ECE408 Fall 2022

Applied Parallel Programming

Lecture 16
Parallel Scan

Course Reminders

- Project Milestone 2: Baseline Convolution Kernel
  - Due Nov 4th
  - Extra credit will be provided on PM3
- Lab 5.1 (reduction) due this week. Lab 5.2 (scan) due next week
- Midterm 1
  - Exam average 64, std dev 17.4
  - Regrade requests due by 12noon CT on Friday Oct 21.

Scan Definition

Definition: *The scan operation takes a binary associative operator* $\oplus$*, and an array of n elements*

$$[x_0, x_1, \ldots, x_{n-1}],$$

and returns the prefix-sum array

$$[x_0, (x_0 \oplus x_1), \ldots, (x_0 \oplus x_1 \oplus \ldots \oplus x_{n-1})].$$

Example: If $\oplus$ is addition, then the scan operation on the array

[3 1 7 0 4 1 6 3],

would return [3 4 11 11 15 16 22 25].

Scan Application Example

- Assume that we have a 100 cm piece of wood
- We need pieces of the following lengths in cm
  - [3 5 2 7 28 4 3 0 8 1]
- How do we cut this piece of wood quickly & how much will be left?

- Method 1:
  - cut the sections sequentially: 3 cm first, 5 cm second, 2 cm third, etc.

- Method 2:
  - calculate prefix-sum array
  - [3, 8, 10, 17, 45, 49, 52, 52, 60, 61] (39 cm left)

Typical Applications of Scan

- Scan is a simple and useful parallel building block
  - Convert recurrences from sequential :

```
for(j=1; j<n; j++)
  out[j] = out[j-1] + f(j);
```

  - into parallel:

```
forall(j) { temp[j] = f(j) };
scan(out, temp);
```

- Useful for many parallel algorithms:
  - radix sort
  - quicksort
  - String comparison
  - Lexical analysis
  - Stream compaction
  - Polynomial evaluation
  - Solving recurrences
  - Tree operations
  - Histograms
  - Memory buffer allocation

Sequential Scan

Given a sequence $[x_0, x_1, x_2, \ldots]$

Calculate output $[y_0, y_1, y_2, \ldots]$

Such that

$$y_0 = x_0$$

$$y_1 = x_0 + x_1$$

$$y_2 = x_0 + x_1 + x_2$$

…

Using a recursive definition

$$y_i = y_{i-1} + x_i$$

A Sequential C Implementation

```
y[0] = x[0];
for (i = 1; i < max_i; i++)
    y[i] = y[i-1] + x[i];
```

Computationally efficient: N additions needed for N elements - O(N)

A Naïve Inclusive Parallel Scan

- Assign one thread to calculate each y element
- Have every thread to add up all x elements needed for the y element

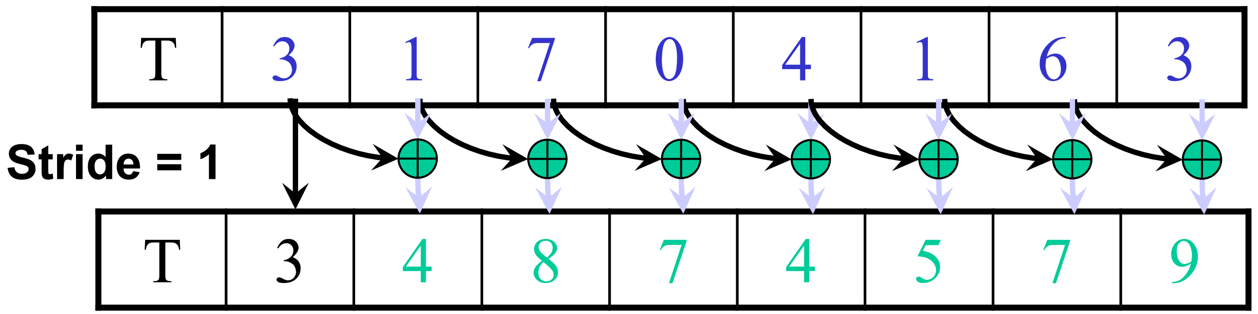
$$y_0 = x_0$$
$$y_1 = x_0 + x_1$$
$$y_2 = x_0 + x_1 + x_2$$

“Parallel programming is easy as long as you do not care about performance.”

