear model in R

Add the regression line to

the scatter plot using geom_abline() Transforming data How do outliers affect the

Fitting a linear model in R

Interpret the model output ## # A tibble: 2×5

```
##
     term
                 estimate std.error statistic
                                               p.value
##
     <chr>
                    <dbl>
                              <dbl>
                                        <dbl>
                                                 <dbl>
    (Intercept) -46.8
                            6.03
                                        -7.75 2.43e - 9
## 2 powerboats
                    0.136
                            0.00764
                                        17.8
                                              5.21e-20
```

- Intercept: The predicted number of deaths if there were no powerboats. But the prediction is negative. Why?
- Powerboats: This is the slope. What does the estimated slope for powerboats mean?

```
## # A tibble: 2 x 5
##
                 estimate std.error statistic
     term
                                                   <dbl>
##
     <chr>>
                    <dbl>
                               <dbl>
                                         <dbl>
     (Intercept)
                  -46.8
                             6.03
                                         -7.75 2.43e- 9
## 2 powerboats
                    0.136
                             0.00764
                                         17.8
                                               5.21e-20
```

- ▶ A one unit change in the number of powerboats registered (X 1,000) is associated with an increase of manatee deaths of 0.1358. That is, an increase in the number of powerboats registered by 1,000 is association with 0.1358 more manatee deaths.
- If powerboat registered increased by 100,000 how many more manatee deaths are expected?

Counfounding

```
mana_data_units<-mana_data%>%mutate(actual_powerboats = powerboats
mana_lm_units <- lm(deaths ~ actual_powerboats, mana_data_units)
tidy(mana_lm_units)</pre>
```

```
## # A tibble: 2 \times 5
##
                                     std.error statistic
                                                            p.value
     term
                          estimate
##
     <chr>
                              <dbl>
                                          <dbl>
                                                    <dbl>
                                                              <dbl>
##
     (Intercept)
                        -46.8
                                    6.03
                                                    -7.75 2.43e- 9
   2 actual powerboats
                          0.000136 0.00000764
                                                    17.8
                                                           5.21e-20
```

What happened to the slope? To the intercept?

Getting the R-squared from vour model

When we run a linear model, the r-squared is also calculated. Here is how to see the r-squared for the manatee data:

```
glance(mana_lm)
```

```
## # A tibble: 1 x 11
##
     r.squared adj.r.squared sigma statistic p.value
                                                           df logLik
         <dbl>
                                        <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl> <dbl
##
                       <dbl> <dbl>
                                                            2 -143.
## 1
         0.893
                       0.890 8.82
                                         316. 5.21e-20
## # ... with 2 more variables: deviance <dbl>, df.residual <int>
```

Focus on:

- Column called r.squared values only.
- Interpretation of r-squared: The fraction of the variation in the values of y that is explained by the line of best fit.

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Fitting a linear model in R

AIC 292. 297

Transforming

How do outliers affect the line of best fit?

```
library(dplyr)
mana_cor <- mana_data %>%
    summarize(corr_mana = cor(powerboats, deaths))
mana_cor

## # A tibble: 1 x 1
```

```
## # A tibble: 1 x
## corr_mana
## <dbl>
## 1 0.945
```

Correlation vs R Squared

```
glance(mana_lm)%>% pull(r.squared)

## [1] 0.8926573

#square the correlation coefficient
.9448054^2

## [1] 0.8926572
```

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Fitting a linear model in R

Add the regression line to the scatter plot using geom_abline()

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Citation of the comment of the I

Add the regression line to the scatter plot using geom_abline()

How do outliers af

line of best fit?

