Optimization-Based Networked Predictive Control of Process Systems with Control and Communication Constraints

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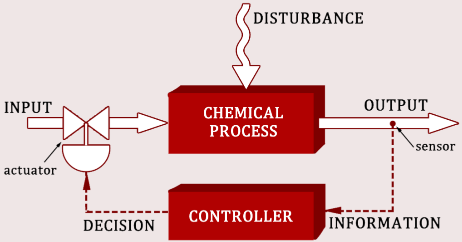
FEEDBACK CONTROL PARADIGMS

Traditional: output transmitted directly and flawlessly to the controller



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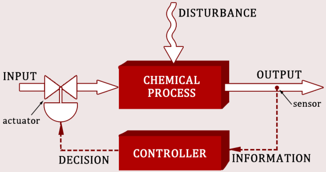
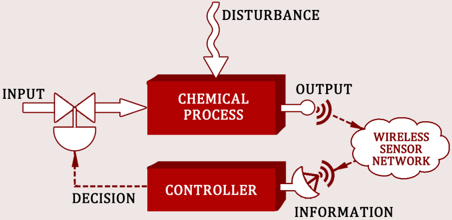


Emerging: increasing complexity of the process/controller interface



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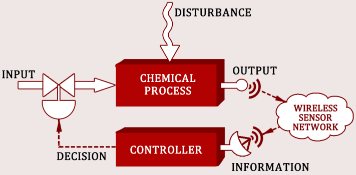


Motivation

- Smart plant operations
  - Advances in actuator and sensor manufacturing technologies
- Economic and operational benefits
  - Reduced installation and maintenance time and costs
  - Flexibility and ease of diagnosis and reconfiguration

Main conflict

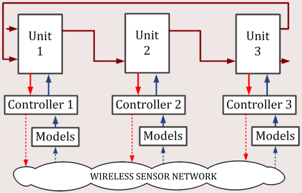
- Energy / resource utilization
- Minimal communication

VS.

- Stability / performance
- Frequent communication

MODEL-BASED NETWORKED CONTROL

Sun & El-Farra (Comput. Chem. Eng., 2008; Ind. Eng. Chem. Res., 2010; Chem. Eng. Sci., 2012)



- Key architectural features:
  - Model of the plant embedded
  - Model state is updated when communication is reestablished
  - Constant rate model update

Closed-loop properties

- Overall stability is guaranteed with reduced communication
- Maximum allowable model update period can be explicitly characterized

Open issues

- Not robust to disturbances
- Controller performance optimization
- Communication cost not accounted for in controller design

OUTLINE OF PRESENT WORK

Scope: Continuous-time process systems

- Plant-model mismatch
- Communication resource constraints

Objectives

- Development of an optimization-based predictive control framework
  - Enforce closed-loop stability
  - Simultaneously optimize control and communication performances
- Application to a representative chemical process example

Approach

- Design of an auxiliary model-based controller
  - Characterization of the maximum allowable model update period in terms of controller design parameters
- Formulation of an optimization-based predictive controller
  - Incorporate communication cost into the cost function
  - Receding horizon implementation

NETWORKED PROCESS SYSTEM DESCRIPTION

Class of systems:

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t)$$

- $\mathbf{x} \in \mathbb{R}^{n_\mathbf{x}}$: vector of process state variables
- $\mathbf{u} \in \mathbb{R}^{n_\mathbf{u}}$: vector of manipulated inputs
- $\mathbf{A}$, $\mathbf{B}$: constant state and input matrices

Approximate dynamic model:

$$\dot{\widehat{\mathbf{x}}}(t) = \widehat{\mathbf{A}}\widehat{\mathbf{x}}(t) + \widehat{\mathbf{B}}\mathbf{u}(t)$$

- $\widehat{\mathbf{x}} \in \mathbb{R}^{n_\mathbf{x}}$: vector of model state variables
- $\widehat{\mathbf{A}} = \mathbf{A} - \delta_\mathbf{A}$: approximate model of $\mathbf{A}$ with uncertainty
- $\widehat{\mathbf{B}} = \mathbf{B} - \delta_\mathbf{B}$: approximate model of $\mathbf{B}$ with uncertainty

STEP 1: MODEL-BASED NETWORKED CONTROL

Design of model-based controller using periodic communication

$$\begin{aligned}\dot{\widehat{\mathbf{x}}}(t) &= \widehat{\mathbf{A}}\widehat{\mathbf{x}}(t) + \widehat{\mathbf{B}}\mathbf{u}(t)\\ \mathbf{u}(t) &= \mathbf{K}\widehat{\mathbf{x}}(t), \quad t \in [t_k,\ t_k + h)\\ \widehat{\mathbf{x}}(t_k) &= \mathbf{x}(t_k)\end{aligned}$$