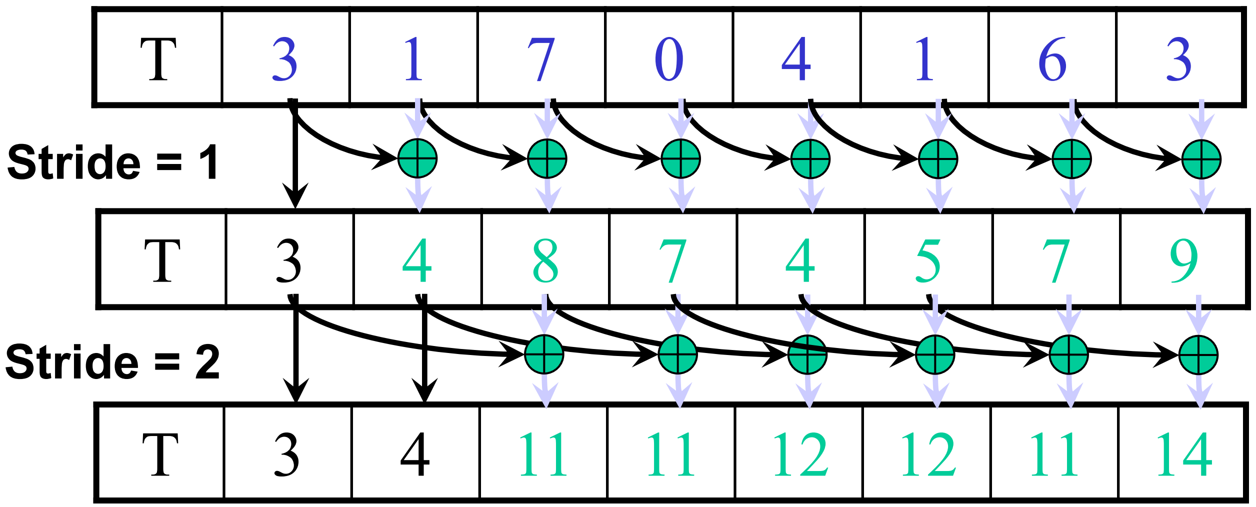
Parallel Inclusive Scan using Reduction Trees

- Calculate each output element as the reduction of all previous elements
  - Some reduction partial sums will be shared among the calculation of output elements
  - Based on hardware added design by Peter Kogge and Harold Stone at IBM in the 1970s – Kogge-Stone Trees