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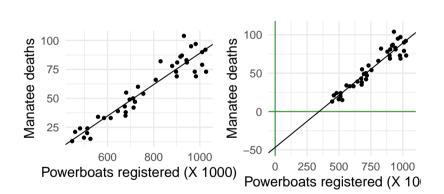
Add the regression line to the scatter plot using geom_abline()

We add a statement to our ggplot geom_abline(intercept = your_intercept, slope = your slope)

so for our manatee data geom_abline(intercept = -46.7520, slope = 0.1358)

Note: by default, ggplot only shows the ploting region that corresponds to the range of data

Add the regression line to the scatter plot using geom_abline()



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How do outliers affect th line of best fit?

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Add the regression line to the scatter plot using geom_abline()

How do outli

line of best fit?

- Countounding
- ▶ When we add the line, we can see the intercept estimate. It is where the line of best fit intersects the y axis. Should we interpret it?
 - ightharpoonup It is far from the bulk of the data, there is no data near powerboats = 0
 - Interpretation would be extrapolation, and is not supported by these data

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Add the regression line

the scatter plot using geom_abline() Transforming data

Manual and

line of best fit?

Transforming data

Transforming data

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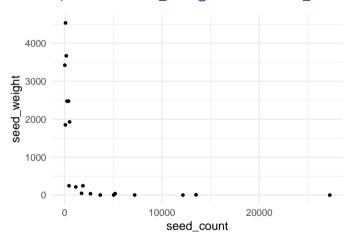
Add the regression line to the scatter plot using

Transforming data

How do outliers affect the line of best fit?

- Sometimes, the data is transformed to another scale so that the relationship between the transformed *x* and *v* is linear
- ▶ Table 3.4 in B&M provides data on the mean number of seeds produced in a year by several common tree species and the mean weight (im milligrams) of the seeds produced.

Scatter plot of seed_weight vs. seed_count



- seed_count and seed_weight both vary widely
- ► Their relationship is not linear

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Fitting a linear model in R Add the regression line to the scatter plot using

Transforming data

How do outliers affect the line of best fit?

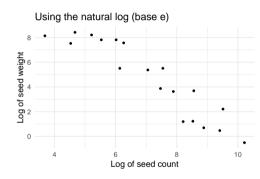
Transforming data

How do outliers affect the line of best fit?

```
Add transformed variables to the dataset using mutate().
```

ightharpoonup We add both log base e and log base 10 variables for illustration

Plot transformed data (log base e)



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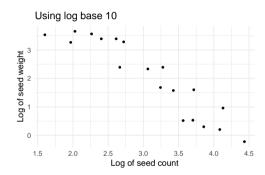
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Plot transformed data (log base 10)



- ▶ You can use either base 10 or base e for class.
- ► The calculations using base *e* are easier

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Fitting a linear model in R

Add the regression line to

Transforming data

How do outliers affect to line of best fit?

```
## # A tibble: 2 x 5
##
     term
                     estimate std.error statistic
                                                    p.value
##
     <chr>
                        <dbl>
                                  <dbl>
                                             <dbl>
                                                      <dbl>
   1 (Intercept)
                        15.5
                                  1.08
                                              14.3 6.37e-11
                                             -10.4 9.28e- 9
## 2 log_seed_count
                        -1.52
                                  0.147
```

seed_mod <- lm(log_seed_weight ~ log_seed_count, data = seed_data)</pre>

```
glance(seed_mod) %>% pull(r.squared)
```

```
## [1] 0.8631177
```

tidy(seed mod)

- Interpret the intercept:
- ► Interpret the slope:

Transforming data

lm() on the log (base 10) variables

```
seed_mod_b10 <- lm(log_b10_weight ~ log_b10_count, data = seed_data) Statistics is Everywhere
tidy(seed mod b10)
## # A tibble: 2 \times 5
##
                     estimate std.error statistic
     term
                                                      p.value
##
     <chr>
                        <dbl>
                                   <dbl>
                                              <dbl>
                                                         <dbl>
   1 (Intercept)
                         6.73
                                   0.469
                                                14.3 6.37e-11
                                   0.147
                                              -10.4 9.28e- 9
   2 log b10 count
                        -1.52
```

```
## [1] 0.8631177
```

