Problem 1

选择相邻的两个数$a, a+1$，此时第二组赢的概率为

$$p=\frac{1}{7}+\frac{6}{7}\times\frac{1}{2}=\frac{4}{7}$$

所以第一组赢的概率为

$$1-p=\frac{3}{7}$$

Problem 2

$$\begin{gathered}\mathbb{P}[I_A=1, I_B=1]=\mathbb{P}[I_A=1]\mathbb{P}[I_B=1] \Leftrightarrow \\ \mathbb{P}[AB]=\mathbb{P}[A]\mathbb{P}[B]\end{gathered}$$

Problem 3

(a)

$$\mathrm{PDF}_M(1)=\frac{1}{n^m}$$

(b)

$$\mathbb{P}[M\leq k]=\left(\frac{k}{n}\right)^m$$

(c)

$$\begin{aligned}\mathrm{PDF}_M(k)&=\mathbb{P}[M\leq k]-\mathbb{P}[M\leq k-1]\\&=\left(\frac{k}{n}\right)^m-\left(\frac{k-1}{n}\right)^m\end{aligned}$$

Problem 4

(a)

$$\begin{aligned}\frac{\mathrm{PDF}_J(k)}{\mathrm{PDF}_J(k-1)}&=\frac{\binom{n}{k}p^k q^{n-k}}{\binom{n}{k-1}p^{k-1}q^{n-k+1}}\\&=\frac{(n-k+1)p}{k(1-p)}\end{aligned}$$

令上式大于1得到

$$
\begin{aligned}
\frac{(n-k+1)p}{k(1-p)} &> 1 \\
(n-k+1)p &> k-kp \\
np+p &> k
\end{aligned}
$$

令上式小于1得到

$$
np+p<k
$$

所以结论成立。

(b)代入$k=np$得到

$$
\begin{aligned}
\binom{n}{np}p^{np}q^{n-np} &= \frac{n!}{(np)!(nq)!}p^{np}q^{nq} \\
&\approx \frac{\sqrt{2\pi n}\left(\frac{n}{e}\right)^n}{\sqrt{2\pi np}\left(\frac{np}{e}\right)^{np}\sqrt{2\pi nq}\left(\frac{nq}{e}\right)^{nq}}p^{np}q^{nq} \\
&= \frac{1}{\sqrt{2\pi npq}}
\end{aligned}
$$

Problem 5

(a)

$$
\mathrm{PDF}_B(i)=\frac{1}{2^{i+1}}
$$

(b)

$$
\begin{aligned}
\mathrm{CDF}_B(i) &= \sum_{j=0}^{i}\mathrm{PDF}_B(i) \\
&= \sum_{j=0}^{i}\frac{1}{2^{i+1}} \\
&= 1-\frac{1}{2^{i+1}}
\end{aligned}
$$