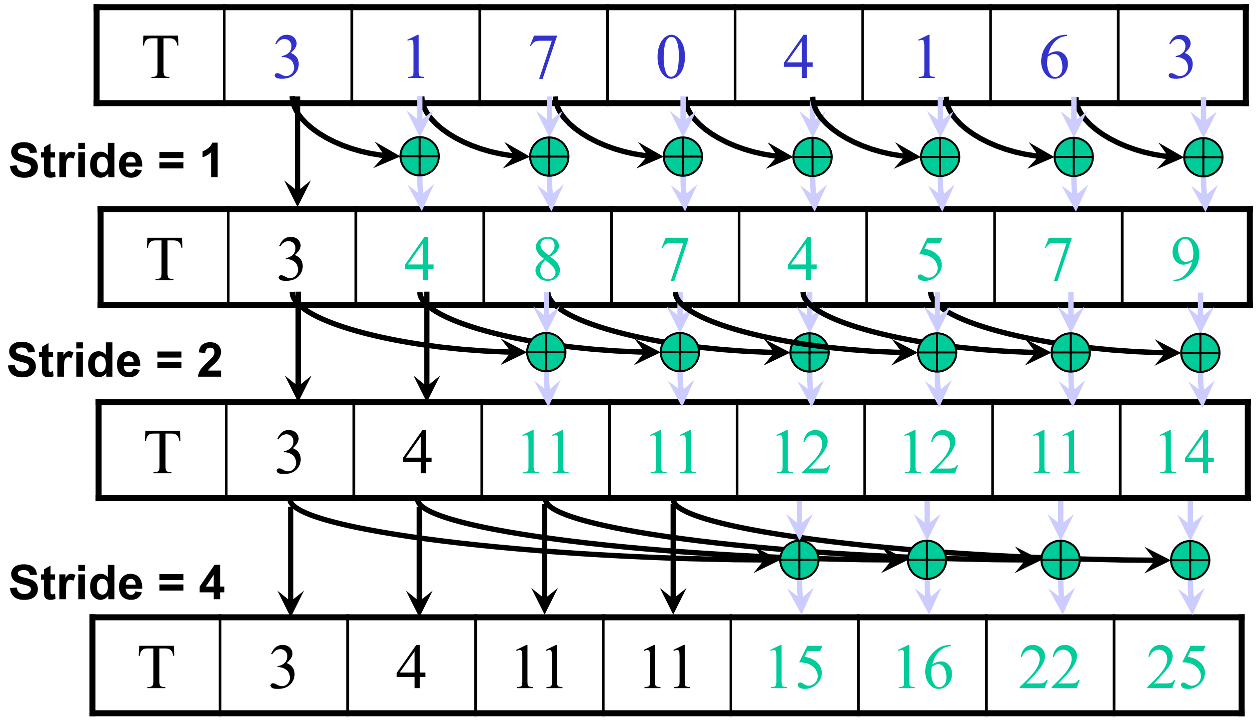
A Kogge-Stone Parallel Scan Algorithm



A Kogge-Stone Parallel Scan Algorithm



A Kogge-Stone Parallel Scan Algorithm



A Kogge-Stone Parallel Scan Algorithm

T	3	1	7	0	4	1	6	3

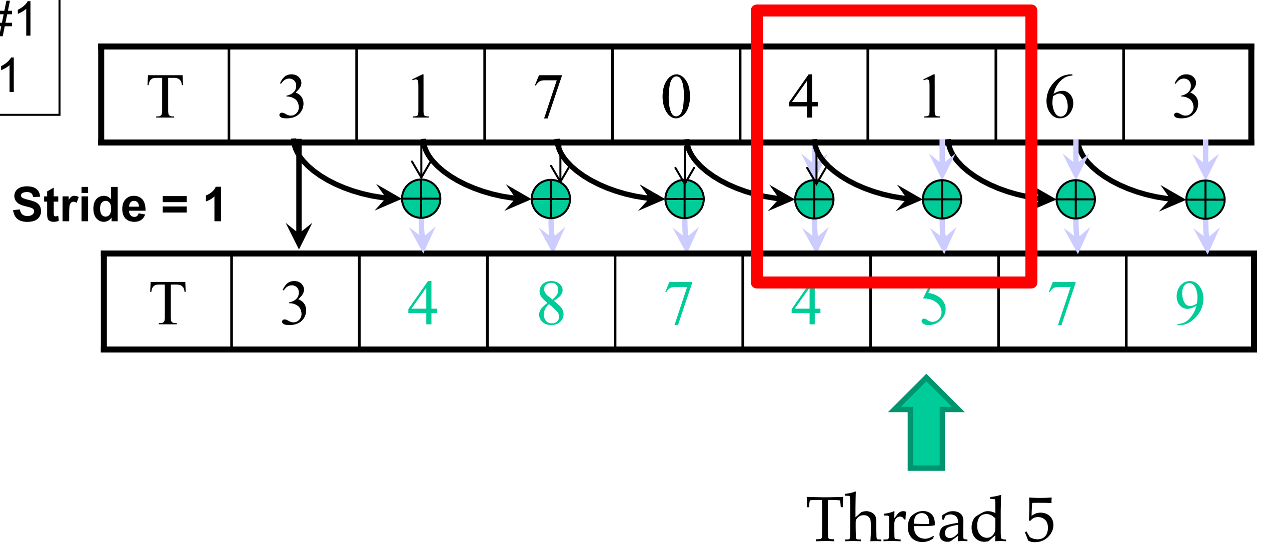
1. Load input from global memory into shared memory array T, size n, which is a power of 2.

Each thread loads one value from the input (global memory) array into shared memory array T.

Assuming that T is a power of 2 in size.

