Problem 1

(a)设事件发生的概率为p，那么示性函数的方差为$p(1-p)$，又因为

$$p(1-p) \leq \frac{1}{4}$$

所以方差最大为$\frac{1}{4}$。

(b)

$$\begin{aligned}\mathbb{P}\left[\left|s_n - p\right| < 0.03\right] &= \mathbb{P}\left[\left|s_n - p\right| < 0.03\right] \\ &\geq 1 - \frac{1}{1928} \times \frac{0.25}{0.03^2} \\ &\approx 0.8559243891194098\end{aligned}$$

(c)由上一题可知，等号成立当且仅当

$$p = \frac{1}{2}$$

所以

$$\begin{aligned}\mathbb{P}\left[\left|s_n - \frac{1}{2}\right| \leq 0.03\right] &= \mathbb{P}\left[\left|1928 \times s_n - 964\right| \leq 0.03 \times 1928\right] \\ &= \mathbb{P}[906 \leq 1928 \times s_n \leq 1021] \\ &= \frac{\sum_{i=906}^{1021} \binom{1928}{i}}{2^{1928}} \\ &\approx 0.9912\end{aligned}$$

(d)不是，0.35 ± 0.03只是置信度，不是概率。

Problem 2

(a)

$$\begin{aligned}\mathbb{E}[E_{ij}] &= \mathbb{P}[B_i = B_j] \\ &= \frac{1}{d} \\ \mathbb{E}[E_{ij}^2] &= \mathbb{P}[B_i = B_j] \\ &= \frac{1}{d} \\ \mathrm{Var}[E_{ij}] &= \frac{1}{d}\left(1 - \frac{1}{d}\right)\end{aligned}$$

(b)因为

$$D=\sum_{i\neq j}E_{ij}$$

以及两两独立，所以

$$\begin{aligned}\mathbb{E}[D]&=\sum_{i\neq j}\mathbb{E}[E_{ij}]\\&=\frac{n(n-1)}{2d}\\\mathrm{Var}[D]&=\sum_{i\neq j}\mathrm{Var}[E_{ij}]\\&=\frac{n(n-1)}{2}\frac{1}{d}\left(1-\frac{1}{d}\right)\end{aligned}$$

(c)根据题意构造随机变量即可，注意此时

$$\begin{aligned}\mathbb{E}[D]&=\frac{n(n-1)}{2d}\\&\approx 17.077\\\mathrm{Var}[D]&=\sum_{i\neq j}\mathrm{Var}[E_{ij}]\\&=\frac{n(n-1)}{2}\frac{1}{d}\left(1-\frac{1}{d}\right)\\&\approx 17.075\end{aligned}$$

结合D为整数可得

$$|D-\mathbb{E}[D]|<6\Leftrightarrow D\in[12,23]$$

由切比雪夫不等式可得

$$\begin{aligned}\mathbb{P}[|D-\mathbb{E}[D]|<6]&\geq 1-\frac{\mathrm{Var}[D]}{36}\\&>1-\frac{18}{36}\\&=\frac{1}{2}\end{aligned}$$

Problem 3

$$\forall\delta>0,\exists n_0,\forall n\geq n_0$$

Problem 4

混淆了置信度和概率，实验结果可能是选择效果好的提交。

Problem 5

根据

$$\mathbb{P}\left[|A_n-\mu_n|\le\epsilon\right]\ge 1-\frac{1}{n}\frac{\sigma^2}{\epsilon^2}$$

只要n足够大即可。

Problem 6

(a)

$$\begin{aligned}\mathbb{P}\left[|A_n-\mu_n|\ge\epsilon\right]&\le\frac{\mathrm{Var}[A_n]}{\epsilon^2}\\&=\frac{\sum_{i=1}^n\mathrm{Var}[X_i]}{n^2\epsilon^2}\\&\le\frac{nb}{n^2\epsilon^2}\\&=\frac{b}{\epsilon^2}\cdot\frac{1}{n}\end{aligned}$$

(b)因为

$$\begin{aligned}\mathbb{P}\left[|A_n-\mu_n|\le\epsilon\right]&=1-\mathbb{P}\left[|A_n-\mu_n|>\epsilon\right]\\&\ge 1-\frac{b}{\epsilon^2}\cdot\frac{1}{n}\end{aligned}$$

所以

$$\lim_{n\to\infty}\mathbb{P}\left[|A_n-\mu_n|\le\epsilon\right]=1$$