- h: model update period
- Asymptotic stabilization of the origin of the closed-loop plant

Closed-loop system formulation

- Model estimation error: $\mathbf{e}(t) = \mathbf{x}(t) - \widehat{\mathbf{x}}(t)$
- Augmented state: $\xi(t) = [\mathbf{x}^T(t)\ \mathbf{e}^T(t)]^T$

$$\begin{aligned}\dot{\xi}(t) &= \Lambda\xi(t)\\ \xi(t) &= e^{\Lambda(t-t_k)}(I_s e^{\Lambda h} I_s)^k \xi_0 = e^{\Lambda(t-t_k)} M^k \xi_0, \quad t \in [t_k,\ t_{k+1})\end{aligned}$$

  - $\xi_0 = \xi(t_0)$, $I_s = \begin{bmatrix} I_{m\times m} & O_{m\times m} \\ O_{m\times m} & O_{m\times m} \end{bmatrix}$

STEP 1: MODEL-BASED NETWORKED CONTROL

Closed-loop system formulation

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Stability condition (Garcia & Antsaklis, 2003, Sun & El-Farra, 2008)

- Eigenvalues of $M(h)$ are strictly inside the unit circle

$$
M(h) = \begin{bmatrix} I_{m\times m} & O_{m\times m} \\ O_{m\times m} & O_{m\times m} \end{bmatrix} e^{h\Lambda} \begin{bmatrix} I_{m\times m} & O_{m\times m} \\ O_{m\times m} & O_{m\times m} \end{bmatrix}
$$

- $\lambda_{\max}\{M(K,\ h,\ \mathbf{A},\ \mathbf{B},\ \widehat{\mathbf{A}},\ \widehat{\mathbf{B}})\} < 1$

STEP 2: OPTIMIZATION-BASED CONTROLLER DESIGN

Finite-horizon optimization problem formulation

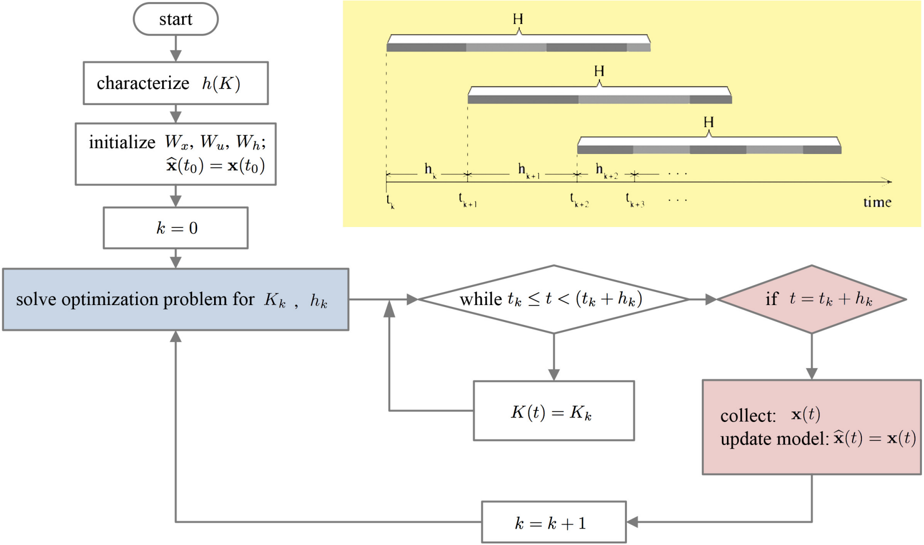
$$\min_{K_k, h_k} \; J = \int_{t_k}^{t_k+H} [\widehat{\mathbf{x}}^T(t) W_{\widehat{\mathbf{x}}} \widehat{\mathbf{x}}(t) + \mathbf{u}^T(t) W_{\mathbf{u}} \mathbf{u}(t)] dt + \underbrace{\frac{W_h}{h_k}}_{\text{communication cost}}$$

Subject to:

Model dynamics:	$\dot{\widehat{\mathbf{x}}}(t) = \widehat{\mathbf{A}}\widehat{\mathbf{x}}(t) + \widehat{\mathbf{B}}\mathbf{u}(t)$
Control action:	$\mathbf{u}(t) = K_k \widehat{\mathbf{x}}(t)$
Stability constraint:	$\lambda_{\max}\{M(K_k,\ h_k)\} < 1$