A Kogge-Stone Parallel Scan Algorithm

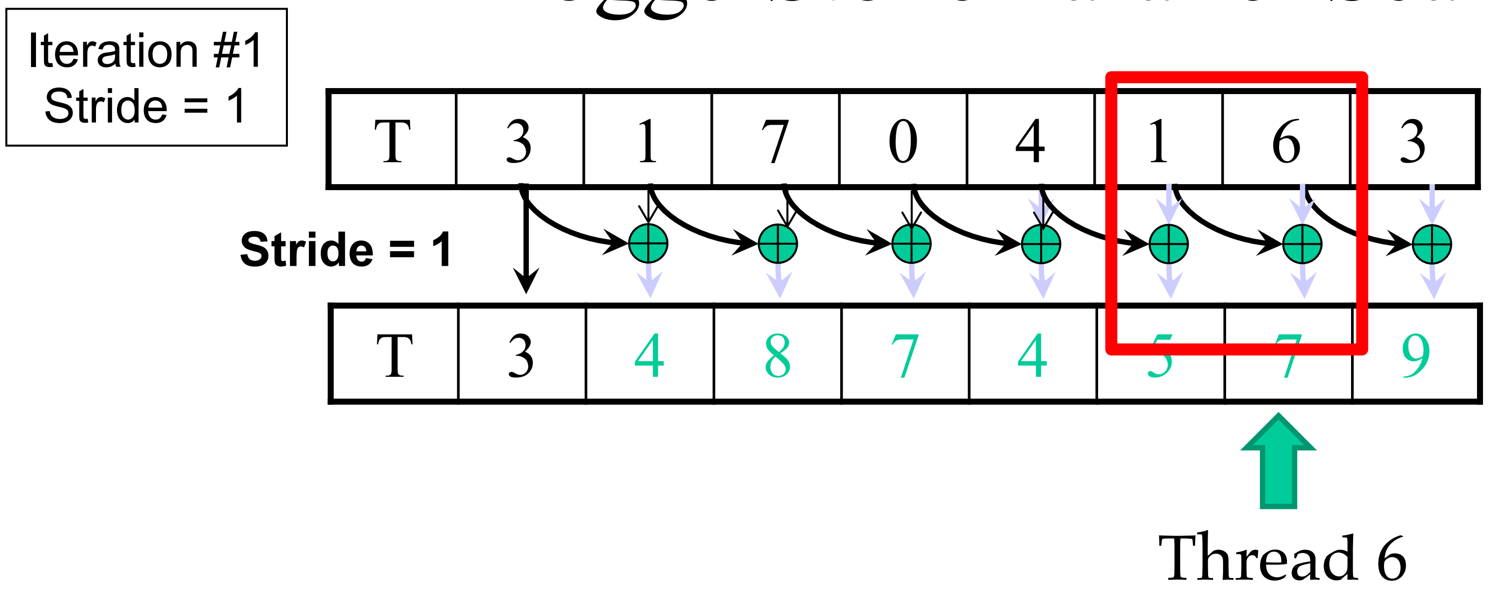
Iteration #1
Stride = 1



1. Load T (previous slide)
2. Iterate log(n) times, stride from 1 to n/2, doubling each time. Add pairs of elements that are *stride* elements apart.

- Active threads: *stride* to *n*-1 (*n* - *stride* active threads)
- Thread *j* adds elements T[*j*] and T[*j-stride*] and writes result into element T[*j*]
- Each iteration requires two syncthreads
  - make sure that input is in place
  - make sure that all input elements have been used

