Question Bank.

MATH-102

Algebra.

Sets and relations.

1. A, B, C are subsets of a universal set S. Prove that $(A\cup B)\cap(A\cup B')\cap(A'\cup B) = A\cap B$.
2. Verify that $A\cup(B\cap C) = (A\cup B)\cap(A\cup C)$, where the sets A, B, C are given by $A = \{x\in Z : 0\le x\le 10\}$, $B = \{x\in Z ; 5\le x\le 15\}$, $C = \{x\in Z : x\geq 5\}$
3. Examine if the relation ρ on the set S is reflexive, symmetric, and transitive where $S = N\times N$ by "$(a,b)\rho(c,d)$ if and only if $ad = bc$" for $(a,b) ; (c,d)\in N\times N$.
4. A relation ρ defined on the set Z by "$a\rho b$ iff $a-b$ is divisible by 5." for $a, b\in Z$. Examine if ρ is an equivalence relation on Z.
5. A relation ρ defined on Z by "$x\rho y$" iff x^2-y^2 is divisible by 5" for $x, y\in Z$. Prove that ρ is an equivalence relation on Z.

Function and Mapping.

1. Show that the mapping f is a bijection and hence determine f^{-1} where $f: R-\{1\}\to R-\{1\}$ is defined by $f(x) = \frac{x+1}{x-1}$, $x\in R-\{1\}$.
2. If $f: A\to B$ is a bijection, describe the process of finding the inverse function f^{-1}. For the bijective function $h(x) = \frac{x-2}{3}$ from R to R find $h^{-1}(y)$.
3. Show that the mapping f is a bijection. Determine f^{-1} where $f: R\to R$ is defined by $f(x) = x^3$, $x\in R$.
4. Define a bijective mapping. What two conditions must a fn. satisfy to be considered bijective? Explain the difference between

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injective and surjective functions. How do these relate to bijective function? Determine if the function $f:(-\pi/2, \pi/2) \to R$ defined by $f(x) = \tan x$ is bijective. Explain why or why not.

5. Find $g \circ f$ and $f \circ g$ if $f: Z \to Z$ is defined by $f(n) = n^2, n \in Z$
   $g: Z \to Z$ is defined by $g(n) = 2n, n \in Z$

6. Let $S = \{x \in R : -1 < x < 1\}$ and $f: R \to S$ be defined as $f(x) = \dfrac{x}{1+|x|}$, $x \in R$, Show that f is a bijection. Determine f^{-1}.

7. Find $g \circ f$ and $f \circ g$ if (i) $f: Z \to Z$ by $f(n) = (-1)^n, n \in Z$ and $g: Z \to Z$ is defined by $g(n) = 2n, n \in Z$.
   (ii) $f: R \to R$ is defined by $f(x) = x + |x|$, $x \in R$, and $g: R \to R$ is defined by $g(x) = |x| - x$, $x \in R$.

Complex Number.

1. If z_1 & z_2 be two complex numbers, then prove that $|z_1 + z_2| \leq |z_1| + |z_2|$. and $|z_1+z_2|^2 + |z_1 - z_2|^2 = 2(|z_1|^2 + |z_2|^2)$.

2. z_1, z_2 are complex numbers such that $|z_1 - 3z_2| = |3 - z_1\bar{z}_2|$ and $|z_2| \neq 1$. Prove that $|z_1| = 3$.

3. z is a complex number satisfying the condition $\left|z - \dfrac{3}{z}\right| = 2$. Find the greatest and least value of $|z|$.

4. Fing arg z where $z = 1 + i\tan\dfrac{3\pi}{5}$.

5. Prove that $\sqrt[n]{i} + \sqrt[n]{-i} = 2\cos\dfrac{\pi}{2n}$, where n is a positive integer > 1 and $\sqrt[n]{z}$ principal nth root of z.

6. Use De-Moivre's theorem to compute $(\sqrt{3}+i)^{10}$ and $(1+i)^5$.

7. Find the cube root of -1.

8. Prove that $\sqrt{i} + \sqrt{-i} = \sqrt{2}$

9. Show that $z = \cos\theta + i\sin\theta$, then $z^n + z^{-n} = 2\cos(n\theta)$.

10. Find the product of all the values of $(1+i)^{4/5}$.

11. If z is a variable complex number such that an amplitude of $\frac{z-i}{z+i}$ is $\pi/4$. Show that the point z lies on a circle in the complex plane.

10. Find the expansion of $\cos n\theta$, $\sin n\theta$, $\tan n\theta$ when n is a positive integer

Polynomials and Equations

1. Find the quotient and remainder when $2x^4+7x^3+x^2+x+4$ is divided by $2x+1$.

2. Find the quotient and remainder when x^6+x^3+1 is divided by $x+1$.

3. If $x^4+px^2+qx+r=0$ has a factor of the form $(x-\alpha)^3$, show that $27q^2+8p^3=0$ and $p^2+12r=0$.

4. Using Remainder theorem find the remainder when $x^4-3x^3+2x^2+x-1$ is divided by x^2-4x+3.

5. Solve the equation $x^4+x^2-2x+6=0$, it is given that $1+i$ is a root.

6. Apply Descarte's rule of signs to find the nature of the roots of the equation $x^4+2x^2+3x-1=0$

7. Find the relation among p, q, r, s so that the product of two roots of the equation $x^4+px^3+qx^2+rx+s=0$ is unity.

8. Solve the equation $2x^4-5x^3-15x^2+10x-15=0$, the root being in geometric progression.

9. Prove that the roots of the equation $\frac{1}{x-1}+\frac{2}{x-2}+\frac{3}{x-3}=x$ are all real.

10. Solve the equation $4x^4-4x^3-21x^2+11x+10=0$ given that the roots are in Arithmatic Progression.

11. Prove that the roots of the equation $(x+4)(x+2)(x-3)+(x+3)(x+1)(x-5)=0$

are all real and different. Separate the intervals in which the roots lies.

12. Solve the equation $2x^3 - x^2 - 18x + 9 = 0$ if two of the roots are equal in magnitude but opposite in sign.