- K_k: controller gain
- h_k: model update period
- H: optimization horizon
- $W_{\widehat{\mathbf{x}}}$: penalty coefficient on the model state
- $W_{\mathbf{u}}$: penalty coefficient on the control action
- W_h: penalty coefficient on communication frequency

RECEDING-HORIZON IMPLEMENTATION STRATEGY

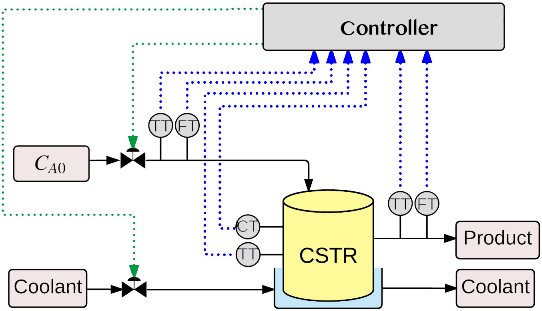


RECEDING-HORIZON IMPLEMENTATION

Key features

- Choice of penalty coefficients
  - Balancing control performance and communication cost
- Characterization of feasible region can be done off-line
  - $\lambda_{\max}\{M(K,\ h)\}$
  - Reduced computation time
- Time-varying model update period h_k
  - Adaptive to changes in operating conditions
- Explicitly incorporates communication cost $\frac{W_h}{h_k}$
  - Other forms of communication cost can be included

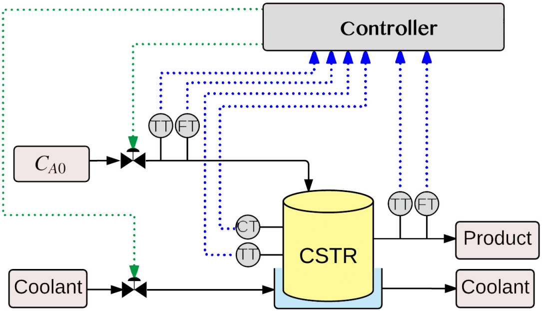
ILLUSTRATIVE EXAMPLE



Problem formulation

- Control objectives:
  - Stabilization near open-loop unstable steady state
  - Minimal information exchange
- State measurements available
- Manipulated input: Q

ILLUSTRATIVE EXAMPLE



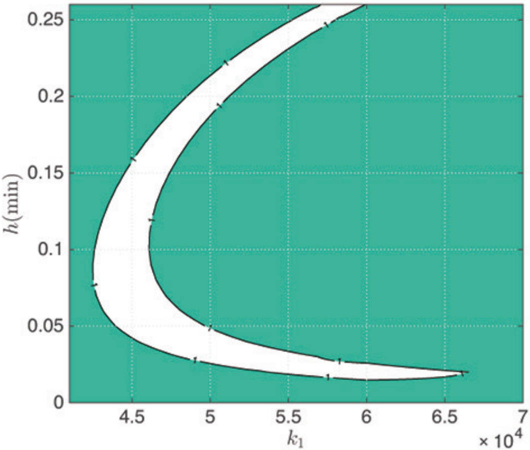
Process dynamic model

$$\dot{T} = \frac{F}{V}(T_0 - T_1) + \frac{-\Delta H}{\rho c_p} k_0 e^{\frac{-E}{RT}} C_A + \frac{Q}{\rho c_p V}$$

$$\dot{C}_A = \frac{F}{V}(C_{A0} - C_A) - k_0 e^{\frac{-E}{RT}} C_A$$

ILLUSTRATIVE EXAMPLE

- Characterization of $\lambda_{\max}\{M(k_1,\ h)\}$, $k_2 = 2000$, $K = [k_1\ k_2]$



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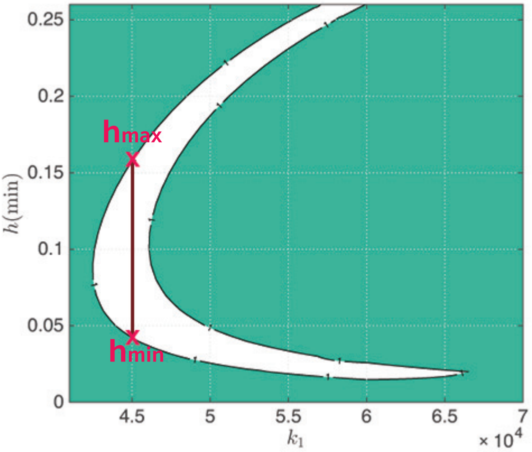