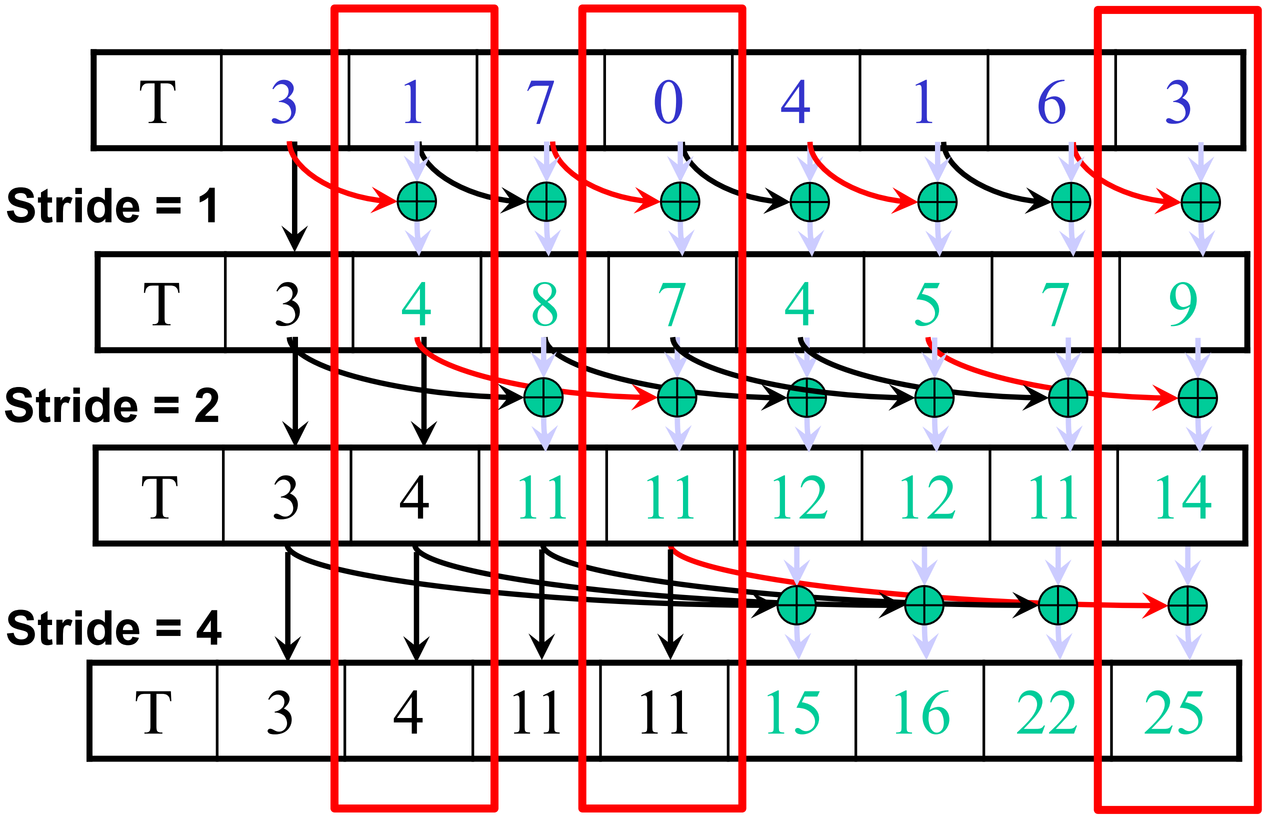
A Kogge-Stone Parallel Scan Algorithm



1. Load T
2. Iterate log(n) times, stride from 1 to n/2, doubling each time. Add pairs of elements that are *stride* elements apart.

- Active threads: *stride* to *n*-1 (*n* - *stride* active threads)
- Thread *j* adds elements T[*j*] and T[*j*-*stride*] and writes result into element T[*j*]
- Each iteration requires two syncthreads
  - syncthreads(); // make sure that input is in place
  - float temp = T[*j*] + T[*j*-*stride*];
  - syncthreads(); // make sure that previous output has been consumed
  - T[*j*] = temp;

Sharing Computation in Kogge-Stone



Iteration #3
Stride = 4

Partial Implementation

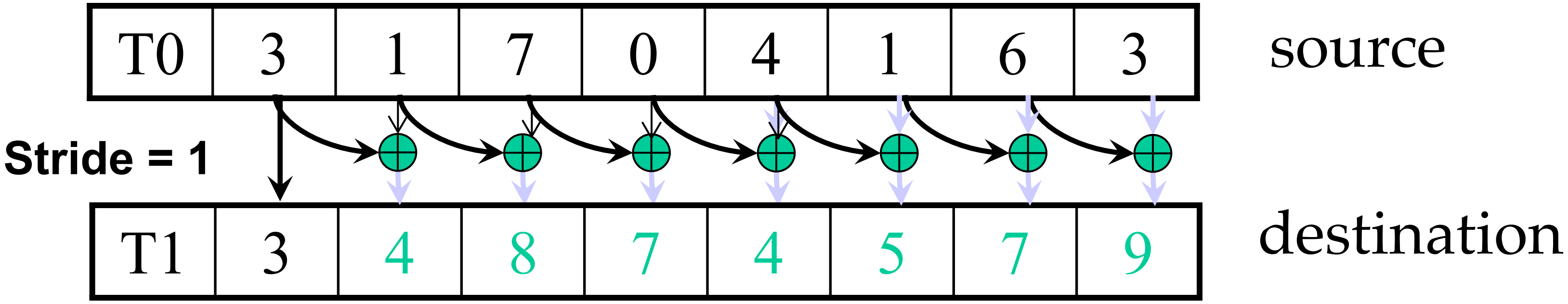
```
__global__
void Kogge_Stone_scan_kernel(float *X, float *Y, int InputSize)
{
 __shared__ float XY[SECTION_SIZE];
 int i = blockIdx.x*blockDim.x + threadIdx.x;
 if (i < InputSize) XY[threadIdx.x] = X[i];

 for (unsigned int stride = 1; stride < blockDim.x; stride *= 2) {
   __syncthreads();
   if (threadIdx.x >= stride)
      // This code has a data race condition
      XY[threadIdx.x] += XY[threadIdx.x-stride];
 }
 Y[i] = XY[threadIdx.x];
}
```