▶ What is different from the log base e output?

glance(seed mod b10) %>% pull(r.squared)

Precitions from lm() when using log (base e) data

- ▶ What seed weight is predicted for a seed count of 2000?
- Worked calculation:
- 1. Write down the line of best fit:

```
log_e(seed.weight) = 15.49130 - 1.522220 \times log_e(seed.count)
```

- 2. Plug in seed.count = 2000 into the line of best fit: $log_e(seed.weight) = 15.49130 1.522220 \times log_e(2000)$
- 3. Solve for seed count by exponentiating both sides:

$$seed.weight = exp(15.49130 - 1.522220 \times log_e(2000))$$

(this uses the property that $e^{log_e(x)} = x$)

$$seed.weight = 50.45$$

4. Interpret: Seeds are expected to weigh 50.45 for trees having a seed count of 2000.

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Regression

Add the regression line to the scatter plot using geom_abline()

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line of best fit?

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Precitions

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Make sure you can do this worked calculation on a calculator that you will bring to the midterm.

No graphing calculators will be permitted.

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Add the regression line to the scatter plot using geom_abline()

How do outliers affect the

line of best fit?

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How do outliers affect the line of best fit?

To study this, we use data from the Organisation for Economic Co-operation and Development (OECD). This dataset was downloaded from $\frac{\text{http:}}{\text{dx.doi.org}} = \frac{10.1787}{888932526084} \text{ and contains information on the health expenditure per capita and the GDP per capita for 40 countries.}$

Have a look

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Next, we want to examine the imported data to see if it is how we expect:

head(spending dat)

atistics is Everywhere gression

dd the regression line to se scatter plot using som_abline()

How do outliers affect the line of best fit?

line of best fit? Counfounding

A tibble: 6 x 4 ## Country Country.code `Health expenditure per capita` `GDP per capita` ## <chr>> <chr>> <dbl> <dbl> 1 Australia AUS 3445 39409 2 Austria 4289 38823 AUT 3 Belgium BEL 3946 36287 4 Brazil BR.A 943 10427 5 Canada CAN 4363 38230 6 Chile CHL 1186 14131

Rename() some variables to use a consistent naming style

If the variable name has spaces, we must use back ticks when referring to it:

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Regression

Add the regression line to the scatter plot using

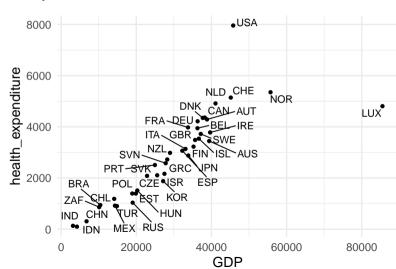
How do outliers affect the

How do outliers affect th line of best fit?

```
library(dplyr)
spending_dat <- spending_dat %>%
    rename(country_code = Country.code,
         health_expenditure = `Health expenditure per capita`, # back ticks
         GDP = `GDP per capita`) # back ticks
```

Examine the relationship

Make a scatter plot of health_expenditure (our response variable) vs. each country's level of GDP:



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Add the regression line to the scatter plot using geom_abline()

How do outliers affect the line of best fit?

Fit a linear model to these data

44.65623

##

Is the relationship linear? Which countries are outliers?

lm(health expenditure ~ GDP, data = spending dat)

0.09399

```
##
## Call:
## Im(formula = health_expenditure ~ GDP, data = spending_dat)
##
## Coefficients:
## (Intercept) GDP
```

Examine the relationship

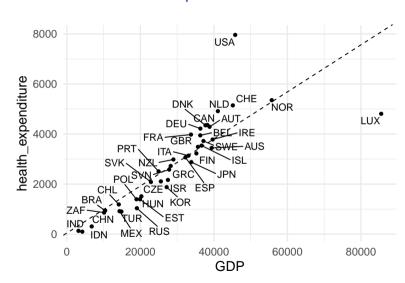
Add the regression line to the graph:

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How do outliers affect the line of hest fit?

```
GDP withline <-ggplot(spending dat, aes(x = GDP, y = health expenditure)) +
  geom point() +
  geom text repel(aes(label = country code)) + # this adds the country code of
  geom abline(intercept = 44.65623, slope = 0.09399, ltv = 2) +
  theme minimal(base size = 15)
```

Examine the relationship



L06: Intro to Linear Regression

How do outliers affect the line of best fit?

lm(health expenditure ~ GDP, data = spending dat no LUX)

```
spending dat no LUX <- spending dat %>% filter(country code != "LUX") | spending data
                                                                                                      How do outliers affect the
                                                                                                     line of best fit?
```

```
##
## Call:
## lm(formula = health expenditure ~ GDP, data = spending dat no LUX)
##
  Coefficients:
   (Intercept)
                         GDP
##
     -785.1044
                     0.1264
```

Examine the relationship without Luxembourg in the data

```
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```

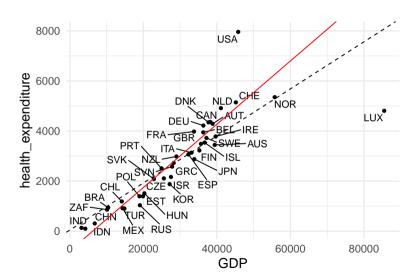
Regression

Add the regression line to the scatter plot using geom_abline()

How do outliers affect the

```
GDP_nolux<-ggplot(spending_dat, aes(x = GDP, y = health_expenditure) geom_text_repel(aes(label = country_code)) + geom_abline(intercept = 44.65623, slope = 0.09399, lty = 2) + geom_abline(intercept = -785.1044, slope = 0.1264, col = "red") + theme_minimal(base_size = 15)
```

Examine the relationship without Luxembourg in the data



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How do outliers affect the line of best fit?

Examine the relationship without USA in the data

```
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```

tistics is Everywhere

```
spending_dat_no_USA <- spending_dat %>% filter(country_code != "USA "Spending_dat (country_code != "USA") the regression line (country_code != "USA") the
```

Examine the relationship without USA in the data

```
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Linear Regression
```

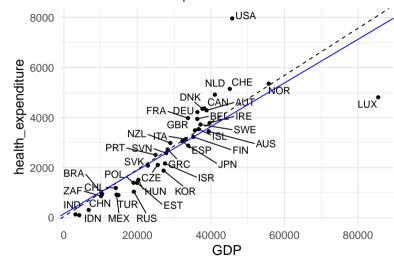
Regression

Add the regression line to the scatter plot using geom_abline()

insforming data

```
GDP_nousa<-ggplot(spending_dat, aes(x = GDP, y = health_expenditure) of the first o
```

Examine the relationship without USA in the data



Examine

the relationship without LUX or USA in the data

Let's write the code together to remove both the USA and LUX and see how it 53/64

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Add the regression line to the scatter plot using

How do outliers affect the

```
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```

```
lm(health expenditure ~ GDP, data = spending dat no USA LUX)
##
## Call:
## lm(formula = health expenditure ~ GDP, data = spending dat no USA LUX)
##
```

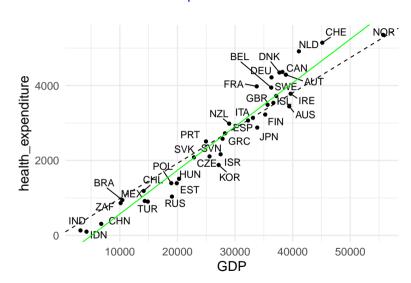
How do outliers affect the

(Intercept) GDP ## -592,6973 0.1166

Coefficients:

```
GDP noluxnousa <-ggplot(spending dat no USA LUX, aes(x = GDP, y = health exper
  geom text repel(aes(label = country code)) +
  geom abline(intercept = 44.65623, slope = 0.09399, lty = 2) +
  geom abline(intercept = -592.6973, slope = 0.1166, col = "green") +
  theme minimal(base size = 15)
```

Examine the relationship without LUX or USA in the data



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Regression

Add the regression line to the scatter plot using geom_abline()

Hanstonning data

How do outliers affect the line of best fit?

Examine the relationship without LUX or USA in the data

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Add the regression line to the scatter plot using

How do outliers affect the

line of best fit?

What would happen if USA's point had actually been along the original line of best fit (say at x = 80000 and y = 7500) and we re-fit the line without USA's point?

Would USA have been an outlier? Would it be considered influential?

- Creating a scatter plot and a simple linear model is an important step in many analyses. It allows you to see the relationship between two quantatitive variables and estimate the line of best fit.
- ▶ Sometimes these relationships will be used to make claims of causality.

Baldi & Moore emphasize that experiments are the best way to study causality. While this is often true, sophisticated causal methods have been developed for the analysis of observational data.

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Statistics is Everywhere

Regression

Fitting a linear mo

Add the regression line to the scatter plot using geom_abline()

Transforming

line of best fit?

Counfounding

Transforming data

line of best fit?

Counfounding

Your book talks about "lurking variables" which Baldi & Moore define as:

A variable that is not among the explanatory or response variables in a study and yet may influence the interpretation of relationships among those variables.

They also (pg 157) define confounding by saying:

Two variables (explanatory or lurking) are confounded when their effects on a response variable cannot be distinguished from each other.

I strongly disagree with this definition. We will use a different definition in this class.

Transforming data

Counfounding

A relationship between your variable of interest (exposure, treatment) and your outcome of interest (disease status, health condition etc) is confounded when there is a variable that is associated with both the exposure and outcome, and is not on the causal pathway between the two.

Variables that are on the causal pathway are those that represent a way in which the exposure acts on the outcome. For example, poor cognitive function would be on the causal pathway between lack of sleep and trying to pay for groceries with your library card.

Example 4.7 "Nature, nuture, and lurking variables" presents an advertisement from the Michigan Symphony:

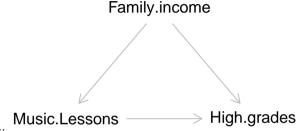
"Question: Which students scored 51 points higher in verbal skills and 39 points higher in math?

Answer: Students who had experience in music."

Marketers often make leading statements that make their product or service sound appealing. The purpose of this ad was to have the target audience impute that music causes higher marks at school because there is an association between enrollment in music and higher marks. However, are students enrolled in music lessons otherwise the same as students not enrolled in music lessons? What else do you expect to differ between these groups of students?

Discussion of some examples from Baldi & Moore

We can encode these differences in a causal diagram. Here is a simple one to



demonstrate the concept:

The direction of the arrows from the "Family Income" node makes explicit that we believe family income to be a confounder of the relationship between taking music lessons and achieving higher grades. It means that not only do these children take music lessons, they also come from families with higher incomes, and higher incomes lead to higher grades in other ways. Of course, family income is not the only possible confounder. What are some others?

L06: Intro to Linear Regression

Statistics is Everywhere

Fitting a linear model in R
Add the regression line to
the scatter plot using
geom_abline()

How do outliers affect the line of best fit?

However, know that causality can be studied using observational data and relies on clever study designs and oftentimes on advanced methods.

You will also have a chance to talk more about confounding in discussion this week.

Regression

Fitting a linear model in R

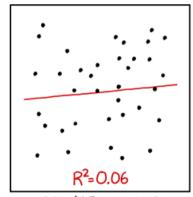
Add the regression line to

Transforming data

How do outliers affect the line of best fit?

Comic Relief

From xkcd.com





I DON'T TRUST LINEAR REGRESSIONS WHEN IT'S HARDER TO GUESS THE DIRECTION OF THE CORRELATION FROM THE SCATTER PLOT THAN TO FIND NEW CONSTELLATIONS ON IT.

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