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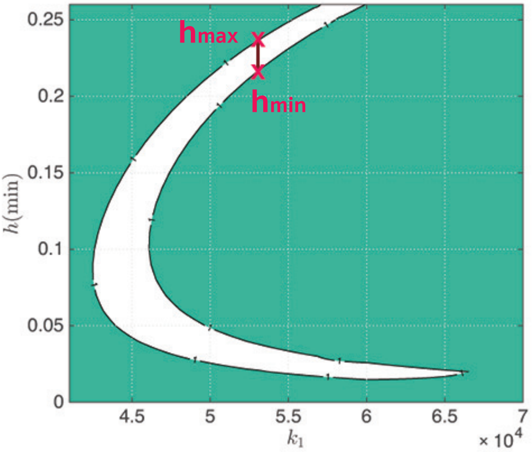
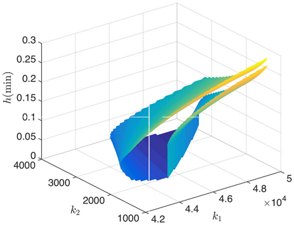
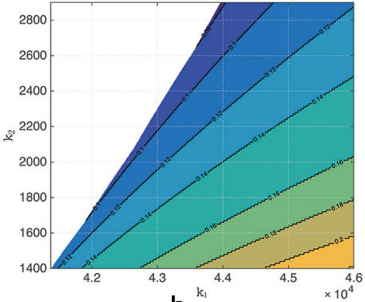


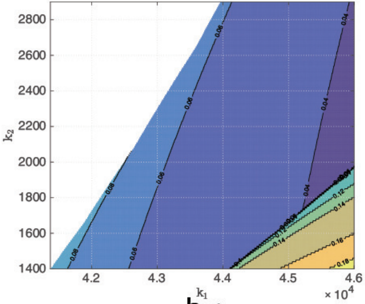
ILLUSTRATIVE EXAMPLE

- Characterization of $\lambda_{\max}\{M(K,h)\}$
- Allowable update period $(h_{\min}, h_{\max})$





h_{max}

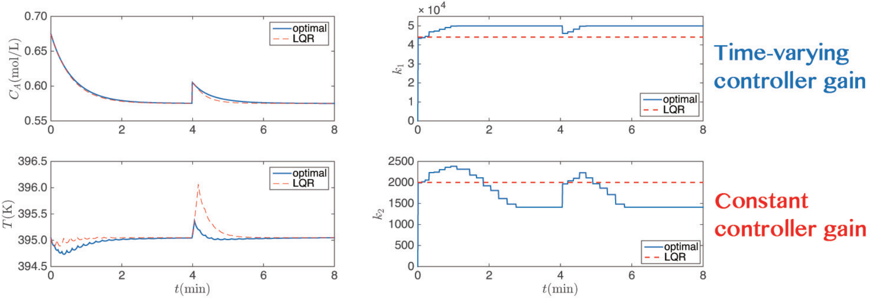


h_{min}

ILLUSTRATIVE EXAMPLE

Comparison of predictive controllers

- With communication cost
  - $W_{\hat{\mathbf{x}}} = W_{\mathbf{u}} = W_h = 1$
  - $H = 6\text{min}$
- Without communication cost
  - $J = \int_0^{\infty} (\hat{\mathbf{x}}^T W_{\hat{\mathbf{x}}} \hat{\mathbf{x}} + \mathbf{u}^T W_{\mathbf{u}} \mathbf{u}) dt$
  - $W_{\hat{\mathbf{x}}} = W_{\mathbf{u}} = 1$
  - $h_{\min} = 0.05\text{min},\ h_{\max} = 0.15\text{min}$



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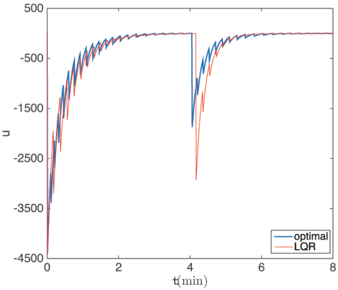

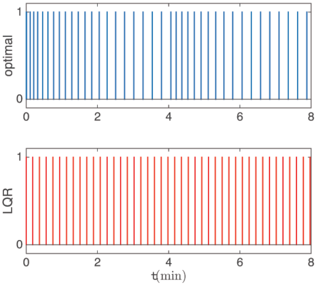

Time-varying update period

Constant update period

SUMMARY

- Continuous-time process systems:
  - Communication resource constraints
  - Plant-model mismatch
- Optimization-based predictive control framework:
  - Auxiliary model-based controller synthesis
    - Characterization of the maximum allowable model update period in terms of controller design parameters
  - Optimization-based controller design
    - Incorporate communication cost into the cost function
    - Receding horizon implementation
- Application to a chemical process example

ACKNOWLEDGEMENTS

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