Double Buffering

- Use two copies of data T0 and T1
- Start by using T0 as input and T1 as output
- Switch input/output roles after each iteration
  - Iteration 0: T0 as input and T1 as output
  - Iteration 1: T1 as input and T0 and output
  - Iteration 2: T0 as input and T1 as output
- This is typically implemented with two pointers, source and destination that swap their contents from one iteration to the next
- This eliminates the need for the second __syncthreads() call

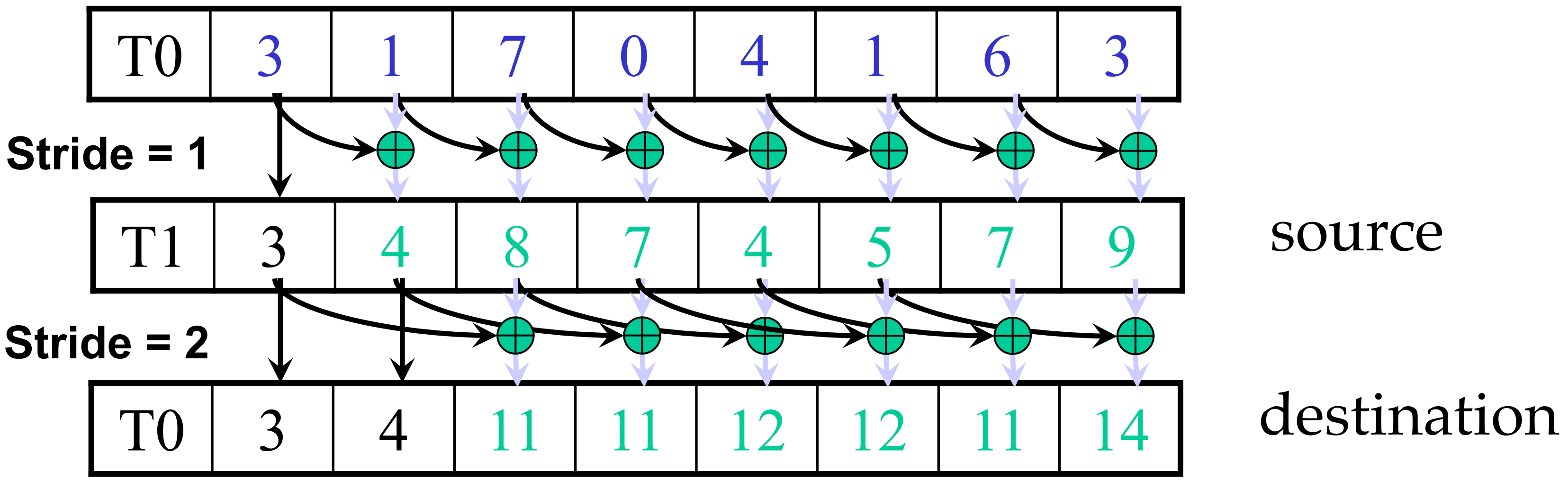
A Double-Buffered Kogge-Stone Parallel Scan Algorithm



- source = &T0[0]; destination = &T1[0];
- Each iteration requires only one syncthreads()
  - syncthreads(); // make sure that input is in place
  - float destination[*j*] = source[*j*] + source[*j-stride*];
  - temp = destination; destination = source; source = temp;
- After the loop, write destination contents to global memory

