Problem 1

(a)略过。

(b)

$$\begin{aligned} p_1 &= \frac{4}{12} \times \frac{4}{12} + \frac{5}{12} \times \frac{7}{12} + \frac{3}{12} \times \frac{11}{12} \\ &= \frac{16+35+33}{144} \\ &= \frac{84}{144} \\ &= \frac{7}{12} \end{aligned}$$

(c)

$$\begin{aligned} p_2 &= \frac{\frac{4}{12} \times \frac{4}{12}}{\frac{7}{12}} \\ &= \frac{4}{21} \end{aligned}$$

(d)因为

$$p_2 \neq \frac{4}{12}$$

(e)

$$p_3 = \frac{\frac{7}{12} \times \frac{5}{12}}{\frac{7}{12}} = \frac{5}{12}$$

Problem 2

(a)略过，直接给出事件发生的概率：

$$\begin{aligned} \mathbb{P}[A] &= \frac{1}{2} \\ \mathbb{P}[B] &= \frac{1}{2} \\ \mathbb{P}[C] &= \frac{1}{2} \\ \mathbb{P}[D] &= \frac{1}{2} \end{aligned}$$

(b)

$$\mathbb{P}[ABCD] = 0 \neq \mathbb{P}[A]\mathbb{P}[B]\mathbb{P}[C]\mathbb{P}[D]$$

(c)显然A, B, C相互独立；另外由对称性，只需验证A, B, D相互独立即可

$$\begin{aligned}
\mathbb{P}[AB] = \mathbb{P}[A]\mathbb{P}[B] = \frac{1}{2} \times \frac{1}{2} \\
\mathbb{P}[AD] = \mathbb{P}[A]\mathbb{P}[D] = \frac{1}{2} \times \frac{1}{2} \\
\mathbb{P}[BD] = \mathbb{P}[B]\mathbb{P}[D] = \frac{1}{2} \times \frac{1}{2} \\
\mathbb{P}[ABD] = \mathbb{P}[A]\mathbb{P}[B]\mathbb{P}[D] = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}
\end{aligned}$$

Problem 3

(a)

$$\exists z, s.t \;\; E(x,z) \text{ and } E(z,y)$$

(b)独立的事件有

$$1, 2, 3, 4,$$

(c)

$$\mathbb{P}[\text{NOT } P(x,y)] = (1-p^2)^{n-2}$$

(d)概率为

$$p(1-r) = p\left(1-(1-p^2)^{n-2}\right)$$

Problem 4

(a)成立

$$\begin{aligned}
\mathbb{P}[A\overline{B}] &= \mathbb{P}[A] - \mathbb{P}[AB] \\
&= \mathbb{P}[A] - \mathbb{P}[A]\mathbb{P}[B] \\
&= \mathbb{P}[A](1-\mathbb{P}[B]) \\
&= \mathbb{P}[A]\mathbb{P}[\overline{B}]
\end{aligned}$$

(b)不一定，考虑投硬币两次的实验：

$$\begin{aligned}
A &= \text{第一次正面} \\
B &= \text{第二次正面} \\
C &= \text{奇数个正面}
\end{aligned}$$

(c)不一定，利用(b)即可。

(d) 成立

$$
\begin{aligned}
\mathbb{P}[A \cap (B \cup C)] &= \mathbb{P}[(A \cap B) \cup (A \cap C)] \\
&= \mathbb{P}[A \cap B] + \mathbb{P}[A \cap C] - \mathbb{P}[A \cap B \cap C] \\
&= \mathbb{P}[A]\mathbb{P}[B] + \mathbb{P}[A]\mathbb{P}[C] - \mathbb{P}[A]\mathbb{P}[B \cap C] \\
&= \mathbb{P}[A] \left(\mathbb{P}[B] + \mathbb{P}[C] - \mathbb{P}[B \cap C]\right) \\
&= \mathbb{P}[A]\mathbb{P}[B \cup C]
\end{aligned}
$$