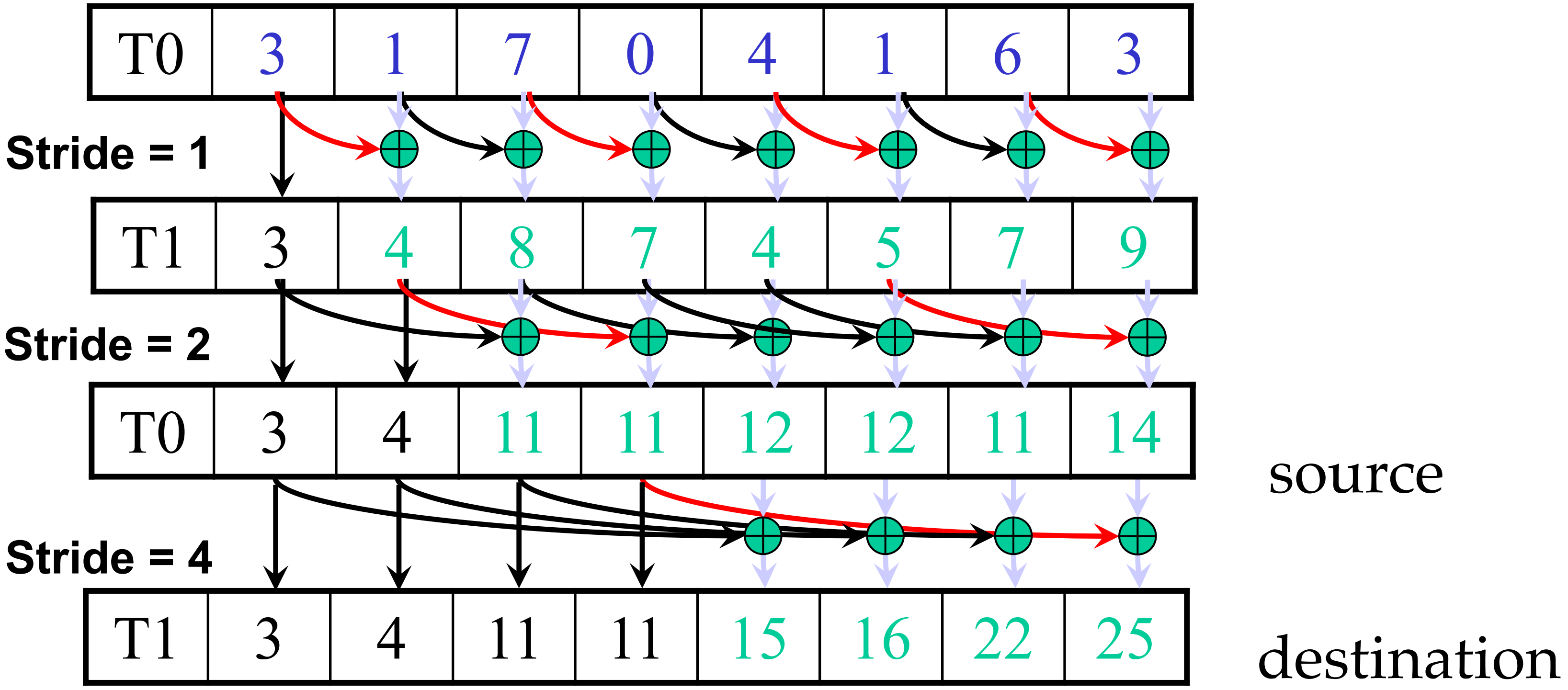
Iteration #1
Stride = 1

A Kogge-Stone Parallel Scan Algorithm



Iteration #2
Stride = 2

Sharing Computation in Kogge-Stone



Iteration #3
Stride = 4

- Each iteration requires only one syncthreads()
  - syncthreads(); // make sure that input is in place
  - float destination[*j*] = source[*j*] + source[*j-stride*];
  - temp = destination; destination = source; source = temp;
- After the loop, write destination contents to global memory

Work Efficiency Analysis

- A Kogge-Stone scan kernel executes log(n) parallel iterations
  - The steps do (n-1), (n-2), (n-4),..(n- n/2) add operations each
  - Total # of add operations: n * log(n) - (n-1) → O(n*log(n)) work
- This scan algorithm is not very work efficient
  - Sequential scan algorithm does *n* adds
  - A factor of log(n) hurts: 20x for 1,000,000 elements!
  - Typically used within each block, where n ≤ 1,024
- A parallel algorithm can be slow when execution resources are saturated due to low work efficiency

ANY MORE QUESTIONS?